KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 12 Differentiation
Booklet

HGS Maths

Dr Frost Course

Name: ______________________________

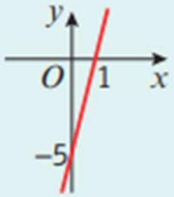
Class: ______________________________

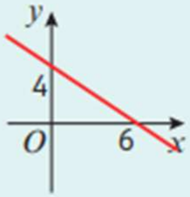
Contents

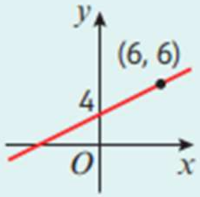
Prior knowledge check

Prior knowledge check

1 Find the gradients of these lines.

a 

b 

c 

← Section 5.1

2 Write each of these expressions in the form x^n where n is a positive or negative real number.

a $x^3 \times x^7$ **b** $\sqrt[3]{x^2}$ **c** $\dfrac{x^2 \times x^3}{x^6}$

d $\sqrt{\dfrac{x^2}{\sqrt{x}}}$

← Sections 1.1, 1.4

3 Find the equation of the straight line that passes through:

a (0, –2) and (6, 1) **b** (3, 7) and (9, 4)

c (10, 5) and (–2, 8)

← Section 5.2

4 Find the equation of the perpendicular to the line $y = 2x - 5$ at the point (2, 1).

← Section 5.3

12.3 Differentiating x^n

✏ If $y = ax^n$ then $\frac{dy}{dx} = nax^{n-1}$ (where a, n are constants)
i.e. multiply by the power and reduce the power by 1

The **operator** for differentiating is: $\frac{d}{dx}(\ldots)$ - which means differentiate with respect to x whatever is in the bracket.

We also use $f(x)$ notation, i.e.

If $y = f(x)$, then $\frac{dy}{dx} = f'(x)$

i.e. $f'(x)$ is the derivative of $f(x)$

Notes

Worked Example

Differentiate with respect to x:

a) x^5

b) $-3x^5$

Worked Example

Find the derivative, $f'(x)$, when $f(x)$ equals:

a) x^6

b) $x^{\frac{1}{2}}$

c) x^{-2}

d) $x^2 \times x^3$

e) $\frac{x}{x^5}$

T.263: 12C Qs 1, P. 94: 12.3 Qs 1

Worked Example

Differentiate with respect to x:

a) $\sqrt[5]{x}$

b) $\frac{3}{5}\sqrt{x}$

Worked Example

Differentiate with respect to x:

a) $\frac{1}{x^4}$

b) $\frac{7}{8x^4}$

Worked Example

Differentiate with respect to x:

$$\frac{3}{5\sqrt{x}}$$

Worked Example

Differentiate with respect to x:

a) $\sqrt{16x^8}$

b) $\sqrt{9}x^8$

T.263: 12C Qs 2, P. 94: 12.3 Qs 2

Worked Example

Find $\frac{dy}{dx}$ when y equals:

a) $7x^3$

b) $-4x^{\frac{1}{2}}$

c) $3x^{-2}$

d) $\frac{8x^7}{3x}$

e) $\sqrt{36x^3}$

T.263: 12C Qs 3+ P. 94: 12.3 Qs 3+

12.4 Differentiating Quadratics

Worked Example

Find $\frac{dy}{dx}$ given that y equals:

a) $x^2 + 3x$

b) $8x - 7$

c) $4x^2 - 3x + 5$

Worked Example

Find the gradient of the curve:
$y = 3x^2 - 2x + 1$ at $(-2, 17)$

T.265: 12D Qs 1-4 P. 94: 12.4 Qs 1-2

Worked Example

Find the coordinates of the point(s) where the gradient is 4:

a) $y = x^2 - 8x + 3$

b) $y = 5x^2 - x + 7$

Worked Example

Let $f(x) = 4x^2 - 8x + 3$

a) Find the gradient of $y = f(x)$ at the point $\left(\frac{1}{2}, 0\right)$

b) Find the coordinates of the point on the graph of $y = f(x)$ where the gradient is 8

c) Find the gradient of $y = f(x)$ at the points where the curve meets the line $y = 4x - 5$

T.265: 12D Qs 5+ P. 94: 12.4 Qs 3+

12.5 Differentiating Functions with Two or More Terms

Notes

Worked Example

Differentiate with respect to x:

$y = 5x^4 - 2x^7 + 12345 - x^5$

Worked Example

Find $\frac{dy}{dx}$ given that y equals:

a) $4x^3 + 2x$

b) $x^3 + x^2 - x^{\frac{1}{2}}$

c) $\frac{1}{3}x^{\frac{1}{2}} + 4x^2$

T.267: 12E Qs 1-3 P. 95: 12.5 Qs 1

Worked Example

Differentiate with respect to x:

$$y = 3\sqrt{x} + 4x^{\frac{5}{3}} - \frac{5}{x} + \frac{1}{\sqrt[3]{x}}$$

Worked Example

Differentiate with respect to x:

$f(x) = x^2(x - 3)$

Worked Example

Differentiate with respect to x:

$$f(x) = \frac{(2x+3)^2}{5x}$$

Worked Example

Differentiate with respect to x:

$$f(x) = \frac{x^2 + 3}{\sqrt{x}}$$

Worked Example

Differentiate:

a) $\frac{1}{4\sqrt{x}}$

b) $x^2(3x + 1)$

c) $\frac{x-2}{x^2}$

Worked Example

Differentiate with respect to x:

$$y = \frac{(x+2)^3}{3x^2}$$

Worked Example

Differentiate with respect to x:

