KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 3 Equations and Inequalities Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

Class: ______________________________

Contents

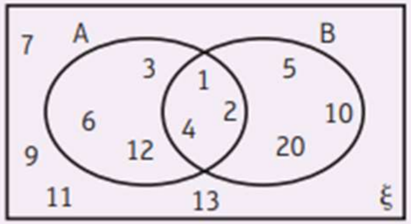
Prior knowledge check

Prior knowledge check

1 A = {factors of 12}

B = {factors of 20}

Write down the numbers in each of these sets:



a $A \cap B$ **b** $(A \cup B)'$

← GCSE Mathematics

2 Simplify these expressions.

a $\sqrt{75}$ **b** $\dfrac{2\sqrt{45} + 3\sqrt{32}}{6}$

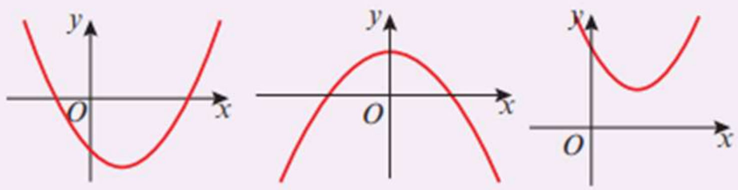
← Section 1.5

3 Match the equations to the correct graph. Label the points of intersection with the axes and the coordinates of the turning point.

a $y = 9 - x^2$ **b** $y = (x - 2)^2 + 4$

c $y = (x - 7)(2x + 5)$

i **ii** **iii**



← Section 2.4

3.2) Quadratic simultaneous equations

The solution(s) to an equation may be:

A single value: $2x + 1 = 5$

Multiple values: $x^2 + 3x + 2 = 0$

An infinitely large set of values: $x > 3$

No (real) values! $x^2 = -1$

Every value! $x^2 + x = x(x + 1)$

The point is that you shouldn't think of the solution to an equation/inequality as an 'answer', but a **<u>set</u>** of values, which might just be a set of 1 value (known as a singleton set), a set of no values (i.e. the empty set $\emptyset$), or an infinite set (in the last example above, this was $\mathbb{R}$)

✎ The solutions to an equation are known as the **solution set**.

Solutions sets

For simultaneous equations, the same is true, except each 'solution' in the solution set is an assignment to **multiple** variables.
All equations have to be satisfied **at the same time, i.e. 'simultaneously'**.

Scenario	Example	Solution Set
A single solution:	$x+y=9$ $x-y=1$	**Solution 1:** $\boldsymbol{x=5, y=4}$ To be precise here, the solution set is of size 1, but this solution is an assignment to multiple variables, i.e. a pair of values.
Two solutions:	$x^2+y^2=10$ $x+y=4$	**Solution 1:** $\boldsymbol{x=3, y=1}$ **Solution 2:** $\boldsymbol{x=1, y=3}$ This time we have two solutions, each an x, y pair.
No solutions:	$x+y=1$ $x+y=3$	The solution set is empty, i.e. ∅, as both equation can't be satisfied at the same time.
Infinitely large set of solutions:	$x+y=1$ $2x+2y=2$	**Solution 1:** $\boldsymbol{x=0, y=1}$ **Solution 2:** $\boldsymbol{x=1, y=0}$ **Solution 3:** $\boldsymbol{x=2, y=-1}$ **…** Infinite possibilities!

Notes

Worked Example

419a: Solve non-linear simultaneous equations/systems of equations where y is the subject of both equations, to give a quadratic equation to be solved.

Solve the following simultaneous equations.

$$\begin{cases} y = x^2 - 2x - 31 \\ y = x - 3 \end{cases}$$

Worked Example

419c: Solve non-linear simultaneous equations/systems of equations with one equation given in the form $x^2 + y^2 = a$ and the other where x or y is the subject.

Solve the following simultaneous equations.

$$\begin{cases} y = 3x + 2 \\ x^2 + y^2 = 2 \end{cases}$$

Worked Example

419d: Solve simultaneous equations/systems of equations given in the form $ax + by = c$ and $x^2 + y^2 = d$

Solve the following simultaneous equations.

