KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
S1 5 Probability Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

Prior knowledge check

Prior knowledge check

1 A bag contains three red balls, four yellow balls and two blue balls. A ball is chosen at random from the bag. Write down the probability that the ball is:

a blue **b** yellow

c not red **d** green.

← **GCSE Mathematics**

2 Three coins are flipped. Write down all the possible outcomes. ← **GCSE Mathematics**

3 Poppy rolls a dice. She keeps rolling until she rolls a 6. Work out the probability that Poppy rolls the dice:

a exactly three times

b fewer than three times

c more than three times.

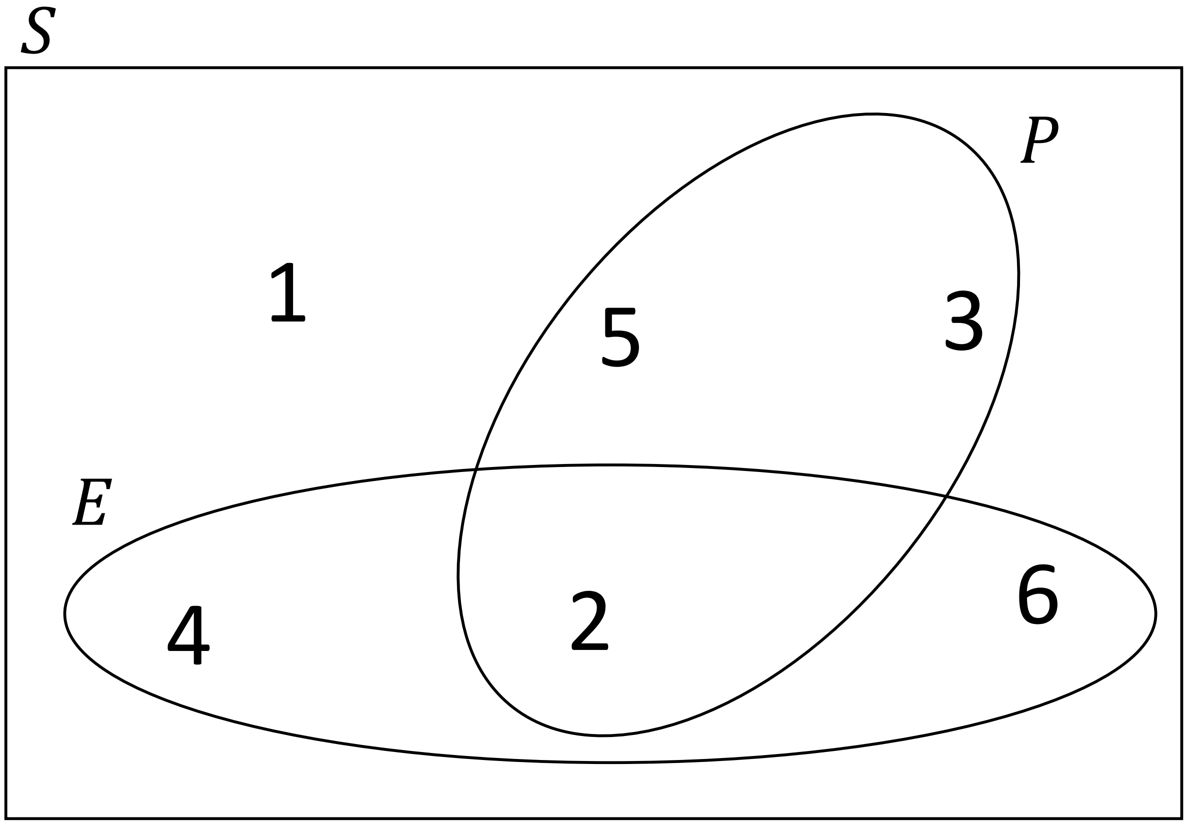
← **GCSE Mathematics**

5.1 Calculating Probabilities

An **experiment** is a repeatable process that gives rise a number a number of **outcomes**.

An **event** is a set of one or more of these outcomes.

(We often use capital letters to represent them)



$E =$ "rolling an even number"
$P =$ "rolling a prime number"

A **sample space** is the set of all possible outcomes.

Because we are dealing with sets, we can use a **Venn diagram**, where

- the numbers are the individual outcomes,
- the sample space is a rectangle and
- the events are sets, each a subset of the sample space.

You do not need to use set notation like ∩ and ∪ in this module (but ordinarily you would!)

Notes

Worked Example

Two fair spinners each have four sectors numbered 1 to 4. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded.

Find the probability of the spinners indicating a sum of:

a) exactly 5

b) more than 5

Your Turn

Two fair spinners each have five sectors numbered 1 to 5. The two spinners are spun together and the sum of the numbers indicated on each spinner is recorded.

Find the probability of the spinners indicating a sum of:

a) exactly 6

b) more than 6

Worked Example

The table shows the times taken, in minutes, for a group of students to complete a number puzzle.

