



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Unit P4: Trigonometry 1

HGS Maths



4

Dr Frost Course



Name: _____

Class: _____

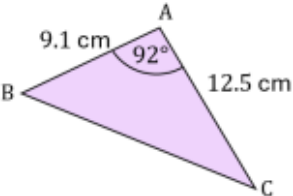
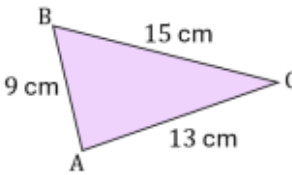
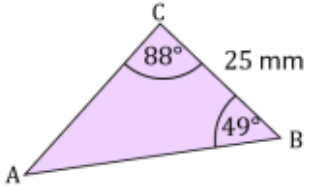
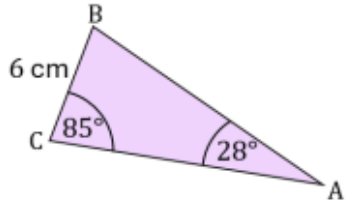
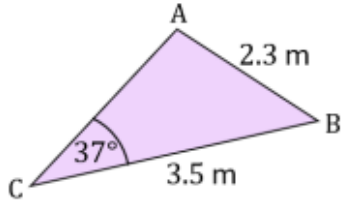
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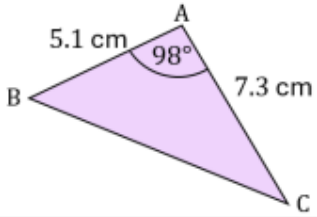
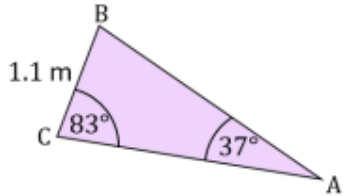
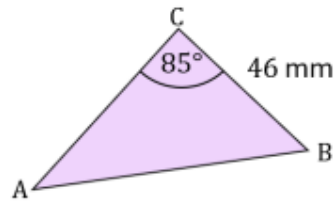
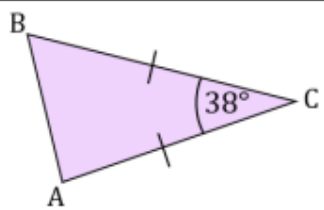
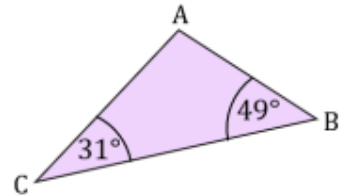
**Italics are summer task prerequisite work*

**Past Paper Practice
Summary**

Prior knowledge check

Triangle	AB	BC	AC	$\widehat{ABC}$	$\widehat{ACB}$	$\widehat{BAC}$
	9.1 cm		12.5 cm			92°
	9 cm	15 cm	13 cm			
		25 mm		49°	88°	
		6 cm			85°	28°
	2.3 m	3.5 m			37°	

Prior knowledge check

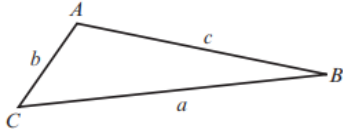
Triangle	Area	AB	BC	AC	$\widehat{ABC}$	$\widehat{ACB}$	$\widehat{BAC}$
		5.1 cm		7.3 cm			98°
			1.1 m			83°	37°
	1489 mm ²		46 mm			85°	
	33.9 cm ²					38°	
	9.67 cm ²				49°	31°	

4.1 Sine Rule, Cosine Rule and Area of Triangles

These are **not** in formulae booklet:

- This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc \cos A$$

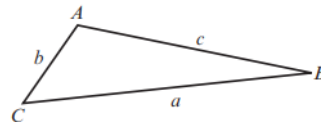


Watch out You can exchange the letters depending on which side you want to find, as long as each side has the same letter as the **opposite** angle.

The sine rule can be used to work out missing sides or angles in triangles.

- This version of the sine rule is used to find the length of a missing side:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

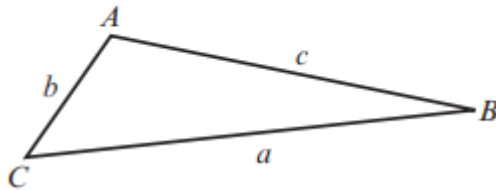


- This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You can use the standard trigonometric ratios for right-angled triangles to prove the sine rule:

- $\text{Area} = \frac{1}{2}ab \sin C$



Notes

Worked Example

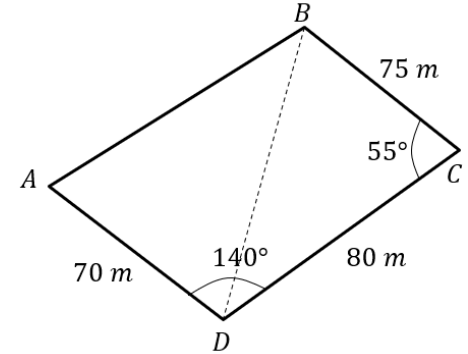
The diagram shows the locations of four mobile phone masts in a field.

$BC = 75\text{ m}$, $CD = 80\text{ m}$, angle $BCD = 55^\circ$ and angle $ADC = 140^\circ$.

In order that the masts do not interfere with each other, they must be at least 70m apart.

Given that A is the minimum distance from D , find:

- The distance A is from B
- The angle BAD
- The area enclosed by the four masts.