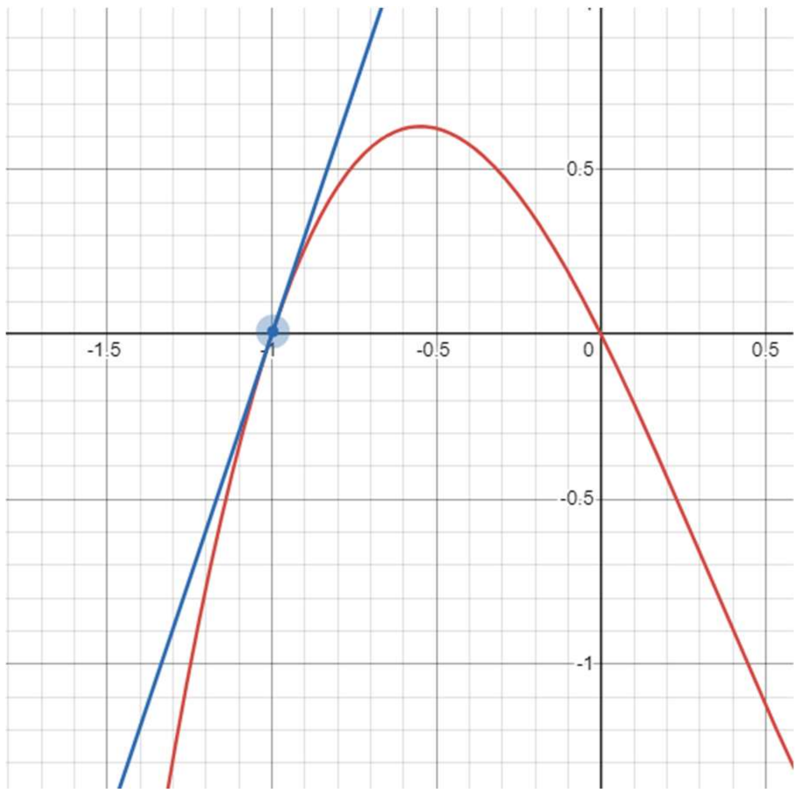
$$y = \frac{1 + 2x}{3x\sqrt{x}}$$

T.267: 12E Qs 4+ P. 95: 12.5 Qs 2+

12.6 Gradients, Tangents and Normals

$\frac{dy}{dx}$ evaluated at a given point $P(x, y)$ gives the value of the tangent at P

Graph shows tangent line to curve. | Desmos



I.e. for the equation of the tangent:

$$y - y_1 = \mathrm{m}(x - x_1)$$

$$m = \frac{dy}{dx} \text{ evaluated at } x_1$$

**for the normal use the negative reciprocal*

On your calculator this is written as:

$$\frac{d}{dx}(\quad)\Big|_{x=}$$

Notes

Worked Example

Find the gradient of the curve:

$y = 5\sqrt{x} - \frac{3}{x}$ at $(16, \frac{317}{16})$

Worked Example

Find the coordinates of the point(s) where the gradient is 2:
$y = x^3 - 3x^2 - 7x + 8$

Worked Example

For the curve $y = f(x)$,
$$\frac{dy}{dx} = \frac{3}{2} - kx^4 + k,$$
where k is a constant.
When $x = -2$, the gradient of the curve is -6. Find k.

Worked Example

Find the equation of the tangent to the curve $y = x^3$ when $x = 2$

Worked Example

520c: **Find an equation of a tangent to a polynomial curve.**

Find an equation of the tangent to the curve $y = x^2 + x + 1$ at the point $(4, 21)$.

Worked Example

520d: **Find an equation of a normal to a polynomial curve.**

Find an equation of the normal to the curve $y = 2x^2 + 4$ at the point $(5, 54)$.

Worked Example

Find the equation of the normal to the curve with equation $y = 8 - 3\sqrt{x}$ at the point where $x = 4$.
Give your answer in the form $ax + by + c = 0$

T.269: 12F Qs 1-2 P. 96: 12.6 Qs 1-5

Worked Example

520f: **Determine the coordinate of a point on a curve given the equation of a line parallel to the tangent or normal.**

A curve $y = f(x)$ has the equation
$f(x) = 3x^3 + 18x^2 + 41x + 33.$

The tangent at point P is parallel to the line $5y - 25x = 27$
.

Find the coordinates of P.

Worked Example

520g: **Determine the equation of a tangent or normal to a curve where the function involves fractional or negative powers.**

The curve $y = f(x)$ is given by $f(x) = 3x^3 + \frac{6}{\sqrt{x}} - 40$

The point P lies on the curve and has coordinates $(1, -31)$.

Find an equation of the tangent to the curve at P.

Worked Example

520h: **Determine the equation of the tangent or normal of a curve at an intercept with the axes.**

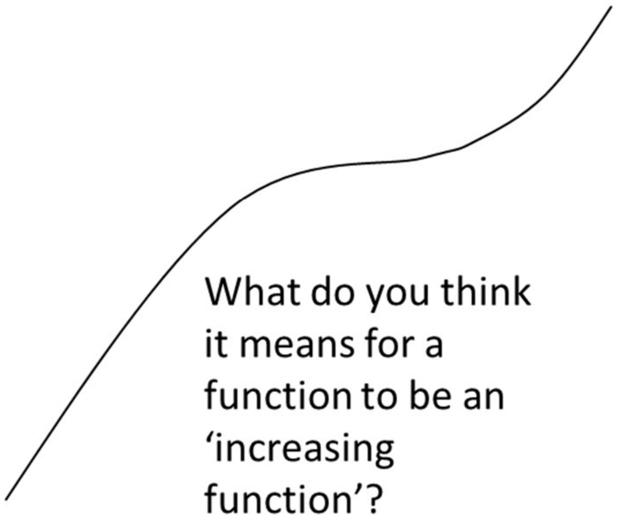
The curve $y = f(x)$ is given by the equation
$f(x) = 7x^3 + 6x^2 - 6x + 26$

Find an equation of the normal to the curve at the point where it intercepts the y-axis.

Worked Example

The point P with x-coordinate $\frac{1}{2}$ lies on the curve with equation $y = 4x^2$.
The normal to the curve at P intersects the curve at points P and Q.
Find the coordinates of Q

T.269: 12F Qs 3+, P. 96: 12.6 Qs 6+

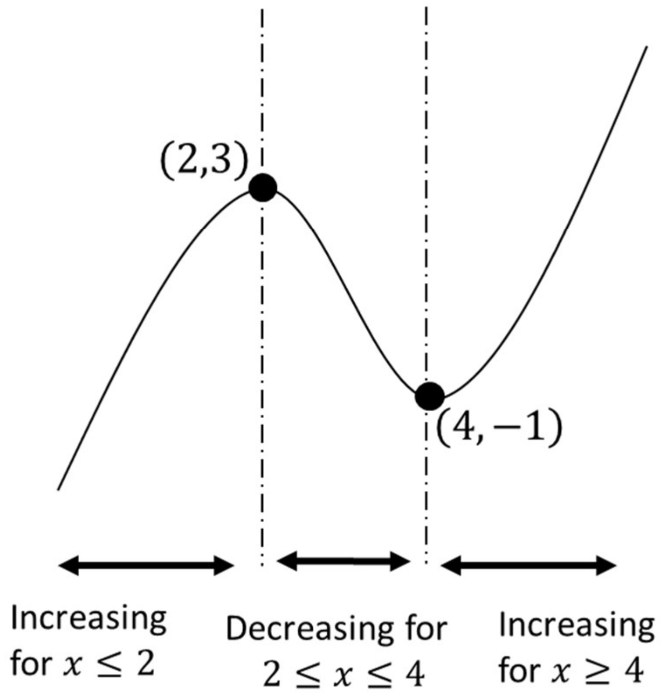


What do you think it means for a function to be an 'increasing function'?

> An increasing function is one whose gradient is always at least 0.
> $f'(x) \geq 0$ for all x.

