



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Unit P3:

Exponentials and Logarithms

HGS Maths



Dr Frost Course



Name: _____

Class: _____

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Prior knowledge check

Prior knowledge check

- 1 Given that $x = 3$ and $y = -1$, evaluate these expressions without a calculator.

a 5^x **b** 3^y **c** 2^{2x-1} **d** 7^{1-y} **e** 11^{x+3y}

← GCSE Mathematics

- 2 Simplify these expressions, writing each answer as a single power.

a $6^8 \div 6^2$ **b** $y^3 \times (y^9)^2$ **c** $\frac{2^5 \times 2^9}{2^8}$ **d** $\sqrt{x^8}$

← Sections 1.1, 1.4

- 3 Plot the following data on a scatter graph and draw a line of best fit.

x	1.2	2.1	3.5	4	5.8
y	5.8	7.4	9.4	10.3	12.8

Determine the gradient and intercept of your line of best fit, giving your answers to one decimal place.

← GCSE Mathematics

3.1 Graphs of Exponential Functions (including gradient)

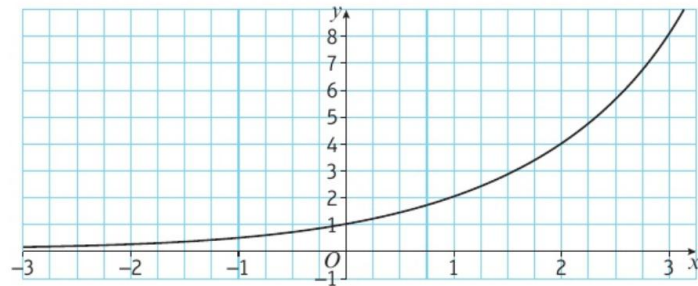
Functions of the form $f(x) = a^x$, where a is a constant, are called **exponential functions**. You should become familiar with these functions and the shapes of their graphs.

For an example, look at a table of values of $y = 2^x$.

x	-3	-2	-1	0	1	2	3
y	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

The value of 2^x tends towards 0 as x decreases, and grows without limit as x increases.

The graph of $y = 2^x$ is a smooth curve that looks like this:



Notation In the expression 2^x , x can be called an **index**, a **power** or an **exponent**.

Links Recall that $2^0 = 1$ and that $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ ← Section 1.4

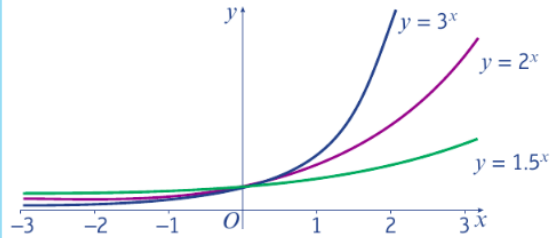
The x -axis is an asymptote to the curve.

Notes

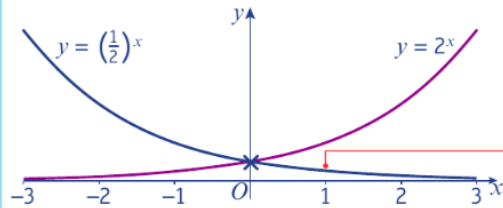
a For all three graphs, $y = 1$ when $x = 0$.

When $x > 0$, $3^x > 2^x > 1.5^x$.

When $x < 0$, $3^x < 2^x < 1.5^x$.



b The graph of $y = \left(\frac{1}{2}\right)^x$ is a reflection in the y-axis of the graph of $y = 2^x$.



$$a^0 = 1$$

Work out the relative positions of the three graphs.

Whenever $a > 1$, $f(x) = a^x$ is an increasing function. In this case, the value of a^x grows without limit as x **increases**, and tends towards 0 as x **decreases**.

Since $\frac{1}{2} = 2^{-1}$, $y = \left(\frac{1}{2}\right)^x$ is the same as $y = (2^{-1})^x = 2^{-x}$.

Whenever $0 < a < 1$, $f(x) = a^x$ is a decreasing function. In this case, the value of a^x tends towards 0 as x **increases**, and grows without limit as x **decreases**.

