KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Prerequisite Mathematics Summer Task Booklet

www.hgsmaths.com

Name: _____

Instructions

This booklet is to be completed by **any student** who wishes to study A level Mathematics at HGS. When you begin lessons, it will be assumed that you are familiar with this material. **You will do a classroom test to check your understanding.**

It is therefore mandatory, i.e. NOT optional.

The content of this booklet is mainly crossover content from Higher GCSE. **Although there will certainly be parts which are unfamiliar to you.**

Using this booklet

1. you should attempt each example yourself where possible
2. Use QR code to watch a video solution for all examples covered in a section (e.g. 2.1)
3. Do exercises in an organised manner (e.g. an exercise book, notepad or folder)
4. Check answers from: https://tinyurl.com/yckeua78 or QR code:

YOUR TEACHER WILL CHECK YOU HAVE A FULLY COMPLETED BOOKLET AT THE START OF YOUR STUDIES

1) Algebraic expressions

1.1) Index laws	https://youtu.be/UO19cAFiii4?si=SV6H1P6w2srlze7U
1.2) Expanding brackets	https://youtu.be/w7PDJMErfvM?si=oKNVDC5g0QCiMGGP
1.3) Factorising	https://youtu.be/Q1Mm9iF83IA?si=5o2BANfOsMd9mdDi

1) Algebraic expressions

1.4) Negative and fractional indices	https://youtu.be/dzPnlZbZRH0?si=5g1PYurpXjkalM8G
1.5) Surds	https://youtu.be/k6CZyMEi89s?si=q-9Tb0CAeh3M1fN3
1.6) Rationalising denominators	https://youtu.be/Ps46XZwzyrU?si=Bh32wJAl59y_unOj

1.1) Index laws

Example 1

Simplify these expressions:

a $x^2 \times x^5$ **b** $2r^2 \times 3r^3$ **c** $\frac{b^7}{b^4}$ **d** $6x^5 \div 3x^3$ **e** $(a^3)^2 \times 2a^2$ **f** $(3x^2)^3 \div x^4$

Example 2

Expand these expressions and simplify if possible:

a $-3x(7x - 4)$ **b** $y^2(3 - 2y^3)$

c $4x(3x - 2x^2 + 5x^3)$ **d** $2x(5x + 3) - 5(2x + 3)$

Example 3

Simplify these expressions:

a $\frac{x^7 + x^4}{x^3}$ **b** $\frac{3x^2 - 6x^5}{2x}$ **c** $\frac{20x^7 + 15x^3}{5x^2}$

Exercise 1A

1 Simplify these expressions:

a $x^3 \times x^4$ **b** $2x^3 \times 3x^2$ **c** $\frac{k^3}{k^2}$

d $\frac{4p^3}{2p}$ **e** $\frac{3x^3}{3x^2}$ **f** $(y^2)^5$

g $10x^5 \div 2x^3$ **h** $(p^3)^2 \div p^4$ **i** $(2a^3)^2 \div 2a^3$

j $8p^4 \div 4p^3$ **k** $2a^4 \times 3a^5$ **l** $\frac{21a^3b^7}{7ab^4}$

m $9x^2 \times 3(x^2)^3$ **n** $3x^3 \times 2x^2 \times 4x^6$ **o** $7a^4 \times (3a^4)^2$

p $(4y^3)^3 \div 2y^3$ **q** $2a^3 \div 3a^2 \times 6a^5$ **r** $3a^4 \times 2a^5 \times a^3$

2 Expand and simplify if possible:

a $9(x - 2)$ **b** $x(x + 9)$ **c** $-3y(4 - 3y)$

d $x(y + 5)$ **e** $-x(3x + 5)$ **f** $-5x(4x + 1)$

g $(4x + 5)x$ **h** $-3y(5 - 2y^2)$ **i** $-2x(5x - 4)$

j $(3x - 5)x^2$ **k** $3(x + 2) + (x - 7)$ **l** $5x - 6 - (3x - 2)$

m $4(c + 3d^2) - 3(2c + d^2)$ **n** $(r^2 + 3t^2 + 9) - (2r^2 + 3t^2 - 4)$

o $x(3x^2 - 2x + 5)$ **p** $7y^2(2 - 5y + 3y^2)$ **q** $-2y^2(5 - 7y + 3y^2)$

r $7(x - 2) + 3(x + 4) - 6(x - 2)$ **s** $5x - 3(4 - 2x) + 6$

t $3x^2 - x(3 - 4x) + 7$ **u** $4x(x + 3) - 2x(3x - 7)$ **v** $3x^2(2x + 1) - 5x^2(3x - 4)$

3 Simplify these fractions:

a $\frac{6x^4 + 10x^6}{2x}$ **b** $\frac{3x^5 - x^7}{x}$ **c** $\frac{2x^4 - 4x^2}{4x}$

d $\frac{8x^3 + 5x}{2x}$ **e** $\frac{7x^7 + 5x^2}{5x}$ **f** $\frac{9x^5 - 5x^3}{3x}$

1.2) Expanding brackets

Example 4

Expand these expressions and simplify if possible:

a $(x + 5)(x + 2)$ **b** $(x - 2y)(x^2 + 1)$ **c** $(x - y)^2$ **d** $(x + y)(3x - 2y - 4)$

Example 5

Expand these expressions and simplify if possible:

a $x(2x + 3)(x - 7)$ **b** $x(5x - 3y)(2x - y + 4)$ **c** $(x - 4)(x + 3)(x + 1)$

Exercise 1B

1 Expand and simplify if possible:

a $(x + 4)(x + 7)$ **b** $(x - 3)(x + 2)$ **c** $(x - 2)^2$

d $(x - y)(2x + 3)$ **e** $(x + 3y)(4x - y)$ **f** $(2x - 4y)(3x + y)$

g $(2x - 3)(x - 4)$ **h** $(3x + 2y)^2$ **i** $(2x + 8y)(2x + 3)$

j $(x + 5)(2x + 3y - 5)$ **k** $(x - 1)(3x - 4y - 5)$ **l** $(x - 4y)(2x + y + 5)$

m $(x + 2y - 1)(x + 3)$ **n** $(2x + 2y + 3)(x + 6)$ **o** $(4 - y)(4y - x + 3)$

p $(4y + 5)(3x - y + 2)$ **q** $(5y - 2x + 3)(x - 4)$ **r** $(4y - x - 2)(5 - y)$

2 Expand and simplify if possible:

a $5(x + 1)(x - 4)$ **b** $7(x - 2)(2x + 5)$ **c** $3(x - 3)(x - 3)$

d $x(x - y)(x + y)$ **e** $x(2x + y)(3x + 4)$ **f** $y(x - 5)(x + 1)$

g $y(3x - 2y)(4x + 2)$ **h** $y(7 - x)(2x - 5)$ **i** $x(2x + y)(5x - 2)$

j $x(x + 2)(x + 3y - 4)$ **k** $y(2x + y - 1)(x + 5)$ **l** $y(3x + 2y - 3)(2x + 1)$

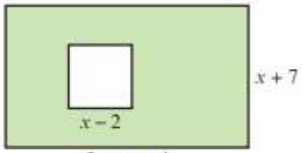
m $x(2x + 3)(x + y - 5)$ **n** $2x(3x - 1)(4x - y - 3)$ **o** $3x(x - 2y)(2x + 3y + 5)$

p $(x + 3)(x + 2)(x + 1)$ **q** $(x + 2)(x - 4)(x + 3)$ **r** $(x + 3)(x - 1)(x - 5)$

s $(x - 5)(x - 4)(x - 3)$ **t** $(2x + 1)(x - 2)(x + 1)$ **u** $(2x + 3)(3x - 1)(x + 2)$

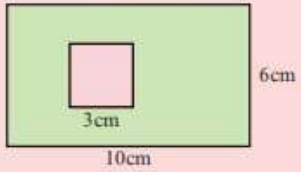
v $(3x - 2)(2x + 1)(3x - 2)$ **w** $(x + y)(x - y)(x - 1)$ **x** $(2x - 3y)^3$

(P) **3** The diagram shows a rectangle with a square cut out.
The rectangle has length $3x - y + 4$ and width $x + 7$.
The square has length $x - 2$.
Find an expanded and simplified expression for the shaded area.



Problem-solving

Use the same strategy as you would use if the lengths were given as numbers:



(P) **4** A cuboid has dimensions $x + 2$ cm, $2x - 1$ cm and $2x + 3$ cm.
Show that the volume of the cuboid is $4x^3 + 12x^2 + 5x - 6$ cm^3.

(E/P) **5** Given that $(2x + 5y)(3x - y)(2x + y) = ax^3 + bx^2y + cxy^2 + dy^3$, where a, b, c and d are constants, find the values of a, b, c and d. **(2 marks)**

1.3) Factorising

Example 6

Factorise these expressions completely:

a $3x + 9$ **b** $x^2 - 5x$ **c** $8x^2 + 20x$ **d** $9x^2y + 15xy^2$ **e** $3x^2 - 9xy$

Example 7

Factorise:

a $x^2 - 5x - 6$ **b** $x^2 + 6x + 8$ **c** $6x^2 - 11x - 10$ **d** $x^2 - 25$ **e** $4x^2 - 9y^2$

Example 8

Factorise completely:

a $x^3 - 2x^2$ **b** $x^3 - 25x$ **c** $x^3 + 3x^2 - 10x$

Exercise 1C

Factorise the quadratic to get thre

1 Factorise these expressions completely:

a $4x + 8$	**b** $6x - 24$	**c** $20x + 15$
d $2x^2 + 4$	**e** $4x^2 + 20$	**f** $6x^2 - 18x$
g $x^2 - 7x$	**h** $2x^2 + 4x$	**i** $3x^2 - x$
j $6x^2 - 2x$	**k** $10y^2 - 5y$	**l** $35x^2 - 28x$
m $x^2 + 2x$	**n** $3y^2 + 2y$	**o** $4x^2 + 12x$
p $5y^2 - 20y$	**q** $9xy^2 + 12x^2y$	**r** $6ab - 2ab^2$
s $5x^2 - 25xy$	**t** $12x^2y + 8xy^2$	**u** $15y - 20yz^2$
v $12x^2 - 30$	**w** $xy^2 - x^2y$	**x** $12y^2 - 4yx$

2 Factorise:

a $x^2 + 4x$	**b** $2x^2 + 6x$	**c** $x^2 + 11x + 24$
d $x^2 + 8x + 12$	**e** $x^2 + 3x - 40$	**f** $x^2 - 8x + 12$
g $x^2 + 5x + 6$	**h** $x^2 - 2x - 24$	**i** $x^2 - 3x - 10$
j $x^2 + x - 20$	**k** $2x^2 + 5x + 2$	**l** $3x^2 + 10x - 8$
m $5x^2 - 16x + 3$	**n** $6x^2 - 8x - 8$	
o $2x^2 + 7x - 15$	**p** $2x^4 + 14x^2 + 24$	
q $x^2 - 4$	**r** $x^2 - 49$	
s $4x^2 - 25$	**t** $9x^2 - 25y^2$	**u** $36x^2 - 4$
v $2x^2 - 50$	**w** $6x^2 - 10x + 4$	**x** $15x^2 + 42x - 9$

Hint For part **n**, take 2 out as a common factor first. For part **p**, let $y = x^2$.