It would be '**<u>strictly</u> increasing**' if $f(x) > 0$ for all x, i.e. is not allowed to go horizontal.

A function can also be increasing and decreasing in certain intervals.



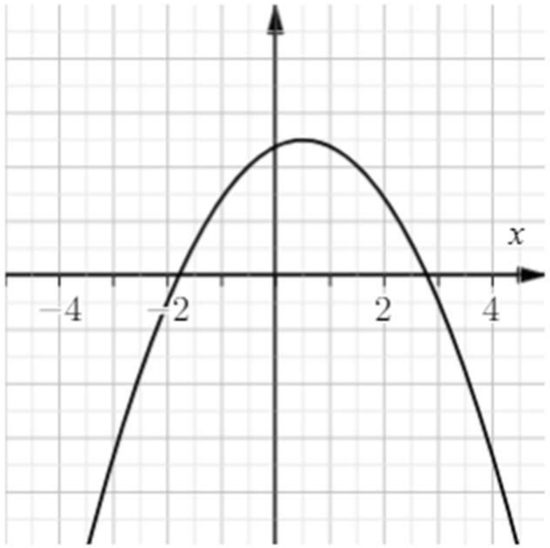
> We could also write "$f(x)$ is decreasing in the interval $[2,4]$"
> **$[a, b]$ represents all the real numbers between a and b inclusive, i.e:**
> $$[a, b] = \{x : a \leq x \leq b\}$$

Notes

Worked Example

516a: Understand the terms increasing and decreasing and apply to a drawn graph.

A sketch of the curve $y = f(x)$ is shown below.

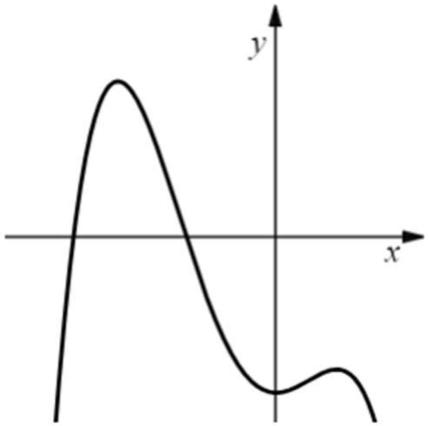


Using the graph, find the interval in which $f(x)$ is decreasing.

Worked Example

516b: **Determine the interval for which a quadratic function is increasing or decreasing.**

A sketch of the curve $y = f(x)$ is shown below.



The curve has the gradient function

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -12x^3 - 13x^2 + 14x.$$

Find the interval in which the curve is increasing.

Worked Example

516d: Determine the interval for which a cubic function is increasing or decreasing.

Find the interval on which the function $y = 3x^3 + 12x^2 - 9x - 8$ is increasing.

Worked Example

516f: Determine the range of values for which a function is increasing/decreasing involving a reciprocal term.

A function is given by the equation

$$f(x) = -3x - \frac{32}{x} - 20, \quad x \neq 0$$

Find the exact interval on which $f(x)$ is decreasing.

T.271: 12G Qs 1-2, P. 97: 12.7 Qs 1-3

Worked Example

Show that the function
$f(x) = x^3 + 6x^2 + 21x + 2$ is increasing for all real values of x.

Worked Example

Show that the function $3 + 4x(-x^2 - 5)$ is decreasing for all $x \in \mathbb{R}$

T.271: 12G Qs 3+, P. 97: 12.7 Qs 4+

12.8 Second Order Derivatives

When you differentiate once, the expression you get is known as the **first derivative**. Unsurprisingly, when we differentiate a second time, the resulting expression is known as the **second derivative**. And so on…

Original Function		First Derivative		Second Derivative
$y = x^4$	Leibniz's	$\frac{dy}{dx} = 4x^3$	→	$\frac{d^2y}{dx^2} = 12x^2$
		$y' = 4x^3$	→	$y'' = 12x^2$
	Newton's	$\dot{y} = 4x^3$	→	$\ddot{y} = 12x^2$
$f(x) = x^4$	Lagrange's →	$f'(x) = 4x^3$	→	$f''(x) = 12x^2$

You can similarly have the third derivative ($\frac{d^3y}{dx^3}$), although this is no longer in the A Level syllabus. We'll see why might use the second derivative soon…

Notes

Notes

Worked Example

515a: Determine the second derivative of a function.

A function is given by the equation

$$y = \left(4x^5 + 3x^2\right)^2$$

Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$.

Worked Example

If $f(x) = 3\sqrt{x} + \frac{1}{2\sqrt{x}}$, find $f''(x)$

Worked Example

515b: Evaluate the second derivative of a function at a specific point.

A function is given by the equation

$$f(x) = \frac{3x^{10} + 2x^5}{x^6}$$

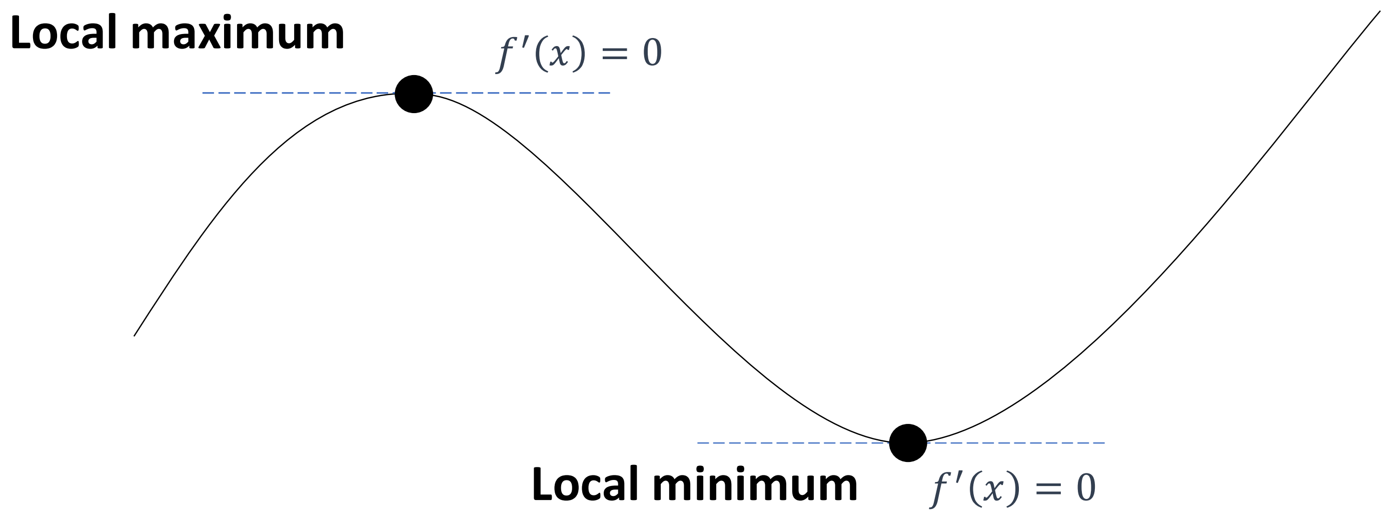
Determine $f''(-4)$.

