Exam–style Practice paper

Section A: Statistics

1 a $0.2 + y + 0.3 + 0.35 = 1$
$y = 1 - 0.85 = 0.15$

b $P(B \text{ and } M) = 0.15$
$P(B) \times P(M) = (0.2 + 0.15) \times (0.3 + 0.15) = 0.35 \times 0.45 = 0.1575$
$P(B \text{ and } M) \neq P(B) \times P(M)$, so 'likes bananas' and 'likes mangoes' are not independent events.

2 a t is a continuous variable, because it is a measured variable which can take any value.

b $\text{mean} = \frac{\sum t}{n} = \frac{140.1}{10} = 14.01$

$\text{standard deviation} = \sqrt{\frac{\sum t^2}{n} - \left(\frac{\sum t}{n}\right)^2} = \sqrt{\frac{1981.33}{10} - \left(\frac{140.1}{10}\right)^2} = 1.36$ (to 3 s.f.)

c 15.8 °C is higher than the current mean so the mean would increase.

d Clare could take a random sample of days from the whole of September for the different locations in the UK.

3 a $0.1 + 0.2 + 0.15 + p + 0.1 + 0.25 = 1$
$p = 1 - 0.8 = 0.2$

b $P(2 \leq X \leq 5) = 1 - P(X = 1) - P(X = 6)$
$= 1 - 0.1 - 0.25 = 0.65$

c i $P(\text{odd}) = 0.1 + 0.15 + 0.1 = 0.35$

$P(\text{odd exactly twice}) = \binom{10}{2} 0.35^2 0.65^8$
$= 0.1757$ (to 4 d.p.)

ii $P(\text{odd more than 6 times}) = \binom{10}{7} 0.35^7 0.65^3 + \binom{10}{8} 0.35^8 0.65^2 + \binom{10}{9} 0.35^9 0.65^1 + 0.35^{10}$
$= 0.0260$ (to 4 d.p.)

4 a The test statistic is the number of plates that are flawed.
$H_0: p = 0.3$, $H_1: p < 0.3$

b $X \sim B(20, 0.3)$
$P(X \leq 2) = 0.0355 < 0.05$
$P(X \leq 3) = 0.1071 > 0.05$
The critical region is $X \leq 2$

c The actual significance level is 0.0355 = 3.55%

d 1 falls into the critical region, therefore there is evidence to support the claim.

5 a The increase in energy released is 3.1 Joules for each degree of temperature.

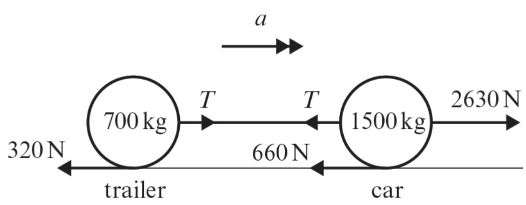
5 b This value of h is a long way from the range of the experimental data: hence the extrapolation is excessive and the predicted value of e would be too unreliable.

c It is not sensible. The regression line predicts a value of e given h, not the other way round.

6
$$P(4.6 \le h \le 6.1) = \frac{0.4\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.1\times10}{0.5\times5+0.5\times10+0.2\times45+0.2\times60+0.2\times80+0.4\times25+0.5\times10}$$
$$= \frac{4+9+12+16+10+1}{2.5+5+9+12+16+10+5}$$
$$= \frac{52}{59.5}$$
$$= 0.87 \text{ (to 2 d.p.)}$$

Section B: Mechanics

7



$F = ma$

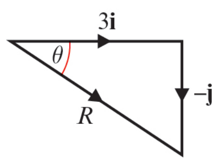
a For the whole system:
$F = 2630 - 660 - 320 = 1650$
$m = 1500 + 700 = 2200$
$1650 = 2200a$
The acceleration of the car is 0.75 m s^{-2}

b For the trailer:
$F = T - 320$, $m = 700$, $a = 0.75$
$T - 320 = 700 \times 0.75 = 525$
$T = 525 + 320$
The tension in the tow-rope is 845 N.

c Since the tow-rope is inextensible, the acceleration of each part of the system is identical and the tension in it is constant throughout.

8 a Resultant force, $F = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$
$F = (3\mathbf{i} - 6\mathbf{j}) + (4\mathbf{i} + 5\mathbf{j}) + (2\mathbf{i} - 2\mathbf{j}) = (9\mathbf{i} - 3\mathbf{j})$
$m = 3$, $\mathbf{a} = ?$
$F = m\mathbf{a}$
$(9\mathbf{i} - 3\mathbf{j}) = 3\mathbf{a}$
The acceleration of the particle is $(3\mathbf{i} - \mathbf{j})$ m s^{-2}

b



$\tan\theta = \frac{1}{3}$

8 b The acceleration acts at an angle of 18.4° below **i**.

c $|\mathbf{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$

The magnitude of the acceleration is $\sqrt{10}$ m s^{-2}

9 a Taking up as positive:

$s = 0$, $a = -9.8$, $t = 5$, $u = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$$0 = (u \times 5) + \tfrac{1}{2}(-9.8 \times 5^2) = 5u - 122.5$$

$$u = \frac{122.5}{5} = 24.5$$

The ball is projected at a speed of 24.5 m s^{-1}

b $u = 24.5$, $a = -9.8$, $v = 0$, $s = ?$

$$v^2 = u^2 + 2as$$

$$0 = 24.5^2 + (2 \times (-9.8s))$$

$$s = \frac{24.5^2}{2 \times 9.8} = \frac{600.25}{19.6} = 30.625$$

The ball reaches a height of 30.6 m above P.

c $s = 15$, $u = 24.5$, $a = -9.8$, $t = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$$15 = 24.5t + \tfrac{1}{2}(-9.8 \times t^2)$$

$$4.9t^2 - 24.5t + 15 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{24.5 \pm \sqrt{24.5^2 - (4 \times 4.9 \times 15)}}{2 \times 4.9}$$

$$= \frac{24.5 \pm \sqrt{306.25}}{9.8}$$

$$= \frac{24.5 \pm 17.5}{9.8}$$

$$= \frac{42}{9.8} \text{ or } \frac{7}{9.8}$$

The ball is at a height of 15 m above P at 0.714 s and 4.29 s after leaving P.

10 $v = 3 + 9t^2 - 4t^3$

When the particle is moving at maximum velocity, $a = \frac{dv}{dt} = 0$

$$0 = \frac{d\left(3 + 9t^2 - 4t^3\right)}{dt}$$

$$= 18t - 12t^2$$

$$= 6t(3 - 2t)$$

10 At $t = 0$, the particle moves at minimum velocity (see graph).
The particle has maximum velocity at $t = \frac{3}{2}$ seconds.

$$s = \int v\,\mathrm{d}t = \int_0^{\frac{3}{2}} \left(3 + 9t^2 - 4t^3\right)\mathrm{d}t$$

$$= \left[3t + \frac{9t^3}{3} - \frac{4t^4}{4}\right]_0^{\frac{3}{2}} = \left[3t + 3t^3 - t^4\right]_0^{\frac{3}{2}}$$

For $t = 0$, all terms are zero, so this becomes:

$$s = 3 \times \left(\frac{3}{2}\right) + 3 \times \left(\frac{3}{2}\right)^3 - \left(\frac{3}{2}\right)^4$$

$$= \frac{9}{2} + \frac{81}{8} - \frac{81}{16} = \frac{153}{16}$$

The particle is moving at maximum velocity when it is $\frac{153}{16}$ m from O.