KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
13 Integration

HGS Maths

Dr Frost Course

Name: _____

Class: _____

Contents

Prior knowledge check

1 Simplify these expressions

a $\frac{x^3}{\sqrt{x}}$ **b** $\frac{\sqrt{x} \times 2x^3}{x^2}$

c $\frac{x^3 - x}{\sqrt{x}}$ **d** $\frac{\sqrt{x} + 4x^3}{x^2}$

← Sections 1.1, 1.4

2 Find $\frac{dy}{dx}$ when y equals

a $2x^3 + 3x - 5$ **b** $\frac{1}{2}x^2 - x$

c $x^2(x + 1)$ **d** $\frac{x - x^5}{x^2}$ ← Section 12.5

3 Sketch the curves with the following equations:

a $y = (x + 1)(x - 3)$

b $y = (x + 1)^2(x + 5)$ ← Chapter 4

13.1) Integrating x^n

Notes

Worked Example

Find y, given that $\frac{dy}{dx} =$

$a)\ 3x^2$ b) $-2x^3$ c) $5x^4$

Worked Example

Find $f(x)$, given that $f'(x) =$

a) $\sqrt{x}$

b) $\sqrt[3]{x}$

c) $\sqrt[4]{x}$

Worked Example

Find y, given that $\frac{dy}{dx} =$

a) $\frac{1}{x^2}$

b) $\frac{2}{x^3}$

c) $\frac{3}{4x^2}$

d) $\frac{6}{5x^3}$

Worked Example

Find y, given that $\frac{dy}{dx} =$

a) $\frac{2}{3}\sqrt{x}$

b) $\frac{4}{7}\sqrt[3]{x}$

c) $\frac{5}{6}\sqrt[4]{x}$

d) $\frac{2}{3\sqrt{x}}$

e) $\frac{4}{7\sqrt[3]{x}}$

f) $\frac{5}{6\sqrt[4]{x}}$

Worked Example

Find y, given that $\frac{dy}{dx} =$

a) $\sqrt{36x^7}$

b) $\sqrt{25}x^7$

c) $2x^{-\frac{7}{10}}$

d) $39x^{\frac{5}{8}}$

e) $(3x-2)^2$

T.289: Ex 13A all Qs, P.104: all Qs

13.2) Indefinite integrals

Notes

Worked Example

521a: Integrate a collection of terms in the form ax^b

Find $\int \left(x^7 + \frac{3}{5}x^4 + x + 4x^{-7}\right)dx$

$I =$ $+c$

Worked Example

521b: **Integrate a collection of terms in the form ax^b requiring rewriting of roots and reciprocals.**

Find $\int \left(-3\sqrt[3]{x} + \frac{4}{x^3}\right) dx$

T.291: Ex 13B Qs1-6, P.105: Qs 1,2

Worked Example

521d: **Integrate an expression written as a fraction with a single term on the denominator.**

Find

$$\int \left(\frac{5x^{\frac{1}{2}} - 3x^{7}}{4x} \right) dx$$

Worked Example

521c: Integrate an expression requiring bracket expansion.

Find $\int \left(3x^4 + x^2\right)\left(2x^5 + 4x\right)dx$

T.291: Ex 13B Qs7-9, P.105: Q8

Worked Example

$$\int \left(\frac{p}{2x^2} + pq\right)\, dx = \frac{2}{x} + 12 + c$$

Find the value of p and the value of q

T.291: Ex 13B Qs10-15, P.105: Qs5-7

13.3) Finding functions

Notes

Worked Example

522a: Determine a function by integrating $\frac{dy}{dx}$

Find the equation of the curve given that $\frac{dy}{dx} = 5x^3 + 9x^2 + 5x$ and that the curve passes through the point $(3, 231)$

T.291: Ex 13C Qs all, P.105: Qs all

13.4) Definite integrals

Notes

Worked Example

524a: **Evaluate a definite integral where the integrand is a collection of terms in the form ax^b**