3 Factorise completely:

a $x^3 + 2x$	**b** $x^3 - x^2 + x$	**c** $x^3 - 5x$
d $x^3 - 9x$	**e** $x^3 - x^2 - 12x$	**f** $x^3 + 11x^2 + 30x$
g $x^3 - 7x^2 + 6x$	**h** $x^3 - 64x$	**i** $2x^3 - 5x^2 - 3x$
j $2x^3 + 13x^2 + 15x$	**k** $x^3 - 4x$	**l** $3x^3 + 27x^2 + 60x$

4 Factorise completely $x^4 - y^4$. **(2 marks)**

Problem-solving

Watch out for terms that can be written as a function of a function: $x^4 = (x^2)^2$

5 Factorise completely $6x^3 + 7x^2 - 5x$. **(2 marks)**

1.4) Negative and fractional indices

Example 9

Simplify:

a $\frac{x^3}{x^{-3}}$ **b** $x^{\frac{1}{2}} \times x^{\frac{3}{2}}$ **c** $(x^3)^{\frac{2}{3}}$ **d** $2x^{1.5} \div 4x^{-0.25}$ **e** $\sqrt[3]{125x^6}$ **f** $\frac{2x^2 - x}{x^5}$

Example 10

Evaluate:

a $9^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $25^{-\frac{3}{2}}$

Example 11

Given that $y = \frac{1}{16}x^2$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{2}}$ **b** $4y^{-1}$

surds.

Exercise 1D

1 Simplify:

a $x^3 \div x^{-2}$ **b** $x^5 \div x^7$ **c** $x^{\frac{3}{2}} \times x^{\frac{5}{2}}$

d $(x^2)^{\frac{3}{2}}$ **e** $(x^3)^{\frac{4}{3}}$ **f** $3x^{0.5} \times 4x^{-0.5}$

g $9x^{\frac{2}{3}} \div 3x^{\frac{1}{6}}$ **h** $5x^{\frac{7}{5}} \div x^{\frac{2}{5}}$ **i** $3x^4 \times 2x^{-5}$

j $\sqrt{x} \times \sqrt[3]{x}$ **k** $(\sqrt{x})^3 \times (\sqrt[3]{x})^4$ **l** $\dfrac{(\sqrt[3]{x})^2}{\sqrt{x}}$

2 Evaluate:

a $25^{\frac{1}{2}}$ **b** $81^{\frac{3}{4}}$ **c** $27^{\frac{1}{3}}$

d 4^{-2} **e** $9^{-\frac{1}{2}}$ **f** $(-5)^{-3}$

g $\left(\frac{3}{4}\right)^0$ **h** $1296^{\frac{3}{4}}$ **i** $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

j $\left(\frac{27}{8}\right)^{\frac{2}{3}}$ **k** $\left(\frac{6}{5}\right)^{-1}$ **l** $\left(\frac{343}{512}\right)^{-\frac{2}{3}}$

3 Simplify:

a $(64x^{10})^{\frac{1}{2}}$ **b** $\dfrac{5x^3 - 2x^2}{x^5}$ **c** $(125x^{12})^{\frac{1}{3}}$ **d** $\dfrac{x + 4x^3}{x^3}$

e $\dfrac{2x + x^2}{x^4}$ **f** $\left(\dfrac{4}{9}x^4\right)^{\frac{3}{2}}$ **g** $\dfrac{9x^2 - 15x^5}{3x^3}$ **h** $\dfrac{5x + 3x^2}{15x^3}$

(E) **4** **a** Find the value of $81^{\frac{1}{4}}$. **(1 mar**

b Simplify $x(2x^{-\frac{1}{3}})^4$. **(2 mark**

(E) **5** Given that $y = \frac{1}{8}x^3$ express each of the following in the form kx^n, where k and n are constants.

a $y^{\frac{1}{3}}$ **(2 mark**

b $\frac{1}{2}y^{-2}$ **(2 mark**

1.5) Surds

Example 12

Simplify:

a $\sqrt{12}$ **b** $\frac{\sqrt{20}}{2}$ **c** $5\sqrt{6} - 2\sqrt{24} + \sqrt{294}$

Example 13

Expand and simplify if possible:

a $\sqrt{2}(5 - \sqrt{3})$ **b** $(2 - \sqrt{3})(5 + \sqrt{3})$

Exercise 1E

1 Do not use your calculator for this exercise. Simplify:

a $\sqrt{28}$ **b** $\sqrt{72}$ **c** $\sqrt{50}$

d $\sqrt{32}$ **e** $\sqrt{90}$ **f** $\frac{\sqrt{12}}{2}$

g $\frac{\sqrt{27}}{3}$ **h** $\sqrt{20}+\sqrt{80}$ **i** $\sqrt{200}+\sqrt{18}-\sqrt{72}$

j $\sqrt{175}+\sqrt{63}+2\sqrt{28}$ **k** $\sqrt{28}-2\sqrt{63}+\sqrt{7}$ **l** $\sqrt{80}-2\sqrt{20}+3\sqrt{45}$

m $3\sqrt{80}-2\sqrt{20}+5\sqrt{45}$ **n** $\frac{\sqrt{44}}{\sqrt{11}}$ **o** $\sqrt{12}+3\sqrt{48}+\sqrt{75}$

2 Expand and simplify if possible:

a $\sqrt{3}(2+\sqrt{3})$ **b** $\sqrt{5}(3-\sqrt{3})$ **c** $\sqrt{2}(4-\sqrt{5})$

d $(2-\sqrt{2})(3+\sqrt{5})$ **e** $(2-\sqrt{3})(3-\sqrt{7})$ **f** $(4+\sqrt{5})(2+\sqrt{5})$

g $(5-\sqrt{3})(1-\sqrt{3})$ **h** $(4+\sqrt{3})(2-\sqrt{3})$ **i** $(7-\sqrt{11})(2+\sqrt{11})$

(E) **3** Simplify $\sqrt{75}-\sqrt{12}$ giving your answer in the form $a\sqrt{3}$, where a is an integer. **(2 marks)**

1.6) Rationalising denominators

Example 14

Rationalise the denominator of:

a $\frac{1}{\sqrt{3}}$ **b** $\frac{1}{3+\sqrt{2}}$ **c** $\frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ **d** $\frac{1}{(1-\sqrt{3})^2}$

Exercise 1F

1 Simplify:

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$ **c** $\frac{1}{\sqrt{2}}$ **d** $\frac{\sqrt{3}}{\sqrt{15}}$

e $\frac{\sqrt{12}}{\sqrt{48}}$ **f** $\frac{\sqrt{5}}{\sqrt{80}}$ **g** $\frac{\sqrt{12}}{\sqrt{156}}$ **h** $\frac{\sqrt{7}}{\sqrt{63}}$

2 Rationalise the denominators and simplify:

a $\frac{1}{1+\sqrt{3}}$ **b** $\frac{1}{2+\sqrt{5}}$ **c** $\frac{1}{3-\sqrt{7}}$ **d** $\frac{4}{3-\sqrt{5}}$ **e** $\frac{1}{\sqrt{5}-\sqrt{3}}$

f $\frac{3-\sqrt{2}}{4-\sqrt{5}}$ **g** $\frac{5}{2+\sqrt{5}}$ **h** $\frac{5\sqrt{2}}{\sqrt{8}-\sqrt{7}}$ **i** $\frac{11}{3+\sqrt{11}}$ **j** $\frac{\sqrt{3}-\sqrt{7}}{\sqrt{3}+\sqrt{7}}$

k $\frac{\sqrt{17}-\sqrt{11}}{\sqrt{17}+\sqrt{11}}$ **l** $\frac{\sqrt{41}+\sqrt{29}}{\sqrt{41}-\sqrt{29}}$ **m** $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}-\sqrt{2}}$

3 Rationalise the denominators and simplify:

a $\frac{1}{(3-\sqrt{2})^2}$ **b** $\frac{1}{(2+\sqrt{5})^2}$ **c** $\frac{4}{(3-\sqrt{2})^2}$

d $\frac{3}{(5+\sqrt{2})^2}$ **e** $\frac{1}{(5+\sqrt{2})(3-\sqrt{2})}$ **f** $\frac{2}{(5-\sqrt{3})(2+\sqrt{3})}$

/P **4** Simplify $\frac{3-2\sqrt{5}}{\sqrt{5}-1}$ giving your answer in the form $p+q\sqrt{5}$, where p and q are rational numbers. **(4 marks)**

Problem-solving

You can check that your answer is in the correct form by writing down the values of p and q and checking that they are rational numbers.

2) Quadratics

2.1) Solving quadratic equations	https://youtu.be/VBweWkrXn1A?si=dB612sGGDvhxUOwE
2.2) Completing the square	https://youtu.be/roOgntGk8SA?si=i5cp4zLg4X5cWZ1C
2.3) Functions	https://youtu.be/vrkcw2WLrg8?si=qnQn52r5fL4-5X9Q

2.1) Solving quadratic equations

Example 1

Solve the following equations:

a $x^2 - 2x - 15 = 0$ **b** $x^2 = 9x$

c $6x^2 + 13x - 5 = 0$ **d** $x^2 - 5x + 18 = 2 + 3x$

Example 2

Solve the following equations

a $(2x - 3)^2 = 25$ **b** $(x - 3)^2 = 7$

Example 3

Solve $3x^2 - 7x - 1 = 0$ by using the formula.

Exercise 2A

1 Solve the following equations using factorisation:

a $x^2 + 3x + 2 = 0$ **b** $x^2 + 5x + 4 = 0$ **c** $x^2 + 7x + 10 = 0$ **d** $x^2 - x - 6 = 0$

e $x^2 - 8x + 15 = 0$ **f** $x^2 - 9x + 20 = 0$ **g** $x^2 - 5x - 6 = 0$ **h** $x^2 - 4x - 12 = 0$

2 Solve the following equations using factorisation:

a $x^2 = 4x$ **b** $x^2 = 25x$ **c** $3x^2 = 6x$ **d** $5x^2 = 30x$

e $2x^2 + 7x + 3 = 0$ **f** $6x^2 - 7x - 3 = 0$ **g** $6x^2 - 5x - 6 = 0$ **h** $4x^2 - 16x + 15 = 0$

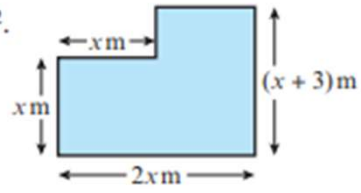
3 Solve the following equations:

a $3x^2 + 5x = 2$ **b** $(2x - 3)^2 = 9$ **c** $(x - 7)^2 = 36$ **d** $2x^2 = 8$ **e** $3x^2 = 5$

f $(x - 3)^2 = 13$ **g** $(3x - 1)^2 = 11$ **h** $5x^2 - 10x^2 = -7 + x + x^2$

i $6x^2 - 7 = 11x$ **j** $4x^2 + 17x = 6x - 2x^2$

(P) 4 This shape has an area of 44 m^2. Find the value of x.



Problem-solving

Divide the shape into two sections:

(P) 5 Solve the equation $5x + 3 = \sqrt{3x + 7}$.

2.2) Completing the square

Example 4

Complete the square for the expressions:

a $x^2 + 8x$ **b** $x^2 - 3x$ **c** $2x^2 + 12x$

Example 5

Write $3x^2 + 6x + 1$ in the form $p(x + q)^2 + r$, where p, q and r are integers to be found.

Example 6

Solve the equation $x^2 + 8x + 10 = 0$ by completing the square.
Give your answers in surd form.

Example 7

Solve the equation $2x^2 - 8x + 7 = 0$. Give your answers in surd form.

