KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Pure Mathematics
P1 7 Algebraic Methods
Booklet

HGS Maths

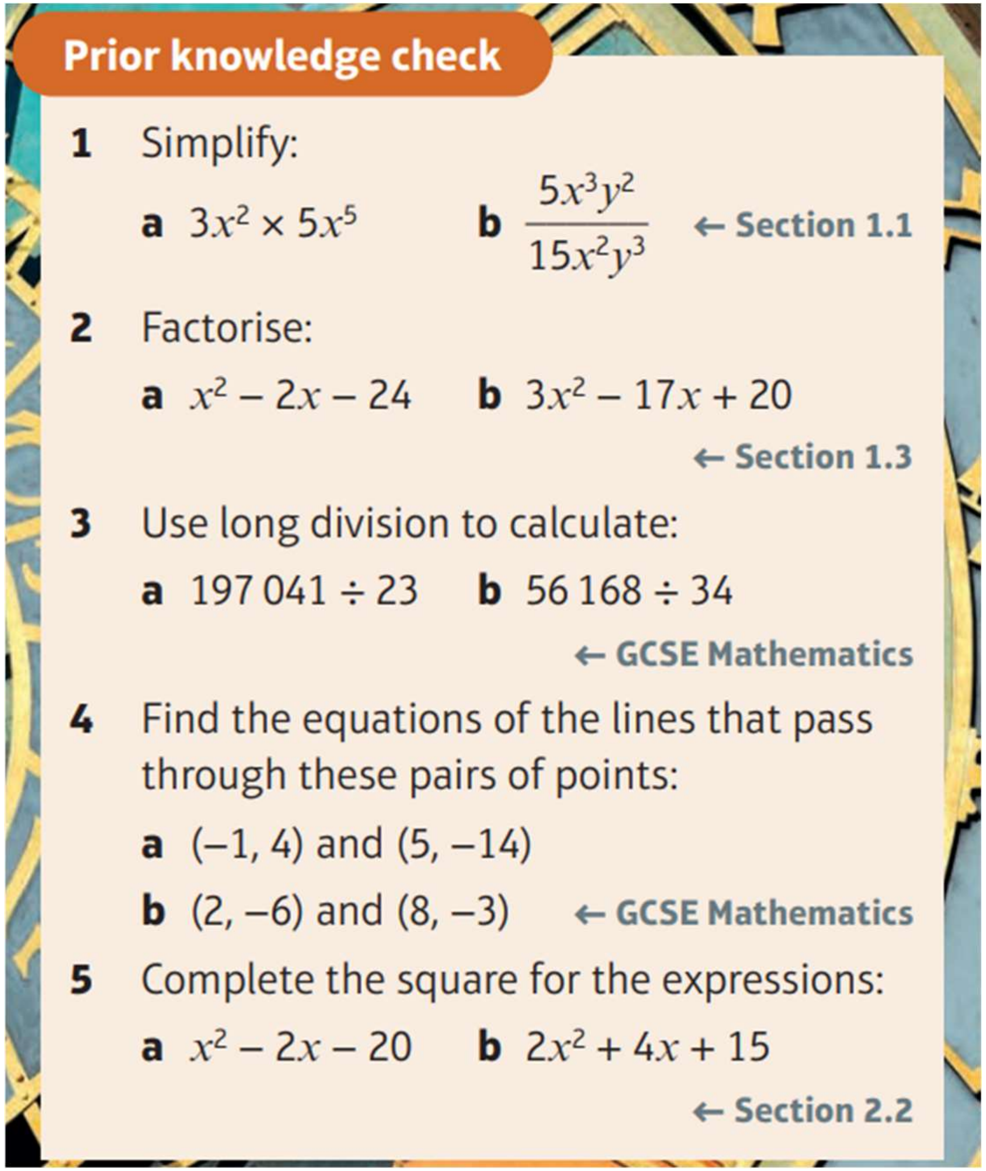
Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