Your Turn

On the same axes, sketch $y = 4^x$, $y = 5^x$ and $y = 3.5^x$

Your Turn

On the same axes, sketch $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$

Worked Example

Sketch $y = 3^{x-2}$

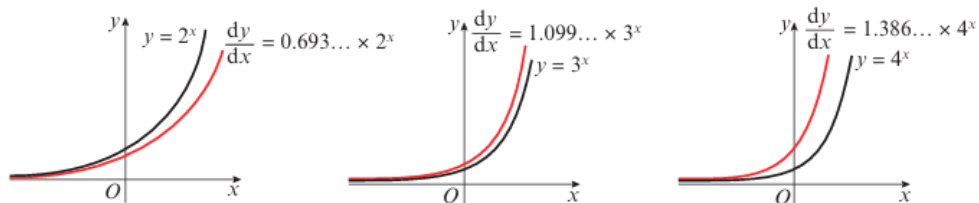
Worked Example

The graph of $y = ka^x$ passes through the points $(4, \frac{16}{3})$ and $(0, \frac{1}{3})$

Find the values of the constants k and a

$$y = e^x$$

Exponential functions of the form $f(x) = a^x$ have a special mathematical property. The graphs of their gradient functions are a similar shape to the graphs of the functions themselves.



In each case $f'(x) = kf(x)$, where k is a constant. As the value of a increases, so does the value of k .

Something unique happens between $a = 2$ and $a = 3$. There is going to be a value of a where the gradient function is exactly the same as the original function. This occurs when a is approximately equal to 2.71828. The exact value is represented by the letter e . Like π , e is both an important mathematical constant and an irrational number.

Function	Gradient function
$f(x) = 1^x$	$f'(x) = 0 \times 1^x$
$f(x) = 2^x$	$f'(x) = 0.693... \times 2^x$
$f(x) = 3^x$	$f'(x) = 1.099... \times 3^x$
$f(x) = 4^x$	$f'(x) = 1.386... \times 4^x$

Online Explore the relationship between exponential functions and their derivatives using GeoGebra.



[14 Exponentials and logarithms, page 314 – GeoGebra](#)

■ **For all real values of x :**

- If $f(x) = e^x$ then $f'(x) = e^x$
- If $y = e^x$ then $\frac{dy}{dx} = e^x$

A similar result holds for functions such as e^{5x} , e^{-x} and $e^{\frac{1}{2}x}$.

■ **For all real values of x and for any constant k :**

- If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$
- If $y = e^{kx}$ then $\frac{dy}{dx} = ke^{kx}$

Worked Example

Differentiate with respect to x :

a) e^{2x}

b) e^{-3x}

c) $4e^{5x}$

d) $6e^{\frac{1}{3}x}$

Worked Example

Sketch:

$$y = 3 + 4e^{\frac{1}{2}x}$$

Worked Example

Sketch:

$$y = e^{-3x} - 2$$

3.2) Exponential modelling

Worked Example

Suppose the population P of a village is modelled by $P = 500e^{2t}$ where t is the numbers of years since February 2009. Find:

- a) The initial population
- b) The initial rate of growth
- c) The population in February 2014

Worked Example

The density of a pesticide in a given section of field, P mg/m², can be modelled by the equation $P = 80e^{-0.003t}$ where t is the time in days since the pesticide was first applied.

- Use this model to estimate the density of pesticide after 30 days.
- Interpret the meaning of the value 80 in this model.
- Show that $\frac{dP}{dt} = kP$, where k is a constant, and state the value of k .
- Interpret the significance of the sign of your answer in part (c).
- Sketch the graph of P against t .

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 30$$

where A and B are constants.

Initially, the temperature of the coffee was 85 °C.

- (a) State the value of A .

(1)

Initially, the coffee was cooling at a rate of 7.5 °C per minute.

- (b) Find a complete equation linking H and t , giving the value of B to 3 decimal places.

(3)

Your Turn

4. Coffee is poured into a cup.

The temperature of the coffee, H °C, t minutes after being poured into the cup is modelled by the equation

$$H = Ae^{-Bt} + 25$$

where A and B are constants.

Initially, the temperature of the coffee was 90 °C.

- (a) State the value of A .

(1)

Initially, the coffee was cooling at a rate of 8 °C per minute.