Time, t (min)	$5 \leq t < 7$	$7 \leq t < 9$	$9 \leq t < 11$	$11 \leq t < 13$	$13 \leq t < 15$
Frequency	6	13	12	5	4

A student is chosen at random. Find the probability that they completed the number puzzle in:

a) under 9 minutes

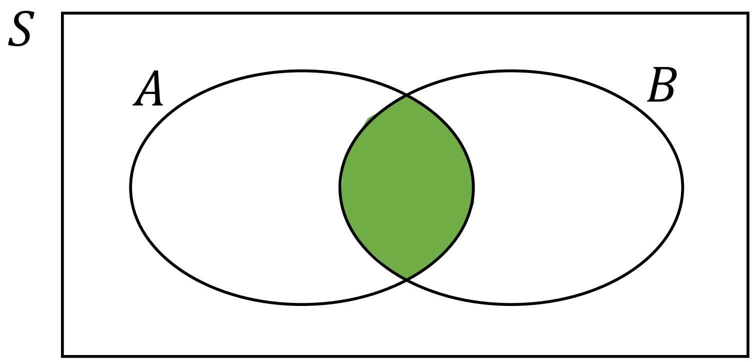
b) over 10.5 minutes

Worked Example

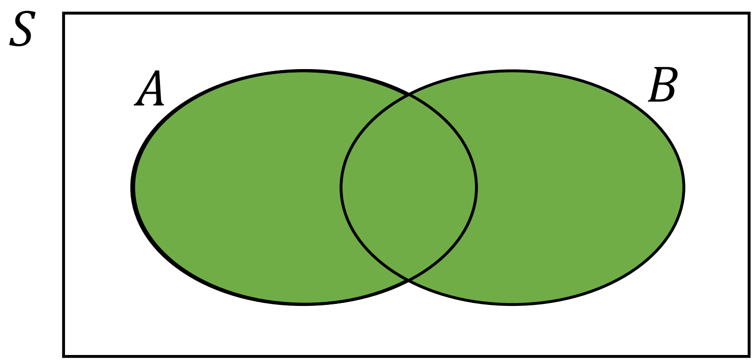
A participant is chosen at random.
What is the probability they weigh more than $14\ kg$?

5.2 Venn Diagrams

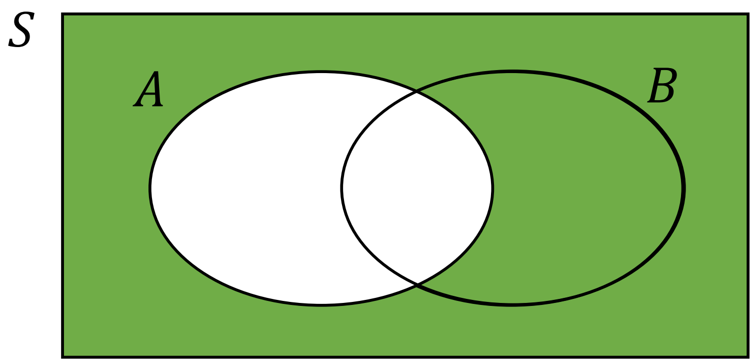
Venn Diagrams allow us to combine events, e.g. "A happened **and** B happened".



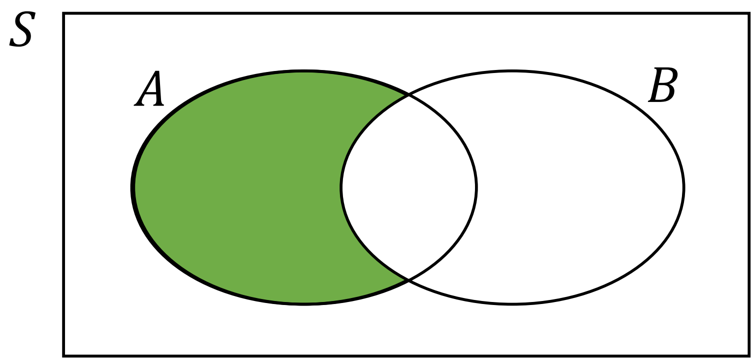
The event "A **and** B"
Known as the **intersection** of A and B.



The event "A **or** B"
Known as the **union** of A and B.



The event "not A"
Known as the **union** of A and B.



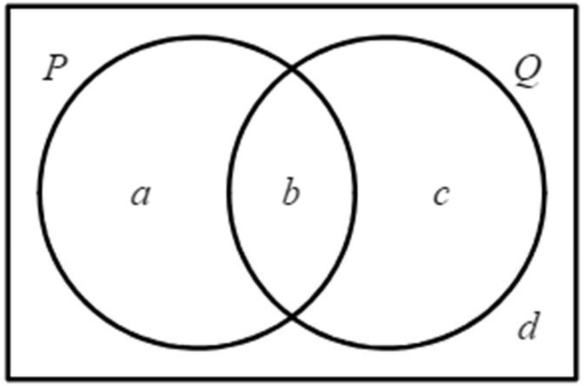
These can be combined,
e.g. "A and not B".

Notes

Worked Example

242a: Understand what is represented by each region or group of regions in a Venn diagram with 2 sets.

Set P represents cars that have a sunroof.
Set Q represents cars that have air-con.