Evaluate

$$\int_1^5 \left(-x^3 + 6x^2 + 4x\right) dx$$

T.297: Ex 13D Qs 1-3, P.106: Qs 1-5

Worked Example

Given that P is a constant and

$$\int_3^7 (4Px + 7)\, dx = 108P^2$$

find the possible values of P

Worked Example

Given that $\int_1^k \frac{1}{\sqrt[4]{x}}\,dx = \frac{28}{3}$,
calculate the value of k

T.297: Ex 13D Qs 4+, P.106: Qs 6+

13.5) Areas under curves

Notes

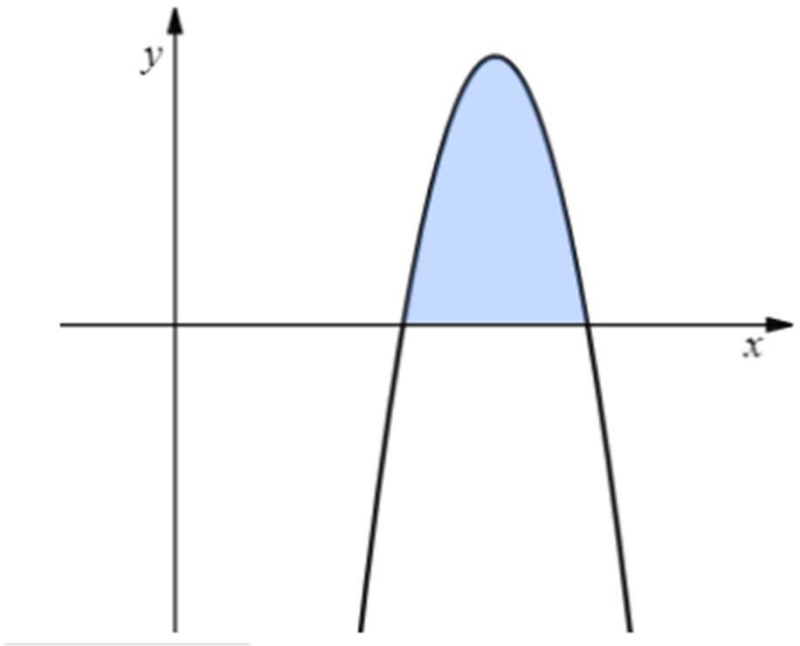
Worked Example

524e: Determine the shaded area enclosed between a curve and the x-axis where the limits are roots.

The diagram below shows the graph of

$$f(x) = -x^2 + 14x - 45.$$

Find the exact area of the shaded region.



T.299: Ex 13E Qs 1,2, P.107: Qs 1-3

Worked Example

Find the area of the finite region bounded by the curve with equation $y = x^2(x + 2)$ and the x-axis

T.299: Ex 13E Qs 3+, P.107: Qs 4+

13.6) Areas under the x-axis

Notes

Worked Example

Find the area of the finite region bounded by the curve with equation $y = x(x - 5)$ and the x-axis

Worked Example

Find the total area bound between the curve $y = x(x-2)(x-4)$ and the x-axis.

T.301: Ex 13F Q 1, P.108: Qs 1,2

Worked Example

Find the total area bound between the curve $y = x^3 + 2x^2 - 15x$ and the x-axis.

T.301: Ex 13F Qs 2,3 , P.109: Qs 3+

13.7) Areas between curves and lines

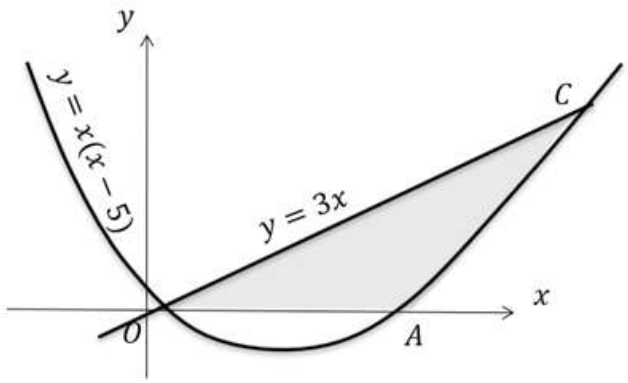
Notes

Worked Example

Determine the area bounded by the curve with equation $y = x(7 - x)$ and the line with equation $y = 2x$

Worked Example

The diagram shows a sketch of the curve with equation $y = x(x - 5)$ and the line with equation $y = 3x$.
Find the area of the shaded region OAC.