Give your answer to three significant figures.

T.272: 12H Qs all, P. 98: 12.8 Qs all

12.9 Stationary Points

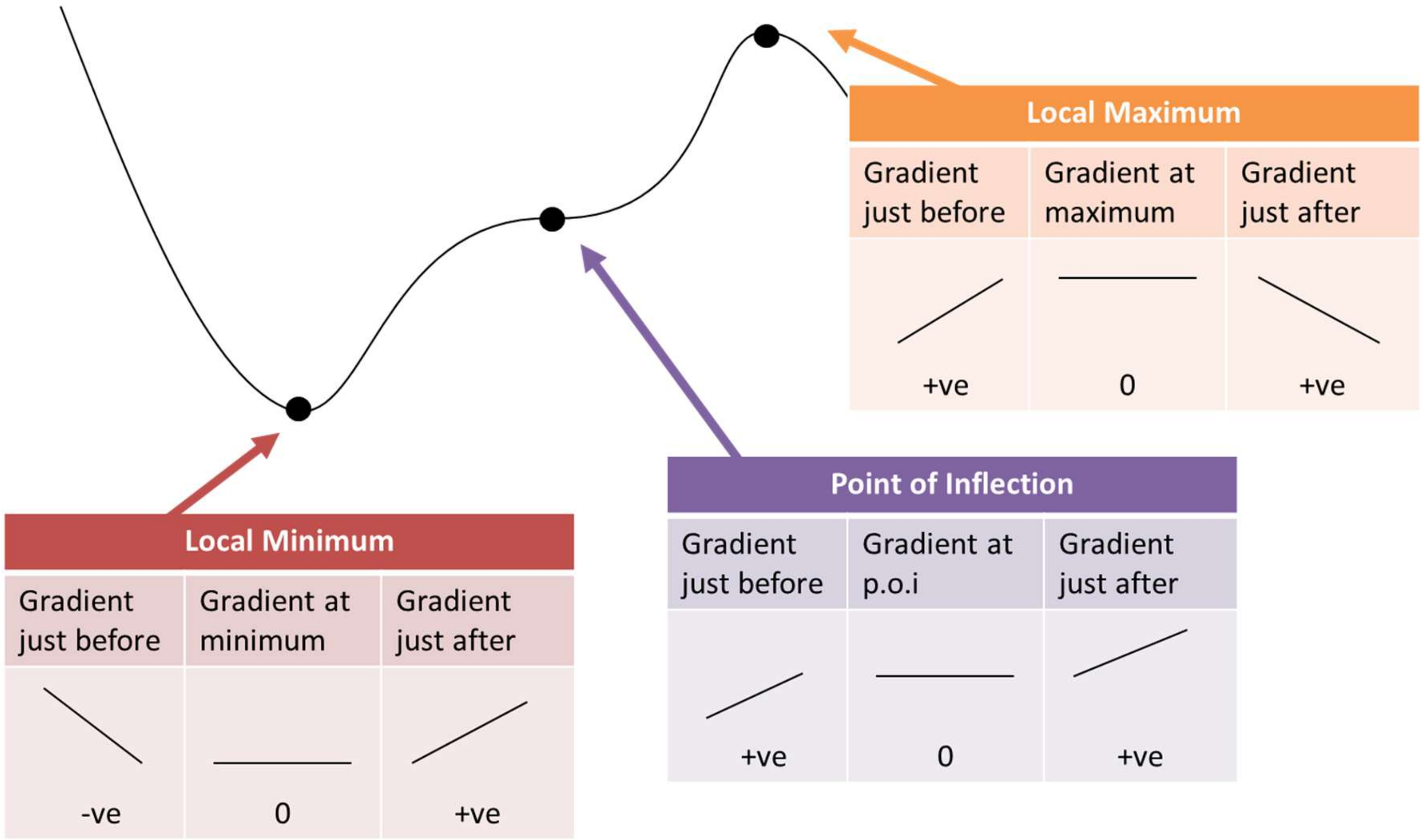
A stationary point is where the gradient is 0, i.e. $f'(x) = 0$.



Note: *It's called a '**local**' maximum because it's the function's largest output within the vicinity. Functions may also have a '**global**' maximum, i.e. the maximum output across the entire function. This particular function doesn't have a global maximum because the output keeps increasing up to infinity. It similarly has no global minimum, as with all cubics.*

How do we tell what type of stationary point?

Method 1: Look at gradient just before and just after point.



Using the second derivative

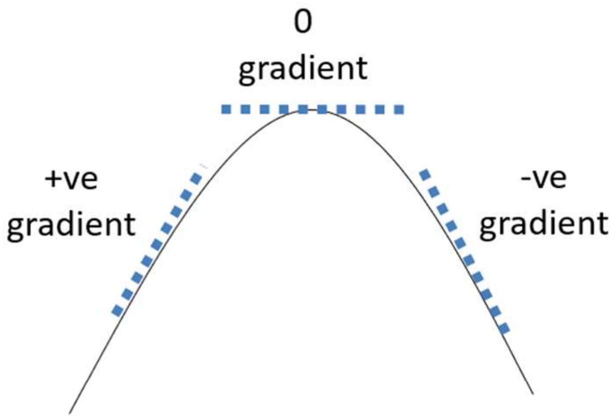
At a stationary point $x = a$:

- If $f''(a) > 0$ the point is a local minimum.
- If $f''(a) < 0$ the point is a local maximum.
- If $f''(a) = 0$ it could be any type of point, so resort to Method 1.

Recall the gradient gives a measure of the **rate of change** of y, i.e. how much the y value changes as x changes.

Thus by differentiating the gradient function, the **second derivative tells us the rate at what the gradient is changing**.

Thus if the second derivative is positive, the gradient is increasing.
If the second derivative is negative, the gradient is decreasing.



At a maximum point, we can see that as x increases, the gradient is decreasing from a positive value to a negative value.

$$\therefore \frac{d^2y}{dx^2} < 0$$

Worked Example
Find the least value of $f(x) = x^2 - 4x + 9$

T.276: 12I Qs 1-2, P. 98: 12.9 Qs 1-3

Worked Example

Find the turning point of
$y = \sqrt{x} - x$

Worked Example

Find the stationary point on the curve with equation
$y = x^4 - 32x$, and determine whether it is a local maximum, a local minimum or a point of inflection.