$$\begin{cases} x + 3y = 4 \\ x^2 + y^2 = 50 \end{cases}$$

Worked Example

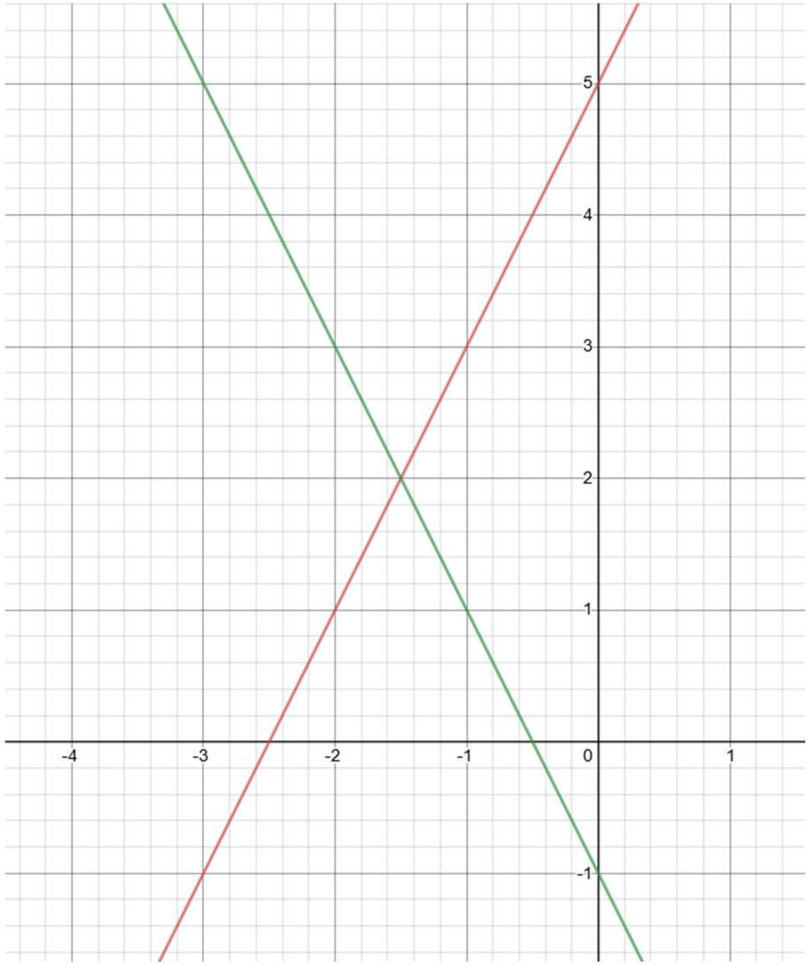
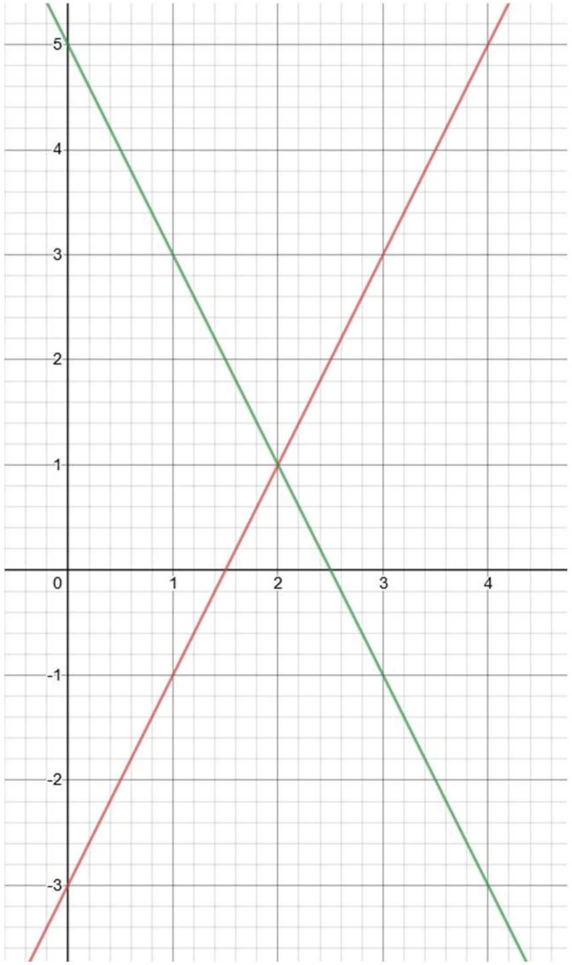
Solve:

$$xy = 12$$
$$x = y - 2$$

T.41 : 3B Qs all, P.17: 3.2 all Qs

3.3) Simultaneous equations on graphs

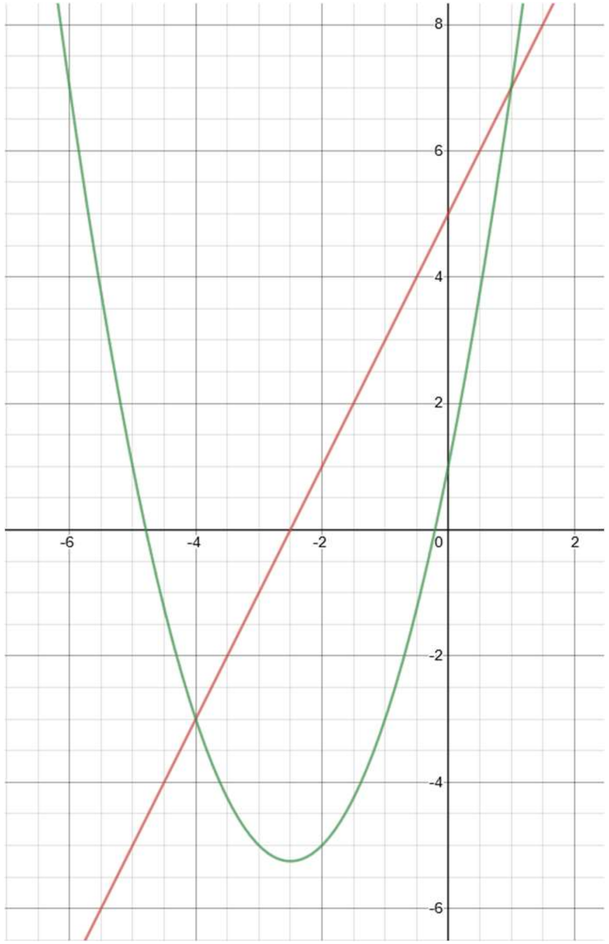
Notes

Worked example	Your turn
Solve: $$y = 2x + 5$$ $$y = -2x - 1$$ 	Solve: $$y = 2x - 3$$ $$y = -2x + 5$$ 