Identify the region that represents cars that do not have a sunroof and do not have air-con.

- ○ a
- ○ b
- ○ c
- ○ d

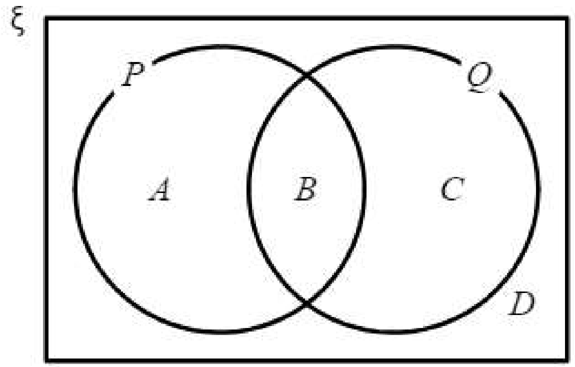
Worked Example

242c: Construct a Venn diagram with 2 sets from a list of elements.

$\xi = \{\text{Prime numbers less than } 20\}$
$P = \{5, 7, 13, 17\}$
$Q = \{5, 7, 19\}$

Complete the Venn diagram to represent this information.

Note: Input answers for each set in ascending order separated by commas. For example 1, 2, 3



$A = \{$ $\}$
$B = \{$ $\}$
$C = \{$ $\}$
$D = \{$ $\}$

Worked Example

Given that $P(A) = 0.6$ and $P(A\ or\ B) = 0.85$, find the probability of:

a) $P(A\ and\ not\ B)$

b) $P(neither\ A\ nor\ B)$

Your Turn

Given that $P(D) = 0.7$ and $P(C\ or\ D) = 0.95$, find the probability of:

a) $P(C\ and\ not\ D)$

b) $P(neither\ C\ nor\ D)$

Worked Example

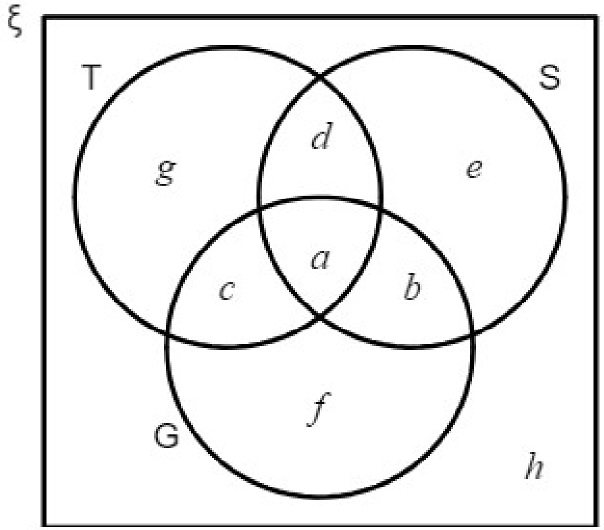
The probability of a person having read book A is 0.37.
The probability that they have read book B is 0.25.
The probability that they have read book A or B or both is 0.54.
A person is chosen at random.
Find the probability that the person has
a) Read both book A and book B
b) Read book A but not book B
c) Read neither book

Worked Example

242g: **Construct a Venn diagram with 3 sets given totals.**

130 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool, the gym and the tennis courts.

21 pupils use the swimming pool, the gym and the tennis courts.
35 pupils use the swimming pool and the gym.
27 pupils use the gym and the tennis courts.
27 pupils use the tennis courts and the swimming pool.
33 pupils use the swimming pool only.
8 pupils use the gym only.
17 pupils use the tennis courts only.

