KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 14 Exponentials and Logarithms – Part 1
Booklet

HGS Maths

Dr Frost Course

Name: _____

Class: _____

Contents

Prior knowledge check

Prior knowledge check

1 Given that $x = 3$ and $y = -1$, evaluate these expressions without a calculator.

a 5^x **b** 3^y **c** 2^{2x-1} **d** 7^{1-y} **e** 11^{x+3y}

← GCSE Mathematics

2 Simplify these expressions, writing each answer as a single power.

a $6^8 \div 6^2$ **b** $y^3 \times (y^9)^2$ **c** $\dfrac{2^5 \times 2^9}{2^8}$ **d** $\sqrt{x^8}$

← Sections 1.1, 1.4

3 Plot the following data on a scatter graph and draw a line of best fit.

x	1.2	2.1	3.5	4	5.8
y	5.8	7.4	9.4	10.3	12.8

Determine the gradient and intercept of your line of best fit, giving your answers to one decimal place.

← GCSE Mathematics

14.4 Logarithms

Notes

Worked Example (DFM 527b)

Write each statement as a logarithm.

a) $3^2 = 9$

b) $2^7 = 128$

c) $64^{\frac{1}{2}} = 8$

Worked Example

Rewrite each statement using a power.

a) $\log_3 81 = 4$

b) $\log_2 \left(\frac{1}{8}\right) = -3$

Worked Example

Without using a calculator, find the value of:

a) $\log_3 81$

b) $\log_4 0.25$

c) $\log_{0.5} 4$

d) $\log_a(a^5)$

Worked Example

Without using a calculator, find the value of:

a) $\log_5 5$

b) $\ln e^2$

c) $\log 1000$

T.320 4D: Qs 1-4, P.116. 4.4: Qs 1,2

Worked Example

Use your calculator to find the following logarithms to 3 decimal places.

a) $\log_3 40$

b) $\ln 8$

c) $\log 75$

T.320 4D: Qs 5+, P.116. 4.4: Qs 3+

14.5 Laws of Logarithms

Notes

Worked Example (DFM 527f)

Write as a single logarithm.

a) $\log_3 6 + \log_3 7$

b) $\log_2 15 - \log_2 3$

c) $2\log_5 3 + 3\log_5 2$

d) $\log_{10} 3 - 4\log_{10}\left(\frac{1}{2}\right)$

T.323 4E: Qs 1-2, P.11. 4.5: Qs 1-2

Worked Example

Write in terms of $\log_a x$, $\log_a y$ and $\log_a z$.

a) $\log_a(x^2yz^3)$

b) $\log_a\left(\frac{x}{y^3}\right)$

c) $\log_a\left(\frac{x\sqrt{y}}{z}\right)$

d) $\log_a\left(\frac{x}{a^4}\right)$

T.323 4E: Qs 3, P.11. 4.5: Qs 3

Worked Example

527c: **Solve logarithmic equations given in the form $\log_a x = b$**

Find the exact solution of

$$2\log_4(9y+8)+3=9$$

$y =$

Worked Example

Solve the equation $\log_{10} 4 + 2 \log_{10} x = 2$.

Worked Example

Solve the equation $\log_3(x + 11) - \log_3(x - 5) = 2$

T.323 4E: Qs 4+, P.11. 4.5: Qs 4+

Worked Example

527h: Solve logarithmic equations given in the form $\log[f(x)] = \log[g(x)]$

Find the exact solution of

$$\log_2(3y-1) = \log_2(y+5) - \log_2(2y-5)$$

Worked Example

527i: Solve equations using logarithm product and quotient laws (excluding power law).

Solve

$$\log_4(x+3)+\log_4(2x+20)=2$$

Worked Example

527j: Solve logarithmic equations by using the power law.

Solve

$$2\log_2(x+7) - \log_2(14x+134) = -1$$

Worked Example

527g: Use laws of logs to write a logarithm as an expression by substitution.

Given that $x = \log 2$ and $y = \log 5$, write

$$\log 20$$

in terms of x and y.

N.B.: Qs Not in textbooks

14.6 Solving Equations using Logarithms

Notes

Worked Example

Solve the following equations, giving your answers to 3 decimal places.

a) $3^x = 20$

b) $5^{4x-1} = 61$

T.325 4F: Qs 1, P.118 4.6: Qs 1-2

Worked Example

Solve the equation $5^{2x} - 12(5^x) + 20 = 0$, giving your answer to 3 significant figures.

T.325 4F: Qs 2, P.118 4.6: Qs 3-4

Worked Example

Find the solution to the equation $3^x = 2^{x+1}$, giving your answer to four decimal places.

Worked Example

Solve the equation $3^{x+1} = 4^{x-1}$. Round your answer to 3 decimal places.

Worked Example

Solve the equation $2^x 3^{x+1} = 5$. Give your answer in exact form.

T.325 4F: Qs 3+, P.118 4.6: Qs 5+

Logarithms and exponentials

$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$e^{x \ln a} = a^x$$

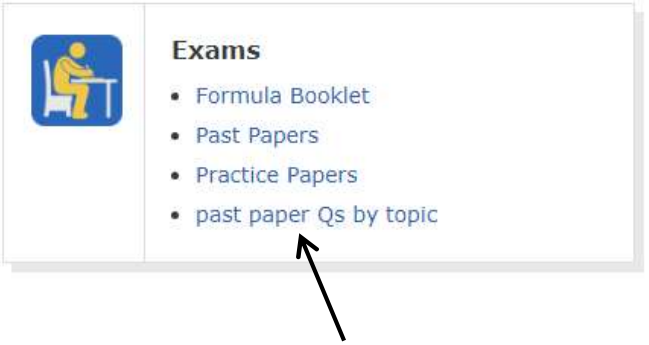
Past Paper Questions

A2 2021 Paper 2 — Exponentials and Logs

3. Using the laws of logarithms, solve the equation

$$\log_3 (12y + 5) - \log_3 (1 - 3y) = 2$$

(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Question	Scheme	Marks	AOs
3	$\log_3(12y+5)-\log_3(1-3y)=2 \Rightarrow \log_3\frac{12y+5}{1-3y}=2$ or c.e. $2=\log_3 9$	B1 M1 no EPEN	1.1b
	$\log_3\frac{12y+5}{1-3y}=2 \Rightarrow \frac{12y+5}{1-3y}=3^2 \Rightarrow 9-27y=12y+5 \Rightarrow y=\ldots$ or c.e. $\log_3(12y+5)=\log_3(3^2(1-3y)) \Rightarrow (12y+5)=3^2(1-3y) \Rightarrow y=\ldots$	M1 [no term]	2.1
	$y=\frac{4}{27}$	A1	1.1b
		(3)	

(3 marks)

3 $\log_a n = x$ is equivalent to $a^x = n$ $\quad (a \neq 1)$

4 The laws of logarithms:

- $\log_a x + \log_a y = \log_a xy$ (the multiplication law)
- $\log_a x - \log_a y = \log_a \left(\frac{x}{y}\right)$ (the division law)
- $\log_a (x^k) = k \log_a x$ (the power law)

5 You should also learn to recognise the following special cases:

- $\log_a \left(\frac{1}{x}\right) = \log_a (x^{-1}) = -\log_a x$ (the power law when $k = -1$)
- $\log_a a = 1$ $\quad (a > 0, a \neq 1)$
- $\log_a 1 = 0$ $\quad (a > 0, a \neq 1)$

6 Whenever $f(x) = g(x)$, $\log_a f(x) = \log_a g(x)$