Exercise 2B

1 Solve the following equations using the quadratic formula.
Give your answers exactly, leaving them in surd form where necessary.

a $x^2 + 3x + 1 = 0$ **b** $x^2 - 3x - 2 = 0$ **c** $x^2 + 6x + 6 = 0$ **d** $x^2 - 5x - 2 = 0$
e $3x^2 + 10x - 2 = 0$ **f** $4x^2 - 4x - 1 = 0$ **g** $4x^2 - 7x = 2$ **h** $11x^2 + 2x - 7 = 0$

2 Solve the following equations using the quadratic formula.
Give your answers to three significant figures.

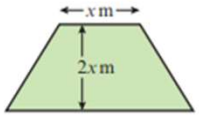
a $x^2 + 4x + 2 = 0$ **b** $x^2 - 8x + 1 = 0$ **c** $x^2 + 11x - 9 = 0$ **d** $x^2 - 7x - 17 = 0$
e $5x^2 + 9x - 1 = 0$ **f** $2x^2 - 3x - 18 = 0$ **g** $3x^2 + 8 = 16x$ **h** $2x^2 + 11x = 5x^2 - 18$

3 For each of the equations below, choose a suitable method and find all of the solutions.
Where necessary, give your answers to three significant figures.

a $x^2 + 8x + 12 = 0$ **b** $x^2 + 9x - 11 = 0$
c $x^2 - 9x - 1 = 0$ **d** $2x^2 + 5x + 2 = 0$
e $(2x + 8)^2 = 100$ **f** $6x^2 + 6 = 12x$
g $2x^2 - 11 = 7x$ **h** $x = \sqrt{8x - 15}$

Hint You can use any method you are confident with to solve these equations.

4 This trapezium has an area of $50\,m^2$.
Show that the height of the trapezium is equal to $5(\sqrt{5} - 1)\,m$.



Problem-solving
Height must be positive. You will have to discard the negative solution of your quadratic equation.

Exercise 2C

1 Complete the square for the expressions:
a $x^2 + 4x$ **b** $x^2 - 6x$ **c** $x^2 - 16x$ **d** $x^2 + x$ **e** $x^2 - 14$

2 Complete the square for the expressions:
a $2x^2 + 16x$ **b** $3x^2 - 24x$ **c** $5x^2 + 20x$ **d** $2x^2 - 5x$ **e** $8x - 2x^2$

Hint In question **3d**, write the expression as $-4x^2 - 16x + 10$ then take a factor of -4 out of the first two terms to get $-4(x^2 + 4x) + 10$.

3 Write each of these expressions in the form $p(x + q)^2 + r$, where p, q and r are constants to be found:
a $2x^2 + 8x + 1$ **b** $5x^2 - 15x + 3$ **c** $3x^2 + 2x - 1$ **d** $10 - 16x - 4x^2$ **e** $2x - 8x^2 + 10$

E **4** Given that $x^2 + 3x + 6 = (x + a)^2 + b$, find the values of the constants a and b. **(2 marks)**

E **5** Write $2 + 0.8x - 0.04x^2$ in the form $A - B(x + C)^2$, where A, B and C are constants to be determined. **(3 marks)**

solutions to quadratic equations quickly.

Exercise 2D

1 Solve these quadratic equations by completing the square. Leave your answers in surd form.
a $x^2 + 6x + 1 = 0$ **b** $x^2 + 12x + 3 = 0$ **c** $x^2 + 4x - 2 = 0$ **d** $x^2 - 10x = 5$

2 Solve these quadratic equations by completing the square. Leave your answers in surd form.
a $2x^2 + 6x - 3 = 0$ **b** $5x^2 + 8x - 2 = 0$ **c** $4x^2 - x - 8 = 0$ **d** $15 - 6x - 2x^2 = 0$

E **3** $x^2 - 14x + 1 = (x + p)^2 + q$, where p and q are constants.
a Find the values of p and q. **(2 marks)**
b Using your answer to part **a**, or otherwise, show that the solutions to the equation $x^2 - 14x + 1 = 0$ can be written in the form $r \pm s\sqrt{3}$, where r and s are constants to be found. **(2 marks)**

E/P **4** By completing the square, show that the solutions to the equation $x^2 + 2bx + c = 0$ are given by the formula $x = -b \pm \sqrt{b^2 - c}$. **(4 marks)**

Problem-solving
Follow the same steps as you would if the coefficients were numbers.

2.3) Functions

Example 8

The functions f and g are given by $f(x) = 2x - 10$ and $g(x) = x^2 - 9$, $x \in \mathbb{R}$.

a Find the values of f(5) and g(10).

b Find the value of x for which $f(x) = g(x)$.

Example 9

The function f is defined as $f(x) = x^2 + 6x - 5$, $x \in \mathbb{R}$.

a Write $f(x)$ in the form $(x + p)^2 + q$.

b Hence, or otherwise, find the roots of $f(x)$, leaving your answers in surd form.

c Write down the minimum value of $f(x)$, and state the value of x for which it occurs.

Example 10

Find the roots of the function $f(x) = x^6 + 7x^3 - 8$, $x \in \mathbb{R}$.

Exercise 2E

1 Using the functions $f(x) = 5x + 3$, $g(x) = x^2 - 2$ and $h(x) = \sqrt{x+1}$, find the values of:

a $f(1)$ **b** $g(3)$ **c** $h(8)$ **d** $f(1.5)$ **e** $g(\sqrt{2})$

f $h(-1)$ **g** $f(4) + g(2)$ **h** $f(0) + g(0) + h(0)$ **i** $\dfrac{g(4)}{h(3)}$

(P) **2** The function $f(x)$ is defined by $f(x) = x^2 - 2x$, $x \in \mathbb{R}$.
Given that $f(a) = 8$, find two possible values for a.

Problem-solving
Substitute $x = a$ into the function and set the resulting expression equal to 8.

3 Find all of the roots of the following functions:

a $f(x) = 10 - 15x$ **b** $g(x) = (x + 9)(x - 2)$ **c** $h(x) = x^2 + 6x - 40$

d $j(x) = 144 - x^2$ **e** $k(x) = x(x + 5)(x + 7)$ **f** $m(x) = x^3 + 5x^2 - 24x$

4 The functions p and q are given by $p(x) = x^2 - 3x$ and $q(x) = 2x - 6$, $x \in \mathbb{R}$.
Find the two values of x for which $p(x) = q(x)$.

5 The functions f and g are given by $f(x) = 2x^3 + 30x$ and $g(x) = 17x^2$, $x \in \mathbb{R}$.
Find the three values of x for which $f(x) = g(x)$.

(E) **6** The function f is defined as $f(x) = x^2 - 2x + 2$, $x \in \mathbb{R}$.

a Write $f(x)$ in the form $(x + p)^2 + q$, where p and q are constants to be found. **(2 marks)**

b Hence, or otherwise, explain why $f(x) > 0$ for all values of x, and find the minimum value of $f(x)$. **(1 mark)**

7 Find all roots of the following functions:

a $f(x) = x^6 + 9x^3 + 8$ **b** $g(x) = x^4 - 12x^2 + 32$

c $h(x) = 27x^6 + 26x^3 - 1$ **d** $j(x) = 32x^{10} - 33x^5 + 1$

e $k(x) = x - 7\sqrt{x} + 10$ **f** $m(x) = 2x^{\frac{2}{3}} + 2x^{\frac{1}{3}} - 12$

Hint The function in part **b** has four roots.

(E/P) **8** The function f is defined as $f(x) = 3^{2x} - 28(3^x) + 27$, $x \in \mathbb{R}$.

a Write $f(x)$ in the form $(3^x - a)(3^x - b)$, where a and b are real constants. **(2 marks)**

b Hence find the two roots of $f(x)$. **(2 marks)**

Problem-solving
Consider $f(x)$ as a function of a function.

3) Equations and inequalities

3.1) Linear simultaneous equations	https://youtu.be/5SpbJOzKGF4?si=XB4k1qamxqYym6F7

3.1) Linear simultaneous equations

Example 1

Solve the simultaneous equations:

a $2x + 3y = 8$
$3x - y = 23$

b $4x - 5y = 4$
$6x + 2y = 25$

Example 2

Solve the simultaneous equations:

$2x - y = 1$
$4x + 2y = -30$

Exercise 3A

1 Solve these simultaneous equations by elimination:

a $2x - y = 6$
$4x + 3y = 22$

b $7x + 3y = 16$
$2x + 9y = 29$

c $5x + 2y = 6$
$3x - 10y = 26$

d $2x - y = 12$
$6x + 2y = 21$

e $3x - 2y = -6$
$6x + 3y = 2$

f $3x + 8y = 33$
$6x = 3 + 5y$

2 Solve these simultaneous equations by substitution:

a $x + 3y = 11$
$4x - 7y = 6$

b $4x - 3y = 40$
$2x + y = 5$

c $3x - y = 7$
$10x + 3y = -2$

d $2y = 2x - 3$
$3y = x - 1$

3 Solve these simultaneous equations:

a $3x - 2y + 5 = 0$
$5(x + y) = 6(x + 1)$

b $\frac{x - 2y}{3} = 4$
$2x + 3y + 4 = 0$

c $3y = 5(x - 2)$
$3(x - 1) + y + 4 = 0$

Hint First rearrange both equations into the same form e.g. $ax + by = c$.

E/P **4** $3x + ky = 8$
$x - 2ky = 5$
are simultaneous equations where k is a constant.

a Show that $x = 3$. **(3 marks)**

b Given that $y = \frac{1}{2}$ determine the value of k. **(1 mark)**

Problem-solving

k is a constant, so it has the same value in both equations.

E/P **5** $2x - py = 5$
$4x + 5y + q = 0$
are simultaneous equations where p and q are constants.
The solution to this pair of simultaneous equations is $x = q$, $y = -1$.
Find the value of p and the value of q. **(5 marks)**

5) Straight line graphs

5.1) $y = mx + c$	https://youtu.be/C7meQTODsnA?si=T57mJnp-X34OnlI2

5.1) $y = mx + c$

Example 1

Work out the gradient of the line joining (−2, 7) and (4, 5)

Example 2

The line joining (2, −5) to (4, a) has gradient −1. Work out the value of a.

Exercise 5A

1 Work out the gradients of the lines joining these pairs of points:

a (4, 2), (6, 3)	**b** (−1, 3), (5, 4)	**c** (−4, 5), (1, 2)
d (2, −3), (6, 5)	**e** (−3, 4), (7, −6)	**f** (−12, 3), (−2, 8)
g (−2, −4), (10, 2)	**h** $(\frac{1}{2}, 2), (\frac{3}{4}, 4)$	**i** $(\frac{1}{4}, \frac{1}{2}), (\frac{1}{2}, \frac{2}{3})$
j (−2.4, 9.6), (0, 0)	**k** (1.3, −2.2), (8.8, −4.7)	**l** (0, 5a), (10a, 0)
m (3b, −2b), (7b, 2b)	**n** $(p, p^2), (q, q^2)$	

2 The line joining (3, −5) to (6, a) has a gradient 4. Work out the value of a.

3 The line joining (5, b) to (8, 3) has gradient −3. Work out the value of b.

4 The line joining (c, 4) to (7, 6) has gradient $\frac{3}{4}$. Work out the value of c.

5 The line joining (−1, 2d) to (1, 4) has gradient $-\frac{1}{4}$. Work out the value of d.

6 The line joining (−3, −2) to (2e, 5) has gradient 2. Work out the value of e.

7 The line joining (7, 2) to (f, 3f) has gradient 4. Work out the value of f.

8 The line joining (3, −4) to (−g, 2g) has gradient −3. Work out the value of g.

(P) **9** Show that the points $A(2, 3)$, $B(4, 4)$ and $C(10,7)$ can be joined by a straight line.

Problem-solving
Find the gradient of the line joining the points A and B and the line joining the points A and C.

(E/P) **10** Show that the points $A(-2a, 5a)$, $B(0, 4a)$ and points $C(6a, a)$ are collinear. **(3 marks)**

Notation Points are collinear if they all lie on the same straight line.

Example 3

Write down the gradient and y-intercept of these lines:

a $y = -3x + 2$ **b** $4x - 3y + 5 = 0$

Example 4

Write these lines in the form $ax + by + c = 0$

a $y = 4x + 3$ **b** $y = -\frac{1}{2}x + 5$

Example 5

The line $y = 4x - 8$ meets the x-axis at the point P. Work out the coordinates of P.

