KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Statistics
S2 2 Conditional Probability
Booklet

HGS Maths

Dr Frost Course

Name: ____________________________________

Class: ____________________________________

Contents

Prior knowledge check

Prior knowledge check

1 Events A and B are mutually exclusive. P(A) = 0.3 and P(B) = 0.45. Find:

- **a** P(A or B)
- **b** P(A and B)
- **c** P(neither A nor B).

← Year 1, Chapter 5

2 Events C and D are independent. P(C) = 0.2 and P(D) = 0.6.

- **a** Find P(C and D).
- **b** Draw a Venn diagram to show events C and D and the whole sample space.
- **c** Find P(neither C nor D).

← Year 1, Chapter 5

3 A bag contains seven counters numbered 1–7. Two counters are selected at random without replacement. Find the probability that:

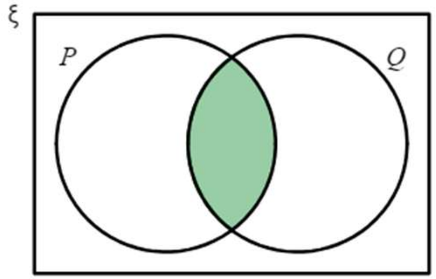
- **a** Both counters are odd-numbered
- **b** At least one counter is odd-numbered.

← Year 1, Chapter 5

Prior knowledge check

243a: **Recognise set notation for the universal set, union, intersection and complement from a shaded Venn diagram with 2 sets.**

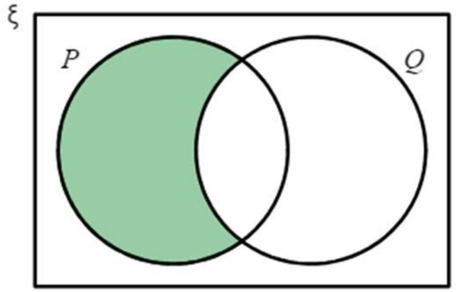
Describe the shaded region using set notation.



- ○ $P \cup Q$
- ○ $P \cap Q$
- ○ $P' \cup Q$
- ○ $P \cup Q'$
- ○ P'
- ○ Q'
- ○ $P' \cap Q$
- ○ $P \cap Q'$
- ○ $P' \cap Q'$

243a: **Recognise set notation for the universal set, union, intersection and complement from a shaded Venn diagram with 2 sets.**

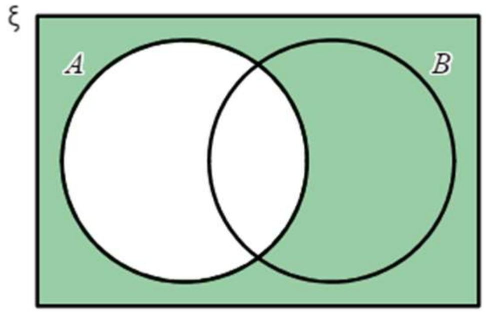
Describe the shaded region using set notation.



- ○ $P \cup Q$
- ○ $P \cap Q$
- ○ $P' \cup Q$
- ○ $P \cup Q'$
- ○ P'
- ○ Q'
- ○ $P' \cap Q$
- ○ $P \cap Q'$
- ○ $P' \cap Q'$

243a: **Recognise set notation for the universal set, union, intersection and complement from a shaded Venn diagram with 2 sets.**

Describe the shaded region using set notation.



- ○ $A \cup B$
- ○ $A \cap B$
- ○ $A' \cup B$
- ○ $A \cup B'$
- ○ A'
- ○ B'
- ○ $A' \cap B$
- ○ $A \cap B'$
- ○ $A' \cap B'$

Prior knowledge check

243g: List elements using set notation.

$\xi = \{\text{odd numbers from 5 to 25}\}$
$C = \{13, 17\}$
$D = \{11, 13, 17\}$

List the elements in $C \cap D$

$C \cap D = \{$ Separate answers with commas $\}$

243h: Determine the number of elements in a set given a list.

