KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12
Pure Mathematics
P1 9 Trigonometric Ratios
Booklet

HGS Maths

Dr Frost Course

Name: ___________________________________

Class: ___________________________________

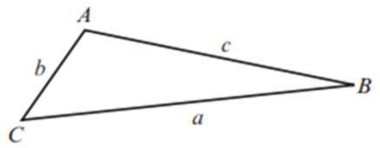
Contents

9.4 Solving Triangle Problems

These are **not** in formulae booklet:

- **This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:**

  $a^2 = b^2 + c^2 - 2bc \cos A$

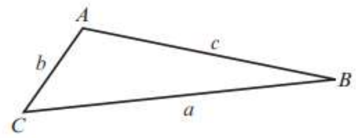


Watch out You can exchange the letters depending on which side you want to find, as long as each side has the same letter as the **opposite** angle.

The sine rule can be used to work out missing sides or angles in triangles.

- **This version of the sine rule is used to find the length of a missing side:**

  $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

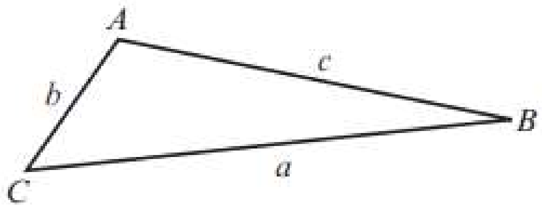


- **This version of the sine rule is used to find a missing angle:**

  $$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You can use the standard trigonometric ratios for right-angled triangles to prove the sine rule:

- **Area** $= \frac{1}{2}ab \sin C$

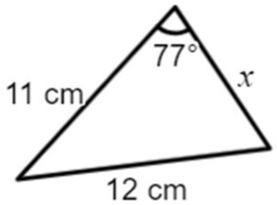


Notes

Worked Example

466e: **Use the sine rule/Law of Sines and cosine rule/Law of Cosines within a single triangle.**

Find the value of x.

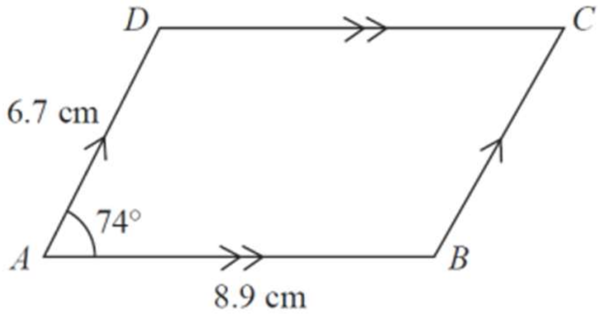


Give your answer correct to 1 decimal place.

T.189 9E: Qs 1-5, P.72 9.4: Qs 1-2

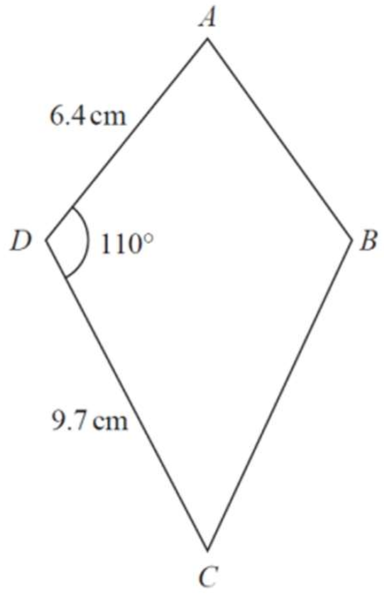
Worked Example

Calculate the area of the parallelogram.



Worked Example

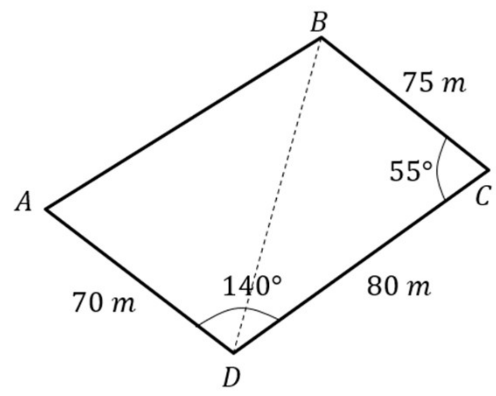
Calculate the area of the kite.