Worked Example

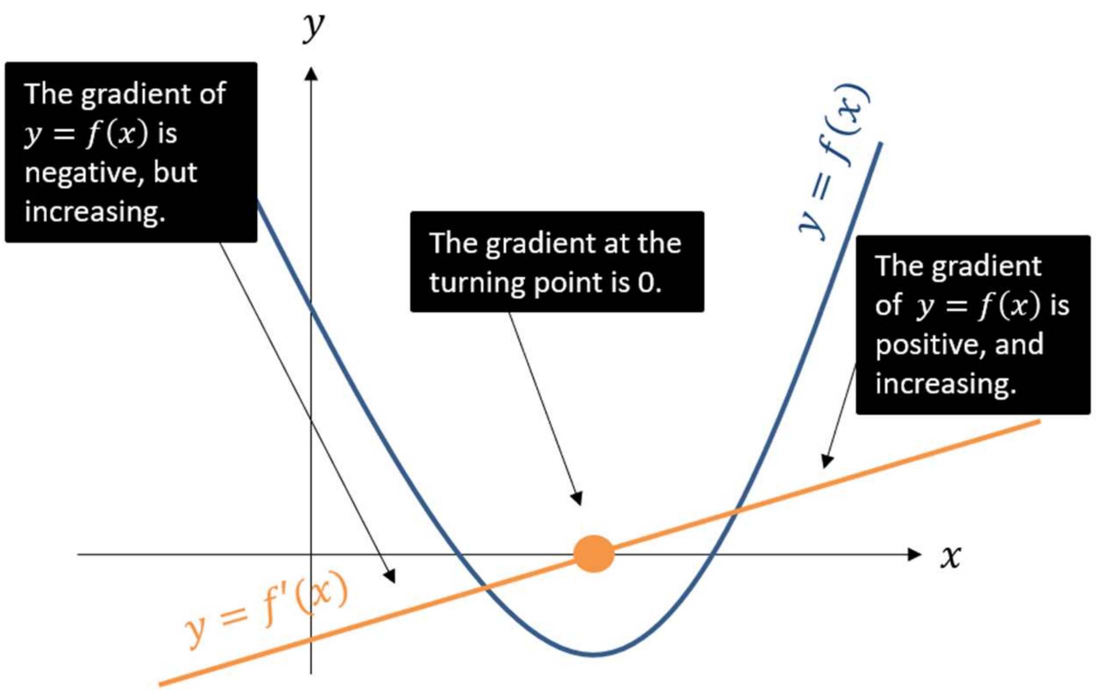
Find the coordinates of the stationary points on the curve with equation $y = 2x^3 - 15x^2 + 24x + 6$ and use the second derivative to determine their nature.

T.276: 12I Qs 3+, P. 98: 12.9 Qs 4+

12.10 Sketching Gradient Functions

The new A Level specification specifically mentions being able to sketch $y = f'(x)$.

If you know the function $f'(x)$ explicitly (e.g. because you differentiated $y = f(x)$), you can use your knowledge of sketching straight line/quadratic/cubic graphs.
But in other cases **you won't be given the function explicitly**, but **just the sketch**.



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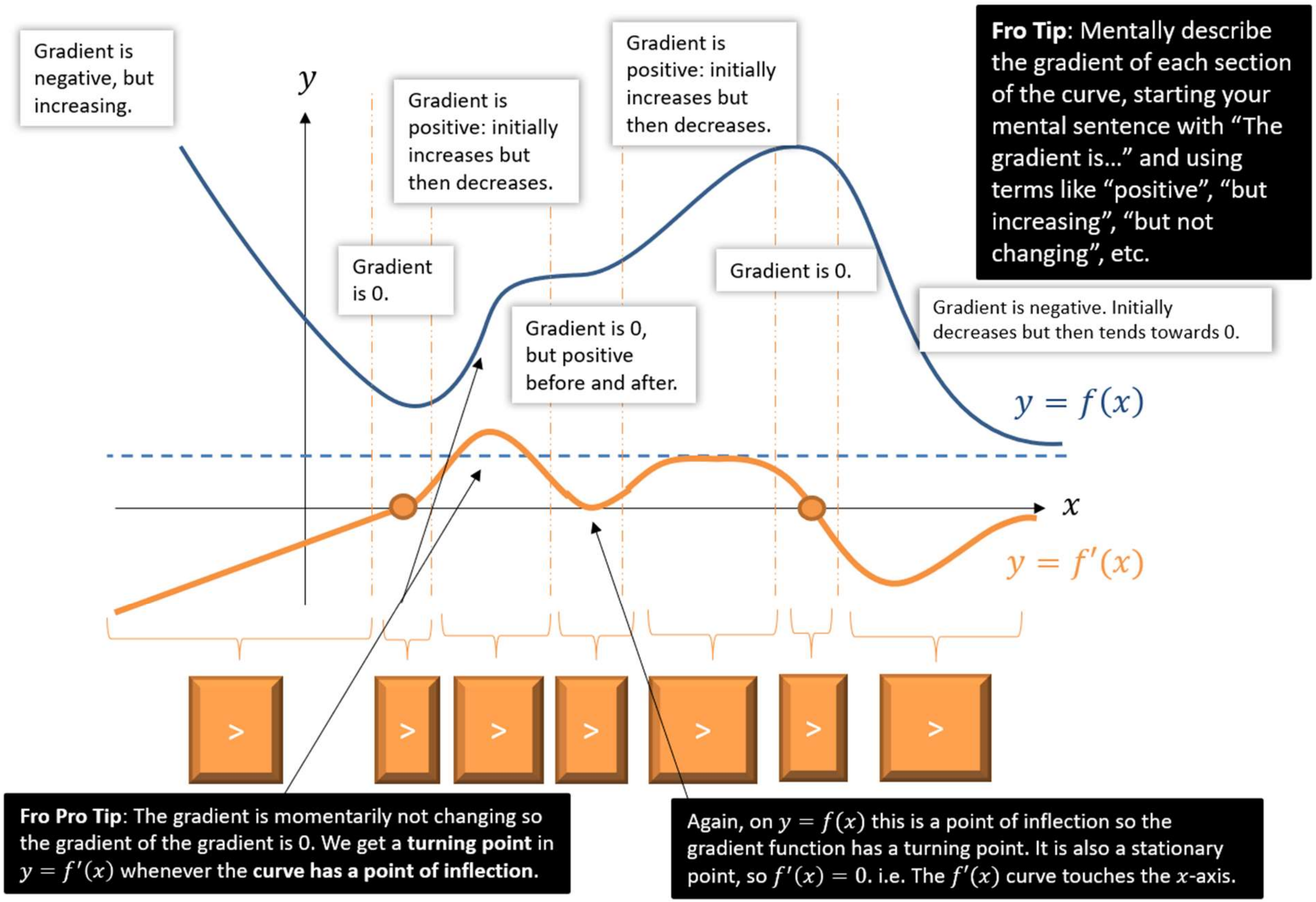
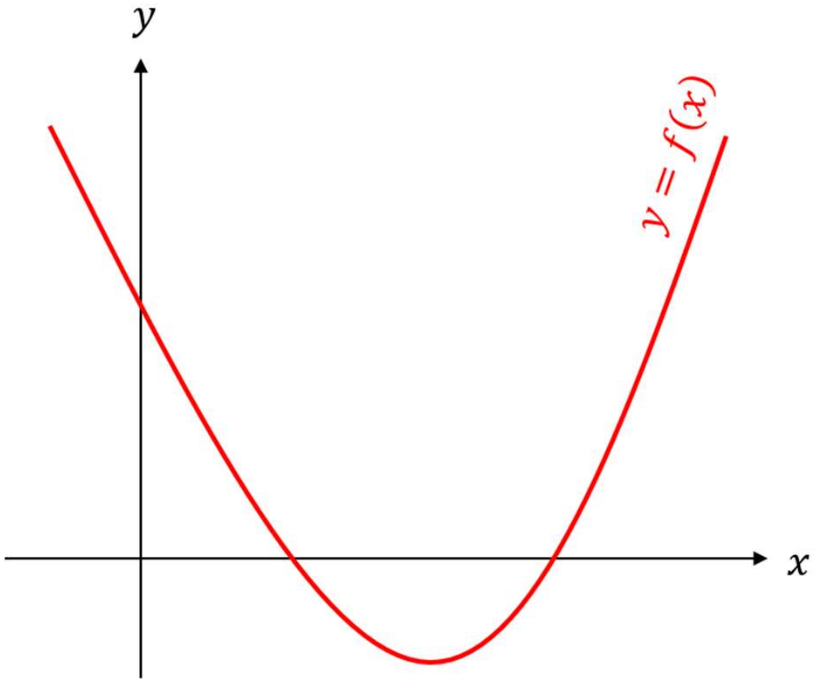