Worked example

Solve:

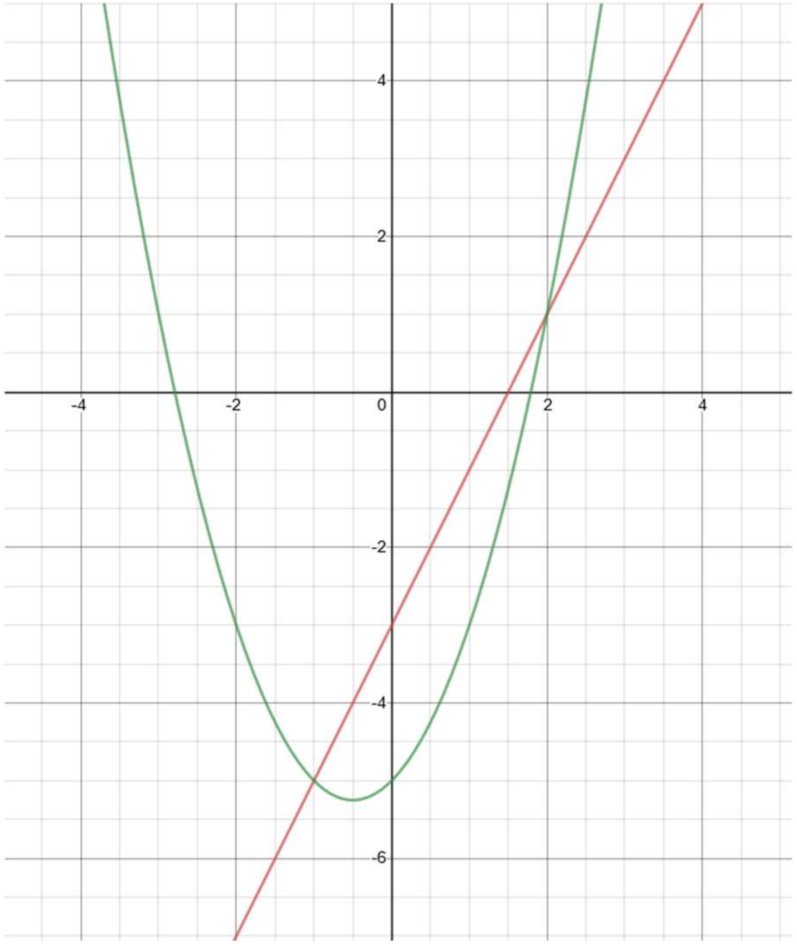
$$y = 2x + 5$$
$$y = x^2 + 5x + 1$$



Your turn

Solve:

$$y = 2x - 3$$
$$y = x^2 + x - 5$$



T.45 : 3C Qs 1-4, P.18: 3.3 all Qs 1-4

Worked Example

By using the discriminant of a subsequent equation, show that the graphs of $4x + y = 3$ and $y = x^2 - 3x + 1$ have two points of intersection

Worked Example

Prove algebraically, and show graphically, that the lines never meet:

$$y = 3x - 3$$
$$y = x^2 + 5x + 4$$

T.45 : 3C Qs 5-7, P.18: 3.3 all Qs 5-6

Worked Example

The line with equation $y = 3x + 4$ meets the curve with equation $kx^2 + 2y + (k - 8) = 0$ at exactly one point. Given that k is a positive constant:

a) Find the value of k.

b) For this value of k, find the coordinates of this point of intersection.

T.45 : 3C Qs 8+, P.18: 3.3 all Qs 7+

3.4) Linear inequalities

Recall that a **set** is a **collection of values** such that:

a) The **order of values does not matter**.

b) There are **no duplicates**.

Recap from GCSE:

- We use curly braces to list the values in a set, e.g. $A = \{1,4,6,7\}$
- If A and B are sets then $A \cap B$ is the **intersection** of A and B, giving a set which has the elements in A **and** B.
- $A \cup B$ is the **union** of A and B, giving a set which has the elements in A **or** in B.
- $\emptyset$ is the empty set, i.e. the set with nothing in it.
- Sets can also be infinitely large. $\mathbb{N}$ is the set of natural numbers (all positive integers), $\mathbb{Z}$ is the set of all integers (including negative numbers and 0) and $\mathbb{R}$ is the set of all real numbers (including all possible decimals).
- We write $x \in A$ to mean "x is a member of the set A". So $x \in \mathbb{R}$ would mean "x is a real number".

$$\{1,2,3\} \cap \{3,4,5\} = \{\mathbf{3}\}$$
$$\{1,2,3\} \cup \{3,4,5\} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}\}$$
$$\{1,2\} \cap \{3,4\} = \emptyset$$

Set Builder Notation

It is possible to construct sets without having to explicitly list its values. We use:

The | or : means "such that".

$$\begin{aligned} &\{expr \mid\ condition\} \\ \text{or}\quad &\{expr:\ condition\} \end{aligned}$$

Can you guess what sets the following give?