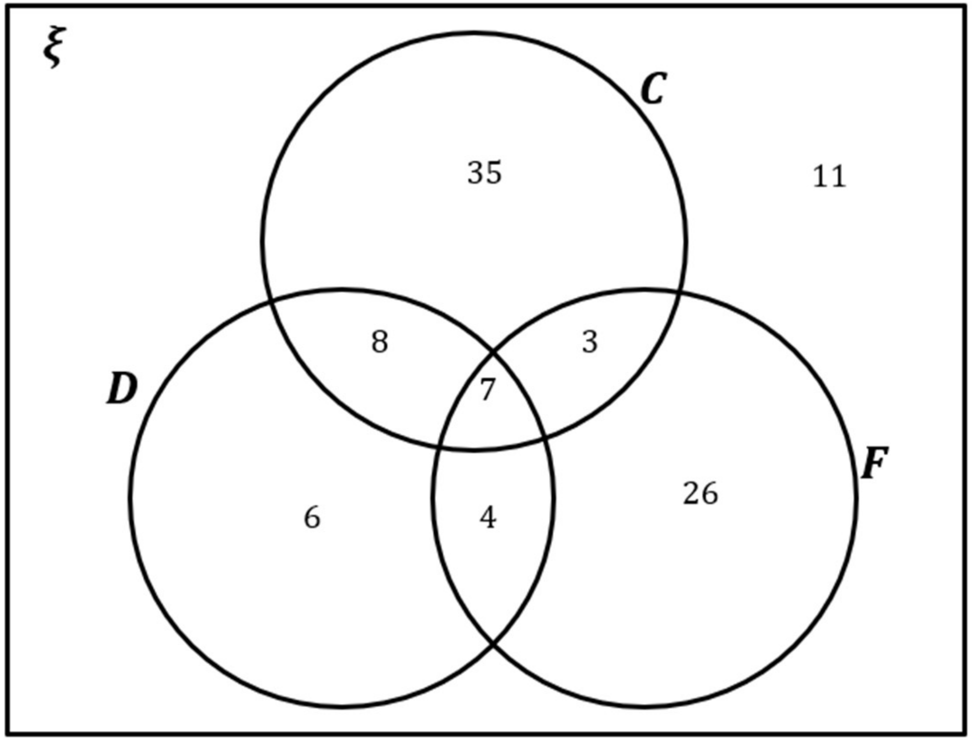


Worked Example

A vet surveys 100 of their clients. They find that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.
Draw a Venn Diagram to represent this data.

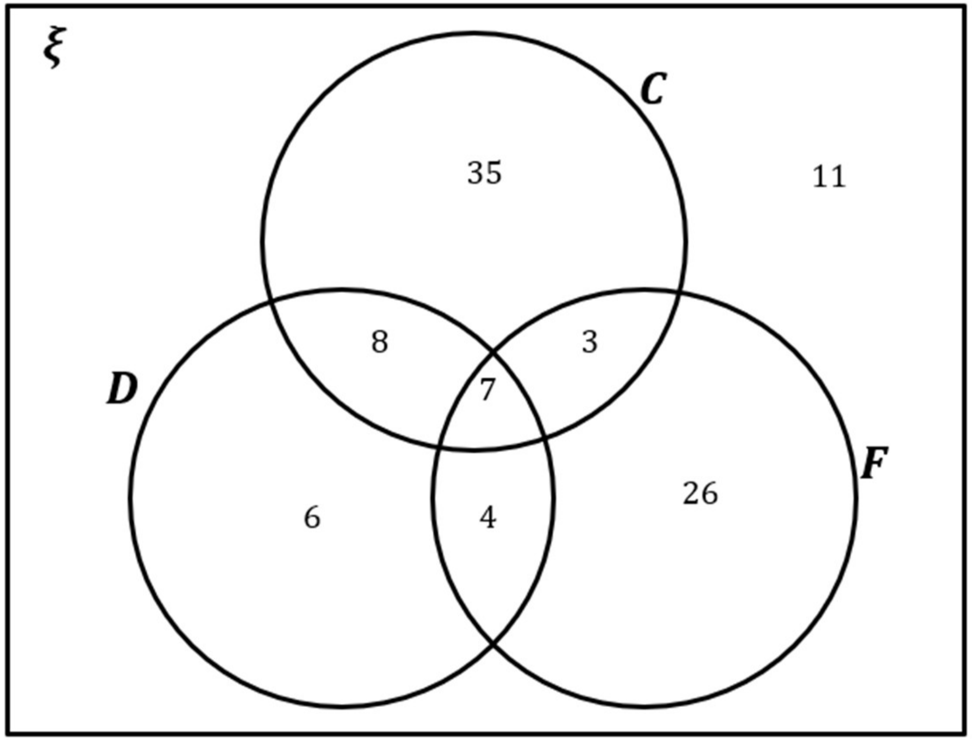
Worked Example

A gym owner surveys 100 of their clients.
A client is chosen at random.
Find the probability that the client:
a) Owns dogs only
b) Does not own fish
c) Does not own dogs, cats or fish
d) Owns fish and cats but not dogs



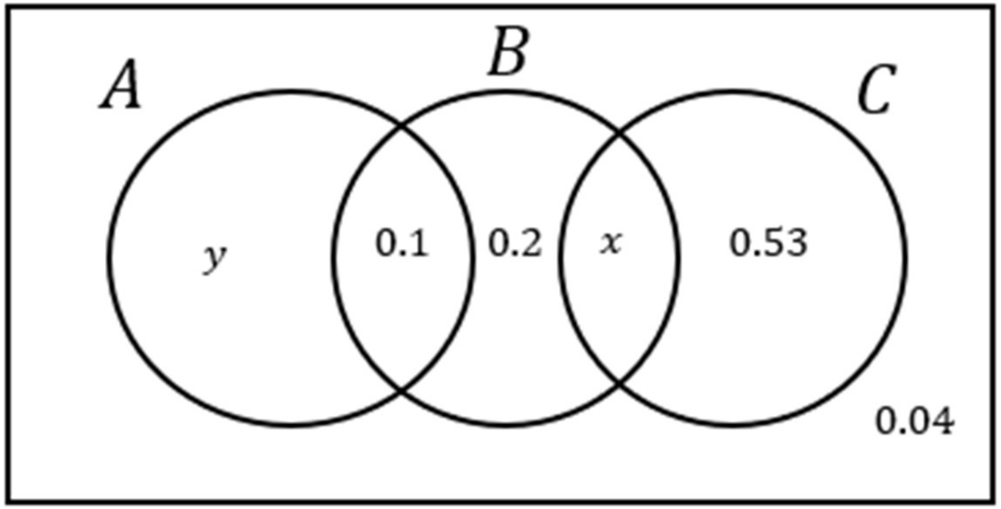
Worked Example

A gym owner surveys 100 of their clients.
A client is chosen at random.
Find the probability that the client:
a) Owns exactly one type of pet.
b) Owns at least two of the types of pet.