Worked Example

The diagram shows the locations of four mobile phone masts in a field.
$BC = 75\ m$, $CD = 80m$, angle $BCD = 55°$ and angle $ADC = 140°$.
In order that the masts do not interfere with each other, they must be at least 70m apart.
Given that A is the minimum distance from D, find:

a) The distance A is from B
b) The angle BAD
c) The area enclosed by the four masts.



Worked Example

T.189 9E: Qs 6+, P.72 9.4: Qs 3+

9.5 Graphs of Sine, Cosine and Tangent

Worked Example

Sketch the graph of $y = \sin x$, $-360° \leq x \leq 360°$

Worked Example

Sketch the graph of $y = \cos x$, $-360° \leq x \leq 360°$

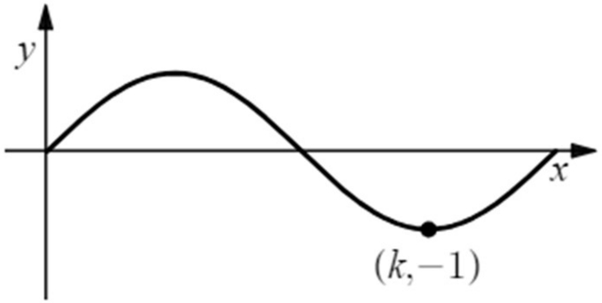
Worked Example

Sketch the graph of $y = \tan x$, $-360° \leq x \leq 360°$

Worked Example

431a: **Plot and recognise graphs of $y = \sin(x)$ and $y = \cos(x)$ in degrees.**

The diagram below shows part of the curve with equation $y = \sin x$.



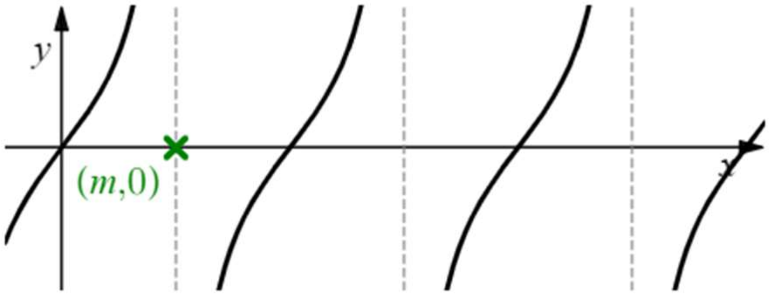
The point $(k, -1)$ is on the curve, as shown on the diagram.

Find the value of k.

Worked Example

431b: **Plot and recognise the graph $y = \tan(x)$ in degrees.**

The diagram below shows part of the curve with equation $y = \tan x$.



The point $(m, 0)$ lies on an asymptote, as shown on the diagram.

Find the value of m.

Worked Example

431c: **Appreciate the relationship between $\sin$ and $\cos$ for complementary angles.**

Given $\sin x = \cos 330$ and $0^\circ \leq x < 90^\circ$, determine the value of x.

Worked Example

a) Sketch the graph of $y = \cos\theta$ in the interval $-360° \leq \theta \leq 360°$

b) i) Sketch the graph of $y = \sin x$ in the interval $-180° \leq x \leq 270°$

ii) $\sin(-30°) = -0.5$. Use your graph to determine two further values of x for which $\sin x = -0.5$

T.194 9E: Qs all, P.73 9.5: Qs all

9.6 Transforming Trigonometric Graphs

Notes

Worked Example

Sketch $y = \sin x - 2, 0 \leq x \leq 360°$

Worked Example

Sketch $y = \cos(x + 45°)$, $0 \leq x \leq 360°$

Worked Example

Sketch $y = 4\sin x$, $0 \leq x \leq 360°$

Worked Example

Sketch $y = -\tan x$, $0 \leq x \leq 360°$

Worked Example

Sketch $y = \sin\left(\frac{x}{2}\right)$, $0 \leq x \leq 360°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = 3\sin x\,, 0 \le x \le 360°$

b) $y = -\tan\theta\,, -180° \le \theta \le 180°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = -1 + \sin x, 0 \leq x \leq 360°$

b) $y = \frac{1}{2} + \cos x, 0 \leq x \leq 360°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = \tan(\theta + 45°)\,, 0 \leq \theta \leq 360°$

b) $y = \cos(\theta - 90°), -360° \leq \theta \leq 360°$

Worked Example

Sketch on separate sets of axes the graphs of:

a) $y = \sin 2x\,, 0 \leq x \leq 360°$

b) $y = \cos\frac{\theta}{3}, -540° \leq \theta \leq 540°$

c) $y = \tan(-x)\,, -360° \leq x \leq 360°$

T.197 9G: Qs all, P.74 9.6: Qs all

4.