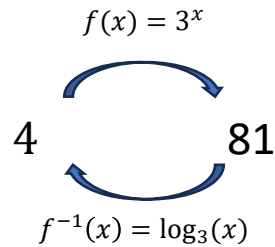
- (b) Find a complete equation linking H and t , giving the value of B to 3 decimal places.

(3)

(Total for Question 4 is 4 marks)

3.3 Logarithms

Logarithms are the inverse of exponentials, i.e.



$\log_a(n) = x$ is equivalent to $a^x = n$

Example	Non-Example
$3^2 = 9 \Leftrightarrow \log_3(9) = 2$	$3^x \neq x^3$

The output of a $\log$ can be a negative, but you cannot $\log$ a negative, i.e.

Example	Non-Example
$\log_4\left(-\frac{1}{16}\right) = -4$	$\log_4(-4) = -1$

Notes

Your Turn

Without using a calculator, find the value of:

- a) $\log_4 16$
- b) $\log_5 125$
- c) $\log_4 1$
- d) $\log_5 5$
- e) $\log_4 4$
- f) $\log_5 \left(\frac{1}{625}\right)$
- g) $\log_4 \left(\frac{1}{64}\right)$
- h) $\log_5 1$
- i) $\log_4(-3)$
- j) $\log_5(-2)$

3.4 Laws of Logarithms

From indices facts: $x^0 = 1$ $x^1 = x$:

$$\log_x(1) = 0$$

$$\log_x(x) = 1$$

By default $\log(x) \equiv \log_{10} x$ and $\ln(x) \equiv \log_e x$

LAWS: *Opposite of indices*

$$\log_x(a) + \log_x(b) \equiv \log_x(ab)$$

$$\log_x(a) - \log_x(b) \equiv \log_x\left(\frac{a}{b}\right)$$

$$\log_a(x^n) \equiv n\log_a(x)$$

ANTI LAWS:

$$\log_x(a \pm b) \neq \log_x(a) \pm \log_x(b)$$

$$(\log_x(a))^2 \neq 2\log_x(a)$$

Notes

Worked Example (DFM 527f)

Write as a single logarithm.

- a) $\log_3 6 + \log_3 7$
- b) $\log_2 15 - \log_2 3$
- c) $2 \log_5 3 + 3 \log_5 2$
- d) $\log_{10} 3 - 4 \log_{10} \left(\frac{1}{2}\right)$

Worked Example

Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$.

a) $\log_a(x^2yz^3)$

b) $\log_a\left(\frac{x}{y^3}\right)$

c) $\log_a\left(\frac{x\sqrt{y}}{z}\right)$

d) $\log_a\left(\frac{x}{a^4}\right)$

Worked Example

Solve the equation $\log_3(x + 11) - \log_3(x - 5) = 2$

Past Paper Q P2 2021 Q3

Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(Total for question = 3 marks)

Your Turn

3. Using the laws of logarithms, solve the equation

$$\log_2(10y + 6) - \log_2(1 - 5y) = 3$$

(3)

(Total for Question 3 is 3 marks)

Past Paper Q AS 2023 Q9

Using the laws of logarithms, solve the equation

$$2\log_5(3x - 2) - \log_5 x = 2$$

(Total for question = 5 marks)

Your Turn

9. (a) Show, using the laws of logarithms, that:

$$\log_b 15x - 2\log_b (2x - 3) = 1, \text{ where } b \text{ is a positive integer}$$

can be written in the form $kx = b(px^2 + qx + r)$ and k, p, q and r are constants to be found.

(3)

- (b) Hence or otherwise, solve $\log_5 15x - 2\log_5 (2x - 3) = 1$

(2)

(Total for Question 9 is 5 marks)

Past Paper Q P1 2023 Q6

$$a = \log_2 x$$

$$b = \log_2(x + 8)$$

Express in terms of a and/or b

(a) $\log_2 \sqrt{x}$

(1)

(b) $\log_2(x^2 + 8x)$

(2)

(c) $\log_2\left(8 + \frac{64}{x}\right)$

Give your answer in simplest form.