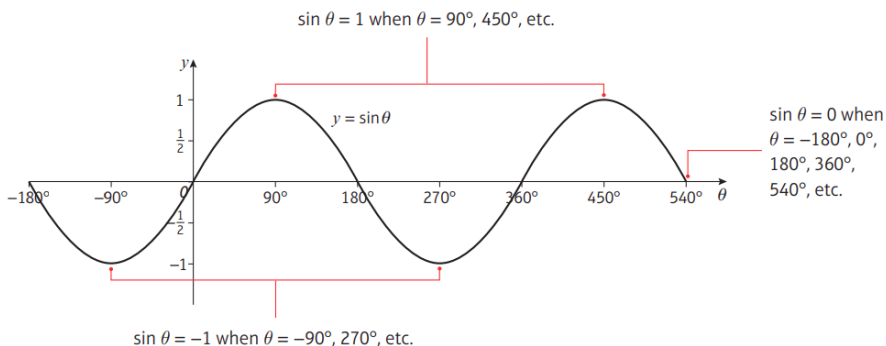
4.2 Trigonometric Graphs

- The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

You need to be able to draw the graphs for a given range of angles.

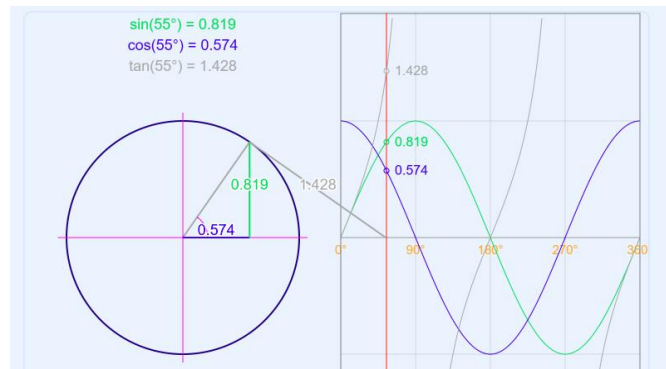
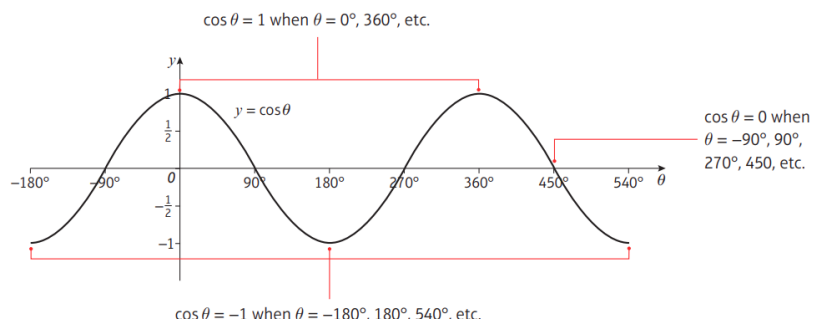
- The graph of $y = \sin \theta$:

- repeats every 360° and crosses the x -axis at $\dots, -180^\circ, 0, 180^\circ, 360^\circ, \dots$
- has a maximum value of 1 and a minimum value of -1 .



- The graph of $y = \cos \theta$:

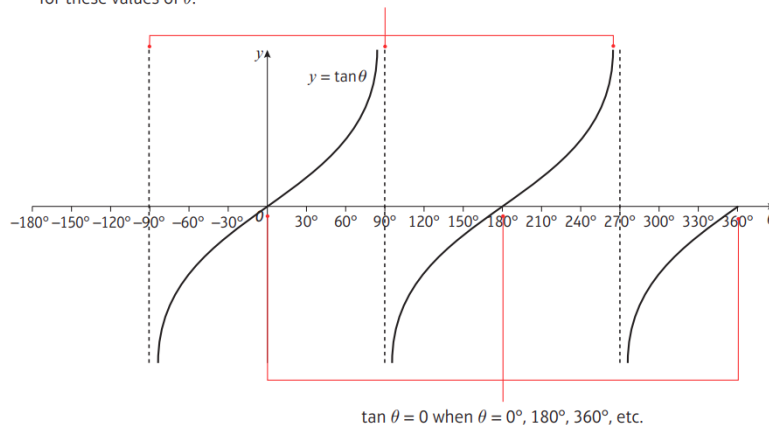
- repeats every 360° and crosses the x -axis at $\dots, -90^\circ, 90^\circ, 270^\circ, 450^\circ, \dots$
- has a maximum value of 1 and a minimum value of -1 .



- The graph of $y = \tan \theta$:

- repeats every 180° and crosses the x -axis at $\dots, -180^\circ, 0^\circ, 180^\circ, 360^\circ, \dots$
- has no maximum or minimum value
- has vertical asymptotes at $x = -90^\circ, x = 90^\circ, x = 270^\circ, \dots$

$\tan \theta$ does **not** have maximum and minimum points but approaches negative or positive infinity as the curve approaches the **asymptotes** at $-90^\circ, 90^\circ, 270^\circ$, etc. $\tan \theta$ is **undefined** for these values of θ .