Gradient is negative, but increasing.
Gradient is positive: initially increases but then decreases.
Gradient is positive: initially increases but then decreases.
Fro Tip: Mentally describe the gradient of each section of the curve, starting your mental sentence with "The gradient is..." and using terms like "positive", "but increasing", "but not changing", etc.
Gradient is 0.
Gradient is 0.
Gradient is 0, but positive before and after.
Gradient is negative. Initially decreases but then tends towards 0.
$y = f(x)$
y
x
$y = f'(x)$
>
>
>
>
>
>
>
Fro Pro Tip: The gradient is momentarily not changing so the gradient of the gradient is 0. We get a turning point in $y = f'(x)$ whenever the curve has a point of inflection.
Again, on $y = f(x)$ this is a point of inflection so the gradient function has a turning point. It is also a stationary point, so $f'(x) = 0$. i.e. The $f'(x)$ curve touches the x-axis.

Notes

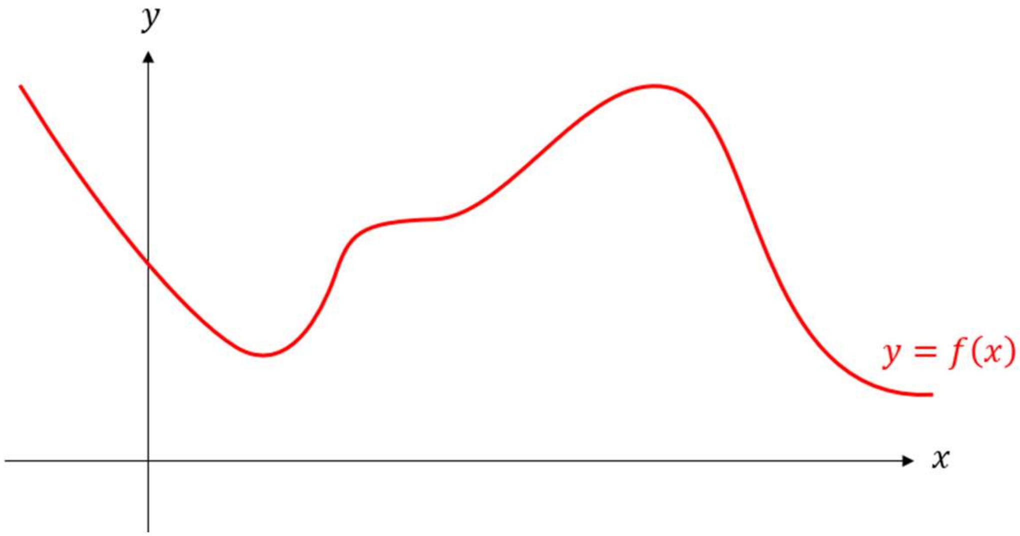
Worked Example

Sketch $y = f'(x)$ on the same axes



Worked Example

Sketch $y = f'(x)$ on the same axes



Worked Example

A positive cubic has the equation $y = f(x)$.
The curve has stationary points at $(-1, 4)$ and $(1, 0)$ and cuts the x-axis at $(-3, 0)$.
Sketch the gradient function, $y = f'(x)$, showing the coordinates of any points where the curve cuts or meets the x-axis.

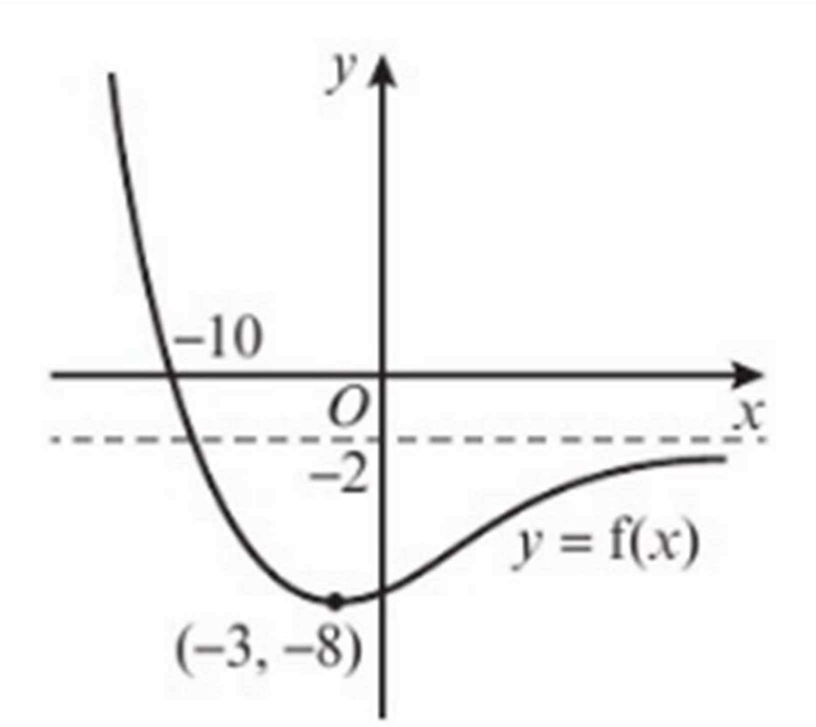
T.278: 12J Qs 1, P. 99: 12.10 Qs 1

Worked Example

The diagram shows the curve with equation $y = f(x)$. The curve has an asymptote at $y = -2$ and a turning point at $(-3, -8)$. It cuts the x-axis at $(-10, 0)$.

a) Sketch the graph of $y = f'(x)$.