(3)

(Total for question = 6 marks)

Your Turn

6. $a = \log_2 x$ $b = \log_2(x + 4)$

Express in terms of a and/or b

(a) $\log_2 \frac{1}{\sqrt[3]{x}}$ (1)

(b) $\log_2(2x^2 + 8x)$ (2)

(c) $\log_2 \left(\frac{16}{x} + 4 \right)$

Give your answer in simplest form. (3)

(Total for Question 6 is 6 marks)

3.5 Solving Equations using Logarithms

Just as you would with undoing a 'trapped' root, you must isolate e^x terms and any $\log_a b$ terms, BEFORE using the inverse operation.

Example:

$$3 + \log_2 x = 5$$

$$\log_2 x = 2$$

$$x = 2^2 = 4$$

Non-Example:

$$2^x - 3 = 5$$

$$\log_2(2^x) - 3 = \log_2 5$$

$$x = 3 + \log_2 5$$

Worked Example

Solve the equation $5^{2x} - 12(5^x) + 20 = 0$, giving your answer to 3 significant figures.

Worked Example

Solve the equation $3^{x+1} = 4^{x-1}$. Round your answer to 3 decimal places.

Worked Example

Solve the equation $2^x 3^{x+1} = 5$. Give your answer in exact form.

Past Paper Q AS 2021 Q2

In this question you should show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x , writing your answer in simplest form.

(Total for question = 3 marks)

Your Turn

2. In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Given

$$\frac{8^{x-1}}{2^{y+2}} = 64$$

express y in terms of x , writing your answer in simplest form.

(3)

(Total for Question 2 is 3 marks)

Past Paper Q 2020 P2

3. (a) Given that

$$2 \log(4 - x) = \log(x + 8)$$

show that

$$x^2 - 9x + 8 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 9x + 8 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log(4 - x) = \log(x + 8)$$

giving a reason for your answer.

(2)

Your Turn

3. (a) Given that

$$2 \log (5 - x) = \log (x + 7)$$

show that

$$x^2 - 11x + 18 = 0$$

(3)

(b) (i) Write down the roots of the equation

$$x^2 - 11x + 18 = 0$$

(ii) State which of the roots in (b)(i) is **not** a solution of

$$2 \log (5 - x) = \log (x + 7)$$

giving a reason for your answer.

(2)

(Total for Question 3 is 5 marks)

Past Paper Q 2020 P2

5. The curve with equation $y = 3 \times 2^x$ meets the curve with equation $y = 15 - 2^{x+1}$ at the point P .

Find, using algebra, the exact x coordinate of P .

(4)

Your Turn

5. The curve with equation $y = 6 \times 3^x$ meets the curve with equation $y = 60 - 3^{x+2}$ at the point P

Find, using algebra, the exact x coordinate of P .

(4)

(Total for Question 5 is 4 marks)

Working with natural logarithms

- The graph of $y = \ln x$ is a reflection of the graph $y = e^x$ in the line $y = x$.

The graph of $y = \ln x$ passes through $(1, 0)$ and does not cross the y -axis.

The y -axis is an asymptote of the graph $y = \ln x$. This means that $\ln x$ is only defined for positive values of x .

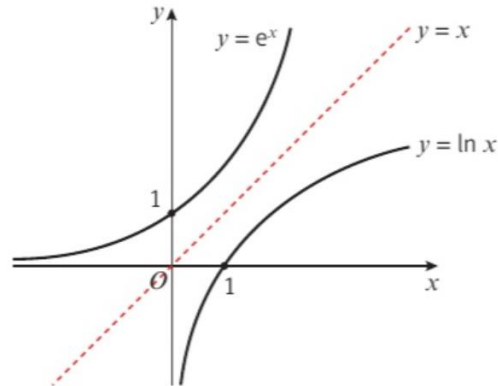
As x increases, $\ln x$ grows without limit, but relatively slowly.

You can also use the fact that logarithms are the inverses of exponential functions to solve equations involving powers and logarithms.

- $e^{\ln x} = \ln(e^x) = x$

Notation

$$\ln x = \log_e x$$



Worked Example

Solve the equation:

$$e^x - 12e^{-x} = -1$$

Worked Example

528c: Solve logarithmic equations involving a single occurrence of $\ln$

Solve for x :

$$3 \ln(-3x - 3) + 3 = 9$$

Give your solution in exact form.