Worked Example

- a) Sketch the graph of $y = \cos \theta$ in the interval $-360^\circ \leq \theta \leq 360^\circ$
- b) i) Sketch the graph of $y = \sin x$ in the interval $-180^\circ \leq x \leq 270^\circ$
ii) $\sin(-30^\circ) = -0.5$. Use your graph to determine two further values of x for which $\sin x = -0.5$

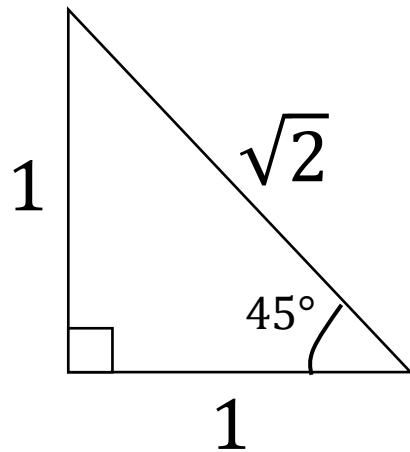
4.3 Trigonometric Identities

Prerequisite Knowledge

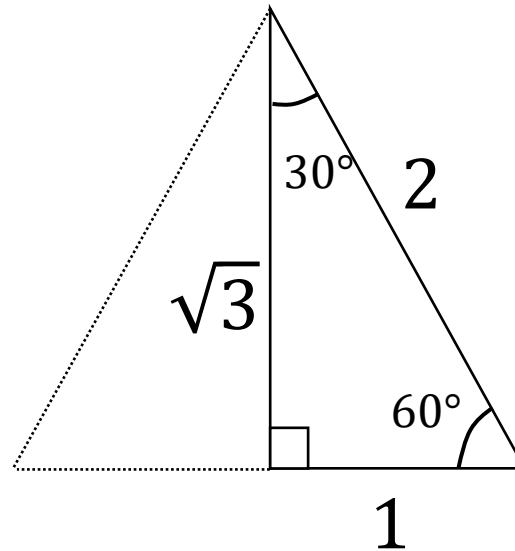
sin/cos/tan of 30° , 45° , 60°

You will frequently encounter angles of 30° , 60° , 45° in geometric problems. Why?

We see these angles in equilateral triangles and half squares:



$$\begin{aligned}\sin(45^\circ) &= \frac{1}{\sqrt{2}} \\ \cos(45^\circ) &= \frac{1}{\sqrt{2}} \\ \tan(45^\circ) &= 1\end{aligned}$$



$$\begin{aligned}\sin(30^\circ) &= \frac{1}{2} \\ \cos(30^\circ) &= \frac{\sqrt{3}}{2} \\ \tan(30^\circ) &= \frac{1}{\sqrt{3}} \\ \sin(60^\circ) &= \frac{\sqrt{3}}{2} \\ \cos(60^\circ) &= \frac{1}{2} \\ \tan(60^\circ) &= \sqrt{3}\end{aligned}$$

Activity

Sort It Out

Exact Trigonometric Values

Sort the trigonometric values into the correct group below.

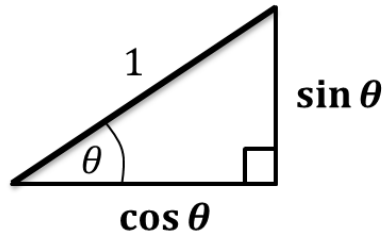
1	sin 45	2	cos 30	3	tan 45
4	cos 0	5	sin 60	6	tan 360
7	sin 90	8	cos 45	9	sin 30
10	cos 60	11	tan 0	12	sin 0
13	sin 360	14	tan 60	15	sin 270
16	tan 180	17	cos 90	18	tan 30
19	sin 135	20	cos 360	21	tan 225
22	cos 180	23	tan 210	24	cos 300
25	tan 240	26	cos 330	27	sin 120
28	sin 150	29	cos 315	30	tan 135

A	Equal to $\frac{1}{2}$	B	Equal to $\frac{\sqrt{3}}{2}$	C	Equal to $\frac{\sqrt{2}}{2}$	D	Equal to -1
E	Equal to 1	F	Equal to 0	G	Equal to $\frac{\sqrt{3}}{3}$	H	Equal to $\sqrt{3}$

Notes

The Unit Circle and Trigonometry

For values of θ in the range $0 < \theta < 90^\circ$, you know that $\sin \theta$ and $\cos \theta$ are lengths on a right-angled triangle:



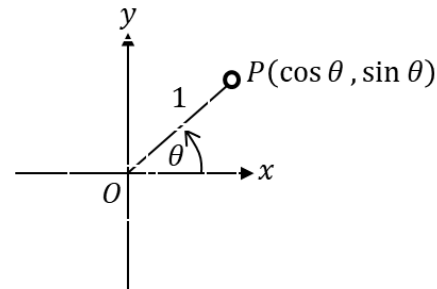
And what would be the **gradient** of the bold line?

$$m = \frac{\Delta y}{\Delta x} = \frac{\sin \theta}{\cos \theta}$$

But also: $\tan \theta = \frac{\text{opp}}{\text{adj}} = \frac{\sin \theta}{\cos \theta}$
 $\therefore m = \tan \theta$

But how do we get the rest of the graph for $\sin, \cos$ and $\tan$ when $90^\circ \leq \theta \leq 360^\circ$?

The point P on a unit circle, such that OP makes an angle θ with the positive x -axis, has coordinates $(\cos \theta, \sin \theta)$.
 OP has gradient $\tan \theta$.



Angles are always measured **anticlockwise**.

(Further Mathematicians will encounter the same when they get to Complex Numbers)

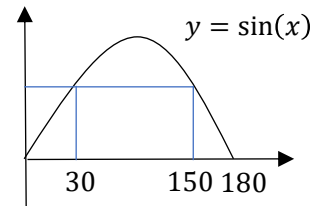
We can consider the coordinate $(\cos \theta, \sin \theta)$ as θ increases from 0 to 360° ...