Exercise 5B

1 Work out the gradients of these lines:

a $y = -2x + 5$ **b** $y = -x + 7$ **c** $y = 4 + 3x$

d $y = \frac{1}{3}x - 2$ **e** $y = -\frac{2}{3}x$ **f** $y = \frac{5}{4}x + \frac{2}{3}$

g $2x - 4y + 5 = 0$ **h** $10x - 5y + 1 = 0$ **i** $-x + 2y - 4 = 0$

j $-3x + 6y + 7 = 0$ **k** $4x + 2y - 9 = 0$ **l** $9x + 6y + 2 = 0$

2 These lines cut the y-axis at $(0, c)$. Work out the value of c in each case.

a $y = -x + 4$ **b** $y = 2x - 5$ **c** $y = \frac{1}{2}x - \frac{2}{3}$

d $y = -3x$ **e** $y = \frac{6}{7}x + \frac{7}{5}$ **f** $y = 2 - 7x$

g $3x - 4y + 8 = 0$ **h** $4x - 5y - 10 = 0$ **i** $-2x + y - 9 = 0$

j $7x + 4y + 12 = 0$ **k** $7x - 2y + 3 = 0$ **l** $-5x + 4y + 2 = 0$

3 Write these lines in the form $ax + by + c = 0$.

a $y = 4x + 3$ **b** $y = 3x - 2$ **c** $y = -6x + 7$

d $y = \frac{4}{5}x - 6$ **e** $y = \frac{5}{3}x + 2$ **f** $y = \frac{7}{3}x$

g $y = 2x - \frac{4}{7}$ **h** $y = -3x + \frac{2}{9}$ **i** $y = -6x - \frac{2}{3}$

j $y = -\frac{1}{3}x + \frac{1}{2}$ **k** $y = \frac{2}{3}x + \frac{5}{6}$ **l** $y = \frac{3}{5}x + \frac{1}{2}$

4 The line $y = 6x - 18$ meets the x-axis at the point P. Work out the coordinates of P.

5 The line $3x + 2y = 0$ meets the x-axis at the point R. Work out the coordinates of R.

6 The line $5x - 4y + 20 = 0$ meets the y-axis at the point A and the x-axis at the point B. Work out the coordinates of A and B.

7 A line l passes through the points with coordinates $(0, 5)$ and $(6, 7)$.

a Find the gradient of the line.

b Find an equation of the line in the form $ax + by + c = 0$.

(E) **8** A line l cuts the x-axis at $(5, 0)$ and the y-axis at $(0, 2)$.

a Find the gradient of the line. **(1 mark)**

b Find an equation of the line in the form $ax + by + c = 0$. **(2 marks)**

(P) **9** Show that the line with equation $ax + by + c = 0$ has gradient $-\frac{a}{b}$ and cuts the y-axis at $-\frac{c}{b}$

Problem-solving

Try solving a similar problem with numbers first:

Find the gradient and y-intercept of the straight line with equation $3x + 7y + 2 = 0$.

(E/P) **10** The line l with gradient 3 and y-intercept $(0, 5)$ has the equation $ax - 2y + c = 0$.
Find the values of a and c. **(2 marks)**

(E/P) **11** The straight line l passes through $(0, 6)$ and has gradient -2. It intersects the line with equation $5x - 8y - 15 = 0$ at point P. Find the coordinates of P. **(4 marks)**

(E/P) **12** The straight line l_1 with equation $y = 3x - 7$ intersects the straight line l_2 with equation $ax + 4y - 17 = 0$ at the point $P(-3, b)$.

a Find the value of b. **(1 mark)**

b Find the value of a. **(2 marks)**

7) Algebraic methods

7.1) Algebraic fractions	https://youtu.be/zkf1lO5EqXA?si=PgGllqb-z09aDnp4

7.1) Algebraic fractions

Example 1

Simplify these fractions:

a $\frac{7x^4 - 2x^3 + 6x}{x}$ **b** $\frac{(x + 7)(2x - 1)}{(2x - 1)}$ **c** $\frac{x^2 + 7x + 12}{(x + 3)}$ **d** $\frac{x^2 + 6x + 5}{x^2 + 3x - 10}$ **e** $\frac{2x^2 + 11x + 12}{(x + 3)(x + 4)}$

Exercise 7A

1 Simplify these fractions:

a $\frac{4x^4 + 5x^2 - 7x}{x}$ **b** $\frac{7x^5 - 5x^5 + 9x^3 + x^2}{x}$ **c** $\frac{-x^4 + 4x^2 + 6}{x}$

d $\frac{7x^5 - x^3 - 4}{x}$ **e** $\frac{8x^4 - 4x^3 + 6x}{2x}$ **f** $\frac{9x^2 - 12x^3 - 3x}{3x}$

g $\frac{7x^3 - x^4 - 2}{5x}$ **h** $\frac{-4x^2 + 6x^4 - 2x}{-2x}$ **i** $\frac{-x^8 + 9x^4 - 4x^3 + 6}{-2x}$

j $\frac{-9x^9 - 6x^6 + 4x^4 - 2}{-3x}$

2 Simplify these fractions as far as possible:

a $\frac{(x + 3)(x - 2)}{(x - 2)}$ **b** $\frac{(x + 4)(3x - 1)}{(3x - 1)}$ **c** $\frac{(x + 3)^2}{(x + 3)}$

d $\frac{x^2 + 10x + 21}{(x + 3)}$ **e** $\frac{x^2 + 9x + 20}{(x + 4)}$ **f** $\frac{x^2 + x - 12}{(x - 3)}$

g $\frac{x^2 + x - 20}{x^2 + 2x - 15}$ **h** $\frac{x^2 + 3x + 2}{x^2 + 5x + 4}$ **i** $\frac{x^2 + x - 12}{x^2 - 9x + 18}$

j $\frac{2x^2 + 7x + 6}{(x - 5)(x + 2)}$ **k** $\frac{2x^2 + 9x - 18}{(x + 6)(x + 1)}$ **l** $\frac{3x^2 - 7x + 2}{(3x - 1)(x + 2)}$

m $\frac{2x^2 + 3x + 1}{x^2 - x - 2}$ **n** $\frac{x^2 + 6x + 8}{3x^2 + 7x + 2}$ **o** $\frac{2x^2 - 5x - 3}{2x^2 - 9x + 9}$

E/P **3** $\frac{6x^3 + 3x^2 - 84x}{6x^2 - 33x + 42} = \frac{ax(x + b)}{x + c}$, where a, b and c are constants.

Work out the values of a, b and c. **(4 marks)**

8) The binomial expansion

8.1) Pascal's triangle	https://youtu.be/0X5tuUJj_Bw?si=mhyvvq8_f_I9VEpX
8.2) Factorial notation	https://youtu.be/ku_ywrJ65Z0?si=xLgHutUTwkkIv84L

8.1) Pascal's triangle

Example 1

Use Pascal's triangle to find the expansions of

a $(x + 2y)^3$ **b** $(2x - 5)^4$

Example 2

The coefficient of x^2 in the expansion of $(2 - cx)^3$ is 294.
Find the possible value(s) of the constant c.

Exercise 8A

1 State the row of Pascal's triangle that would give the coefficients of each expansion:

a $(x + y)^3$ **b** $(3x - 7)^{15}$ **c** $\left(2x + \frac{1}{2}\right)^n$ **d** $(y - 2x)^{n+4}$

2 Write down the expansion of:

a $(x + y)^4$ **b** $(p + q)^5$ **c** $(a - b)^3$ **d** $(x + 4)^3$

e $(2x - 3)^4$ **f** $(a + 2)^5$ **g** $(3x - 4)^4$ **h** $(2x - 3y)^4$

3 Find the coefficient of x^3 in the expansion of:

a $(4 + x)^4$ **b** $(1 - x)^5$ **c** $(3 + 2x)^3$ **d** $(4 + 2x)^5$

e $(2 + x)^6$ **f** $\left(4 - \frac{1}{2}x\right)^4$ **g** $(x + 2)^5$ **h** $(3 - 2x)^4$

(P) **4** Fully expand the expression $(1 + 3x)(1 + 2x)^3$.

Problem-solving

Expand $(1 + 2x)^3$, then multiply each term by 1 and by $3x$.

(P) **5** Expand $(2 + y)^3$. Hence or otherwise, write down the expansion of $(2 + x - x^2)^3$ in ascending powers of x.

(P) **6** The coefficient of x^2 in the expansion of $(2 + ax)^3$ is 54. Find the possible values of the constant a.

(P) **7** The coefficient of x^3 in the expansion of $(2 - x)(3 + bx)^3$ is 45. Find possible values of the constant b.

(P) **8** Work out the coefficient of x^2 in the expansion of $(p - 2x)^3$. Give your answer in terms of p.

(P) **9** After 5 years, the value of an investment of £500 at an interest rate of $X\%$ per annum is given by:

$$500\left(1 + \frac{X}{100}\right)^5$$

Find an approximation for this expression in the form $A + BX + CX^2$, where A, B and C are constants to be found. You can ignore higher powers of X.

8.2) Factorial notation

Example 3

Calculate:

a 5! **b** ${}^5C_2$ **c** the 6th entry in the 10th row of Pascal's triangle

Exercise 8B

1 Work out:

a $4!$ **b** $9!$ **c** $\frac{10!}{7!}$ **d** $\frac{15!}{13!}$

2 Without using a calculator, work out:

a $\binom{4}{2}$ **b** $\binom{6}{4}$ **c** ${}^{6}C_{3}$ **d** $\binom{5}{4}$ **e** ${}^{10}C_{8}$ **f** $\binom{9}{5}$

3 Use a calculator to work out:

a $\binom{15}{6}$ **b** ${}^{10}C_{7}$ **c** $\binom{20}{10}$ **d** $\binom{20}{17}$ **e** ${}^{14}C_{9}$ **f** ${}^{18}C_{5}$

4 Write each value a to d from Pascal's triangle using ${}^{n}C_{r}$ notation:

1
1 1
1 2 1
1 3 3 1
1 (a) 6 4 1
1 5 (b) 10 5 1
1 6 (c) (d) 15 6 1

5 Work out the 5th number on the 12th row from Pascal's triangle.

6 The 11th row of Pascal's triangle is shown below.

1 10 45 … …

a Find the next two values in the row.

b Hence find the coefficient of x^3 in the expansion of $(1 + 2x)^{10}$.

7 The 14th row of Pascal's triangle is shown below.

1 13 78 … …

a Find the next two values in the row.

b Hence find the coefficient of x^4 in the expansion of $(1 + 3x)^{13}$.

8 The probability of throwing exactly 10 heads when a fair coin is tossed 20 times is given by $\binom{20}{10}0.5^{20}$. Calculate the probability and describe the likelihood of this occurring.

(P) **9** Show that:

a ${}^{n}C_{1} = n$ **b** ${}^{n}C_{2} = \frac{n(n-1)}{2}$

(E) **10** Given that $\binom{50}{13} = \frac{50!}{13!a!}$, write down the value of a. **(1 mark)**

(E) **11** Given that $\binom{35}{p} = \frac{35!}{p!18!}$, write down the value of p. **(1 mark)**

9) Trigonometric ratios

9.1) The cosine rule	https://youtu.be/LBvY4APYguo?si=roV11p-XjoQDBZl8
9.2) The sine rule	https://youtu.be/yZqpDMZ1EsU?si=k81MU8bXoQ8MSJJk
9.3) Areas of triangles	https://youtu.be/fphNJlonm1I?si=zBvvpW9TKWNf6yJ8

9.1) The cosine rule

Example 1

Calculate the length of the side AB of the triangle ABC in which $AC = 6.5\,\text{cm}$, $BC = 8.7\,\text{cm}$ and $\angle ACB = 100°$.