Prior knowledge check



Prior knowledge check

1 Simplify:

a $3x^2 \times 5x^5$ **b** $\dfrac{5x^3y^2}{15x^2y^3}$ ← **Section 1.1**

2 Factorise:

a $x^2 - 2x - 24$ **b** $3x^2 - 17x + 20$

← **Section 1.3**

3 Use long division to calculate:

a 197 041 ÷ 23 **b** 56 168 ÷ 34

← **GCSE Mathematics**

4 Find the equations of the lines that pass through these pairs of points:

a (–1, 4) and (5, –14)

b (2, –6) and (8, –3) ← **GCSE Mathematics**

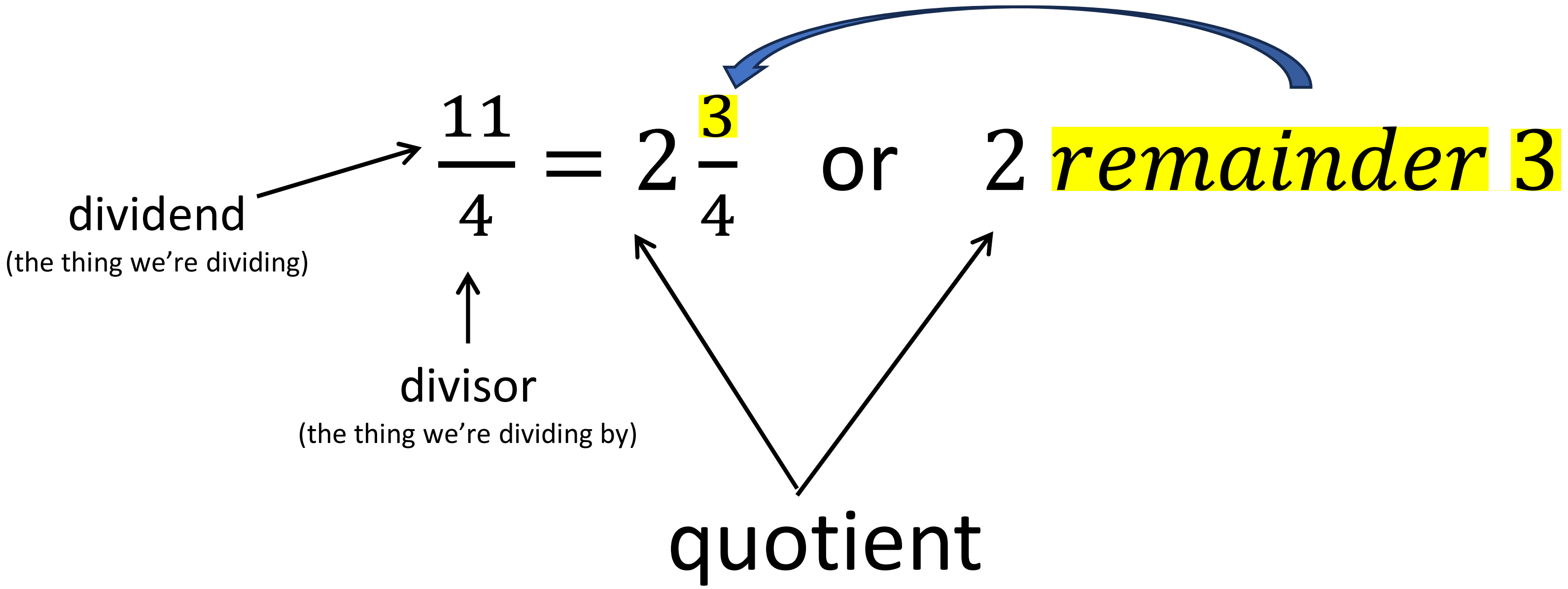
5 Complete the square for the expressions:

a $x^2 - 2x - 20$ **b** $2x^2 + 4x + 15$

← **Section 2.2**

PR work from 7.1 on DFM:
Use K201a, K201b, K201c, K201d and K201e

7.2 Dividing Polynomials



$$\frac{11}{4} = 2\frac{3}{4} \quad \text{or} \quad 2\ remainder\ 3$$

dividend
(the thing we're dividing)

divisor
(the thing we're dividing by)

quotient

Notes

$$\frac{3x+6}{3} = x+2 \quad , \quad \text{why?}$$

think multiplication:

$$\begin{array}{c|l} & x \quad +2 \\ \hline 3 & 3x + 6 \end{array} \qquad r=0$$

(remainder)