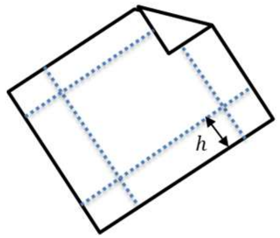
b) State the equation of the asymptote of $y = f'(x)$.



T.278: 12J Qs 2, P. 99: 12.10 Qs 2,3

12.11 Modelling with Differentiation

1. Optimisation Problems/Modelling, *e.g.*

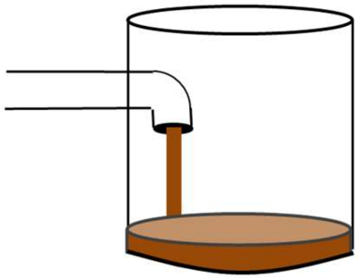


We have a sheet of A4 paper, which we want to fold into a cuboid. What height should we choose for the cuboid in order to maximise the volume?

2. Rate of change:

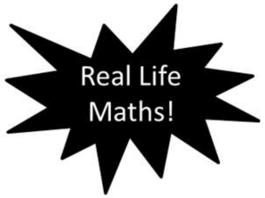
Up to now we've had y in terms of x, where $\frac{dy}{dx}$ means "the **rate** at which y changes with respect to x".

But we can use similar gradient function notation for other **physical quantities**.



A sewage container fills at a rate of 20 cm³ per second.

How could we use appropriate notation to represent this?



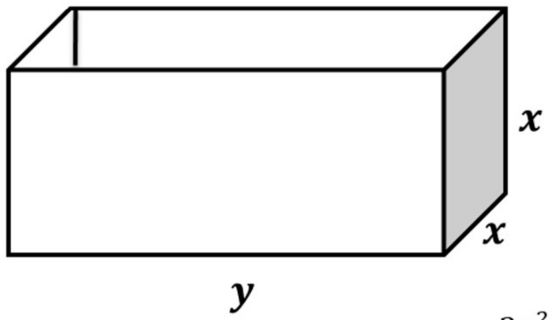
$$\frac{dV}{dt} = 20\ cm^3/s$$

"The rate at which the volume V changes with respect to time t."

Example Optimisation Problem

Optimisation problems in an exam usually follow the following pattern:

- There are 2 variables involved (you may have to introduce one yourself), typically lengths.
- There are expressions for **two different physical quantities**:
  - One is a **constraint**, e.g. "the surface area is 20cm²".
  - The **other we wish to maximise/minimise**, e.g. "we wish to maximise the volume".
- We use the constraint to **eliminate one of the variables** in the latter equation, so that it is then **just in terms of one variable**, and we can then use differentiation to find the turning point.



[Textbook] A large tank in the shape of a cuboid is to be made from 54m² of sheet metal. The tank has a horizontal base and no top. The height of the tank is x metres. Two of the opposite vertical faces are squares.

a) Show that the volume, V m³, of the tank is given by

$$V = 18x - \frac{2}{3}x^3.$$

We need to introduce a second variable ourselves so that we can find expressions for the surface area and volume.

These are the two equations mentioned in the guidance: one for surface area and one volume.

$$2x^2 + 3xy = 54$$
$$V = x^2 y$$

But we want V just in terms of x:

$$y = \frac{54 - 2x^2}{3x} \quad \rightarrow \quad V = x^2\left(\frac{54 - 2x^2}{3x}\right)$$

$$V = \frac{54x^2 - 2x^4}{3x} = 18x - \frac{2}{3}x^3$$

b) Given that x can vary, use differentiation to find the maximum or minimum value of V.

$$\frac{dV}{dt} = 18 - 2x^2 = 0 \quad \therefore x = 3$$

$$V = 18(3) - \frac{2}{3}(3)^3 = 36$$

Once we have the 'optimal' value of x, we sub it back into V to get the best possible volume.

Notes

Worked Example

Given that the volume, $V\ cm^3$, of an expanding sphere is related to its radius, $r\ cm$, by the formula $V = \frac{4}{3}\pi r^3$, find the rate of change of volume with respect to radius at the instant when the radius is $5\ cm$.

Worked Example

A cuboid is to be made from $54m^2$ of sheet metal.
The cuboid has a horizontal base and no top.
The height of the cuboid is x metres.
Two of the opposite vertical faces are squares.

a) Show that the volume, V m^3, of the tank is given by $V = 18x - \frac{2}{3}x^3$.

b) Given that x can vary, use differentiation to find the maximum or minimum value of V.