Notes

A Few Trigonometric Angle Laws

1 $\sin(x) = \sin(180^\circ - x)$

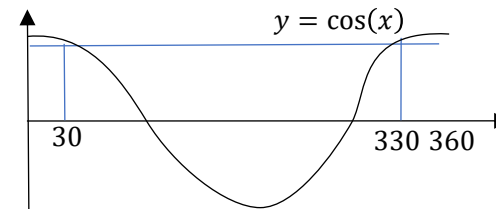
e.g. $\sin(150^\circ) = \sin(30^\circ)$



We saw this in the previous chapter when covering the 'ambiguous case' when using the sine rule.

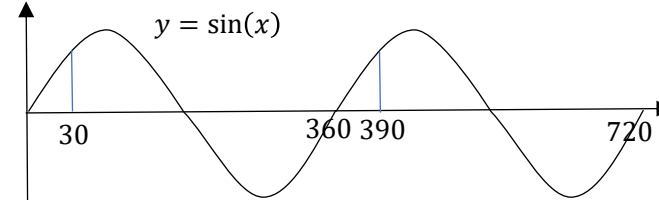
2 $\cos(x) = \cos(360^\circ - x)$

e.g. $\cos(330^\circ) = \cos(30^\circ)$



3 $\sin$ and $\cos$ repeat every 360°
but $\tan$ every 180°

e.g. $\sin(390^\circ) = \sin(30^\circ)$



4 $\sin(x) = \cos(90^\circ - x)$

e.g. $\sin(50^\circ) = \cos(40^\circ)$

Remember from the previous chapter that "cosine" by definition is the sine of the "complementary" angle. This was/is never covered in the textbook but caught everyone by surprise when it came up in a C3 exam.

Examples

$$\begin{aligned} \tan(225^\circ) &= \tan(45^\circ) = 1 \\ \tan(210^\circ) &= \tan(30^\circ) = \frac{1}{\sqrt{3}} \\ \sin(150^\circ) &= \sin(30^\circ) = \frac{1}{2} \\ \cos(300^\circ) &= \cos(60^\circ) = \frac{1}{2} \end{aligned}$$

$\tan$ repeats every 180°
so can subtract 180°

For $\sin$ we can subtract
from 180° .

For $\cos$ we can subtract
from 360° .

$$\sin(-45^\circ) = -\sin(45^\circ) = -\frac{1}{\sqrt{2}}$$

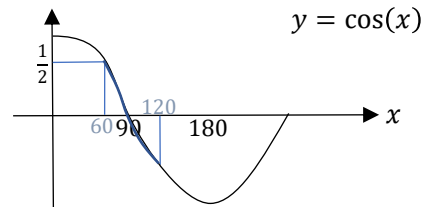
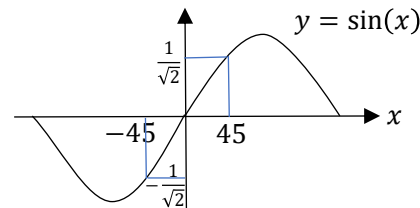
We have to resort to a sketch for this one.

$$\cos(750^\circ) = \cos(30^\circ) = \frac{\sqrt{3}}{2}$$

$\cos$ repeats
every 360° .

$$\cos(120^\circ) = -\cos(60^\circ) = -\frac{1}{2}$$

Again, let's just use a graph.



Use the 'laws' where you can,
but otherwise just draw out a
quick sketch of the graph.

- $\sin(x) = \sin(180 - x)$
- $\cos(x) = \cos(360 - x)$
- $\sin, \cos$ repeat every 360°
but $\tan$ every 180°

reflections: It's not hard to see from the
graph that in general, $\sin(-x) = -\sin(x)$.
Even more generally, a function f is known as
an '**odd function**' if $f(-x) = -f(x)$.
 $\tan$ is similarly 'odd' as $\tan(-x) = -\tan(x)$.

A function is **even** if $f(-x) = f(x)$. Examples
are $f(x) = \cos(x)$ and $f(x) = x^2$ as
 $\cos(-x) = \cos(x)$ and $(-x)^2 = (x)^2$. You do
not need to know this for the exam.

The graph is rotationally
symmetric about 90° . Since 120°
is 30° above 90° , we get the same
 y value for $90^\circ - 30^\circ = 60^\circ$,
except negative.

Test Your Understanding

Without a calculator, work out the value of each below.

$$\cos(315^\circ) =$$

$$\sin(420^\circ) =$$

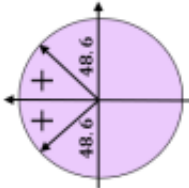
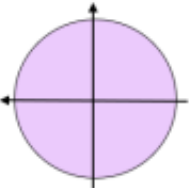
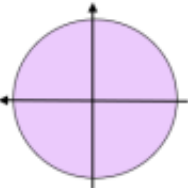
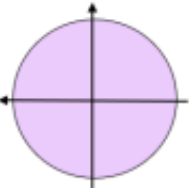
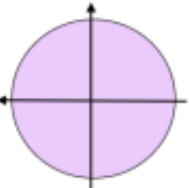
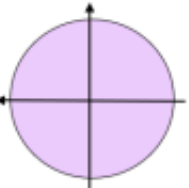
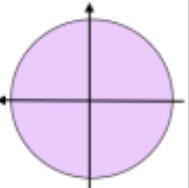
$$\tan(-120^\circ) =$$