Example 2

Find the size of the smallest angle in a triangle whose sides have lengths 3 cm, 5 cm and 6 cm.

Example 3

Coastguard station B is 8 km, on a bearing of 060°, from coastguard station A. A ship C is 4.8 km, on a bearing of 018°, away from A. Calculate how far C is from B.

Example 4

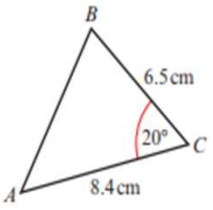
In $\triangle ABC$, $AB = x$ cm, $BC = (x + 2)$ cm, $AC = 5$ cm and $\angle ABC = 60°$.
Find the value of x.

Exercise 9A

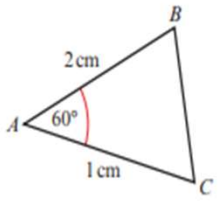
Give answers to 3 significant figures, where appropriate.

1 In each of the following triangles calculate the length of the missing side.

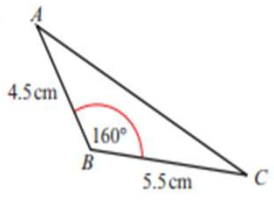
a



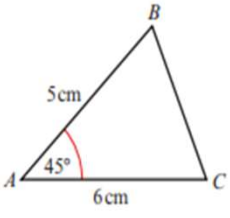
b



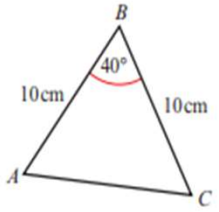
c



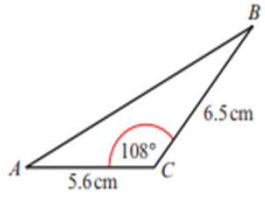
d



e

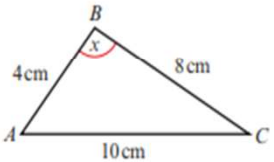


f

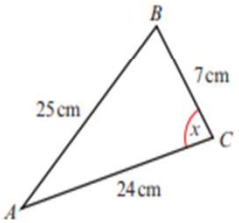


2 In the following triangles calculate the size of the angle marked x:

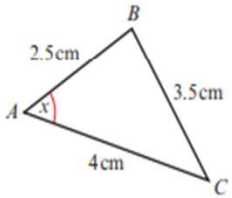
a



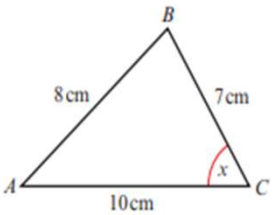
b



c



d



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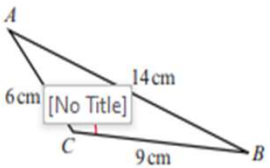

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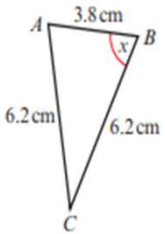
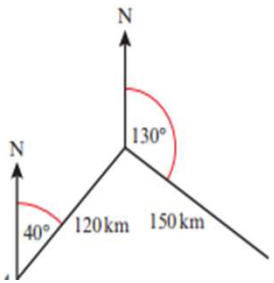


3 A plane flies from airport A on a bearing of 040° for 120 km and then on a bearing of 130° for 150 km. Calculate the distance of the plane from the airport.

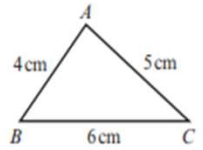


4 From a point A a boat sails due north for 7 km to B. The boat leaves B and moves on a bearing of 100° for 10 km until it reaches C. Calculate the distance of C from A.

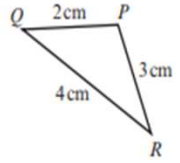
5 A helicopter flies on a bearing of 080° from A to B, where $AB = 50$ km.
It then flies for 60 km to a point C.
Given that C is 80 km from A, calculate the bearing of C from A.

6 The distance from the tee, T, to the flag, F, on a particular hole on a golf course is 494 yards.
A golfer's tee shot travels 220 yards and lands at the point S, where $\angle STF = 22°$.
Calculate how far the ball is from the flag.

7 Show that $\cos A = \frac{1}{8}$



8 Show that $\cos P = -\frac{1}{4}$



9 In $\triangle ABC$, $AB = 5$ cm, $BC = 6$ cm and $AC = 10$ cm.
Calculate the size of the smallest angle.

10 In $\triangle ABC$, $AB = 9.3$ cm, $BC = 6.2$ cm and $AC = 12.7$ cm.
Calculate the size of the largest angle.

11 The lengths of the sides of a triangle are in the ratio 2 : 3 : 4.
Calculate the size of the largest angle.

12 In $\triangle ABC$, $AB = (x - 3)$ cm, $BC = (x + 3)$ cm, $AC = 8$ cm and $\angle BAC = 60°$.
Use the cosine rule to find the value of x.

13 In $\triangle ABC$, $AB = x$ cm, $BC = (x - 4)$ cm, $AC = 10$ cm and $\angle BAC = 60°$.
Calculate the value of x.

14 In $\triangle ABC$, $AB = (5 - x)$ cm, $BC = (4 + x)$ cm, $\angle ABC = 120°$ and $AC = y$ cm.

a Show that $y^2 = x^2 - x + 61$.

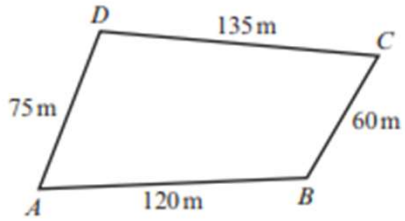
b Use the method of completing the square to find the minimum value of y^2, and give the value of x for which this occurs.

(P) **15** In $\triangle ABC$, $AB = x$ cm, $BC = 5$ cm, $AC = (10 - x)$ cm.

a Show that $\cos \angle ABC = \dfrac{4x - 15}{2x}$

b Given that $\cos \angle ABC = -\frac{1}{7}$, work out the value of x.

(P) **16** A farmer has a field in the shape of a quadrilateral as shown.



Problem-solving

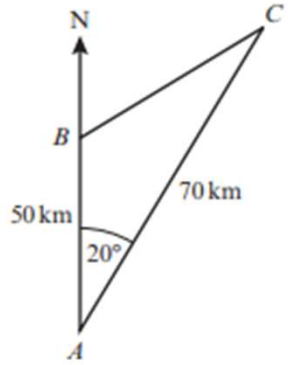
You will have to use the cosine rule twice. Copy the diagram and write any angles or lengths you work out on your copy.

The angle between fences AB and AD is 74°. Find the angle between fences BC and CD.

(E/P) **17** The diagram shows three cargo ships, A, B and C, which are in the same horizontal plane. Ship B is 50 km due north of ship A and ship C is 70 km from ship A. The bearing of C from A is 020°.

a Calculate the distance between ships B and C, in kilometres to 3 s.f. **(3 marks)**

b Calculate the bearing of ship C from ship B. **(4 marks)**



9.2) The sine rule

Example 5

In $\triangle ABC$, $AB = 8\,\text{cm}$, $\angle BAC = 30°$ and $\angle BCA = 40°$. Find BC.

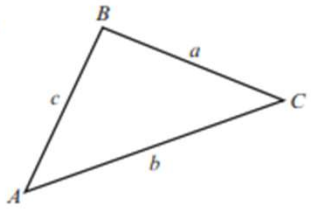
Example 6

In $\triangle ABC$, $AB = 3.8\,\text{cm}$, $BC = 5.2\,\text{cm}$ and $\angle ABC = 35°$. Find $\angle ABC$.

Exercise 9B

Give answers to 3 significant figures, where appropriate.

1 In each of parts **a** to **d**, the given values refer to the general triangle.



a Given that $a = 8\,\text{cm}$, $A = 30°$, $B = 72°$, find b.

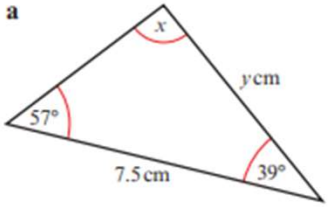
b Given that $a = 24\,\text{cm}$, $A = 110°$, $C = 22°$, find c.

c Given that $b = 14.7\,\text{cm}$, $A = 30°$, $C = 95°$, find a.

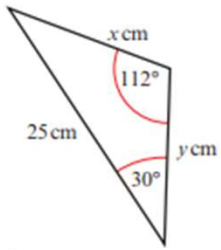
d Given that $c = 9.8\,\text{cm}$, $B = 68.4°$, $C = 83.7°$, find a.

2 In each of the following triangles calculate the values of x and y.

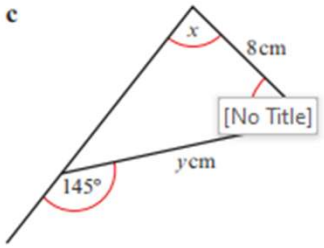
a



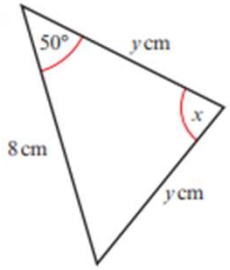
b



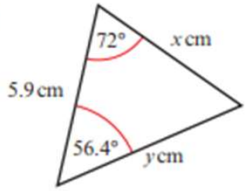
c



d



e



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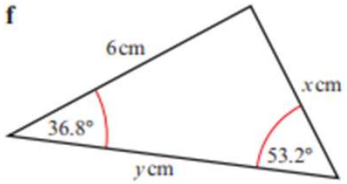
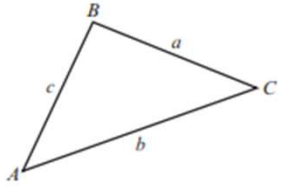


Hint In parts **c** and **d**, start by finding the size of the third angle.

3 In each of the following sets of data for a triangle ABC, find the value of x.



a $AB = 6\,\text{cm}$, $BC = 9\,\text{cm}$, $\angle BAC = 117°$, $\angle ACB = x$

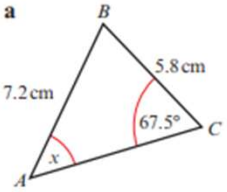
b $AC = 11\,\text{cm}$, $BC = 10\,\text{cm}$, $\angle ABC = 40°$, $\angle CAB = x$

c $AB = 6\,\text{cm}$, $BC = 8\,\text{cm}$, $\angle BAC = 60°$, $\angle ACB = x$

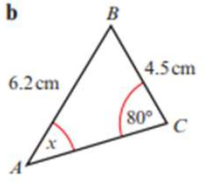
d $AB = 8.7\,\text{cm}$, $AC = 10.8\,\text{cm}$, $\angle ABC = 28°$, $\angle BAC = x$

4 In each of the diagrams shown below, work out the size of angle x.

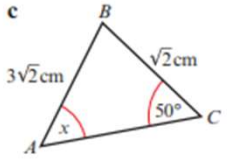
a



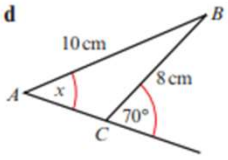
b



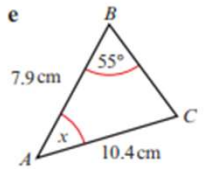
c



d



e



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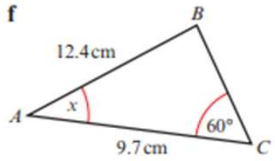
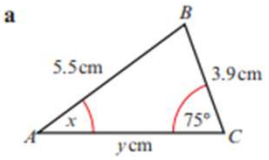


5 In $\triangle PQR$, $QR = \sqrt{3}\,\text{cm}$, $\angle PQR = 45°$ and $\angle QPR = 60°$. Find **a** PR and **b** PQ.

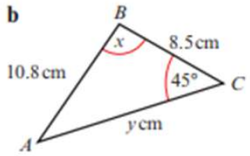
6 In $\triangle PQR$, $PQ = 15\,\text{cm}$, $QR = 12\,\text{cm}$ and $\angle PRQ = 75°$. Find the two remaining angles.

