KING EDWARD VI HANDSWORTH GRAMMAR SCHOOL FOR BOYS

KING EDWARD VI ACADEMY TRUST BIRMINGHAM

Year 12
Pure Mathematics
P1 5 Straight Line graphs
Booklet

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

5.2) Equations of straight lines
5.3) Parallel and perpendicular lines
5.4) Length and area
5.5) Modelling with straight lines

Past Paper Practice
Summary

Prior knowledge check

Prior knowledge check

1 Find the point of intersection of the following pairs of lines.

a $y = 4x + 7$ and $5y = 2x - 1$

b $y = 5x - 1$ and $3x + 7y = 11$

c $2x - 5y = -1$ and $5x - 7y = 14$

← GCSE Mathematics

2 Simplify each of the following:

a $\sqrt{80}$ **b** $\sqrt{200}$ **c** $\sqrt{125}$

← Section 1.5

3 Make y the subject of each equation:

a $6x + 3y - 15 = 0$ **b** $2x - 5y - 9 = 0$

c $3x - 7y + 12 = 0$

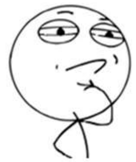
← GCSE Mathematics

5.2) Equations of straight lines

notes

Just for your interest…

Why might we want to put a straight line equation in the form $ax + by + c = 0$?

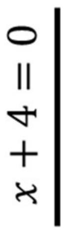


$y = mx + c$
"Slope-Intercept Form"

$ax + by + c = 0$
"Standard Form"

Coverage

$y = mx + c$ doesn't allow you to represent vertical lines. Standard form allows us to do this by just making b zero.



Symmetry

In general, the '**linear combination**' of two variables x and y is $ax + by$, i.e. "some amount of x and some amount of y". There is a greater elegance and symmetry to this form over $y = mx + c$ because x and y appear similarly within the expression.

Usefulness

This more 'elegant' form also means it ties in with vectors and matrices. In FM, you will learn about the '**dot product**' of two vectors:

$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = ax + by$$

thus since $ax + by + c = 0$, we can represent a straight line using:

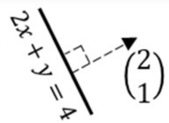
$$\begin{pmatrix} a \\ b \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} + c = 0 \qquad (1)$$

We can extend to 3D points to get the equation of a **plane**:

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} + d = 0 \qquad (2)$$

Conveniently, in equation (1), the vector $\begin{pmatrix} a \\ b \end{pmatrix}$ is **perpendicular to the line**. And in equation (2), the vector $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is perpendicular to the plane. Nice!

$$2x + y = 4 \;\Rightarrow\; \begin{pmatrix} 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = 4 \;\Rightarrow$$



Worked Example

$$2x + 3y = 6$$

Gradient:

y-intercept:

x-intercept:

Sketch:

Fill in the blank

Line	x-intercept	y-intercept
$y = 2x + 3$		
$y = 3x + 2$		
$y = 3x - 2$		
$y = 2x - 3$		
$y = 3 - 2x$		
$y = 2 - 3x$		
$2x + 3y = 6$		
$3x + 2y = 6$		
$y = ax + b$		

Worked Example

495a: Determine an equation of a straight line given the gradient and one point using $y - y_1 = m(x - x_1)$

Find an equation of the line with gradient $\frac{1}{4}$ and that passes through the point $\left(-\frac{9}{2}, 10\right)$.

Worked Example

495c: Determine an equation of a straight line, in the form $ax + by = c$, given two points using $y - y_1 = m(x - x_1)$

Determine an equation of the line that passes through the points $(4, 3)$ and $\left(5, \frac{11}{3}\right)$.

Write your answer in the form $ax + by = c$, where a, b and c are **integers**.

T.94: 5B Qs 1-3, P.38: 5.2 Qs 1-4

Worked Example

The lines $y = 2x - 7$ and $3x + 2y - 21 = 0$ intersect at the point A.
The point B has coordinates $(2, -8)$.
Find the equation of the line that passes through the points A and B.
Write your answer in the form $ax + by + c = 0$, where a, b and c are integers.