(In words "All numbers $2x$ such that x is an integer)

$$\{2x : x \in \mathbb{Z}\} = \{0, 2, -2, 4, -4, 6, -6, \ldots\}$$

i.e. The set of all even numbers!

$$\{2^x : x \in \mathbb{N}\} = \{2, 4, 8, 16, 32, \ldots\}$$

$$\{xy:\ x, y\ are\ prime\} = \{4, 6, 10, 14, 15, \ldots\}$$

i.e. All possible products of two primes.

We previously talked about 'solutions sets', so set builder notation is very useful for specifying the set of solutions!

Examples

All odd numbers.	$\{2x + 1 \ : \ x \in \mathbb{Z}\}$
All (real) numbers greater than 5.	$\{x: \ x > 5\}$ Technically it should be $\{x: \ x > 5, x \in \mathbb{R}\}$ but the $x > 5$ by default implies **real numbers** greater than 5.
All (real) numbers less than 5 **or** greater than 7.	$\{x: \ x < 5\} \cup \{x: x > 7\}$ We combine the two sets together.
All (real) numbers between 5 and 7 inclusive.	$\{x: \ 5 \leq x \leq 7\}$ While we could technically write $\{x: x \geq 5\} \cap \{x: x \leq 7\}$, we tend to write multiple required conditions within the same set.

Notes

Worked Example

If $x < 3$ and $2 \leq x < 4$, what is the combined solution set?

Worked Example

Use set notation to describe the set of values for which:

$$10(9x+8) < 7 \text{ or } 6(5x-4) \geq \frac{3-2x}{4}$$

T.48: 3D Q3 , P.19: 3.4 Q5-8

3.5) Quadratic inequalities

Notes

Worked Example

458a: Solve quadratic inequalities given in the form $x^2 + bx + c > 0$ or $x^2 + bx + c < 0$

Solve:

$$x^2 + 4x \geq 0$$

Worked Example

458d: Solve inequalities requiring rearrangement to the form $x^2 + bx + c > 0$ or $x^2 + bx + c < 0$

Solve the inequality

$$z(z-3) < 2(z-2)$$

Worked Example

458g: Solve quadratic inequalities requiring rearrangement to the form $ax^2 + bx + c > 0$ or $ax^2 + bx + c < 0$

Solve the inequality

$$m(2m - 5) + 3 < 0$$

Worked Example

458j: Solve quadratic inequalities given in the form $ax^2 > b$ or $ax^2 < b$

Solve the inequality

$$5z^2 \geq 20$$

Worked Example

458k: Solve quadratic inequalities given in the form $-x^2 + bx + c > 0$ or $-x^2 + bx + c < 0$

Solve:

$$5k - 6 - k^2 \leq 0$$

T.50: 3E Qs1-3 , P.20: 3.5 Qs 1-2

Worked Example

Find the set of values for which $\frac{10}{x} > 5, x \neq 0$

Worked Example

Find the set of values for which $\frac{5}{x-3} < 2$

T.50: 3E Q 4 , P.20: 3.5 Q3

Worked Example

458m: Solve quadratic inequalities involving a division by a positive algebraic expression.

Solve

$$1 < \frac{7x}{2x^2+3}$$

Worked Example

4580: Combine a linear and a quadratic inequality.

n is an integer such that $7n - 4 \geq 24$ and $\dfrac{5n}{n^2+4} \geq 1$

Find all the possible values of n.

Worked Example

The equation $kx^2 - 5kx + 50 = 0$, where k is a constant, has no real roots.
Prove that k satisfies the inequality $0 \leq k < 8$

T.50: 3E Qs4+, P.20: 3.5 Qs 4+

3.6) Inequalities on graphs

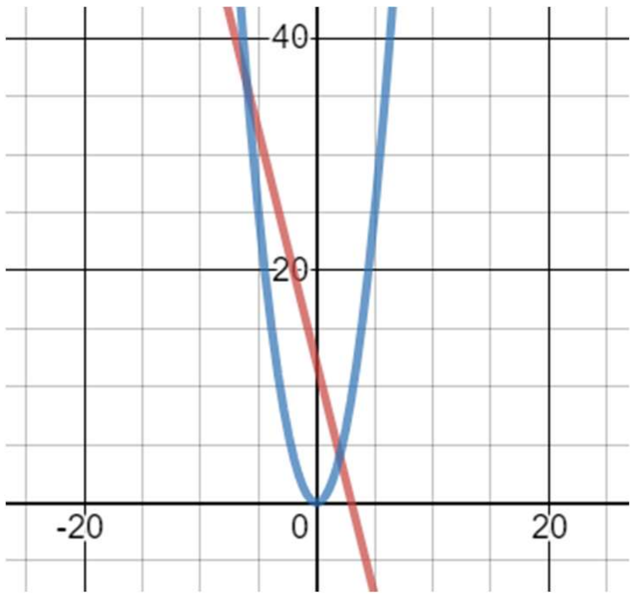
Notes

Worked Example

L_1 has equation $y = 12 - 4x$.
L_2 has equation $y = x^2$.
The diagram shows a sketch of L_1 and L_2 on the same axes.