Worked Example

Find the exact coordinates of the points where the graph with equation $y = 6 + \ln(5 - x)$ intersects the axes

Worked Example

Solve the equation:

$$3^x e^{x+4} = 2$$

Give your answer as an exact value

3.6 Logarithmic Graphs

Case 1: $y = ax^n$

Start with a non-linear relationship ————— $y = ax^n$
Take logs of both sides ($\log = \log_{10}$) ————— $\log y = \log ax^n$
Use the multiplication law ————— $\log y = \log a + \log x^n$
Use the power law ————— $\log y = \log a + n \log x$

Compare this equation to the straight line equation, $Y = MX + C$.

$\log y$ variable	=	n constant (gradient)	$\log x$ variable	+	$\log a$ constant (intercept)
Y variable	=	M constant (gradient)	X variable	+	C constant (intercept)

- If $y = ax^n$ then the graph of $\log y$ against $\log x$ will be a straight line with gradient n and vertical intercept $\log a$.

Notes

Case 2: $y = ab^x$

Start with a non-linear relationship ————— $y = ab^x$

Take logs of both sides ($\log = \log_{10}$) ————— $\log y = \log ab^x$

Use the multiplication law ————— $\log y = \log a + \log b^x$

Use the power law ————— $\log y = \log a + x \log b$

Compare this equation to the straight line equation, $Y = MX + C$.

$\log y$ variable	=	$\log b$ constant (gradient)	x variable	+	$\log a$ constant (intercept)
Y variable	=	M constant (gradient)	X variable	+	C constant (intercept)

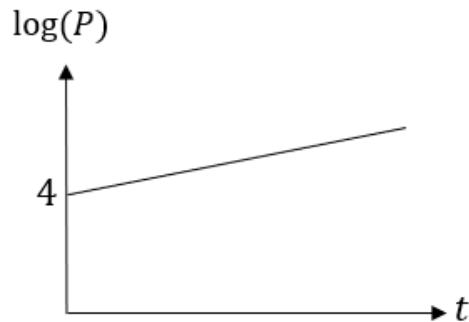
Worked Example

The graph represents the growth of a population of bacteria, P , over t hours.

The graph has a gradient of 0.3 and meets the vertical axis at $(0,4)$.

A scientist suggest that this growth can be modelled by the equation $P = ab^t$, where a and b are constants to be found.

- Write down an equation for the line.
- Find the values of a and b , giving them to 3 sf where necessary.
- Interpret the meaning of the constant a in this model.



Worked Example

The table below gives the rank (by size) and population of a country's largest cities and districts (the capital city is number 1 but has been excluded as an outlier).

City	A	B	C	D	E
Rank, R	2	3	4	5	6
Population	2 000 000	1 400 000	1 200 000	1 000 000	900 000

The relationship between the rank and population can be modelled by the formula:

$P = aR^n$ where a and n are constants.

- Draw a table giving values of $\log R$ and $\log P$ to 2dp.
- Plot a graph of $\log R$ against $\log P$ using the values from your table and draw the line of best fit.
- Use your graph to estimate the values of a and n to two significant figures.

Worked Example

A population is increasing exponentially according to the model $P = ab^t$, where a, b are constants to be found.

The population is recorded as follows:

Years t after 2016	1.4	2.6	4.4
Population P	4706	7346	14324

- a) Draw a table giving values of t and $\log P$ (to 3dp).
- b) A line of best fit is drawn for the data in your new table, and it happens to go through the first data point above (where $t = 1.4$) and last (where $t = 4.4$).
Determine the equation of this line of best fit.
- c) Hence, determine the values of a and b in the model.
- d) Estimate the population in 2020

8. The temperature, $\theta^\circ\text{C}$, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \quad t \geq 0$$

Find, according to the model,

- (a) the temperature of the cup of tea when it was placed on the table,
- (b) the value of t , to one decimal place, when the temperature of the cup of tea was 35°C .
- (c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C .

(1)

(3)

(1)

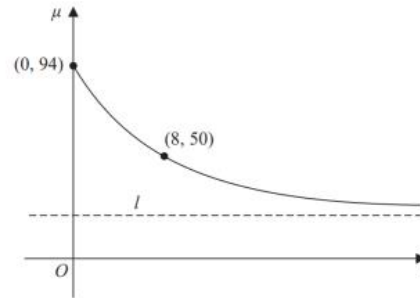


Figure 2

The temperature, $\mu^\circ\text{C}$, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \quad t \geq 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l , also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

- (d) find an equation for the asymptote l .