$\xi = \{7, 8, 9, 10, 11, 12, 13, 14, 15\}$
$P = \{8, 9, 10, 11, 13, 14, 15\}$
$Q = \{7, 8, 11, 12, 13, 14, 15\}$

Find $n(P \cap Q)$

$n(P \cap Q) =$

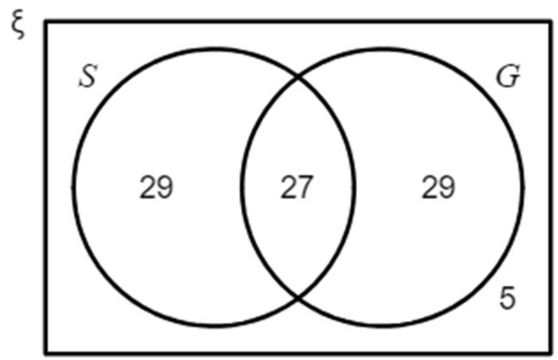
2.1 Set Notation

Notes

Worked Example

356b: **Use set notation to calculate probabilities of non-mutually exclusive events from a Venn diagram filled with frequencies.**

90 pupils in a sports centre are surveyed, who use the swimming pool (S) and the gym (G), as shown on the Venn diagram below.

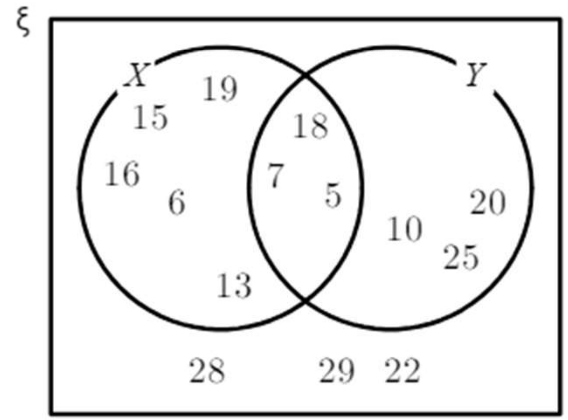


Find $P\left(S' \cap G\right)$

356c: **Use set notation to calculate probabilities of non-mutually exclusive events from a Venn filled with elements.**

The Venn diagam represents the information from the following sets.

$\xi = \{5, 6, 7, 10, 13, 15, 16, 18, 19, 20, 22, 25, 28, 29\}$
$X = \{5, 6, 7, 13, 15, 16, 18, 19\}$
$Y = \{5, 7, 10, 18, 20, 25\}$



Find $P(X \cup Y)$

$P(X \cup Y) =$

Worked Example

A card is selected at random from a pack of 52 playing cards.
Let A be the event that the card is an ace.
Let D be the event that the card is a diamond. Find:

a) $P(A \cap D)$
b) $P(A \cup D)$
c) $P(A')$
d) $P(A' \cap D)$

Worked Example

Given that:
$P(A) = 0.3$
$P(B) = 0.4$
$P(A \cap B) = 0.25$
Explain why events A and B are not independent.

Worked Example

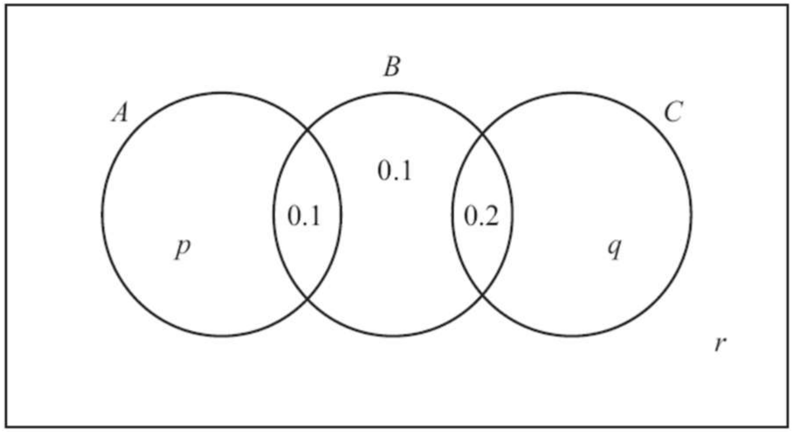
Given that:
$P(A) = 0.3$
$P(B) = 0.4$
$P(A \cap B) = 0.25$
$P(C) = 0.2$
A and C are mutually exclusive.
Events B and C are independent.

a) Draw a Venn diagram to illustrate the events A, B and C, showing the probabilities for each region.
b) Find $P\big((A \cap B') \cup C\big)$

Worked Example

The events *A* and *B* are independent.