a) Find the coordinates of the points of intersection.

b) Hence write down the solution to the inequality $12 - 4x > x^2$



T.52: 3F all Q+, P.21: 3.6 all Qs

3.7) Regions

Notes

Worked Example

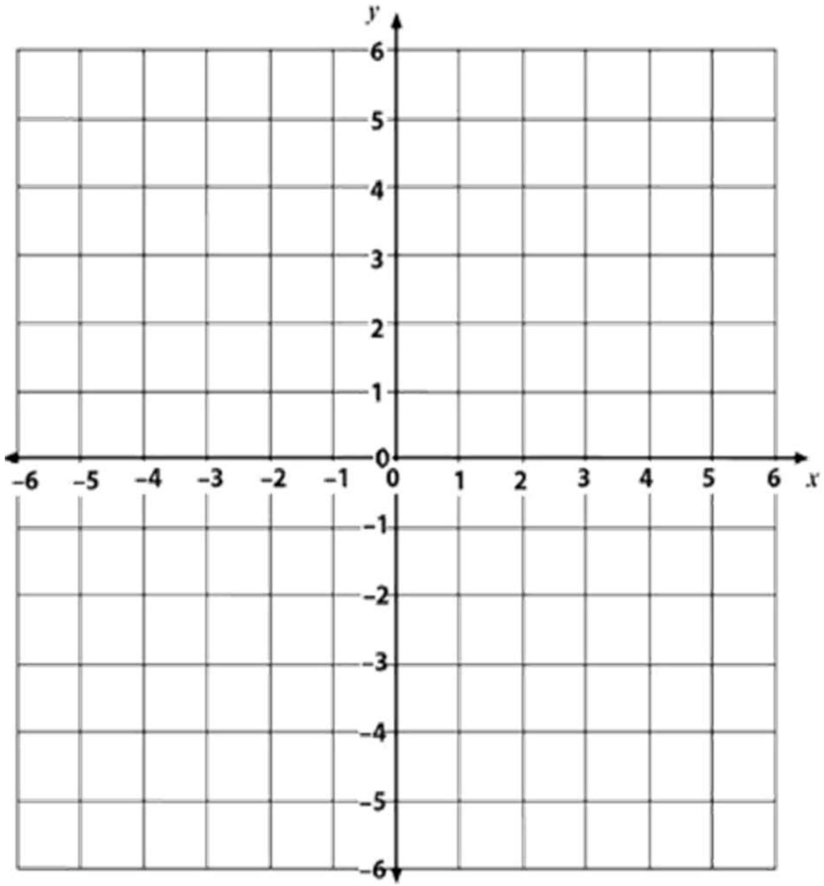
Shade the region that satisfies the inequalities:

$$4y + x \leq 12$$
$$y > x^2 - 5x - 6$$

Worked example

Shade the region which satisfies the inequalities. Label it R.