c) Justify that the value of V you have found is a maximum

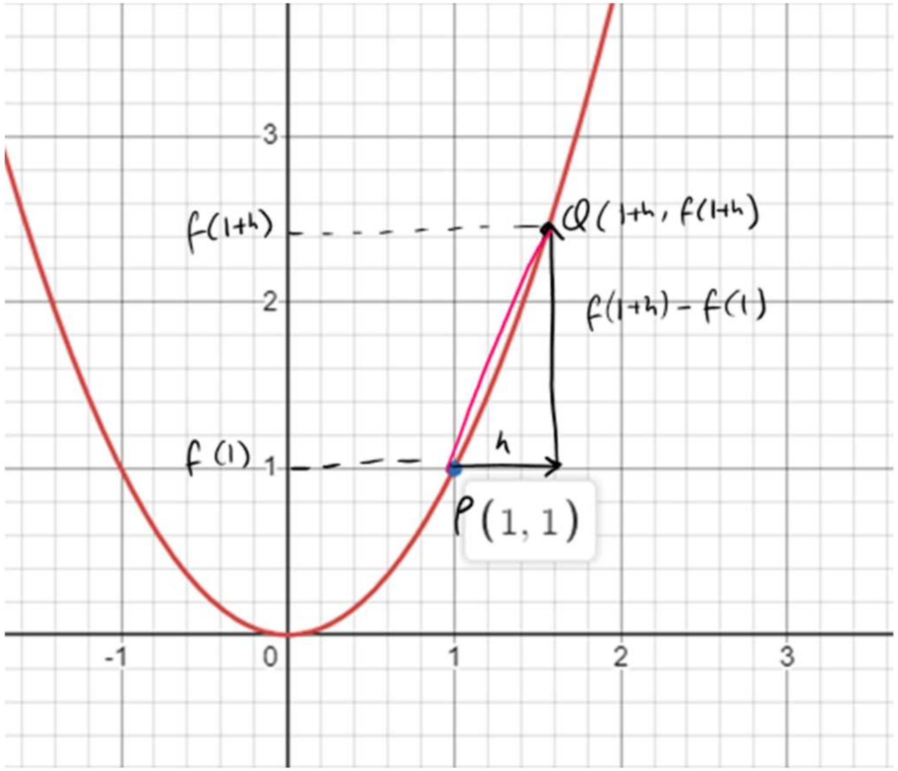
T.281: 12K Qs all, P. 101: 12.11 Qs all

12.2 Finding the Derivative

Where does the rule/method we have been using come from?

At GCSE you learnt how to *estimate* the gradient of a curve by *sketching* the tangent and then finding the gradient of the tangent at that point.

We can generalise this idea to the gradient of a *chord* from the point $P(x, f(x))$ we are interested in to a point some distance away $\mathrm{Q}(x+h, f(x+h))$, i.e.



So the gradient of the chord PQ is

$$\frac{f(x+h)-f(x)}{h}$$

Notes

The gradient of this chord is not the actual gradient at P *(i.e. the gradient of the tangent to $y = f(x)$ at P)*. It is an approximation.

The approximation will get better as the distance between P and Q reduces, i.e. h gets smaller, which we write as $h \to 0$ *(read as h tends to zero)*

Therefore, you need to know and use:

The gradient function, or derivative, of the curve $y = f(x)$ is written as $f'(x)$ or $\frac{dy}{dx}$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

The gradient function can be used to find the gradient of the curve for any value of x.

Practically you don't have to use the $\lim_{h \to 0}$ bit, just use $h \to 0$. The above is just how it is stated in formulae books/textbooks.

Notes

Worked Example

The point A with coordinates $(4, 16)$ lies on the curve with equation $y = x^2$.
At point A the curve has gradient g.

a) Show that $g = \lim_{h \to 0}(8 + h)$

b) Deduce the value of g.

T.261: 12B Qs 1-4, P. 93: 2.2 Qs 1,2

Worked Example

Prove from first principles that the derivative of $5x$ is 5

Worked Example

Prove from first principles that the derivative of $5x^2$ is $10x$

Worked Example

Prove from first principles that the derivative of x^3 is $3x^2$

T.261: 12B Qs 5+, P. 93: 2.2 Qs 3+

Extract from Formulae book

Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

1. The curve C has equation

$$y = 3x^4 - 8x^3 - 3$$

(a) Find (i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Verify that C has a stationary point when $x = 2$

(2)

(c) Determine the nature of this stationary point, giving a reason for your answer.

(2)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

1(a)	(i) $\frac{dy}{dx} = 12x^3 - 24x^2$	M1 A1	1.1b 1.1b
	(ii) $\frac{d^2y}{dx^2} = 36x^2 - 48x$	A1ft	1.1b
		(3)	
(b)	Substitutes $x = 2$ into their $\frac{dy}{dx} = 12 \times 2^3 - 24 \times 2^2$	M1	1.1b
	Shows $\frac{dy}{dx} = 0$ and states "hence there is a stationary point"	A1	2.1
		(2)	
(c)	Substitutes $x = 2$ into their $\frac{d^2y}{dx^2} = 36 \times 2^2 - 48 \times 2$	M1	1.1b
	$\frac{d^2y}{dx^2} = 48 > 0$ and states "hence the stationary point is a minimum"	A1ft	2.2a
		(2)	
			(7 marks)

Summary of Key Points

1 The **gradient** of a **curve** at a given point is defined as the gradient of the **tangent** to the curve at that point.

2 The **gradient function**, or **derivative**, of the curve $y = \mathrm{f}(x)$ is written as $\mathrm{f}'(x)$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

The gradient function can be used to find the gradient of the curve for any value of x.

3 For all real values of n, and for a constant a:

- If $\mathrm{f}(x) = x^n$ then $\mathrm{f}'(x) = nx^{n-1}$
- If $y = x^n$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = nx^{n-1}$
- If $\mathrm{f}(x) = ax^n$ then $\mathrm{f}'(x) = anx^{n-1}$
- If $y = ax^n$ then $\frac{\mathrm{d}y}{\mathrm{d}x} = anx^{n-1}$

4 For the quadratic curve with equation $y = ax^2 + bx + c$, the derivative is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 2ax + b$$

5 If $y = \mathrm{f}(x) \pm \mathrm{g}(x)$, then $\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}'(x) \pm \mathrm{g}'(x)$.

6 The tangent to the curve $y = \mathrm{f}(x)$ at the point with coordinates $(a, \mathrm{f}(a))$ has equation

$$y - \mathrm{f}(a) = \mathrm{f}'(a)(x - a)$$

7 The normal to the curve $y = \mathrm{f}(x)$ at the point with coordinates $(a, \mathrm{f}(a))$ has equation

$$y - \mathrm{f}(a) = -\frac{1}{\mathrm{f}'(a)}(x - a)$$

8
- The function $\mathrm{f}(x)$ is **increasing** on the interval $[a, b]$ if $\mathrm{f}'(x) \geqslant 0$ for all values of x such that $a < x < b$.
- The function $\mathrm{f}(x)$ is **decreasing** on the interval $[a, b]$ if $\mathrm{f}'(x) \leqslant 0$ for all values of x such that $a < x < b$.

9 Differentiating a function $y = \mathrm{f}(x)$ twice gives you the second order derivative, $\mathrm{f}''(x)$ or $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$

10 Any point on the curve $y = \mathrm{f}(x)$ where $\mathrm{f}'(x) = 0$ is called a **stationary point**. For a small positive value h:

Type of stationary point	$\mathrm{f}'(x-h)$	$\mathrm{f}'(x)$	$\mathrm{f}'(x+h)$
Local maximum	Positive	0	Negative
Local minimum	Negative	0	Positive
Point of inflection	Negative	0	Negative
	Positive	0	Positive

11 If a function $\mathrm{f}(x)$ has a stationary point when $x = a$, then:

- if $\mathrm{f}''(a) > 0$, the point is a local minimum
- if $\mathrm{f}''(a) < 0$, the point is a local maximum.

If $\mathrm{f}''(a) = 0$, the point could be a local minimum, a local maximum or a point of inflection. You will need to look at points on either side to determine its nature.

T.282: mixed ex., P. 103: BSG