T.304: Ex 13G Qs 1-3 , P.110: Qs 1,2

Worked Example

Determine the area bounded by the curve with equation $y = 5x - x^2 - 3$ and the line with equation $y = 5 - x$

T.304: Ex 13G Qs 4+ , P.110: Qs 3+

Past Paper Questions

AS 2019

Integration

3. (a) Given that k is a constant, find

$$\int \left(\frac{4}{x^3} + kx \right) \mathrm{d}x$$

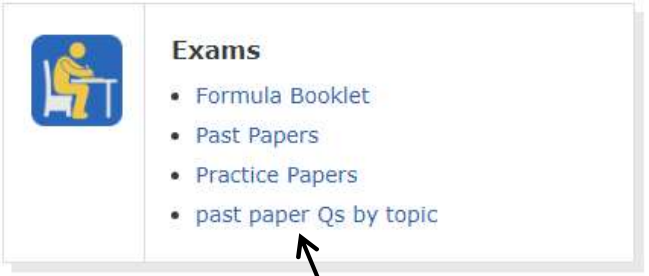
simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2} \left(\frac{4}{x^3} + kx \right) \mathrm{d}x = 8$$

(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$\int 4x^{-3} + kx \ \mathrm{d}x = -2x^{-2} + \frac{1}{2}kx^2$ [No Tilde]	M1	This mark is given for recognising that x^n becomes x^{n+1} when integrating
		A1	This mark is given for two correctly integrated terms (without c)
	$-\frac{2}{x^2} + \frac{kx^2}{2} + c$	A1	This mark is given for a full answer with a constant (in any correct form)
(b)	$\left[-\frac{2}{x^2} + \frac{k}{2}x^2 \right]_{0.5}^{2} =$	M1	This mark is given for substituting the limits 2 and 0.5 and setting equal to 8
	$\left(-\frac{2}{2^2} + \frac{4k}{2} \right) - \left(-\frac{2}{0.5^2} + \frac{0.5^2 k}{2} \right) = 8$		
	$\left(-\frac{1}{2} + 2k \right) - \left(-8 + \frac{k}{8} \right) = 8$	M1	This mark is given for a method to solve a linear equation in k
	$7.5 + \frac{15}{8}k = 8$		
	$k = \frac{4}{15}$	A1	This mark is given for finding a correct value for k

Summary of Key Points

Summary of key points

1 If $\frac{dy}{dx} = x^n$, then $y = \frac{1}{n+1}x^{n+1} + c, n \neq -1$.

Using function notation, if $f'(x) = x^n$, then $f(x) = \frac{1}{n+1}x^{n+1} + c, n \neq -1$.

2 If $\frac{dy}{dx} = kx^n$, then $y = \frac{k}{n+1}x^{n+1} + c, n \neq -1$.

Using function notation, if $f'(x) = kx^n$, then $f(x) = \frac{k}{n+1}x^{n+1} + c, n \neq -1$.

When integrating polynomials, apply the rule of integration separately to each term.

3 $\int f'(x)dx = f(x) + c$

4 $\int(f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$

5 To find the constant of integration, c

- Integrate the function
- Substitute the values (x, y) of a point on the curve, or the value of the function at a given point $f(x) = k$ into the integrated function
- Solve the equation to find c

6 If $f'(x)$ is the derivative of $f(x)$ for all values of x in the interval $[a, b]$, then the definite integral is defined as $\int_a^b f'(x)dx = [f(x)]_a^b = f(b) - f(a)$

7 The area between a positive curve, the x-axis and the lines $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b y\,dx$$

where $y = f(x)$ is the equation of the curve.

8 When the area bounded by a curve and the x-axis is below the x-axis, $\int y\,dx$ gives a negative answer.

9 You can use definite integration together with areas of trapeziums and triangles to find more complicated areas on graphs.