T.94: 5C Qs 3+ and 5D all Qs, P.38: 5.2 Qs 5+

5.3) Parallel and perpendicular lines

notes

Worked Example

Determine whether the pairs of lines are parallel, perpendicular or neither:

$5x - 2y - 3 = 0$

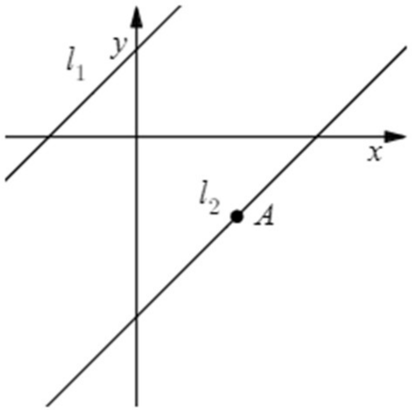
$y = \frac{5}{2}x$

$5x + 3y - 21 = 0$

$3x - 5y + 2 = 0$

495b: Determine an equation of a straight line, in the form $ax + by = c$, parallel to another using $y - y_1 = m(x - x_1)$

The line l_1 has the equation $6x - 10y + 55 = 0$.
The line l_2 is parallel to l_1 and passes through the point $A\left(\frac{21}{2}, -5\right)$ as shown in the diagram below.



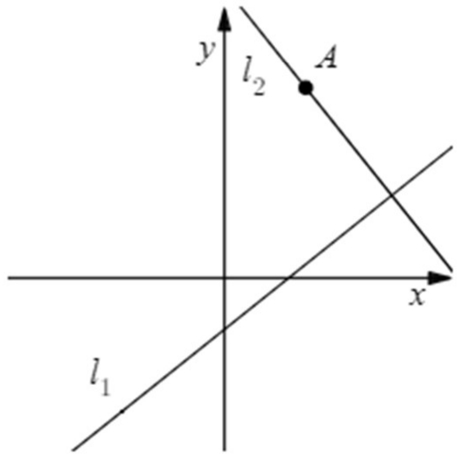
Determine the equation of l_2.
Give your answer in the form $ax + by = c$, where a, b and c are **integers**.

Worked Example

495d: Find an equation of a straight line, in the form $ax + by = c$, perpendicular to another using $y - y_1 = m(x - x_1)$

The line l_1 has the equation $8x - 10y - 25 = 0$.

The line l_2 is perpendicular to l_1 and passes through the point $A\left(4, \frac{19}{2}\right)$ as shown in the diagram below.



Find the equation of l_2.
Give your answer in the form $ax + by = c$, where a, b and c are **integers**.

T.99: 5F Qs 1-6, P.39: 5.3 Qs 1-4

Worked Example

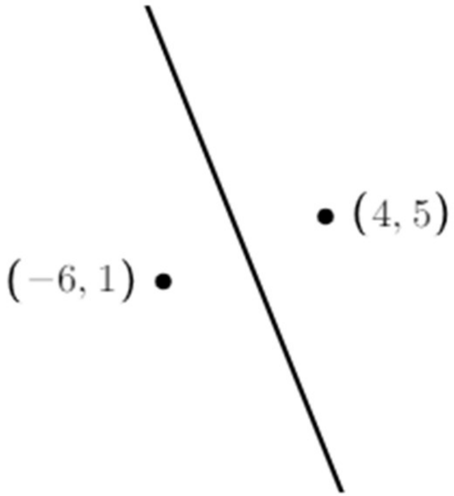
495e: Determine the perpendicular bisector of a line using $y - y_1 = m(x - x_1)$

A straight line passes through the points $R(4, 5)$ and $S(-6, 1)$.

Find the equation of the perpendicular bisector of RS.

Give your answer in the form $ax + by + c = 0$, where a, b and c are **integers.**

Simplify your answer where possible.