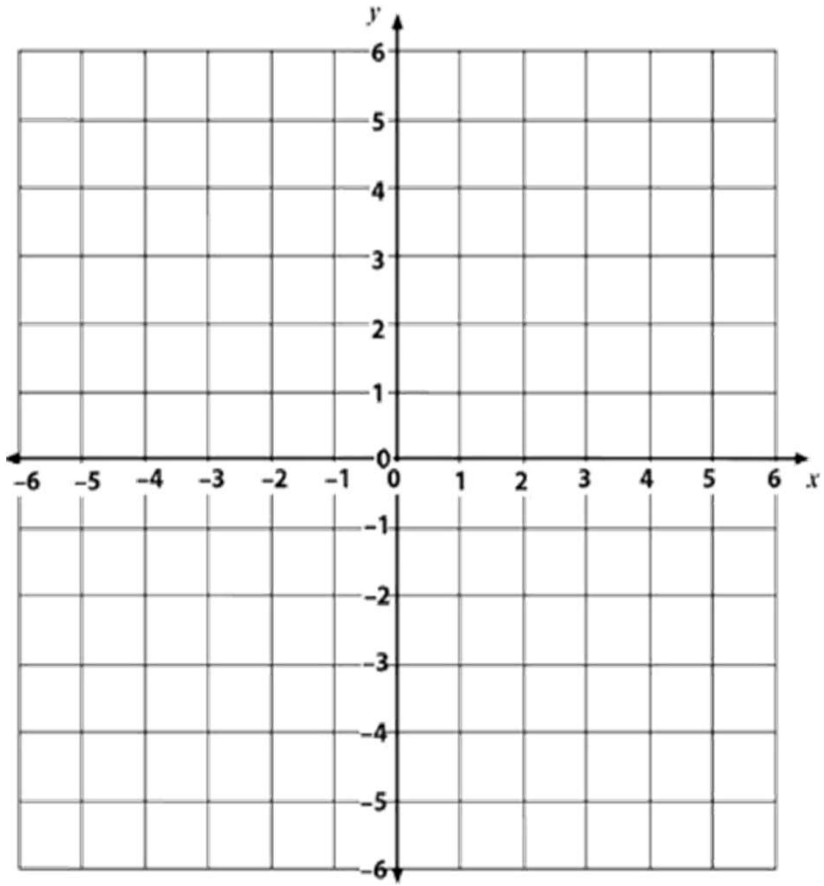
$2 \leq x \leq 5$ and $1 < y < 3$



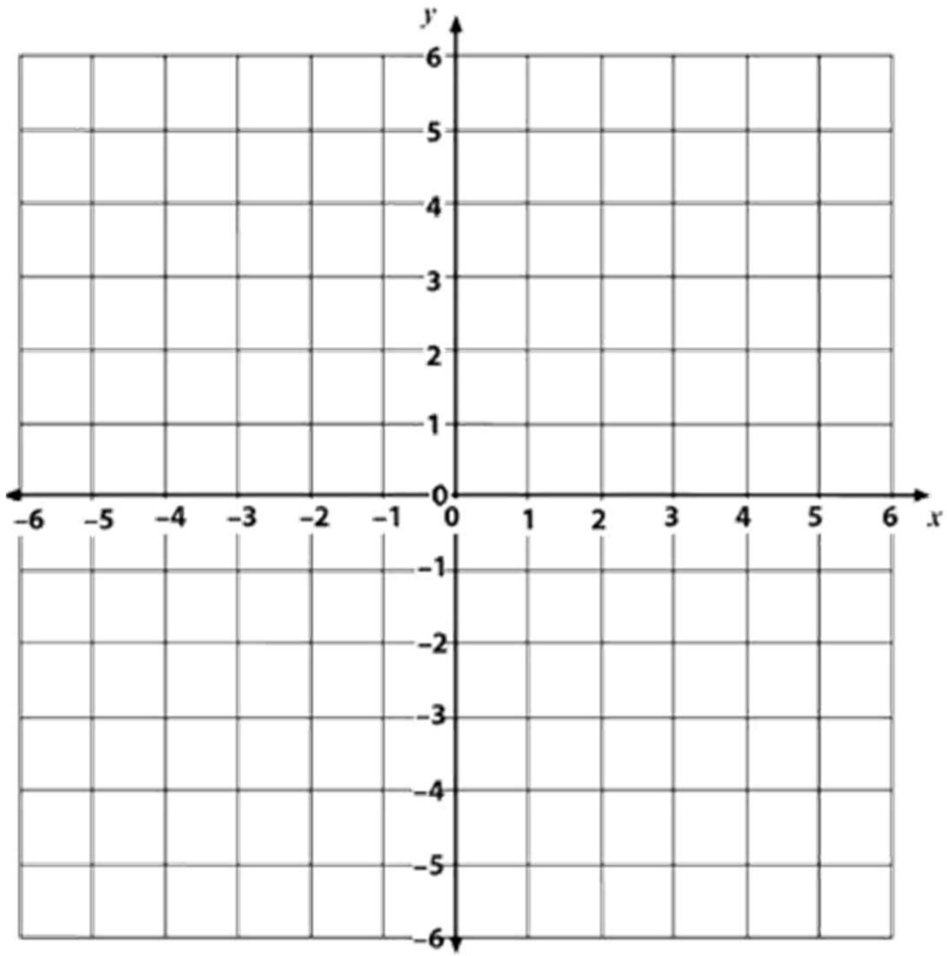