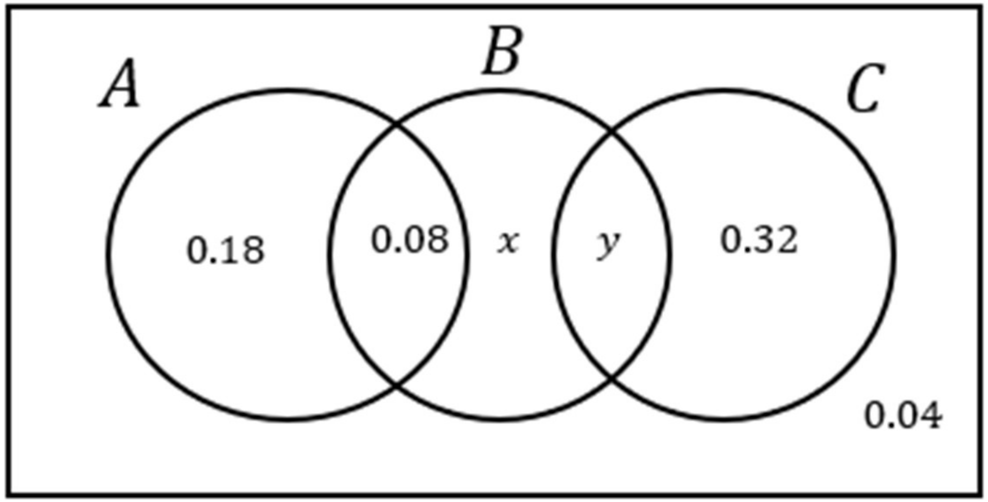
Worked Example

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = 0.35$, find $P(A)$



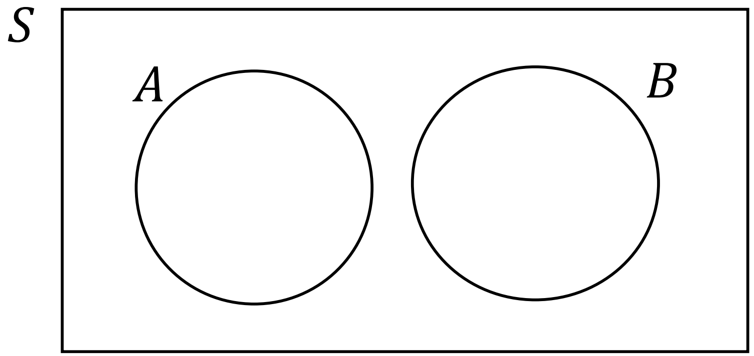
Worked Example

The Venn diagram shows the probabilities of group members taking part in activities A, B and C.
Given that $P(B) = P(C)$, find the values of x and y



5.3 Mutually Exclusive and Independent Events

- If two events are mutually exclusive **they can't happen at the same time**.
- If A and B are mutually exclusive then:
  - $\boldsymbol{P(A\ and\ B) = 0}$
  - $\boldsymbol{P(A\ or\ B) = P(A) + P(B)}$
- The Venn Diagram would look like:



Since $P(A\ and\ B) = 0$, there can't be any outcomes in the overlap, so we don't have an overlap!

- If two events are independent **then whether one event happens does not affect the probability of the other happening**.
- If A and B are independent then:
  - $\boldsymbol{P(A\ and\ B) = P(A) \times P(B)}$

Note: Independence does not affect how the circles interact in a Venn Diagram.

Notes

Worked Example

250d: **Determine probabilities of mutually exclusive events with a giver relationship.**

The table shows the probability a pupil chosen at random studies Spanish, Portuguese, Chinese or German.

The probability for German is twice as likely as for Chinese.

A pupil is chosen at random.

Work out the probability that the pupil studies Chinese.

Subject	Spanish	Portuguese	Chinese	German
Probability	0.1	0.3		

Worked Example

Events A and B are mutually exclusive and $P(A) = 0.2$ and $P(B) = 0.4$

a) Find $P(A\ or\ B)$

b) Find $P(A\ but\ not\ B)$

c) Find $P(neither\ A\ nor\ B)$

Worked Example

Events A and B are independent.
$P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{5}$
Find $P(A\ and\ B)$

Worked Example

633c: **Determine whether two events are independent given probabilities.**

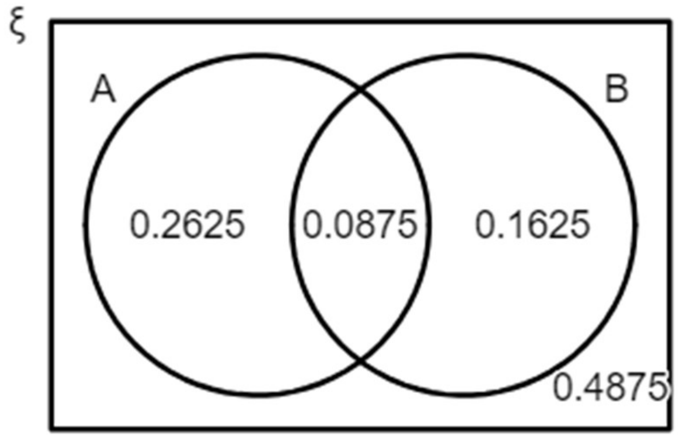
A and B are two events.