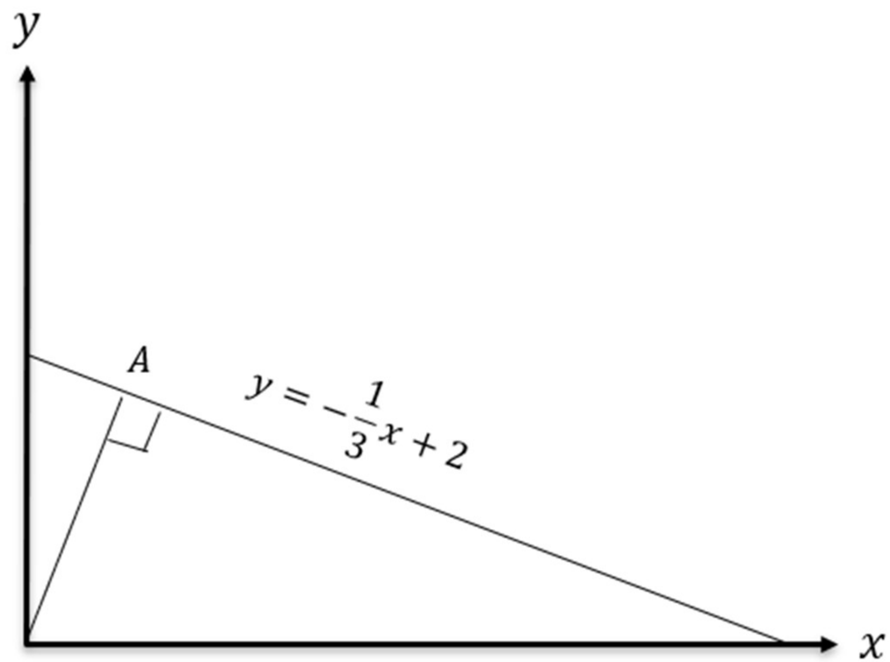


Worked Example

The points A, B and C have coordinates $(0, 12)$, $(-3, 0)$ and $(0, c)$ respectively.
The line through points A and B is perpendicular to the line through points B and C.
Find the value of c

Worked Example

Determine the coordinates of A



T.99: 5F Qs 7+, P.39: 5.3 Qs 5+

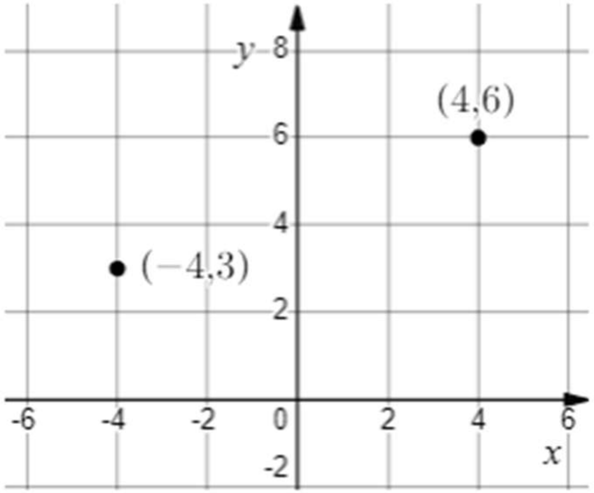
5.4) Length and area

notes

Worked Example

289a: Use Pythagoras' theorem to find the distance between two points.

The points $(-4, 3)$ and $(4, 6)$ are plotted on the coordinate grid.