Your Turn

- 8 The temperature, θ °C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 16 + 69e^{-\frac{t}{6}} \quad t \geq 0$$

Find, according to the model,

- (a) the temperature of the cup of tea when it was placed on the table, (1)
- (b) the value of t , to two decimal places, when the temperature of the cup of tea was 40 °C. (3)
- (c) Explain why, according to this model, the temperature of the cup of tea could not fall to 10 °C. (1)

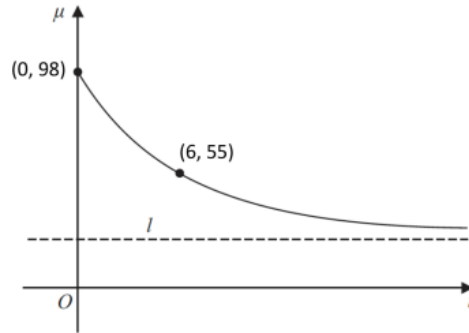


Figure 2

The temperature, μ °C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{6}} \quad t \geq 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l , also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

- (d) find an equation for the asymptote l . (4)

(Total for Question 8 is 9 marks)

12. An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \quad 1 \leq t \leq 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V , in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(

(b) Interpret, with reference to the model, the value of ab .

(

Using this model, calculate

(c) the total number of views of the advert in the first 20 days after the advert went live.
Give your answer to 2 significant figures.

(

Your Turn

- 12** An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_8 V = 0.068t + 2.589 \quad 1 \leq t \leq 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V , in the first t days after the advert went live.

- (a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

- (b) Interpret, with reference to the model, the value of ab .

(1)

Using this model, calculate

- (c) the total number of views of the advert in the first 15 days after the advert went live.
Give your answer to 2 significant figures.

(2)

(Total for Question 12 is 7 marks)

5. The mass, A kg, of algae in a small pond, is modelled by the equation

$$A = pq^t$$

where p and q are constants and t is the number of weeks after the mass of algae was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} A$ given by the equation

$$\log_{10} A = 0.03t + 0.5$$

- (a) Use this relationship to find a complete equation for the model in the form

$$A = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

- (b) With reference to the model, interpret

- (i) the value of the constant p ,
- (ii) the value of the constant q .

(2)

- (c) Find, according to the model,

- (i) the mass of algae in the pond when $t = 8$, giving your answer to the nearest 0.5 kg,
- (ii) the number of weeks it takes for the mass of algae in the pond to reach 4 kg.

(3)

- (d) State one reason why this may not be a realistic model in the long term.

(1)

Your Turn

5. The mass, M grams of a colony of bacteria, is modelled by the equation

$$M = pq^t$$

where p and q are constants and t is the number of days after the mass was first recorded.

Data recorded indicates that there is a linear relationship between t and $\log_{10} M$ given by the equation

$$\log_{10} M = 0.02t + 0.6$$

- (a) Use this relationship to find a complete equation for the model in the form

$$M = pq^t$$

giving the value of p and the value of q each to 4 significant figures.

(4)

- (b) With reference to the model, interpret

- (i) the value of the constant p ,
- (ii) the value of the constant q .

(2)

- (c) Find, according to the model,

- (i) the mass of the colony when $t = 5$, giving your answer to the nearest gram,
- (ii) the number of days it takes for the mass of the colony to reach 7 grams.

(3)

- (d) State one reason why this may not be a realistic model in the long term.

(1)

(Total for Question 5 is 10 marks)

8.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The air pressure, P kg/cm², inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \quad t \in \mathbb{R} \quad t \geq 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm²

(a) state the value of k .

(1)

From the instant when the tyre developed the puncture,

(b) find the time taken for the air pressure to fall to 1 kg/cm²

Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.

Give your answer in kg/cm² per minute to 3 significant figures.

Your Turn

8. In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The volume of water, V litres, in a paddling pool, t minutes from the instant when it was punctured is given by the equation

$$V = k + 1900e^{-0.1t} \quad t \in \mathbb{R} \quad t \geq 0$$

where k is a constant.

Given that the initial volume of water in the paddling pool was 2000 litres,

(a) state the value of k .