$P(A) = 0.3$
$P(B) = 0.45$
$P(A \text{ and } B) = 0.23$

Determine whether A and B are ir

633d: **Determine whether two events are independent given a Venn diagram.**

A and B are two events.

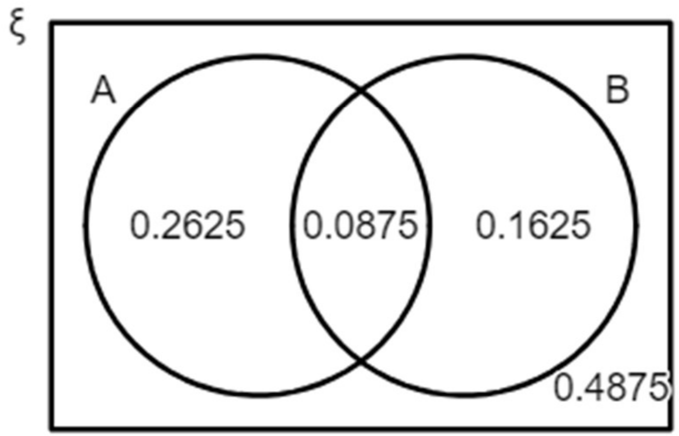


Determine whether A and B are independent.

Worked Example

633d: **Determine whether two events are independent given a Venn diagram.**

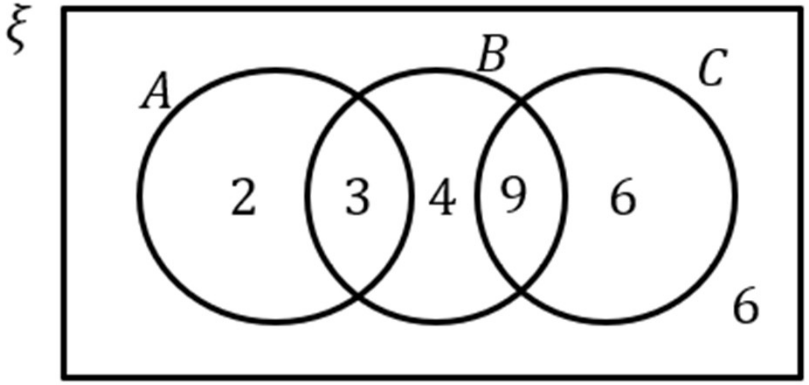
A and B are two events.



Determine whether A and B are independent.

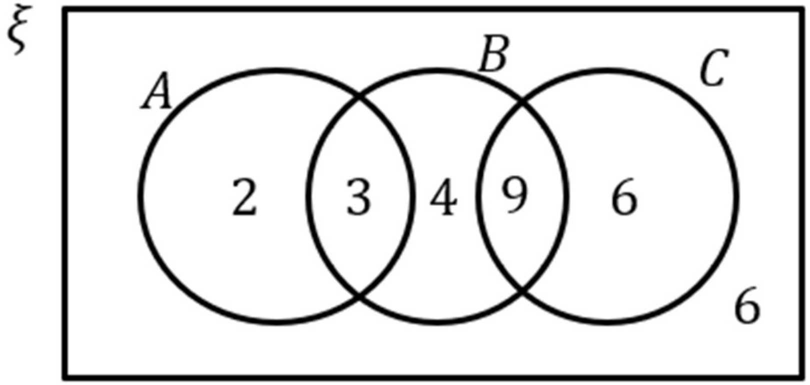
Worked Example

The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes.
Find the probability that a student chosen at random watches B or C or both.



Worked Example

The Venn diagram shows the number of students in a particular class who watch any of three popular TV programmes. Determine whether watching A and watching B are statistically independent.



Worked Example

355b: **Calculate probabilities of mutually exclusive events from given probabilities.**

Events C and D are mutually exclusive.