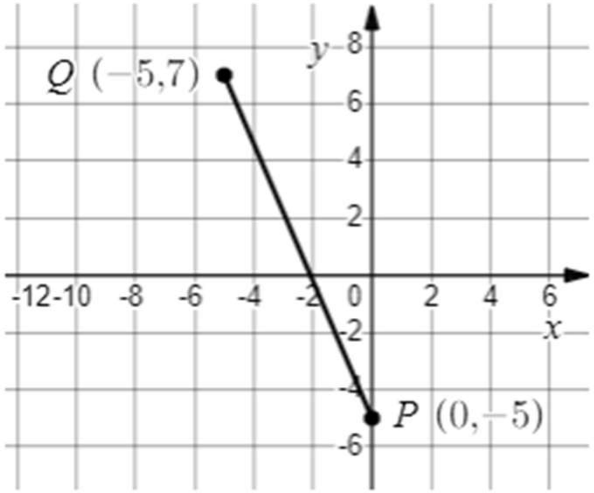


Find the distance between the two points.
Give your answer correct to 1 decimal place.

Worked Example

289b: Use Pythagoras' theorem to determine the perimeter of a rectangle given the coordinates of its vertices.

The line segment that connects $P\,(0,-5)$ and $Q\,(-5,7)$ is drawn on the coordinate grid.



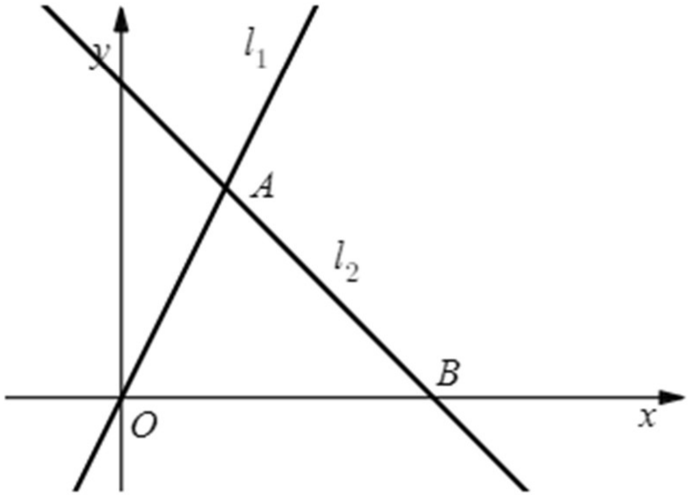
Determine the length PQ.
Give your answer correct to 1 decimal place.

Worked Example

495f: Determine the area of a triangle enclosed by an axis and two intersecting lines.

The line l_1 has equation $y = 3x$
The line l_2 has equation $2y + 3x = 8$

The lines l_1 and l_2 intersect at A.
The line l_2 intersects the x-axis at B.



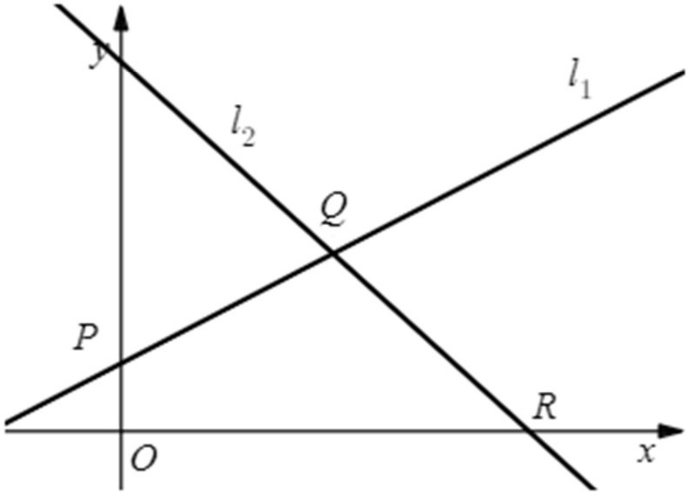
Find the exact area of triangle OAB.

Worked Example

495g: Determine the area of a quadrilateral enclosed by both axes and two intersecting lines.