$$\tan(-45^\circ) =$$

$$\sin(135^\circ) =$$

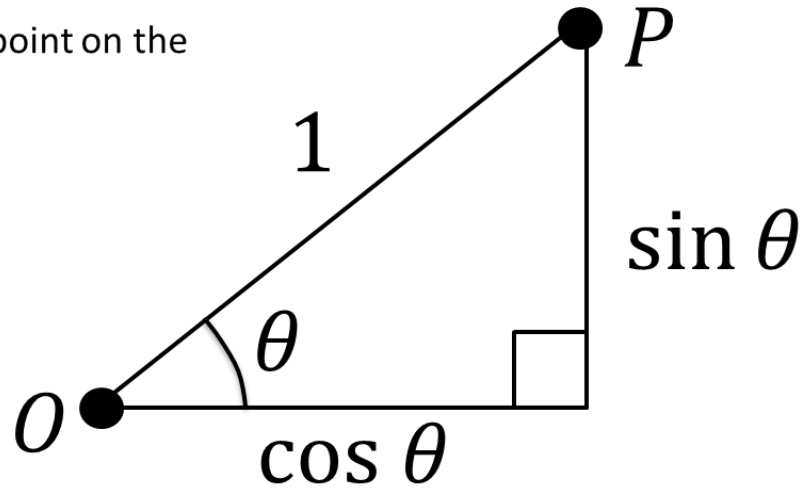
Prerequisite Knowledge

Fill in the Blanks Solving Trigonometric Equations

Question	Rearranged Equation	Acute Angle	Unit Circle	Solutions
Solve $4 \sin \theta = 3$ for $0^\circ \leq \theta < 360^\circ$	$\sin \theta = \frac{3}{4}$	$\theta = \sin^{-1}\left(\frac{3}{4}\right)$ $\theta = 48.6^\circ$		
Solve $5 \cos \theta = 1$ for $0^\circ \leq \theta < 360^\circ$				
Solve $5 \tan \theta = -8$ for $0^\circ \leq \theta < 360^\circ$				
Solve $\tan \theta - 6 = 0$ for $-180^\circ \leq \theta < 180^\circ$				
Solve $3 \sin \theta + 2 = 0$ for $-180^\circ \leq \theta < 180^\circ$				
Solve $16 \cos^2 \theta = 9$ for $0^\circ \leq \theta < 360^\circ$				
Solve $4 \tan^2 \theta = 25$ for $-180^\circ \leq \theta < 180^\circ$				

10.3 Trigonometric Identities

Returning to our point on the unit circle...



1

Then $\tan \theta = \frac{\sin \theta}{\cos \theta}$

2

Pythagoras gives
you...



$$\sin^2 \theta + \cos^2 \theta = 1$$

$\sin^2 \theta$ is a shorthand for $(\sin \theta)^2$. It does NOT mean the sin is being squared – this does not make sense as sin is a function, and not a quantity that we can square!

Worked Example

Simplify:

$$\sin^2 3x + \cos^2 3x$$

Worked Example

Prove that $1 - \tan \theta \sin \theta \cos \theta \equiv \cos^2 \theta$

Worked Example

Prove that $\tan \theta + \frac{1}{\tan \theta} \equiv \frac{1}{\sin \theta \cos \theta}$

Worked Example

Simplify $5 - 5 \sin^2 \theta$

Worked Example

Simplify:

$$\frac{\sin 2\theta}{\sqrt{1 - \sin^2 2\theta}}$$

Worked Example

Prove that

$$\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \equiv 1 - \tan^2 \theta$$

Worked Example

Prove that

$$\frac{\tan x \cos x}{\sqrt{1 - \cos^2 x}} \equiv 1$$

Worked Example

Prove that

$$\tan^2 \theta \equiv \frac{1}{\cos^2 \theta} - 1$$

Worked Example K502a

Given that $\sin \theta = \frac{2}{5}$ and that θ is obtuse, find the exact value of $\cos \theta$

Worked Example

502b: Determine exact values of $\sin$, $\cos$ and $\tan$ using another known trigonometric ratio, for obtuse or reflex angles.

It is given that B is reflex and that $\sin B = -\frac{1}{2}$

Find the exact value of $\cos B$.

Worked Example

502c: Determine exact values of $\sin$, $\cos$ and $\tan$ using another known trigonometric ratio in terms of an algebraic expression.

Given that

$$\sin \theta = \frac{k-1}{k+1}$$

where k is a positive constant and θ is acute.

Find an expression for $\cos \theta$ in terms of k .

$\cos \theta =$ 

Worked Example

502f: Write an expression given in the form $a \sin^2 x + b \cos^2 x$ in terms of just sin or cos

Given that:

$$2 \sin^2 3\alpha + 8 \cos^2 3\alpha \equiv A + B \sin^2 3\alpha$$

where A and B are integers.

Work out the values of A and B .

Worked Example

502i: Simplify a trigonometric expression by using the identity

$$\sin^2 x + \cos^2 x \equiv 1 \text{ and } \tan x = \frac{\sin x}{\cos x}$$

Simplify

$$1 - \sin \alpha \cos \alpha \tan \alpha$$

giving your answer as a single trigonometric function.

Worked Example

502j: Simplify a trigonometric expression involving $\tan^2 x$ or $\tan^3 x$

Simplify

$$1 - \cos^2 \theta \tan^2 \theta$$

giving your answer as a single trigonometric function.

Worked Example

502k: Simplify an algebraic fraction in terms of $\sin$ and $\cos$ using quadratic factorisation.