Find the value of *p*.



Worked Example

Events A and B are independent.
$P(A) = x$
$P(B) = y$

Find:
a) $P(A \cap B)$
b) $P(A \cup B')$

Worked Example

ξ = $\{Months\ of\ the\ year\}$
A = $\{Months\ starting\ with\ A\}$
B = $\{Months\ with\ six\ letters\}$
Draw a Venn diagram to represent this information.

Worked Example

Represent as a Venn diagram:

ξ =Positive integers between 10 and 20 inclusive

$\mathrm{A} = \{$Multiples of $3\}$

$\mathrm{B} = \{$Multiples of $6\}$

Worked Example

In a group of 30 mathematicians:

- 15 have studied Calculus.
- 22 have studied Topology.
- Some have studied both.
- 3 mathematicians have not yet studied either Calculus or topology

Find the number of mathematicians who have studied both Calculus and Topology.

2.2 Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

- This will work if A and B are NOT independent

In fact, if A and B are independent then:

$$P(B|A) = P(B)$$

Notes

Worked Example

A group is made up of 42 men and 68 women.
36 of the women and 24 of the men are left-handed.

a) Draw a two-way table to show this information.

b) One person is chosen at random. Find:

   i) P(left-handed)

   ii) P(left-handed | man)

   iii) P(woman | left-handed)

Worked Example

635g: Use a Venn Diagram to find the probability of a conjunction using the conditional probability formula.

For the events A and B,

$$P(A' \cap B) = 0.49$$
$$P(A|B) = 0.3$$

Find $P(A \cap B)$.

$P(A \cap B) =$

Worked Example

The following two-way table shows what foreign language students in Year 9 study.
B is the event that the student is a boy.
F is the event they chose French as their language.

	$\boldsymbol{B}$	$\boldsymbol{B'}$
$\boldsymbol{F}$	14	38
$\boldsymbol{F'}$	26	22

Determine:

a) $P(F|B')$

b) $P(B|F')$

Worked Example

A school has 75 students in year 12. Of these students, 25 study only humanities subjects (H) and 37 study only science subjects (S). 11 students study both science and humanities subjects.

a) Draw a two-way table to show this information.

b) Find:

i) $P(S' \cap H')$

ii) $P(S|H)$

iii) $P(H|S')$

Worked Example

Two four-sided dice are thrown together, and the sum of the numbers shown is recorded.

a) Draw a sample-space diagram showing the possible outcomes.

b) Given that at least one dice lands on a 3, find the probability that the sum on the two dice is exactly 5.

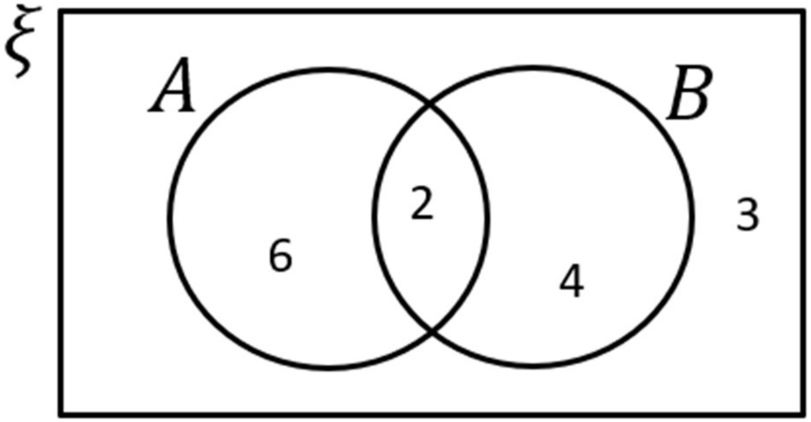
c) State one modelling assumption used in your calculations.