$P(C) = 0.7$ and $P(D) = 0.05$

Find $P(D \text{ but not } C)$

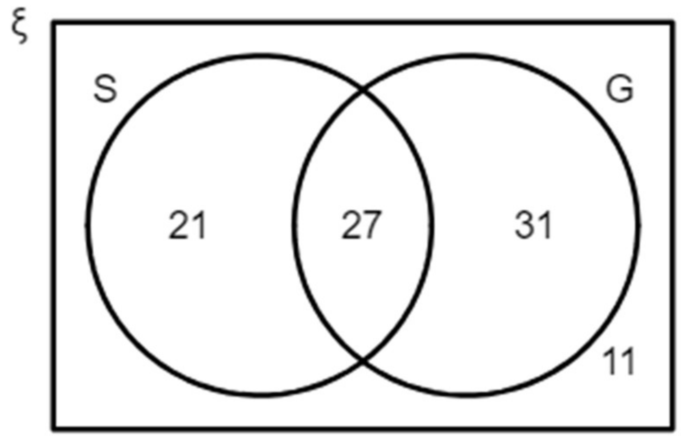
$P(D \text{ but not } C) =$

Worked Example

355c: **Calculate probabilities of a single region using a Venn diagram with two non-mutually exclusive sets.**

90 pupils in a sports centre are surveyed.
The pupils can only use the swimming pool and the gym.

27 pupils use the swimming pool and the gym.
48 pupils use the swimming pool.
58 pupils use the gym.



Find the probability to select a pupil that uses the gym but not the swimming pool.

Worked Example

355f: **Calculate probabilities by drawing a Venn diagram with two non-mutually exclusive sets.**

There are 90 pupils in a group.
The only languages available for the group to study are German and Spanish.

45 pupils study German.
53 pupils study Spanish.
17 pupils study neither German nor Spanish.

Find the probability to select a pupil that studies German but not Spanish.

Worked Example

355g: Calculate probabilities by drawing a Venn diagram with three non-mutually exclusive sets.

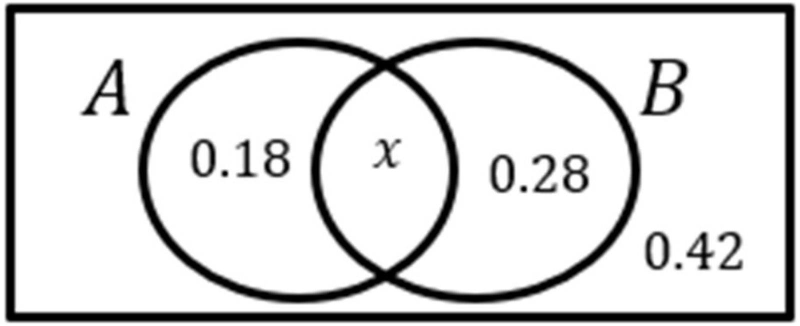
There are 190 pupils in a group.
The only languages available for the group to study are Italian, Spanish and German.

34 pupils study the three languages.
59 pupils study both Italian and Spanish.
52 pupils study both Spanish and German.
47 pupils study both German and Italian.
91 pupils study Italian.
105 pupils study Spanish.
84 pupils study German.

Find the probability to select a pupil that studies both Italian and Spanish, but not German.

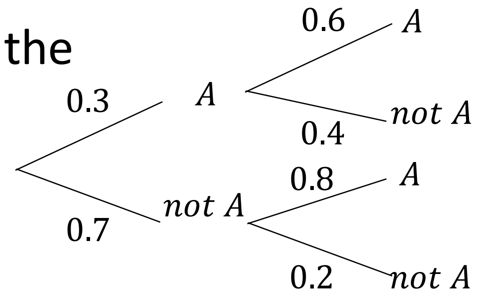
Worked Example

Determine if events A and B are independent.



5.4 Tree Diagrams

At GCSE we saw that tree diagrams were an effective way of showing the outcome of two events which happen **in succession**.
(though you can avoid them in many problems!)



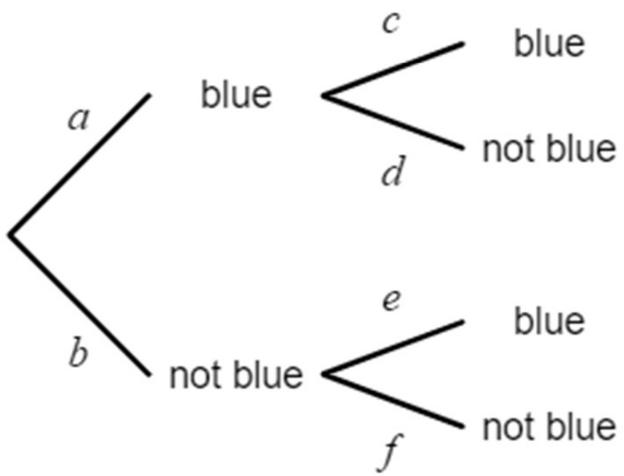
Notes

Worked Example

354a: **Draw a tree diagram to represent successive dependent events.**

Louis has 2 blue socks, 1 red sock and 2 green socks in a bag.

Louis takes one sock at random from the bag, keeps it, and takes another sock from the bag.



Complete the tree diagram.

Worked Example

354b: **Deal with a single chain of outcomes involving successive dependent events.**

Dave has 2 red balls, 1 white ball and 2 yellow balls in a box.

Dave takes one ball at random from the box, keeps it, and takes another ball from the box.

Find the probability that Dave takes two red balls.

Worked Example

354c: **Deal with multiple different chains of outcomes involving successive dependent events.**

Libby has 2 green socks, 1 red sock and 7 purple socks in a bag.

Libby takes one sock at random from the bag, keeps it, and takes another sock from the bag.