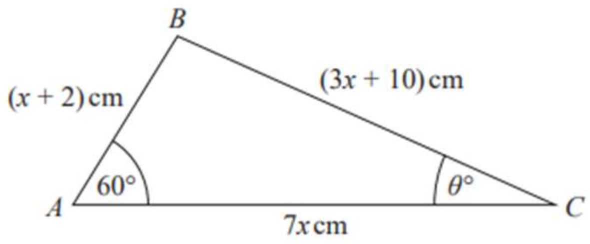


Figure 1

Figure 1 shows a sketch of triangle ABC with $AB = (x + 2)$ cm, $BC = (3x + 10)$ cm, $AC = 7x$ cm, angle $BAC = 60°$ and angle $ACB = \theta°$

(a) (i) Show that $17x^2 - 35x - 48 = 0$

(3)

(ii) Hence find the value of x.

(1)

(b) Hence find the value of θ giving your answer to one decimal place.

(2)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

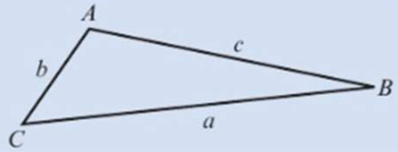
Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

4(a)(i)	$(3x+10)^2=(x+2)^2+(7x)^2-2(x+2)(7x)\cos 60°$ oe	M1	3.1a
	Uses $\cos 60° = \frac{1}{2}$, expands the brackets and proceeds to a 3 term quadratic equation	dM1	1.1b
	$17x^2-35x-48=0$ *	A1*	2.1
		(3)	
(ii)	$x=3$	B1	3.2a
		(1)	
(b)	$\frac{\sin ACB}{5}=\frac{\sin 60°}{19}\Rightarrow \sin ACB = ...\left(\frac{5\sqrt{3}}{38}\right)$ or e.g. $5^2=21^2+19^2-2\times19\times21\cos ACB\Rightarrow \cos ACB = ...\left(\frac{37}{38}\right)$	M1	1.1b
	$\theta =$ awrt 13.2	A1	1.1b
		(2)	
			(6 marks)

Summary of Key Points

1 This version of the cosine rule is used to find a missing side if you know two sides and the angle between them:

$$a^2 = b^2 + c^2 - 2bc\cos A$$



2 This version of the cosine rule is used to find an angle if you know all three sides:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

3 This version of the sine rule is used to find the length of a missing side:

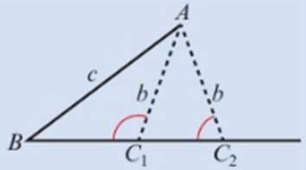
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

4 This version of the sine rule is used to find a missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

5 The sine rule sometimes produces two possible solutions for a missing angle:

$$\sin\theta = \sin(180° - \theta)$$



6 Area of a triangle = $\frac{1}{2}ab\sin C$.

7 The graphs of sine, cosine and tangent are **periodic**. They repeat themselves after a certain interval.

- The graph of $y = \sin\theta$: repeats every 360° and crosses the x-axis at …, –180°, 0, 180°, 360°, …
  has a maximum value of 1 and a minimum value of –1.
- The graph of $y = \cos\theta$: repeats every 360° and crosses the x-axis at …, –90°, 90°, 270°, 450°, …
  has a maximum value of 1 and a minimum value of –1
- The graph of $y = \tan\theta$: repeats every 180° and crosses the x-axis at … –180°, 0°, 180°, 360°, …
  has no maximum or minimum value
  has vertical asymptotes at $x = -90°$, $x = 90°$, $x = 270°$, …

T.198 mixed ex, P.76 BSG