Your turn

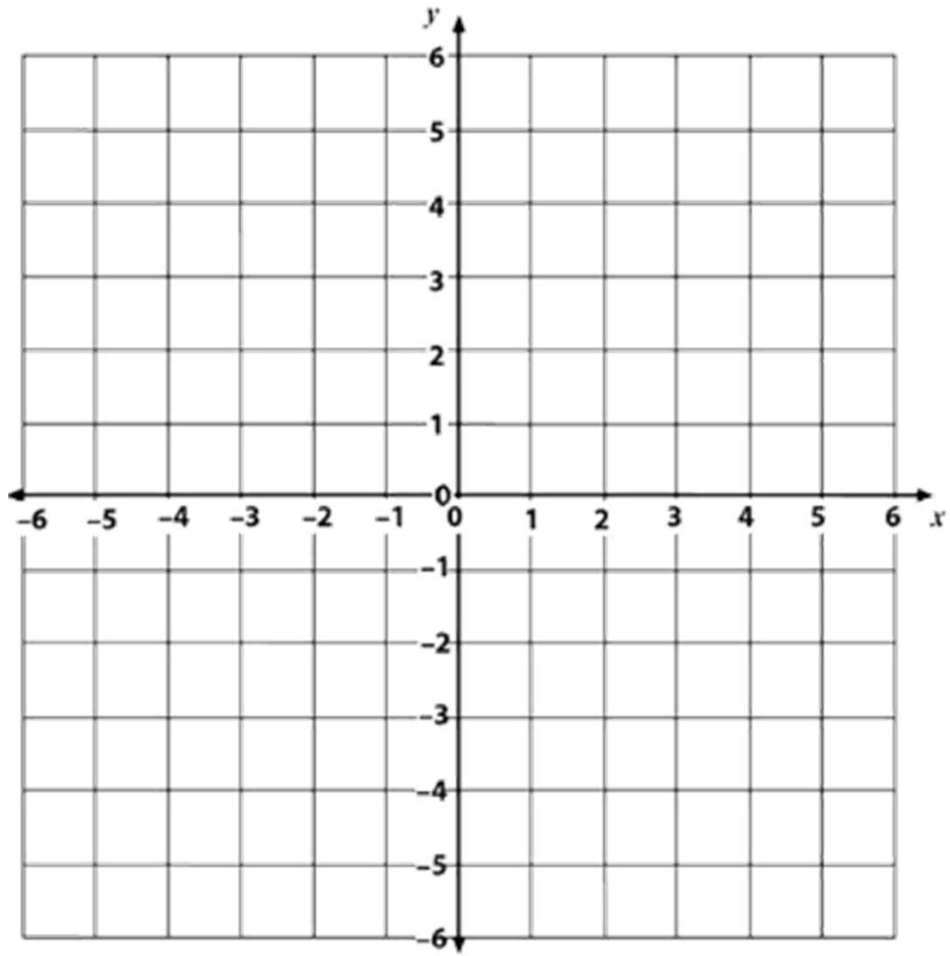
Shade the region which satisfies the inequalities. Label it R