Find the probability that Libby takes at least one green sock.

Worked Example

354f: **Determine the probability of successive events by grouping outcomes (e.g. into odd and even).**

There are 10 tiles in a bowl.

There is a number on each tile.

(3) (5) (5) (5) (5) (6) (6) (7) (7) (7)

Zoe takes at random three of the tiles.

She works out the **product** of the numbers on the three tiles.

Work out the probability that the product is an **even** number.

Worked Example

The probability I hit a target on each shot is 0.3. I keep firing until I hit the target. Determine the probability I hit the target on the 5^{th} shot.

Worked Example

463e: **Form an equation using sampling without replacement.**

Stacey has 6 marbles, of which k are red. The remainder of the marbles are blue.

Stacey takes a marble, does not replace it, and then takes another marble.

The probability that she takes two red marbles is $\frac{2}{5}$.

Show that $k^2 + ak + b = 0$, where a and b are constants to be found.

Extension Questions

1 [STEP I 2010 Q12] Prove that, for any real numbers x and y, $x^2 + y^2 \geq 2xy$.

(i) Carol has two bags of sweets. The first bag contains a red sweets and b blue sweets, whereas the second bag contains b red sweets and a blue sweets. Carol shakes the bags and picks one sweet from each bag without looking. Prove that the probability that the sweets are of the same colour cannot exceed the probability that they are of different colours.

(ii) Simon has three bags of sweets. The first bag contains a red sweet, b white sweets and c yellow sweets. The second bag contains b red sweets, c white sweets and a yellow sweets. The third bag contains c red sweets, a white sweets and b yellow sweets. Simon shakes the bags and picks one sweet from each bag without looking. Show that the probability that exactly two of the sweets are of the same colour is

$$\frac{3(a^2b + b^2c + c^2a + ab^2 + bc^2 + ca^2)}{(a+b+c)^3}$$

and find the probability that the sweets are all of the same colour. Deduce that the probability that exactly two of the sweets are of the same colour is at least 6 times the probability that the sweets are all of the same colour.

2 [STEP I 2011 Q12] I am selling raffle tickets for £1 per ticket. In the queue for tickets, there are m people each with a single £1 coin and n people each with a single £2 coin. Each person in the queue wants to buy a single raffle ticket and each arrangement of people in the queue is equally likely to occur. Initially, I have no coins and a large supply of tickets. I stop selling tickets if I cannot give the required change.

(i) In the case $n = 1$ and , $m \geq 1$, find the probability that I am able to sell one ticket each person in the queue.

(ii) By considering the first people in the queue, show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 2$ and $m \geq 2$ is $\frac{m-1}{m+1}$

(iii) Show that the probability that I am able to sell one ticket to each person in the queue in the case $n = 3$ and $m \geq 3$ is $\frac{m-2}{m+1}$.

3 I have an unfair coin with a fixed probability p of heads. Determine how the unfair coin could be used to simulate a fair coin, i.e. you declare "Heads" or "Tails" each with probability 0.5.

?

Extract from Formulae book

Statistics

Probability

$$P(A') = 1 - P(A)$$

2. A factory buys 10% of its components from supplier A, 30% from supplier B and the rest from supplier C. It is known that 6% of the components it buys are faulty.

Of the components bought from supplier A, 9% are faulty and of the components bought from supplier B, 3% are faulty.

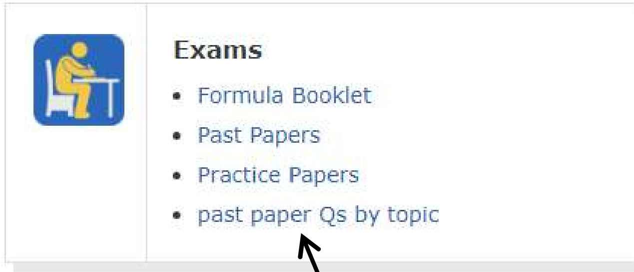
(a) Find the percentage of components bought from supplier C that are faulty.

(3)

A component is selected at random.

(b) Explain why the event "the component was bought from supplier B" is not statistically independent from the event "the component is faulty".

(1)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Qn	Scheme	Marks	AO
2 (a)	[Let $p = P(F \mid C)$]		
	Tree diagram or some other method to find an equation for p	M1	2.1
	$0.1\times0.09+0.3\times0.03+0.6\times p=0.06$	A1	1.1b
	$p=0.07$ i.e. 7%	A1	1.1b
		(3)	
(b)	e.g. $P(B \text{ and } F)=0.3\times0.03=0.009$ but $P(B)\times P(F)=0.3\times0.06=0.018$	B1	2.4
	These are not equal so not independent		
		(1)	
		(4 marks)	

Summary of Key Points

1 A **Venn diagram** can be used to represent events graphically. Frequencies or probabilities can be placed in the regions of the Venn diagram.

2 For **mutually exclusive** events, P(A or B) = P(A) + P(B).

3 For **independent** events, P(A and B) = P(A) × P(B).

4 A **tree diagram** can be used to show the outcomes of two (or more) events happening in succession.