2.3 Conditional Probabilities in Venn Diagrams

Notes

Worked Example

Using the Venn diagram, determine:



a) $P(A|B)$

b) $P(A'|B')$

c) $P(B|A \cup B)$

Worked Example

Given that $P(A) = 0.5$ and $P(A \cap B) = 0.3$, determine:

$P(B|A)$

Worked Example

Given that $P(Y) = 0.6$ and $P(X \cap Y) = 0.4$,
determine:
$P(X'|Y)$

Worked Example

Given that $P(A) = 0.5$, $P(B) = 0.5$ and $P(A \cap B) = 0.4$, determine:
$P(B|A')$

Worked Example

Given that
$P(E) = 0.28$
$P(E \cup F) = 0.76$
$P(E \cap F') = 0.11$
Draw a Venn diagram to illustrate the probabilities of each region.

Worked Example

Given that

$P(A \cap B') = 0.4$

$P(A \cup B) = 0.75$

Determine:

a) $P(B)$

b) $P(A' \cap B')$

Worked Example

Given that
$P(A') = 0.7,$
$P(B') = 0.2$
$P(A \cap B') = 0.1$
Determine:
a) $P(A \cup B')$
b) $P(B|A')$

Worked Example

The events A and B are independent.

$$P(B|C) = \frac{5}{11}$$

a) Find the values of p, q and r

b) Find $P(A \cup C|B)$

Worked Example

A and B are two events such that $P(A) = 0.55$, $P(B) = 0.4$ and $P(A \cap B) = 0.15$

a) Draw a Venn diagram showing the probabilities for events A and B

b) Find:

i) $P(A|B)$

ii) $P(B|(A \cup B))$

iii) $P(A'|B')$

2.4 Probability Formulae

If events are NOT mutually exclusive, then:

$$\mathrm{P}(\mathrm{A} \cup B) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - P(A \cap B)$$

Addition formulae

In fact, if the events were mutually exclusive then $P(A \cap B) = 0$, so we get back to

$$\mathrm{P}(\mathrm{A} \cup B) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B})$$

Full Laws of Probability

If events A and B are <u>independent</u>.

$$P(A \cap B) = P(A) \times P(B)$$
$$P(A|B) = P(A)$$

If events A and B are <u>mutually exclusive</u>:

$$P(A \cap B) = 0$$
$$P(A \cup B) = P(A) + P(B)$$

In general:

We first encountered this in the previous section.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

This is known as the **<u>Addition Law</u>.** **Informal Proof**: If we added the probabilities in the A and B sets in the Venn Diagram, we'd be double counting the intersection, so subtract so that it's only counted once.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

SUPER IMPORTANT TIPS

If I were to identify two tips that will possible help you the most in probability questions:

If you see the words '**given that**', <u>Immediately</u> write out the law for conditional probability.

Example: "Given Bob walks to school, find the probability that he's not late…"

First thing you should write: $P(L'|W) = \frac{P(L' \cap W)}{P(W)} = \cdots$

If you see the words '**are independent**', <u>Immediately</u> write out the laws for independence.
(Even before you've finished reading the question!)

Example: "A is independent from B…"

First thing you should write:

$$P(A \cap B) = P(A)P(B)$$
$$P(A|B) = P(A)$$

If you're stuck on a question where you have to find a probability given others, it's probably because you've failed to take into account that two events are independent or mutually exclusive, or you need to use the conditional probability or additional law.

Notes

Worked Example

A and B are two events, with $P(A) = 0.6$, $P(B) = 0.7$ and $P(A \cup B) = 0.9$

Find $P(A \cap B)$

Worked Example

Two events A and B are independent.