7 In each of the following diagrams work out the values of x and y.

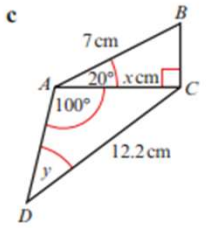
a



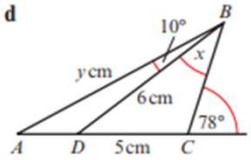
b



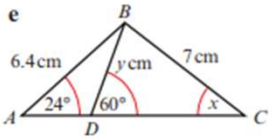
c



d



e



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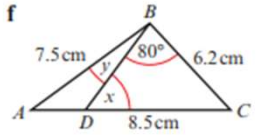


8 Town B is 6 km, on a bearing of 020°, from town A. Town C is located on a bearing of 055° from town A and on a bearing of 120° from town B. Work out the distance of town C from:

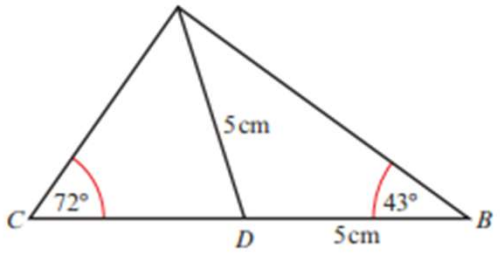
a town A **b** town B

Problem-solving Draw a sketch to show the information.

9 In the diagram $AD = DB = 5$ cm, $\angle ABC = 43°$ and $\angle ACB = 72°$.
Calculate:

a AB

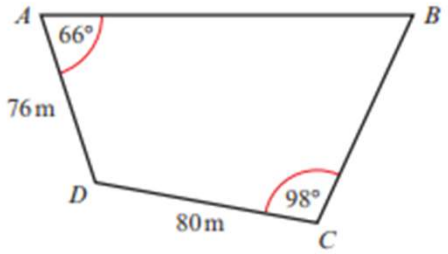
b CD



10 A zookeeper is building an enclosure for some llamas.
The enclosure is in the shape of a quadrilateral as shown.
If the length of the diagonal BD is 136 m

a find the angle between the fences AB and BC

b find the length of fence AB



E/P **11** In $\triangle ABC$, $AB = x$ cm, $BC = (4 - x)$ cm, $\angle BAC = y$ and $\angle BCA = 30°$.

Given that $\sin y = \frac{1}{\sqrt{2}}$, show that

$x = 4(\sqrt{2} - 1)$ **(5 marks)**

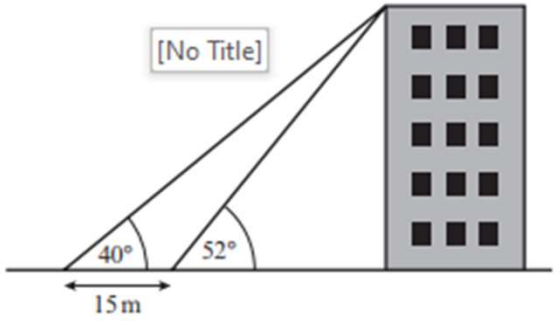
Problem-solving

You can use the value of sin y directly in your calculation. You don't need to work out the value of y.

E/P **12** A surveyor wants to determine the height of a building. She measures the angle of elevation of the top of the building at two points 15 m apart on the ground.

a Use this information to determine the height of the building. **(4 marks)**

b State one assumption made by the surveyor in using this mathematical model. **(1 mark)**



Example 7

In $\triangle ABC$, $AB = 4\,\text{cm}$, $AC = 3\,\text{cm}$ and $\angle ABC = 44°$. Work out the two possible values of $\angle ACB$.

Exercise 9C

Give answers to 3 significant figures, where appropriate.

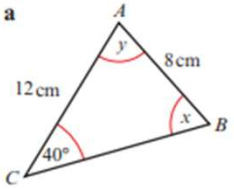
1 In $\triangle ABC$, $BC = 6\,\text{cm}$, $AC = 4.5\,\text{cm}$ and $\angle ABC = 45°$.

a Calculate the two possible values of $\angle BAC$.

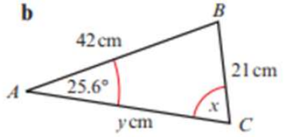
b Draw a diagram to illustrate your answers.

2 In each of the diagrams shown below, calculate the possible values of x and the corresponding values of y.

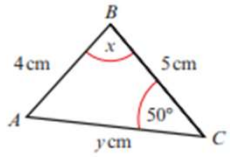
a



b



c



3 In each of the following cases $\triangle ABC$ has $\angle ABC = 30°$ and $AB = 10\,\text{cm}$.

a Calculate the least possible length that AC could be.

b Given that $AC = 12\,\text{cm}$, calculate $\angle ACB$.

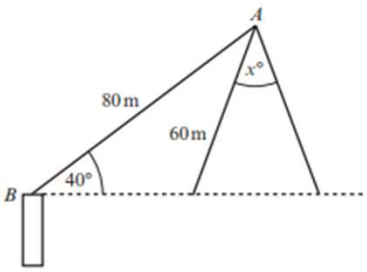
c Given instead that $AC = 7\,\text{cm}$, calculate the two possible values of $\angle ACB$.

(P) **4** Triangle ABC is such that $AB = 4\,\text{cm}$, $BC = 6\,\text{cm}$ and $\angle ACB = 36°$. Show that one of the possible values of $\angle ABC$ is 25.8° (to 3 s.f.). Using this value, calculate the length of AC.

(P) **5** Two triangles ABC are such that $AB = 4.5\,\text{cm}$, $BC = 6.8\,\text{cm}$ and $\angle ACB = 30°$. Work out the value of the largest angle in each of the triangles.

(E/P) **6 a** A crane arm AB of length 80 m is anchored at point B at an angle of 40° to the horizontal. A wrecking ball is suspended on a cable of length 60 m from A. Find the angle x through which the wrecking ball rotates as it passes the two points level with the base of the crane arm at B. **(6 marks)**

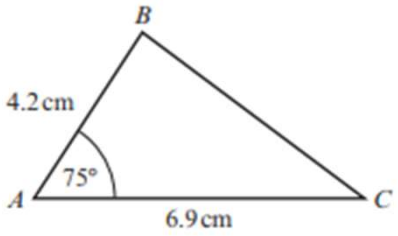
b Write down one modelling assumption you have made. **(1 mark)**



9.3) Areas of triangles

Example 8

Work out the area of the triangle shown below.



Example 9

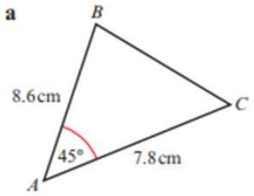
In $\triangle ABC$, $AB = 5\,\text{cm}$, $BC = 6\,\text{cm}$ and $\angle ABC = x$. Given that the area of $\triangle ABC$ is $12\,\text{cm}^2$ and that AC is the longest side, find the value of x.

53.13…° and 126.86…°, but here you know B is the largest angle because AC is the largest side.

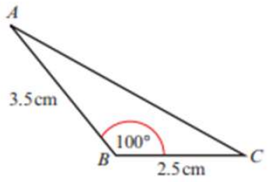
Exercise 9D

1 Calculate the area of each triangle.

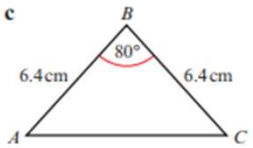
a



b

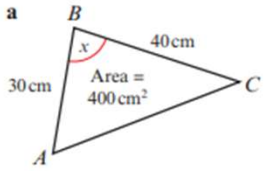


c

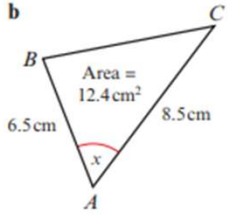


2 Work out the possible sizes of x in the following triangles.

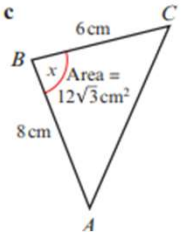
a



b

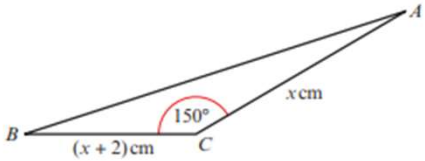


c



3 A fenced triangular plot of ground has area 1200 m². The fences along the two smaller sides are 60 m and 80 m respectively and the angle between them is θ. Show that $\theta = 150°$, and work out the total length of fencing.

(P) **4** In triangle ABC, $BC = (x + 2)$ cm, $AC = x$ cm and $\angle BCA = 150°$. Given that the area of the triangle is 5 cm², work out the value of x, giving your answer to 3 significant figures.



(E/P) **5** In $\triangle PQR$, $PQ = (x + 2)$ cm, $PR = (5 - x)$ cm and $\angle QPR = 30°$. The area of the triangle is A cm².

a Show that $A = \frac{1}{4}(10 + 3x - x^2)$. **(3 marks)**

b Use the method of completing the square, or otherwise, to find the maximum value of A, and give the corresponding value of x. **(4 marks)**

(E/P) **6** In $\triangle ABC$, $AB = x$ cm, $AC = (5 + x)$ cm and $\angle BAC = 150°$. Given that the area of the triangle is $3\frac{3}{4}$ cm²

Problem-solving

x represents a length so it must be positive.

a Show that x satisfies the equation $x^2 + 5x - 15 = 0$. **(3 marks)**

b Calculate the value of x, giving your answer to 3 significant figures. **(3 marks)**

10) Trigonometric identities and equations

10.1) Angles in all four quadrants	https://youtu.be/5intm9ZNP4M?si=YC6ftnI-7xKU3-fl
10.2) Exact values of trigonometrical ratios	https://youtu.be/yZqpDMZ1EsU?si=k81MU8bXoQ8MSJJk

10.1) Angles in all four quadrants

Example 1

Write down the values of:

a $\sin 90°$ **b** $\sin 180°$ **c** $\sin 270°$

d $\cos 180°$ **e** $\cos(-90)°$ **f** $\cos 450°$

Example 2

Write down the values of:

a $\tan 45°$ **b** $\tan 135°$ **c** $\tan 225°$

d $\tan(-45°)$ **e** $\tan 180°$ **f** $\tan 90°$

Example 3

Find the signs of $\sin\theta$, $\cos\theta$ and $\tan\theta$ in the second quadrant.

Example 4

Express in terms of trigonometric ratios of acute angles:

a $\sin(-100°)$ **b** $\cos 330°$ **c** $\tan 500°$

Exercise 10A

1 Draw diagrams to show the following angles. Mark in the acute angle that OP makes with the x-axis.

a $-80°$ **b** $100°$ **c** $200°$ **d** $165°$ **e** $-145°$

f $225°$ **g** $280°$ **h** $330°$ **i** $-160°$ **j** $-280°$

2 State the quadrant that OP lies in when the angle that OP makes with the positive x-axis is:

a $400°$ **b** $115°$ **c** $-210°$ **d** $255°$ **e** $-100°$

3 Without using a calculator, write down the values of:

a $\sin(-90°)$ **b** $\sin 450°$ **c** $\sin 540°$ **d** $\sin(-450°)$ **e** $\cos(-180°)$

f $\cos(-270°)$ **g** $\cos 270°$ **h** $\cos 810°$ **i** $\tan 360°$ **j** $\tan(-180°)$

4 Express the following in terms of trigonometric ratios of acute angles:

a $\sin 240°$ **b** $\sin(-80°)$ **c** $\sin(-200°)$ **d** $\sin 300°$ **e** $\sin 460°$

f $\cos 110°$ **g** $\cos 260°$ **h** $\cos(-50°)$ **i** $\cos(-200°)$ **j** $\cos 545°$

k $\tan 100°$ **l** $\tan 325°$ **m** $\tan(-30°)$ **n** $\tan(-175°)$ **o** $\tan 600°$

5 Given that θ is an acute angle, express in terms of $\sin\theta$:

a $\sin(-\theta)$ **b** $\sin(180° + \theta)$ **c** $\sin(360° - \theta)$

d $\sin(-(180° + \theta))$ **e** $\sin(-180° + \theta)$ **f** $\sin(-360° + \theta)$

g $\sin(540° + \theta)$ **h** $\sin(720° - \theta)$ **i** $\sin(\theta + 720°)$

Hint The results obtained in questions **5** and **6** are true for all values of θ.