The line l_1 has equation $y = 4x + 5$
The line l_2 has equation $y = -7x + 27$

The line l_1 passes through Q and intersects the y-axis at P.
The line l_2 passes through Q and intersects the x-axis at R

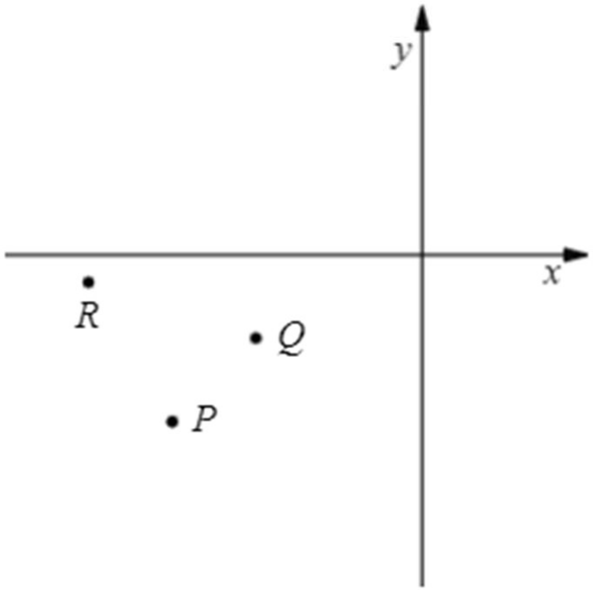


Find the exact area of the quadrilateral $OPQR$.

Worked Example

495i: Determine the area of a triangle given by 3 coordinates, given the equation of a line between two of them.

The line l_1 passes through the points $P(-9, -6)$ and $Q(-6, -3)$.
The line l_2 passes through the point $R(-12, -1)$ and is perpendicular to l_1.
The lines l_1 and l_2 intersect at the point S.



By first using the equations of l_1 and l_2 to find the coordinates of S, work out the area of the triangle PQR.

T.102: 5G Qs all, P.40: 5.4 Qs all

5.5) Modelling with straight lines

notes

Worked Example

495h: Determine the equation of a line for a given context using $y - y_1 = m(x - x_1)$

The distance that a car can travel in a journey starting with a full tank of fuel was investigated.

From a full tank of fuel, 105.8 litres of fuel were consummed after the car had travelled 80 km
From a full tank of fuel, 304.6 litres of fuel were consummed after the car had travelled 360 km

Using a linear model, with V litres being the volume of fuel consummed and d km being the distance the car had travelled, find an equation linking V with d.

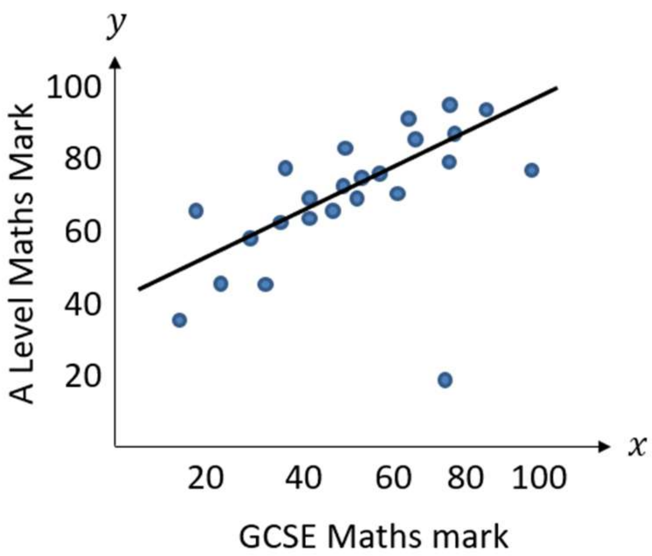
Worked Example

The A Level Maths mark, y %, and GCSE Maths mark, x %, is recorded for several students.
Assume the line goes through $(0, 40)$ and $(60, 80)$.

a) Write a linear model

b) Interpret the gradient and y-intercept in this context

c) Predict the A Level Maths mark of a student who got 100% for their GCSE Maths mark



Worked Example

In 2010 the population of rabbits in an area was 200. Locals projected that the number of rabbits would increase by 4 per year.

a) Write a linear model for the population, p, of rabbits t years after 2010

b) Write down a reason why this might not be a realistic model.

