KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
S1 6.1 Probability Distributions

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Probability distributions

You are already familiar with the concept of **variable** in statistics: a collection of values (e.g. favourite colour of students in the room):

$\boldsymbol{x}$	red	green	blue	orange
$\boldsymbol{P(X = x)}$	0.3	0.4	0.1	0.2

If each is assigned a probability of occurring, it becomes a **random variable**.

A **random variable X represents a single experiment/trial**. It consists of outcomes with a probability for each.

i.e. X is a random variable (capital letter), but x is a particular outcome.

$$P(X = x)$$

"The probability that…

…the outcome of the random variable X…

…was the specific outcome x"

A shorthand for $P(X = x)$ is $p(x)$ (note the lowercase p).
It's like saying "the probability that the outcome of my coin throw was heads" ($P(X = heads)$) vs "the probability of heads" ($p(heads)$). In the latter the coin throw was implicit, so we can skip the '$X =$'.

Notes

Worked Example

Let $X =$ number of tails when a fair coin is tossed 4 times.
Describe the probability distribution of X:
a) As a list   b) as a table   c) as a diagram   d) as a mass function

Worked Example

548f: Determine a constant in a function for a probability distribution.

The discrete random variable X has the probability function

$$P(X=x)=\begin{cases}k\left(x^2+2\right) & x=1,2,3,4\\ 0, & \text{otherwise}\end{cases}$$

where k is a constant.

Find the value of k.

Worked Example

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^3}, \qquad x = 1, 2, 3, 4$$

Find the value of k

Your Turn

The random variable X has a probability function

$$P(X = x) = \frac{k}{x^2}, \qquad x = 1, 2, 3, 5$$

Find the value of k

$$k = \frac{900}{1261}$$

Worked Example

A biased six-sided dice with faces numbered $1, 2, 3, 4, 5$ and 6 is rolled. The number on the bottom-most face is modelled as a random variable X.
Given that $P(X = x) = \frac{k}{x}$,

a) Find the value of k
b) Give the probability distribution of X in table form
c) Find the probability that:

i) $X \geq 2$
ii) $1 \leq X < 4$
iii) $X < 1$
iv) $2X + 1 > 11$

Worked Example

The random variable X has a probability function

$$P(X = x) = \begin{cases} kx & x = 1, 3 \\ k(x - 2) & x = 2, 4 \end{cases}$$

a) Find the value of k

b) Find $P(X > 1)$

Worked Example

The random variable X has a probability function

$$P(X = x) = \begin{cases} k(2-x)^2 & x = -2, -1, 0, 1, 2, 3 \\ 0 & otherwise \end{cases}$$

Find the value of k

Worked Example

A spinner has six equally-sized sections.
Four contain the letter G. 2 contain the letter Y.
The spinner is spun until it lands on Y or has been spun five times in total.
Find the probability distribution of the random variable S, the number of times the spinner is spun.

Worked Example

The random variable X can taken any integer value from 1 to 30. Given that X has a discrete uniform distribution, find:

a) $P(X = 5)$

b) $P(X \geq 20)$

c) $P(12 < X < 21)$

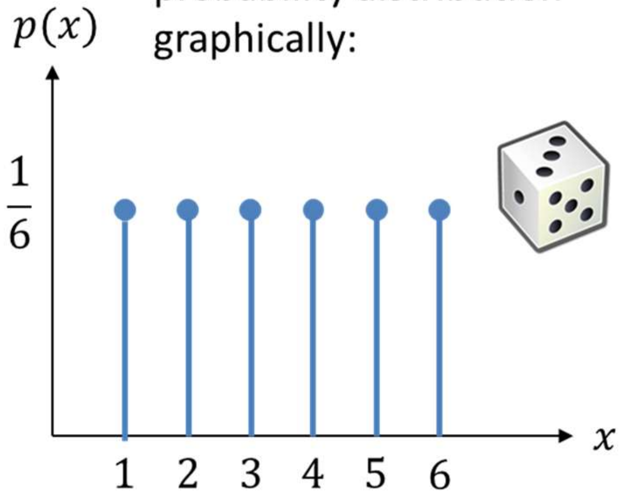
Worked Example

A discrete random variable has a probability distribution as shown in the table. Find the value of a

$\boldsymbol{x}$	0	1	2	3
$\boldsymbol{P(X = x)}$	a	$a - \frac{1}{4}$	$a + \frac{1}{3}$	$3a$

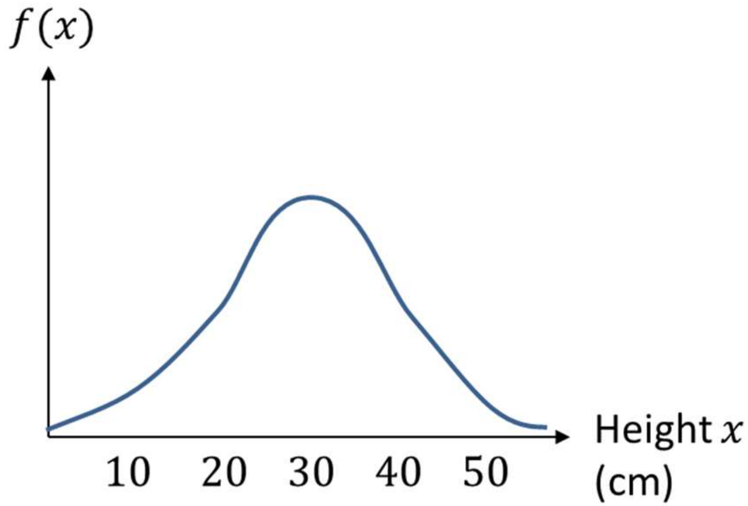
A few last things…

We can also represent a probability distribution graphically:



The throw of a die is an example of a **discrete uniform distribution** because the probability of each outcome is the same.

$p(x)$ for discrete random variables is known as a **probability mass function**, because the probability of each outcome represents an actual 'amount' (i.e. mass) of probability.



We can also have probability distributions for **continuous** variables, e.g. height.
However, the probability that something has a height of say **exactly** 30cm, is infinitely small (effectively 0).
$p(x)$ (written $f(x)$) for continuous random variables is known as a **probability density function**. $p(30)$ wouldn't give us the probability of being 30cm tall, but the amount of probability **per unit height**, i.e. the density. This is similar to histograms where frequency density is the "frequency per unit value". Just as an area in a histogram would then give a frequency, and area under a probability density graph would give a probability (mass).

You will encounter the **Normal Distribution** in Year 2, which is an example of a continuous probability distribution.

1. A discrete random variable X has the probability function

$$P(X = x) = \begin{cases} k(1-x)^2 & x = -1,\ 0,\ 1 \text{ and } 2 \\ 0 & \text{otherwise.} \end{cases}$$

(*a*) Show that $k = \dfrac{1}{6}$.

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

x	-1	0	1	2
$p(x)$	$k(1--1)^2 = 4k$	k	0	k

Since $\sum p(x) = 1$, $4k + k + 0 + k = 1$

$6k = 1 \rightarrow k = \frac{1}{6}$