6 Given that θ is an acute angle, express in terms of $\cos\theta$ or $\tan\theta$:

a $\cos(180° - \theta)$ **b** $\cos(180° + \theta)$ **c** $\cos(-\theta)$ **d** $\cos(-(180° - \theta))$

e $\cos(\theta - 360°)$ **f** $\cos(\theta - 540°)$ **g** $\tan(-\theta)$ **h** $\tan(180° - \theta)$

i $\tan(180° + \theta)$ **j** $\tan(-180° + \theta)$ **k** $\tan(540° - \theta)$ **l** $\tan(\theta - 360°)$

10.2) Exact values of trigonometrical ratios

Example 5

Find the exact value of $\sin(-210°)$.

Exercise 10B

1 Express the following as trigonometric ratios of either 30°, 45° or 60°, and hence find their exact values.

a $\sin 135°$	**b** $\sin(-60°)$	**c** $\sin 330°$	**d** $\sin 420°$	**e** $\sin(-300°)$
f $\cos 120°$	**g** $\cos 300°$	**h** $\cos 225°$	**i** $\cos(-210°)$	**j** $\cos 495°$
k $\tan 135°$	**l** $\tan(-225°)$	**m** $\tan 210°$	**n** $\tan 300°$	**o** $\tan(-120°)$

11) Vectors

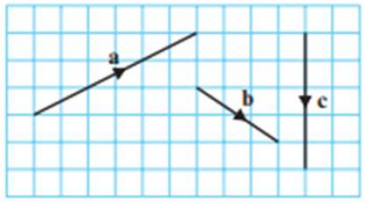
11.1) Vectors	https://youtu.be/Oao432GabNQ?si=0VZxY688r1LTz9RC
11.2) Representing vectors	https://youtu.be/yZqpDMZ1EsU?si=k81MU8bXoQ8MSJJk

11.1) Vectors

Example 1

The diagram shows vectors **a**, **b** and **c**.

Draw a diagram to illustrate the vector addition **a** + **b** + **c**.



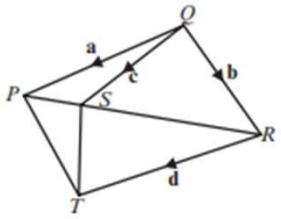
Example 2

In the diagram, $\overrightarrow{QP} = \mathbf{a}$, $\overrightarrow{QR} = \mathbf{b}$, $\overrightarrow{QS} = \mathbf{c}$ and $\overrightarrow{RT} = \mathbf{d}$.

Find in terms of **a**, **b**, **c** and **d**:

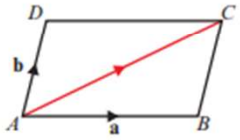
a $\overrightarrow{PS}$ **b** $\overrightarrow{RP}$

c $\overrightarrow{PT}$ **d** $\overrightarrow{TS}$



Example 3

$ABCD$ is a parallelogram. $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{b}$. Find $\overrightarrow{AC}$.



Notation This is called the **parallelogram law** for vector addition.

Example 4

Show that the vectors $6\mathbf{a} + 8\mathbf{b}$ and $9\mathbf{a} + 12\mathbf{b}$ are parallel.

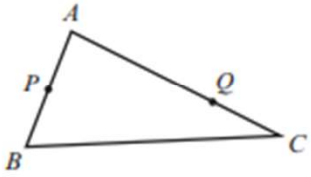
Example 5

In triangle ABC, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$.

P is the midpoint of AB.

Q divides AC in the ratio $3:2$.

Write in terms of $\mathbf{a}$ and $\mathbf{b}$:



a $\overrightarrow{BC}$ **b** $\overrightarrow{AP}$ **c** $\overrightarrow{AQ}$ **d** $\overrightarrow{PQ}$

Exercise 11A

1 The diagram shows the vectors **a**, **b**, **c** and **d**.

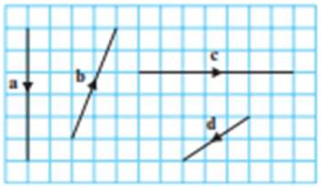
Draw a diagram to illustrate these vectors:

a $\mathbf{a}+\mathbf{c}$ **b** $-\mathbf{b}$

c $\mathbf{c}-\mathbf{d}$ **d** $\mathbf{b}+\mathbf{c}+\mathbf{d}$

e $\mathbf{a}-2\mathbf{b}$ **f** $2\mathbf{c}+3\mathbf{d}$

g $\mathbf{a}+\mathbf{b}+\mathbf{c}+\mathbf{d}$



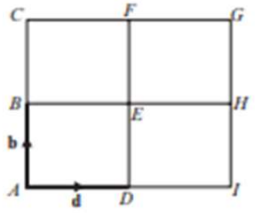
2 $ACGI$ is a square, B is the midpoint of AC, F is the midpoint of CG, H is the midpoint of GI, D is the midpoint of AI.

$\overrightarrow{AB}=\mathbf{b}$ and $\overrightarrow{AD}=\mathbf{d}$. Find, in terms of **b** and **d**:

a $\overrightarrow{AC}$ **b** $\overrightarrow{BE}$ **c** $\overrightarrow{HG}$ **d** $\overrightarrow{DF}$

e $\overrightarrow{AE}$ **f** $\overrightarrow{DH}$ **g** $\overrightarrow{HB}$ **h** $\overrightarrow{FE}$

i $\overrightarrow{AH}$ **j** $\overrightarrow{BI}$ **k** $\overrightarrow{EI}$ **l** $\overrightarrow{FB}$



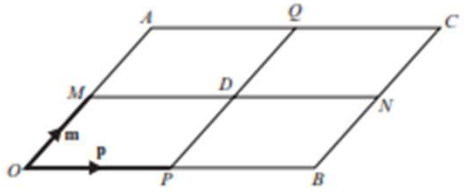
3 $OACB$ is a parallelogram. M, Q, N and P are the midpoints of OA, AC, BC and OB respectively.

Vectors **p** and **m** are equal to $\overrightarrow{OP}$ and $\overrightarrow{OM}$ respectively. Express in terms of **p** and **m**.

a $\overrightarrow{OA}$ **b** $\overrightarrow{OB}$ **c** $\overrightarrow{BN}$ **d** $\overrightarrow{DQ}$

e $\overrightarrow{OD}$ **f** $\overrightarrow{MQ}$ **g** $\overrightarrow{OQ}$ **h** $\overrightarrow{AD}$

i $\overrightarrow{CD}$ **j** $\overrightarrow{AP}$ **k** $\overrightarrow{BM}$ **l** $\overrightarrow{NO}$

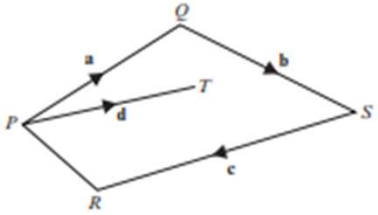


4 In the diagram, $\overrightarrow{PQ}=\mathbf{a}$, $\overrightarrow{QS}=\mathbf{b}$, $\overrightarrow{SR}=\mathbf{c}$ and $\overrightarrow{PT}=\mathbf{d}$.

Find in terms of **a**, **b**, **c** and **d**:

a $\overrightarrow{QT}$ **b** $\overrightarrow{PR}$

c $\overrightarrow{TS}$ **d** $\overrightarrow{TR}$



5 In the triangle PQR, $PQ=2\mathbf{a}$ and $QR=2\mathbf{b}$.

The midpoint of PR is M. Find, in terms of **a** and **b**:

a $\overrightarrow{PR}$ **b** $\overrightarrow{PM}$ **c** $\overrightarrow{QM}$

(P) **6** $ABCD$ is a trapezium with AB parallel to DC and $DC=3AB$.

M divides DC such that $DM:MC=2:1$. $\overrightarrow{AB}=\mathbf{a}$ and $\overrightarrow{BC}=\mathbf{b}$.

Find, in terms of **a** and **b**:

a $\overrightarrow{AM}$ **b** $\overrightarrow{BD}$ **c** $\overrightarrow{MB}$ **d** $\overrightarrow{DA}$

Problem-solving

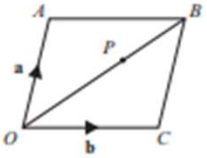
Draw a sketch to show the information given in the question.

7 $OABC$ is a parallelogram. $\overrightarrow{OA}=\mathbf{a}$ and $\overrightarrow{OC}=\mathbf{b}$.

The point P divides OB in the ratio 5:3.

Find, in terms of **a** and **b**:

a $\overrightarrow{OB}$ **b** $\overrightarrow{OP}$ **c** $\overrightarrow{AP}$



8 State with a reason whether each of these vectors is parallel to the vector $\mathbf{a}-3\mathbf{b}$:

a $2\mathbf{a}-6\mathbf{b}$ **b** $4\mathbf{a}-12\mathbf{b}$ **c** $\mathbf{a}+3\mathbf{b}$ **d** $3\mathbf{b}-\mathbf{a}$ **e** $9\mathbf{b}-3\mathbf{a}$ **f** $\frac{1}{2}\mathbf{a}-\frac{2}{3}\mathbf{b}$

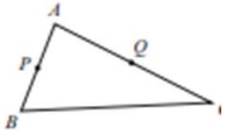
(P) **9** In triangle ABC, $\overrightarrow{AB}=\mathbf{a}$ and $\overrightarrow{AC}=\mathbf{b}$.

P is the midpoint of AB and Q is the midpoint of AC.

a Write in terms of **a** and **b**:

i $\overrightarrow{BC}$ **ii** $\overrightarrow{AP}$ **iii** $\overrightarrow{AQ}$ **iv** $\overrightarrow{PQ}$

b Show that PQ is parallel to BC.

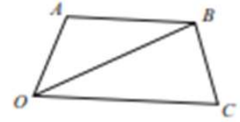


(P) **10** $OABC$ is a quadrilateral. $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OC}=3\mathbf{b}$ and $\overrightarrow{OB}=\mathbf{a}+2\mathbf{b}$.

a Find, in terms of **a** and **b**:

i $\overrightarrow{AB}$ **ii** $\overrightarrow{CB}$

b Show that AB is parallel to OC.



(P) **11** The vectors $2\mathbf{a}+k\mathbf{b}$ and $5\mathbf{a}+3\mathbf{b}$ are parallel. Find the value of k.

11.2) Representing vectors

Example 6

$\mathbf{a} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Find **a** $\frac{1}{3}\mathbf{a}$ **b** $\mathbf{a} + \mathbf{b}$ **c** $2\mathbf{a} - 3\mathbf{b}$

Example 7

$\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$, $\mathbf{b} = 2\mathbf{i} + 7\mathbf{j}$

Find **a** $\frac{1}{2}\mathbf{a}$ **b** $\mathbf{a} + \mathbf{b}$ **c** $3\mathbf{a} - 2\mathbf{b}$

Example 8

a Draw a diagram to represent the vector $-3\mathbf{i} + \mathbf{j}$

b Write this as a column vector.