T.106: 5H Qs all, P.42: 5.5 Qs all

Extract from Formulae book

8.

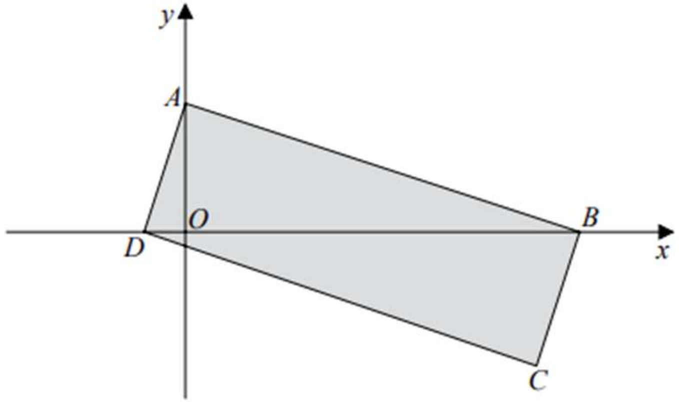


Figure 1

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation $5y + 2x = 10$

(a) show that the straight line through the points *A* and *D* has equation $2y - 5x = 4$

(4)

(b) find the area of the rectangle *ABCD*.

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

8 (a)	Gradient $AB = -\frac{2}{5}$	B1	2.1
	y coordinate of A is 2	B1	2.1
	Uses perpendicular gradients $y = +\frac{5}{2}x + c$	M1	2.2a
	$\Rightarrow 2y - 5x = 4$ *	A1*	1.1b
		(4)	
(b)	Uses Pythagoras' theorem to find AB or AD Either $\sqrt{5^2 + 2^2}$ or $\sqrt{\left(\frac{4}{5}\right)^2 + 2^2}$	M1	3.1a
	Uses area $ABCD = AD \times AB = \sqrt{29} \times \sqrt{\frac{116}{25}}$	M1	1.1b
	area $ABCD = 11.6$	A1	1.1b
		(3)	

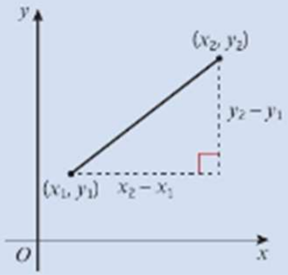
(7 marks)

Summary of Key Points

Summary of key points

1 The gradient m of the line joining the point with coordinates (x_1, y_1) to the point with coordinates (x_2, y_2) can be calculated using the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



2 • The equation of a straight line can be written in the form

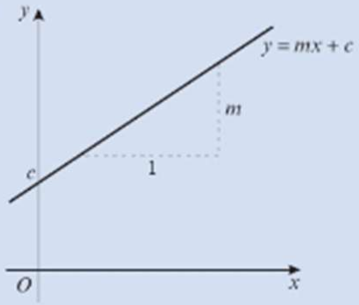
$$y = mx + c,$$

where m is the gradient and $(0, c)$ is the y-intercept.

• The equation of a straight line can also be written in the form

$$ax + by + c = 0,$$

where a, b and c are integers.



3 The equation of a line with gradient m that passes through the point with coordinates (x_1, y_1) can be written as $y - y_1 = m(x - x_1)$.

4 Parallel lines have the same gradient.

5 If a line has a gradient m, a line perpendicular to it has a gradient of $-\frac{1}{m}$

6 If two lines are perpendicular, the product of their gradients is -1.

7 You can find the distance d between (x_1, y_1) and (x_2, y_2) by using the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

8 The point of intersection of two lines can be found using simultaneous equations.

9 Two quantities are in direct proportion when they increase at the same rate. The graph of these quantities is a straight line through the origin.

10 A mathematical model is an attempt to represent a real-life situation using mathematical concepts. It is often necessary to make assumptions about the real-life problems in order to create a model.

T.108: mixed ex. , P.44: BSG