Simplify

$$\frac{12 + 3 \sin x}{2 \cos^2 x - 5 \sin x + 10}$$

Giving your answer in the form $\frac{a}{b + c \sin x}$

Worked Example

Given that $p = 3 \cos \theta$ and $q = 2 \sin \theta$, show that $4p^2 + 9q^2 = 36$

13.**In this question you must show all stages of your working.****Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta \equiv \frac{\cos \theta}{1 - \sin \theta} \quad \theta \neq (2n + 1)90^\circ \quad n \in \mathbb{Z} \quad (3)$$

Given that $\cos 2x \neq 0$

Your Turn

13.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\sin \theta} + \frac{1}{\tan \theta} \equiv \frac{\sin \theta}{1 - \cos \theta} \quad \theta \neq 180n^\circ \quad n \in \mathbb{Z}$$

(3)

12.**In this question you must show detailed reasoning.****Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0$$

(3)

Your Turn

12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

can be written as

$$\tan^2 x - \tan x - 1 = 0$$

(3)

10.4 Simple Trigonometric Equations

You need to be able to solve simple trigonometric equations of the form $\sin \theta = k$ and $\cos \theta = k$ (where $-1 \leq k \leq 1$) and $\tan \theta = p$ (where $p \in \mathbb{R}$) for given intervals of θ .

- **Solutions to $\sin \theta = k$ and $\cos \theta = k$ only exist when $-1 \leq k \leq 1$.**
- **Solutions to $\tan \theta = p$ exist for all values of p .**

Worked Example

503a: Solve a trigonometric equation given in the form $\sin x = k$ where x is in degrees.

Solve $\sin x = -0.8$ in the interval $0^\circ \leq x \leq 720^\circ$

Give your solution(s) correct to 1 decimal place where appropriate.

 $x =$ $^\circ$

 $x =$ $^\circ$

 $x =$ $^\circ$

 $x =$ $^\circ$

503f: Solve trigonometric equations given in the form $\sin^2 x = a$ where x is in degrees.

Solve $\sin^2 x = 0.49$ in the interval $0^\circ \leq x \leq 360^\circ$

Give your solution(s) correct to 1 decimal place where appropriate.

 $x =$ $^\circ$

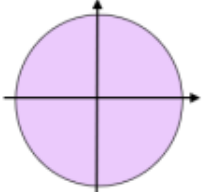
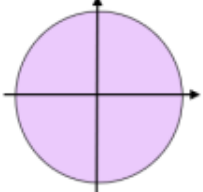
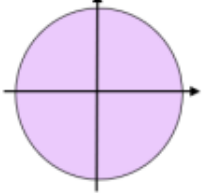
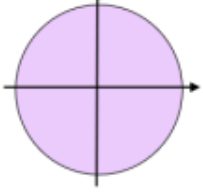
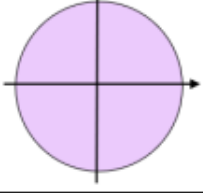
 $x =$ $^\circ$

 $x =$ $^\circ$

 $x =$ $^\circ$

10.5 Harder Trigonometric Equations

You need to be able to solve equations of the form $\sin n\theta = k$, $\cos n\theta = k$ and $\tan n\theta = p$.

Question	Substitute $x = \dots$	Rearrange Equation	Acute Angle	Range for x	Unit Circle	Solutions for x	Solutions for θ
Solve $7 \cos(2\theta) = 5$ for $0^\circ \leq \theta < 180^\circ$	$x = 2\theta$ $7 \cos x = 5$	$\cos x = \frac{5}{7}$	$x = \cos^{-1}\left(\frac{5}{7}\right)$ $x = 44.415^\circ$	$0^\circ \leq x < 360^\circ$			
Solve $8 \sin(3\theta) - 7 = 0$ for $-90^\circ \leq \theta < 90^\circ$	$x = 3\theta$ $8 \sin x - 7 = 0$						
Solve $\frac{4}{\tan(\theta + 25)} = 3$ for $-180^\circ \leq \theta < 180^\circ$							
$9 \cos(2\theta - 15) = 4$ for $0^\circ \leq \theta < 360^\circ$							
$\frac{\tan(3\theta + 70)}{2} + 3 = 0$ for $-90^\circ \leq \theta < 90^\circ$							

Worked Example

503e: Solve a trigonometric equation given in the form $\sin(ax + b) = k$ where x is in degrees and k is negative.

Solve $\sin(2x + 90) = -0.4$ in the interval $0^\circ \leq x \leq 360^\circ$

Give your solution(s) correct to 1 decimal place where appropriate.

 $x =$		$^\circ$
 $x =$		$^\circ$
 $x =$		$^\circ$
 $x =$		$^\circ$

Worked Example

503i: Solve a trigonometric equation given in the form

$$p \sin(ax + b) = q \cos(ax + b)$$

Solve $4 \sin(\frac{1}{2}\theta + 60) = \sqrt{3} \cos(\frac{1}{2}\theta + 60)$ in the interval $0^\circ < \theta < 720^\circ$

Give your solutions correct to 2 decimal places where appropriate.

 $\theta =$ $^\circ$

 $\theta =$ $^\circ$

Worked Example

503n: Solve a trigonometric equation from a modelled scenario.

The depth of water, H meters, in a harbour on a particular day is modelled by the formula

$$H = 10 + 2 \sin\left(\frac{1}{2}\theta\right), \quad 0 \leq \theta \leq 1440$$

where θ is the number of minutes after midnight.

Freddie needs to leave the harbour by 1 pm at the latest. Find the last time the water is at a depth of the water is **11.96** meters before Freddie needs to leave. Give your answer to the nearest minute in 24 hour clock.