$P(A) = \frac{1}{3}$

$P(B) = \frac{1}{4}$

Find:

a) $P(A \cap B)$

b) $P(A|B)$

c) $P(A \cup B)$

Worked Example

C and D are two events such that

$P(C) = 0.2$

$P(D) = 0.6$

$P(C|D) = 0.3$

Find:

a) $P(C \cap D)$

b) $P(D|C)$

c) $P(C \cup D)$

Worked Example

A and B are two independent events such that

$P(A) = \frac{1}{4}$

$P(A \cup B) = \frac{2}{3}$

Find:

a) $P(B)$

b) $P(A' \cap B)$

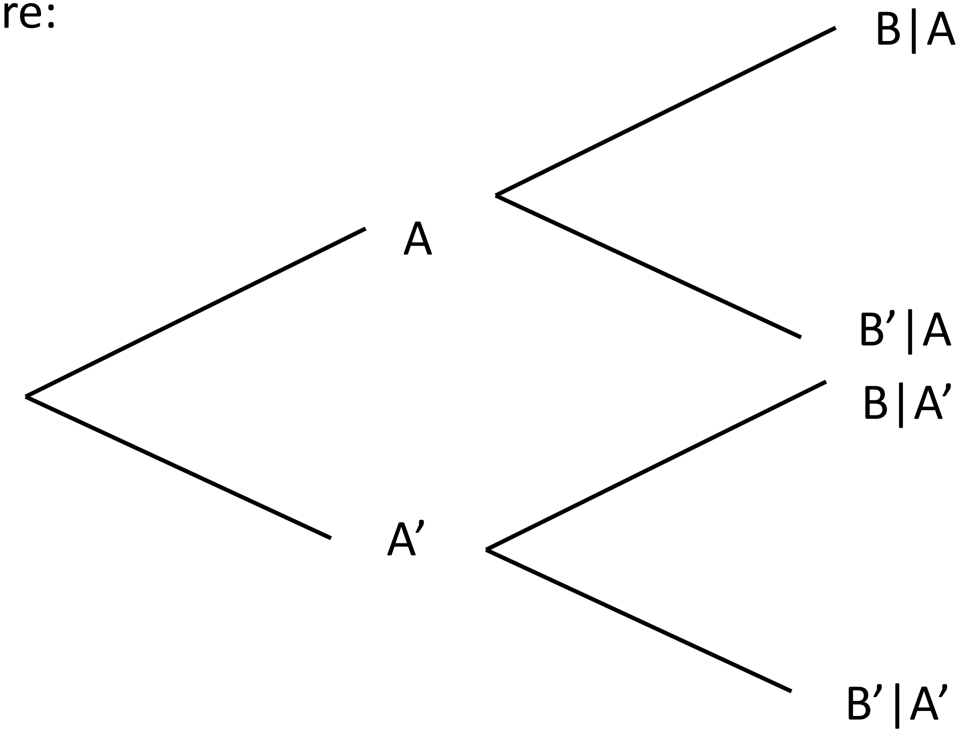
c) $P(B'|A)$

Worked Example

There are three events: A, B and C.
A and B are mutually exclusive.
A and C are independent.
$P(A) = 0.2$
$P(B) = 0.4$
$P(A \cup C) = 0.7$
Find:
a) $P(A|C)$
b) $P(A \cup B)$
c) $P(C)$

2.5 Tree Diagrams

General structure:



Notes

Worked Example

A bag contains 6 green beads and 4 yellow beads.
A bead is taken from the bag at random, the colour is recorded and it is not replaced.
A second bead is then taken from the bag and its colour recorded.
Given that both balls are the same colour, find the probability that they are both yellow.

Worked Example

There are two bags.
Bag A contains 5 red balls and 5 blue balls
Bag B contains 3 red balls and 6 blue balls.
One ball is taken from bag A and placed in bag B. Then one ball is taken from bag B.
Find the probability that:

a) A red ball is taken from bag B.
b) Given that a red ball is taken from bag B, the ball taken from bag A was also red.