Example 9

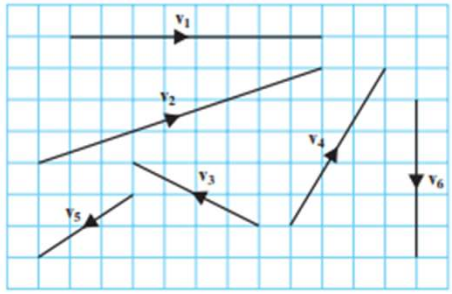
Given that $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$, $\mathbf{b} = 12\mathbf{i} - 10\mathbf{j}$ and $\mathbf{c} = -3\mathbf{i} + 9\mathbf{j}$, find $\mathbf{a} + \mathbf{b} + \mathbf{c}$, using column vector notation in your working.

Example 10

Given $\mathbf{a} = 5\mathbf{i} + 2\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$, find $2\mathbf{a} - \mathbf{b}$ in terms of $\mathbf{i}$ and $\mathbf{j}$.

Exercise 11B

1 These vectors are drawn on a grid of unit squares.
Express the vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$, $\mathbf{v}_5$ and $\mathbf{v}_6$ in:
(i) **i, j** notation (ii) column vector form



2 Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 4\mathbf{i} - \mathbf{j}$, find these vectors in terms of **i** and **j**.

a $4\mathbf{a}$ **b** $\frac{1}{2}\mathbf{a}$ **c** $-\mathbf{b}$ **d** $2\mathbf{b} + \mathbf{a}$

e $3\mathbf{a} - 2\mathbf{b}$ **f** $\mathbf{b} - 3\mathbf{a}$ **g** $4\mathbf{b} - \mathbf{a}$ **h** $2\mathbf{a} - 3\mathbf{b}$

3 Given that $\mathbf{a} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -8 \\ -1 \end{pmatrix}$ find:

a $5\mathbf{a}$ **b** $-\frac{1}{2}\mathbf{c}$ **c** $\mathbf{a} + \mathbf{b} + \mathbf{c}$ **d** $2\mathbf{a} - \mathbf{b} + \mathbf{c}$

e $2\mathbf{b} + 2\mathbf{c} - 3\mathbf{a}$ **f** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

(P) **4** Given that $\mathbf{a} = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - \mathbf{j}$, find:

a λ if $\mathbf{a} + \lambda\mathbf{b}$ is parallel to the vector **i** **b** μ if $\mu\mathbf{a} + \mathbf{b}$ is parallel to the vector **j**

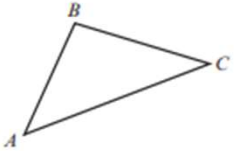
(P) **5** Given that $\mathbf{c} = 3\mathbf{i} + 4\mathbf{j}$ and $\mathbf{d} = \mathbf{i} - 2\mathbf{j}$, find:

a λ if $\mathbf{c} + \lambda\mathbf{d}$ is parallel to $\mathbf{i} + \mathbf{j}$ **b** μ if $\mu\mathbf{c} + \mathbf{d}$ is parallel to $\mathbf{i} + 3\mathbf{j}$

c s if $\mathbf{c} - s\mathbf{d}$ is parallel to $2\mathbf{i} + \mathbf{j}$ **d** t if $\mathbf{d} - t\mathbf{c}$ is parallel to $-2\mathbf{i} + 3\mathbf{j}$

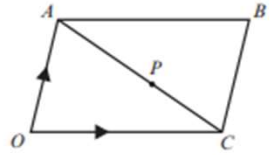
(E) **6** In triangle ABC, $\overrightarrow{AB} = 4\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{AC} = 5\mathbf{i} + 2\mathbf{j}$.
Find BC.

(2 marks)



(P) **7** $OABC$ is a parallelogram.
P divides AC in the ratio 3 : 2. $\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = 7\mathbf{i}$.
Find in **i**, **j** format and column vector format:

a $\overrightarrow{AC}$ **b** $\overrightarrow{OP}$ **c** $\overrightarrow{AP}$



(E/P) **8** $\mathbf{a} = \begin{pmatrix} j \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 10 \\ k \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

Given that $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$, find the values of j and k.

(2 marks)

Problem-solving

You can consider $\mathbf{b} - 2\mathbf{a} = \mathbf{c}$ as two linear equations. One for the x-components and one for the y-components.

(E/P) **9** $\mathbf{a} = \begin{pmatrix} p \\ -q \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} q \\ p \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 7 \\ 4 \end{pmatrix}$

Given that $\mathbf{a} + 2\mathbf{b} = \mathbf{c}$, find the values of p and q. **(2 marks)**

(E/P) **10** The resultant of the vectors $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = p\mathbf{i} - 2p\mathbf{j}$ is parallel to the vector $\mathbf{c} = 2\mathbf{i} - 3\mathbf{j}$.
Find:

a the value of p **(4 marks)**

b the resultant of vectors **a** and **b**. **(1 mark)**

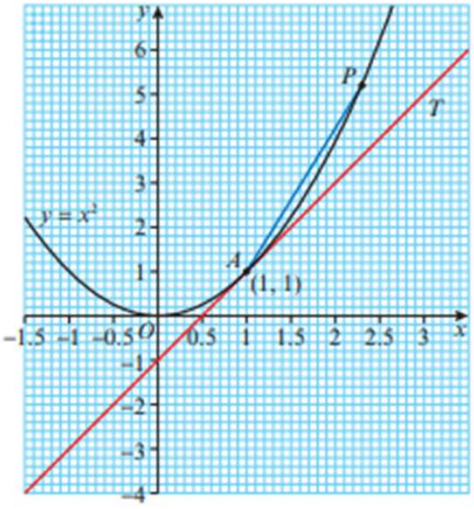
12) Differentiation

12.1) Gradients of curves	https://youtu.be/hzdHcgFasT4?si=x0HFNCjGhZFZ49mW

12.1) Gradients of curves

Example 1

The diagram shows the curve with equation $y = x^2$.
The tangent, T, to the curve at the point $A(1, 1)$ is shown.
Point A is joined to point P by the chord AP.



a Calculate the gradient of the tangent, T.

b Calculate the gradient of the chord AP when P has coordinates:

i $(2, 4)$

ii $(1.5, 2.25)$

iii $(1.1, 1.21)$

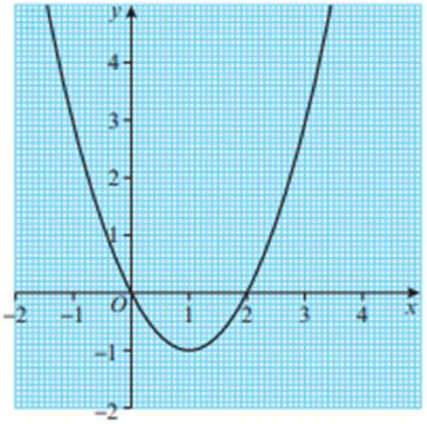
iv $(1.01, 1.0201)$

v $(1 + h, (1 + h)^2)$

c Comment on the relationship between your answers to parts **a** and **b**.

Exercise 12A

1 The diagram shows the curve with equation $y = x^2 - 2x$.



a Copy and complete this table showing estimates for the gradient of the curve.

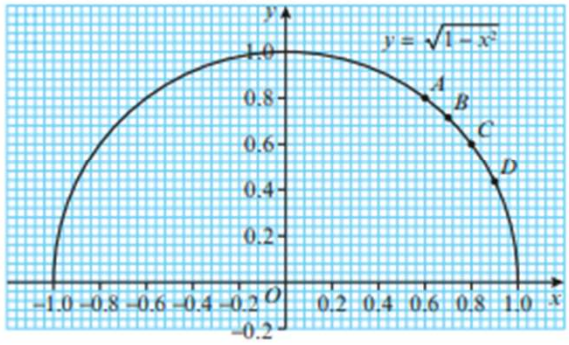
x-coordinate	−1	0	1	2	3
Estimate for gradient of curve					

b Write a hypothesis about the gradient of the curve at the point where $x = p$.

c Test your hypothesis by estimating the gradient of the graph at the point (1.5, –0.75).

Hint Place a ruler on the graph to approximate each tangent.

2 The diagram shows the curve with equation $y = \sqrt{1 - x^2}$.
The point A has coordinates (0.6, 0.8).
The points B, C and D lie on the curve with x-coordinates 0.7, 0.8 and 0.9 respectively.



a Verify that point A lies on the curve.

b Use a ruler to estimate the gradient of the curve at point A.

c Find the gradient of the line segments:
i AD
ii AC
iii AB

Hint Use algebra for part **c**.

d Comment on the relationship between your answers to parts **b** and **c**.

3 F is the point with coordinates (3, 9) on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point F to the points with coordinates:
i (4, 16) **ii** (3.5, 12.25) **iii** (3.1, 9.61)
iv (3.01, 9.0601) **v** $(3 + h, (3 + h)^2)$

b What do you deduce about the gradient of the tangent at the point (3, 9)?

4 G is the point with coordinates (4, 16) on the curve with equation $y = x^2$.

a Find the gradients of the chords joining the point G to the points with coordinates:
i (5, 25) **ii** (4.5, 20.25) **iii** (4.1, 16.81)
iv (4.01, 16.0801) **v** $(4 + h, (4 + h)^2)$

b What do you deduce about the gradient of the tangent at the point (4, 16)?

14) Exponentials and logarithms

14.1) Exponential functions	https://youtu.be/45xMAKFIpJQ?si=4p1vKryOFJojh-Ip

14.1) Exponential functions

Example 1

a On the same axes sketch the graphs of $y = 3^x$, $y = 2^x$ and $y = 1.5^x$.

b On another set of axes sketch the graphs of $y = \left(\frac{1}{2}\right)^x$ and $y = 2^x$.

Example 2

Sketch the graph of $y = \left(\frac{1}{2}\right)^{x-3}$. Give the coordinates of the point where the graph crosses the y-axis.

the graph of $y = \left(\frac{1}{2}\right)^x$ with scale factor 8.

Exercise 14A

1 **a** Draw an accurate graph of $y = (1.7)^x$, for $-4 \leqslant x \leqslant 4$.

b Use your graph to solve the equation $(1.7)^x = 4$.

2 **a** Draw an accurate graph of $y = (0.6)^x$, for $-4 \leqslant x \leqslant 4$.

b Use your graph to solve the equation $(0.6)^x = 2$.

3 Sketch the graph of $y = 1^x$.

(P) **4** For each of these statements, decide whether it is true or false, justifying your answer or offering a counter-example.

a The graph of $y = a^x$ passes through (0, 1) for all positive real numbers a.

b The function $f(x) = a^x$ is always an increasing function for $a > 0$.

c The graph of $y = a^x$, where a is a positive real number, never crosses the x-axis.

5 The function $f(x)$ is defined as $f(x) = 3^x$, $x \in \mathbb{R}$. On the same axes, sketch the graphs of:

a $y = f(x)$ **b** $y = 2f(x)$ **c** $y = f(x) - 4$ **d** $y = f\left(\frac{1}{2}x\right)$

Write down the coordinates of the point where each graph crosses the y-axis, and give the equations of any asymptotes.

(P) **6** The graph of $y = ka^x$ passes through the points (1, 6) and (4, 48). Find the values of the constants k and a.

Problem-solving

Substitute the coordinates into $y = ka^x$ to create two simultaneous equations. Use division to eliminate one of the two unknowns.

(P) **7** The graph of $y = pq^x$ passes through the points (−3, 150) and (2, 0.048).

a By drawing a sketch or otherwise, explain why $0 < q < 1$.

b Find the values of the constants p and q.