 :

10.6 Equations and Identities

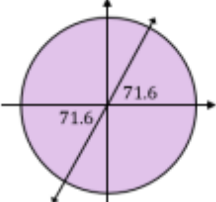
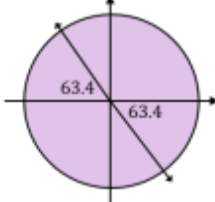
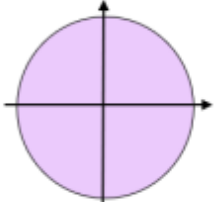
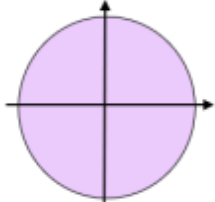
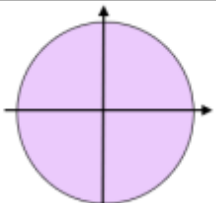
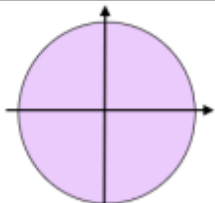
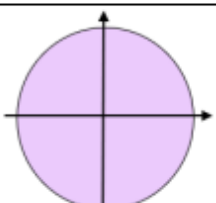
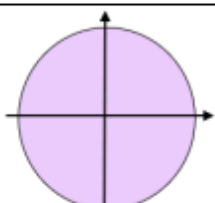
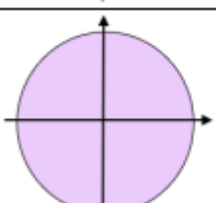
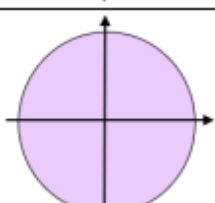
Know your trig identities!

For all values of θ , $\sin^2 \theta + \cos^2 \theta \equiv 1$

For all values of θ such that $\cos \theta \neq 0$, $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$

Also worth knowing:

$$\begin{aligned}\sin(90^\circ - \theta) &= \cos(\theta) \\ \cos(90^\circ - \theta) &= \sin(\theta)\end{aligned}$$

Question	Factorise and Solve	Solve 1 st Trig Equation	Solve 2 nd Trig Equation
Solve $\tan^2 \theta - \tan \theta - 6 = 0$ for $0^\circ \leq \theta < 360^\circ$	$(\tan \theta - 3)(\tan \theta + 2) = 0$ $\tan \theta = 3$ or $\tan \theta = -2$	 $\theta = \tan^{-1}(3)$ $\theta = 71.6^\circ$ $\theta = 71.6^\circ, 251.6^\circ$	 $\theta = \tan^{-1}(2)$ $\theta = 63.4^\circ$ $\theta = 116.6^\circ, 296.6^\circ$
Solve $6 \sin^2 \theta - \sin \theta - 1 = 0$ for $-180^\circ \leq \theta < 180^\circ$			
Solve $2 \cos^2 \theta + 7 \cos \theta + 3 = 0$ for $0^\circ \leq \theta < 360^\circ$			
Solve $3 \tan^2 \theta - 8 \tan \theta + 4 = 0$ for $-180^\circ \leq \theta < 180^\circ$			
Solve $9 \sin^2 \theta - 1 = 0$ for $0^\circ \leq \theta < 360^\circ$			

Worked Example

Solve in the interval $0 \leq x < 360^\circ$:

$$5 \sin^2 x + 3 \sin x - 2 = 0$$

Worked Example

Solve in the interval $0 \leq x \leq 360^\circ$:

$$\sin^2(x - 30^\circ) = \frac{1}{2}$$

13.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\cos \theta} + \tan \theta = \frac{\cos \theta}{1 - \sin \theta} \quad \theta \neq (2n + 1)90^\circ \quad n \in \mathbb{Z} \quad (3)$$

Given that $\cos 2x \neq 0$

(b) solve for $0 < x < 90^\circ$

$$\frac{1}{\cos 2x} + \tan 2x = 3 \cos 2x$$

giving your answers to one decimal place.

(5)

Your Turn

13.

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that

$$\frac{1}{\sin \theta} + \frac{1}{\tan \theta} \equiv \frac{\sin \theta}{1 - \cos \theta} \quad \theta \neq 180n^\circ \quad n \in \mathbb{Z}$$

(3)

Given that $\sin 2x, \tan 2x \neq 0$

(b) solve for $0 \leq x \leq 180^\circ$

$$\frac{1}{\sin 2x} + \frac{1}{\tan 2x} = 4 \sin 2x$$

(5)

(Total for Question 13 is 8 marks)

12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0$$

(3)

(b) Hence solve, for $0 < x \leq 360^\circ$

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval $0 < x \leq 1800^\circ$, explaining briefly the reason for your answer.

(2)

Your Turn

12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

can be written as

$$\tan^2 x - \tan x - 1 = 0$$

(3)

(b) Hence solve, for $-90^\circ < x < 90^\circ$

$$\tan x \cos x - \sin x \cos^2 x = \frac{\sin x + \cos^3 x}{\tan x}$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$\tan \frac{1}{2}x \cos \frac{1}{2}x - \sin \frac{1}{2}x \cos^2 \frac{1}{2}x = \frac{\sin \frac{1}{2}x + \cos^3 \frac{1}{2}x}{\tan \frac{1}{2}x}$$

in the interval $-900^\circ < x < 900^\circ$, explaining briefly the reason for your answer.