Worked Example

On a randomly chosen day the probability that a person travels to school by car, bicycle or on foot is $\frac{1}{2}, \frac{1}{6}$ and $\frac{1}{3}$ respectively.

The probability of being late when using these methods of travel is $\frac{1}{5}, \frac{2}{5}$ and $\frac{1}{10}$ respectively.

Given that the person is late, find the probability that they did not travel on foot.

Worked Example

A bag contains 9 blue balls and 3 red balls.
A ball is selected at random from the bag and its colour is recorded.
The ball is not replaced.
A second ball is selected at random and its colour is recorded.
Find the probability that:

a) The second ball selected is red

b) Both balls selected are red, given that the second ball selected is red.

Worked Example

In bag A there are 5 white and 2 red counters.
In bag B there are 3 white counters and 7 red counters.
A person takes at random one counter from A and one counter from B.
Find the probability that the counters are the same colour.

Worked Example

In bag A there are 5 white and 2 red counters.
In bag B there are 3 white counters and 7 red counters.
A person takes at random one counter from A and one counter from B.
Find the probability that the counters are different colours.

Worked Example

A person plays a game of tennis and then a game of golf.
They can only win or lose each game.
The probability of winning tennis is 0.6
The probability of winning golf is 0.35
The results of each game are independent of each other.
Calculate the probability that the person wins at least one game.

Worked Example

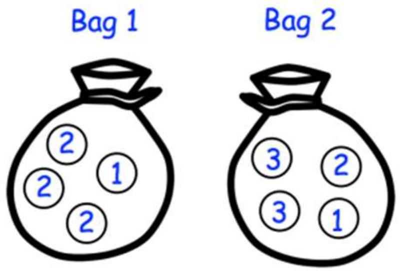
The table shows 50 students, who each study one language.
Two students are chosen at random.

	Japanese	**Spanish**
Female	13	15
Male	5	17

Calculate the probability that the two chosen students study the same language.

Worked Example

There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.
If it is labelled 1, he puts the disc in bag 2.
If it is labelled 2, he does not put the disc in bag 2.
He then chooses a disc at random from bag 2.
He then adds the numbers of the two discs he selected to give his score.
Find the probability that his score is 4.

4. Given that

$$P(A) = 0.35 \qquad P(B) = 0.45 \qquad \text{and} \qquad P(A \cap B) = 0.13$$

find

(a) $P(A' \mid B')$

(2)

(b) Explain why the events A and B are not independent.

(1)

The event C has $P(C) = 0.20$

The events A and C are mutually exclusive and the events B and C are statistically independent.

(c) Draw a Venn diagram to illustrate the events A, B and C, giving the probabilities for each region.

(5)

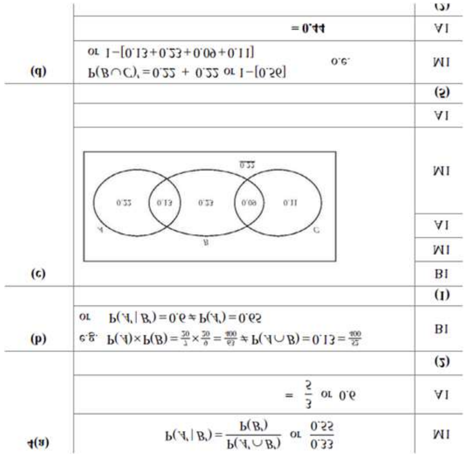
(d) Find $P([B \cup C]')$

(2)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



Summary of Key Points

1 The event A and B can be written as $A \cap B$. The '$\cap$' symbol is the symbol for **intersection**.
The event A or B can be written as $A \cup B$. The '$\cup$' symbol is the symbol for **union**.
The event not A can be written as A'. This is also called the **complement** of A.

2 The probability that B occurs given that A has already occurred is written as $P(B|A)$.
For independent events, $P(A|B) = P(A|B') = P(A)$, and $P(B|A) = P(B|A') = P(B)$.

3 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4 $P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ so $P(B \cap A) = P(B|A) \times P(A)$