(1)

From the instant when the pool was punctured,

(b) find the time taken for the volume to fall to 500 litres

Give your answer in minutes to one decimal place.

(3)

(c) Find the rate at which the volume of water in the pool is decreasing exactly 20 minutes after the instant when it was punctured.

Give your answer in litres per minute to 3 significant figures.

(2)

(Total for Question 8 is 6 marks)

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

Past Paper Questions

A2 2019 Paper 1

Exponentials and Logs

7. In a simple model, the value, £ V , of a car depends on its age, t , in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A , a possible equation linking V with t .

(4)

The value of car A is monitored over a 10-year period.

Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B .

(1)



Exams

- Formula Booklet
- Past Papers
- Practice Papers
- [past paper Qs by topic](#)

Past paper practice by topic. Both new and old specification can be found via this link on [hgsmaths.com](https://www.hgsmaths.com)

(c) $V = 20000e^{-0.223t} + 5000$ A constant should be added (e.g. $V = 20000e^{-0.223t}$) The value of V should be increased For example:	BI	suggesting a linear relationship This mark is given for a statement car after 10 years
£2120 is close to £2000 This model is reliable since the value	AI	comparing the two possible values of the This mark is given for a valid statement when $t = 10$
When $t = 10$, $V = £2120$	MI	This mark is given for finding a value for correct exponential model
$V = 20000e^{-0.223t}$ $k = \ln 0.8 = -0.223$ $10000 = 20000e^{-k}$ When $t = 1$ and $V = 10000$	AI	the car after one year to find a value for k This mark is given for using the model to show the initial value for $V = 20000$
When $t = 0$ and $V = 20000$, $V = 20000$	MI	This mark is given for stating the model is of a
$V = 20000e^{-k}$ adjusted to see	MI	suitable exponential model for V in terms This mark is given for suggesting a
Working or answers are essential marks	MI	marks

Summary of Key Points

Summary of key points

1 For all real values of x :

- If $f(x) = e^x$ then $f'(x) = e^x$
- If $y = e^x$ then $\frac{dy}{dx} = e^x$

2 For all real values of x and for any constant k :

- If $f(x) = e^{kx}$ then $f'(x) = ke^{kx}$
- If $y = e^{kx}$ then $\frac{dy}{dx} = ke^{kx}$

3 $\log_a n = x$ is equivalent to $a^x = n$ ($a \neq 1$)

4 The laws of logarithms:

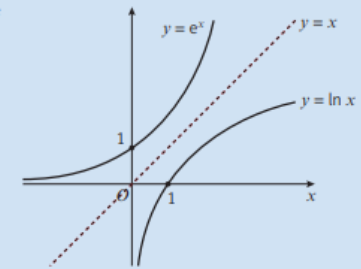
- $\log_a x + \log_a y = \log_a xy$ (the multiplication law)
- $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ (the division law)
- $\log_a (x^k) = k \log_a x$ (the power law)

5 You should also learn to recognise the following special cases:

- $\log_a \left(\frac{1}{x}\right) = \log_a (x^{-1}) = -\log_a x$ (the power law when $k = -1$)
- $\log_a a = 1$ ($a > 0, a \neq 1$)
- $\log_a 1 = 0$ ($a > 0, a \neq 1$)

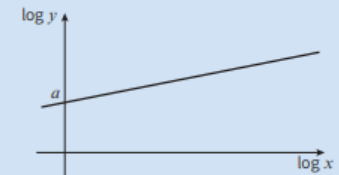
6 Whenever $f(x) = g(x)$, $\log_a f(x) = \log_a g(x)$

7 The graph of $y = \ln x$ is a reflection of the graph $y = e^x$ in the line $y = x$.

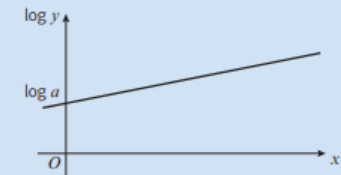


8 $e^{\ln x} = \ln(e^x) = x$

9 If $y = ax^n$ then the graph of $\log y$ against $\log x$ will be a straight line with gradient n and vertical intercept $\log a$.



10 If $y = ab^x$ then the graph of $\log y$ against x will be a straight line with gradient $\log b$ and vertical intercept $\log a$.



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