(2)

(Total for Question 12 is 9 marks)

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 60 m
- a carriage starts a circuit at a vertical height of 2 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 20)^2$$

where a and b are positive constants.

(a) Find a complete equation for the model.

(3)

(b) Use the model to determine the height of the carriage above the ground when $t = 40$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 29 \cos(9t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where α and β are constants.

(c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

(d) give a reason why the alternative model would be more appropriate.

(1)

Your Turn

13. On a roller coaster ride, passengers travel in carriages around a track.

On the ride, carriages complete multiple circuits of the track such that

- the maximum vertical height of a carriage above the ground is 42 m
- a carriage starts a circuit at a vertical height of 1.5 m above the ground
- the ground is horizontal

The vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is modelled by the equation

$$H = a - b(t - 15)^2$$

where a and b are positive constants.

- (a) Find a complete equation for the model.

(3)

- (b) Use the model to determine the height of the carriage above the ground when $t = 25$

(1)

In an alternative model, the vertical height, H m, of a carriage above the ground, t seconds after the carriage starts the first circuit, is given by

$$H = 20.25 \cos(7t + \alpha)^\circ + \beta \quad 0 \leq \alpha < 360^\circ$$

where α and β are constants.

- (c) Find a complete equation for the alternative model.

(2)

Given that the carriage moves continuously for 2 minutes,

- (d) give a reason why the alternative model would be more appropriate.

(1)

(Total for Question 13 is 7 marks)

Past Paper Questions

12. (a) Solve, for $-180^\circ \leq x < 180^\circ$, the equation

$$3 \sin^2 x + \sin x + 8 = 9 \cos^2 x$$

giving your answers to 2 decimal places.

(6)

(b) Hence find the smallest positive solution of the equation

$$3 \sin^2(2\theta - 30^\circ) + \sin(2\theta - 30^\circ) + 8 = 9 \cos^2(2\theta - 30^\circ)$$

giving your answer to 2 decimal places.

(2)



Exams

- [Formula Booklet](#)
- [Past Papers](#)
- [Practice Papers](#)
- [past paper Qs by topic](#)

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

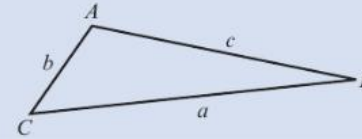
1.1	IM	$(x^2 \sin^2 - 1) \theta = 8 + x \sin^2 + x^2 \sin^2 \theta \leq x^2 \sin^2 - 1 = x^2 \cos^2 \theta \geq 0$	(6) 1
1.1	IA	$0 = 1 - x \sin^2 + x^2 \sin^2 \leq 1 \Leftrightarrow$	
1.1	IM	$0 = (1 + x \sin^2)(1 - x \sin^2) \Leftrightarrow$	
1.1	IA	$\frac{1}{x} - \frac{1}{x^2} = x \sin^2 \Leftrightarrow$	
1.1	IM	Use series to obtain two correct values	
1.1	IA	All four of $x = 1.448^\circ, 1.622^\circ, -1.047^\circ, -1.002^\circ$	
	(d)		
1.1	IM	Answer $5.9 - 3.0 = 2.9$ is correct	(d)
1.1	IA	$2.9 \leq x \leq 0 \Leftrightarrow$	
	(5)		

(8 marks)

Summary of Key Points

- 1 This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc \cos A$$



- 2 This version of the cosine rule is used to find an angle if you know all three sides:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

- 3 This version of the sine rule is used to find the length of a missing side:

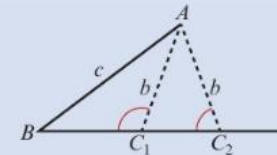
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- 4 This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- 5 The sine rule sometimes produces two possible solutions for a missing angle:

$$\sin \theta = \sin (180^\circ - \theta)$$



- 6 Area of a triangle = $\frac{1}{2}ab \sin C$.

- 7 The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

- The graph of $y = \sin \theta$: repeats every 360° and crosses the x -axis at $\dots, -180^\circ, 0, 180^\circ, 360^\circ, \dots$ has a maximum value of 1 and a minimum value of -1 .
- The graph of $y = \cos \theta$: repeats every 360° and crosses the x -axis at $\dots, -90^\circ, 90^\circ, 270^\circ, 450^\circ, \dots$ has a maximum value of 1 and a minimum value of -1 .
- The graph of $y = \tan \theta$: repeats every 180° and crosses the x -axis at $\dots, -180^\circ, 0^\circ, 180^\circ, 360^\circ, \dots$ has no maximum or minimum value has vertical asymptotes at $x = -90^\circ, x = 90^\circ, x = 270^\circ, \dots$

Summary of Key Points

- 5** For all values of θ , $\sin^2 \theta + \cos^2 \theta \equiv 1$
- 6** For all values of θ such that $\cos \theta \neq 0$, $\tan \theta \equiv \frac{\sin \theta}{\cos \theta}$
- 7** • Solutions to $\sin \theta = k$ and $\cos \theta = k$ only exist when $-1 \leq k \leq 1$
• Solutions to $\tan \theta = p$ exist for all values of p .
- 8** When you use the inverse trigonometric functions on your calculator, the angle you get is called the **principal value**.
- 9** Your calculator will give principal values in the following ranges:
- $\cos^{-1}$ in the range $0 \leq \theta \leq 180^\circ$
 - $\sin^{-1}$ in the range $-90^\circ \leq \theta \leq 90^\circ$
 - $\tan^{-1}$ in the range $-90^\circ \leq \theta \leq 90^\circ$