$1 \leq x \leq 4$ and $2 < y < 6$

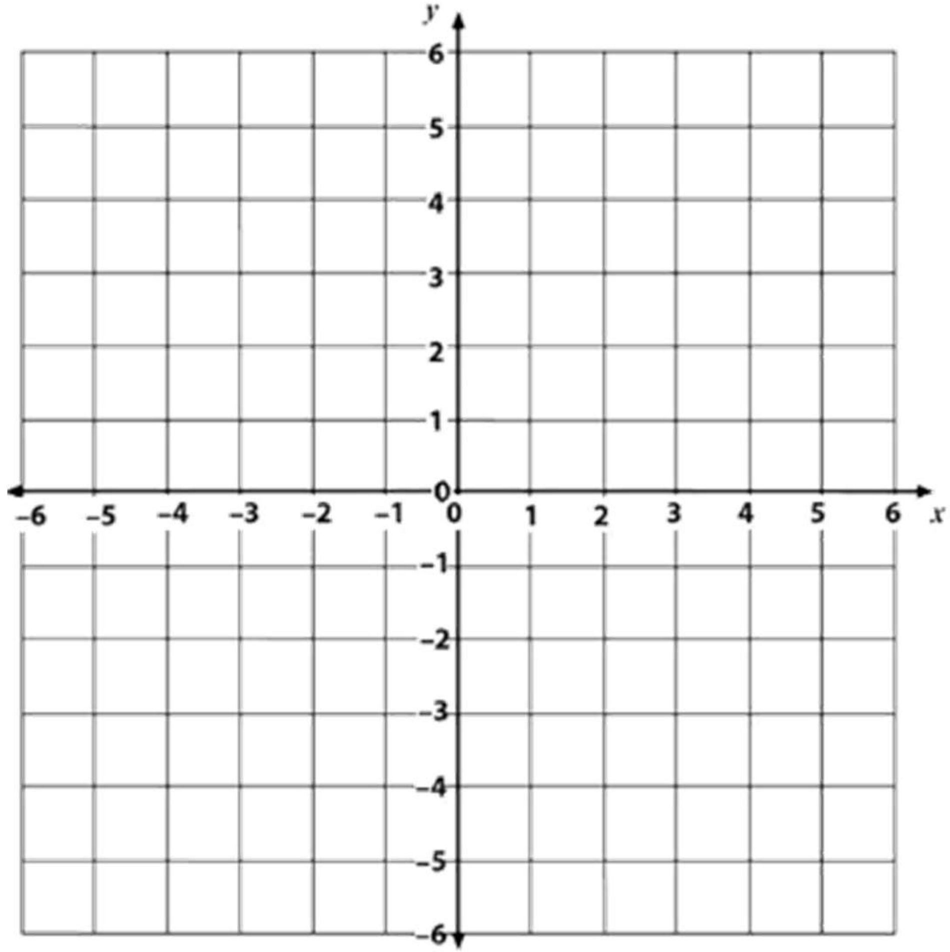


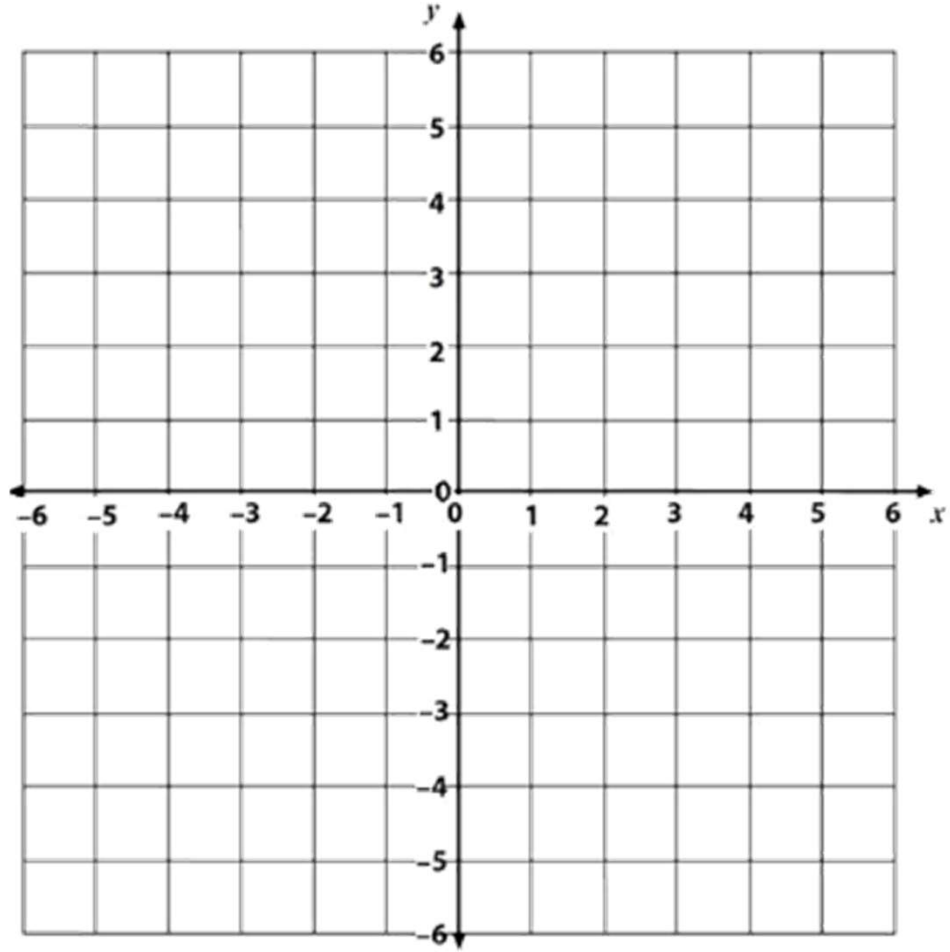
Worked example	Your turn
Shade the region which satisfies the inequalities: $x \geq -2, y < 1$ and $y < x - 1$	Shade the region which satisfies the inequalities. Label it R. $x > -3, y \leq 4$ and $y < x - 2$





Worked example	Your turn
Shade the region which satisfies the inequalities: $x \geq 2, y > -1$ and $x + y \leq 5$	Shade the region which satisfies the inequalities. Label it R. $x \geq 2, y > 1$ and $x + y \leq 6$





Past Paper Questions

1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

Question	Scheme	Marks	AOs
1	Finds critical values $x^2 - x > 20 \Rightarrow x^2 - x - 20 > 0 \Rightarrow x = (5, -4)$	M1	1.1b
	Chooses outside region for their values Eg. $x > 5,\ x < -4$	M1	1.1b
	Presents solution in set notation $\{x : x < -4\} \cup \{x : x > 5\}$ oe	A1	2.5
		(3)	
(3 marks)			

Summary of Key Points

Summary of key points

1 Linear simultaneous equations can be solved using elimination or substitution.

2 Simultaneous equations with one linear and one quadratic equation can have up to two pairs of solutions. You need to make sure the solutions are paired correctly.

3 The solutions of a pair of simultaneous equations represent the points of intersection of their graphs.

4 For a pair of simultaneous equations that produce a quadratic equation of the form $ax^2 + bx + c = 0$:

- $b^2 - 4ac > 0$ two real solutions
- $b^2 - 4ac = 0$ one real solution
- $b^2 - 4ac < 0$ no real solutions

5 The solution of an inequality is the set of all real numbers x that make the inequality true.

6 To solve a quadratic inequality:

- Rearrange so that the right-hand side of the inequality is 0
- Solve the corresponding quadratic equation to find the critical values
- Sketch the graph of the quadratic function
- Use your sketch to find the required set of values.

7 The values of x for which the curve $y = f(x)$ is **below** the curve $y = g(x)$ satisfy the inequality $f(x) < g(x)$.

The values of x for which the curve $y = f(x)$ is **above** the curve $y = g(x)$ satisfy the inequality $f(x) > g(x)$.

8 $y < f(x)$ represents the points on the coordinate grid below the curve $y = f(x)$.

$y > f(x)$ represents the points on the coordinate grid above the curve $y = f(x)$.

9 If $y > f(x)$ or $y < f(x)$ then the curve $y = f(x)$ is not included in the region and is represented by a dotted line.

If $y \geqslant f(x)$ or $y \leqslant f(x)$ then the curve $y = f(x)$ is included in the region and is represented by a solid line.