What about: $\frac{3x+7}{3} \Rightarrow$

$$\begin{array}{c|l} & x \quad +2 \\ \hline 3 & 3x \quad +6 \end{array} \qquad r=1 \leftarrow \text{remainder.}$$

step ①: need $3 \times x$ as we require $3x$ term

step②: choose "+2" because two 'whole' 3's go into 7

---

now

$$\begin{array}{c|l} & x \quad +2 \\ \hline 3 & \underbrace{3x + 6}_{\text{dividend}} \end{array} \qquad r=1 \quad \Rightarrow \quad \frac{3x+7}{3} = \underbrace{x+2}_{q} + \frac{1}{3}$$

quotient q.

divisor (d)

$\leftarrow r$

$\leftarrow d$

Notes

similarly : $\dfrac{3x+7}{x+1} \Rightarrow$

①

$$\begin{array}{c|c} & 3 \\ \hline x & 3x \\ +1 & +3 \end{array}$$

$\Rightarrow$

②

$$\begin{array}{c|c} & 3 \\ \hline x & 3x \\ +1 & +3 \end{array}$$

no more terms (already at x^0)

$\Downarrow$

$$\begin{array}{c|c} & 3 \\ \hline x & 3x \\ +1 & +3 \end{array} \quad r=4$$

$$\frac{3x+7}{x+1} = 3 + \frac{4}{x+1} \Leftarrow$$

---

then : $\dfrac{3x^2+7x}{x+1} \Rightarrow$

①

$$\begin{array}{c|c} & 3x \\ \hline x & 3x^2 \\ +1 & +3x \end{array}$$

$\Rightarrow$

②

$$\begin{array}{c|cc} & 3x & +4 \\ \hline x & 3x^2 & +4x \\ +1 & +3x & +4 \end{array}$$

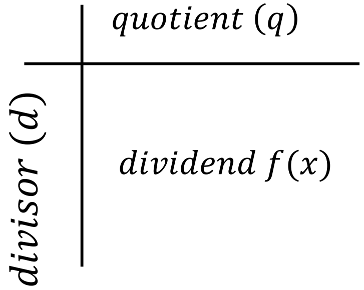
③ $r=3$

So $$\frac{3x^2+7x}{x+1} = 3x+4 + \frac{3}{x+1}$$

Notes

Generally:

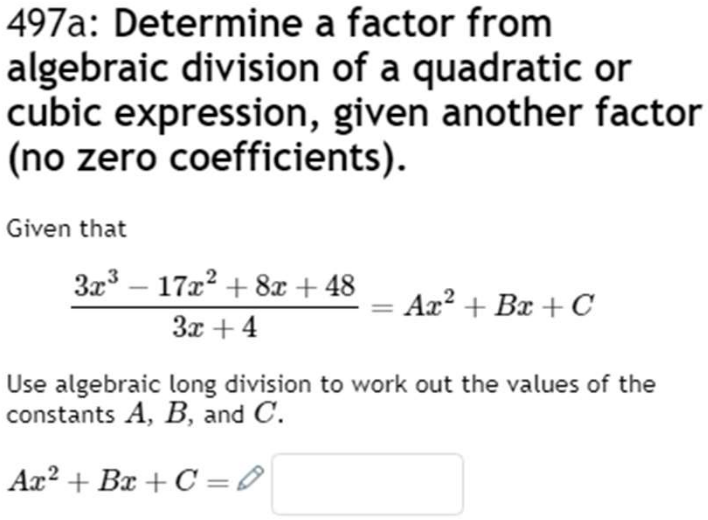
$$\frac{f(x)}{divisor(d)} = quotient\ (q) + \frac{remainder\ (r)}{divisor\ (d)}$$



STEPS:

1. Pick first part of q by making highest power
2. Fill in rest of column
3. Pick next part of q to balance next highest power
4. Repeat until q is a constant term

Worked Example



497a: Determine a factor from algebraic division of a quadratic or cubic expression, given another factor (no zero coefficients).

Given that

$$\frac{3x^3 - 17x^2 + 8x + 48}{3x + 4} = Ax^2 + Bx + C$$

Use algebraic long division to work out the values of the constants A, B, and C.

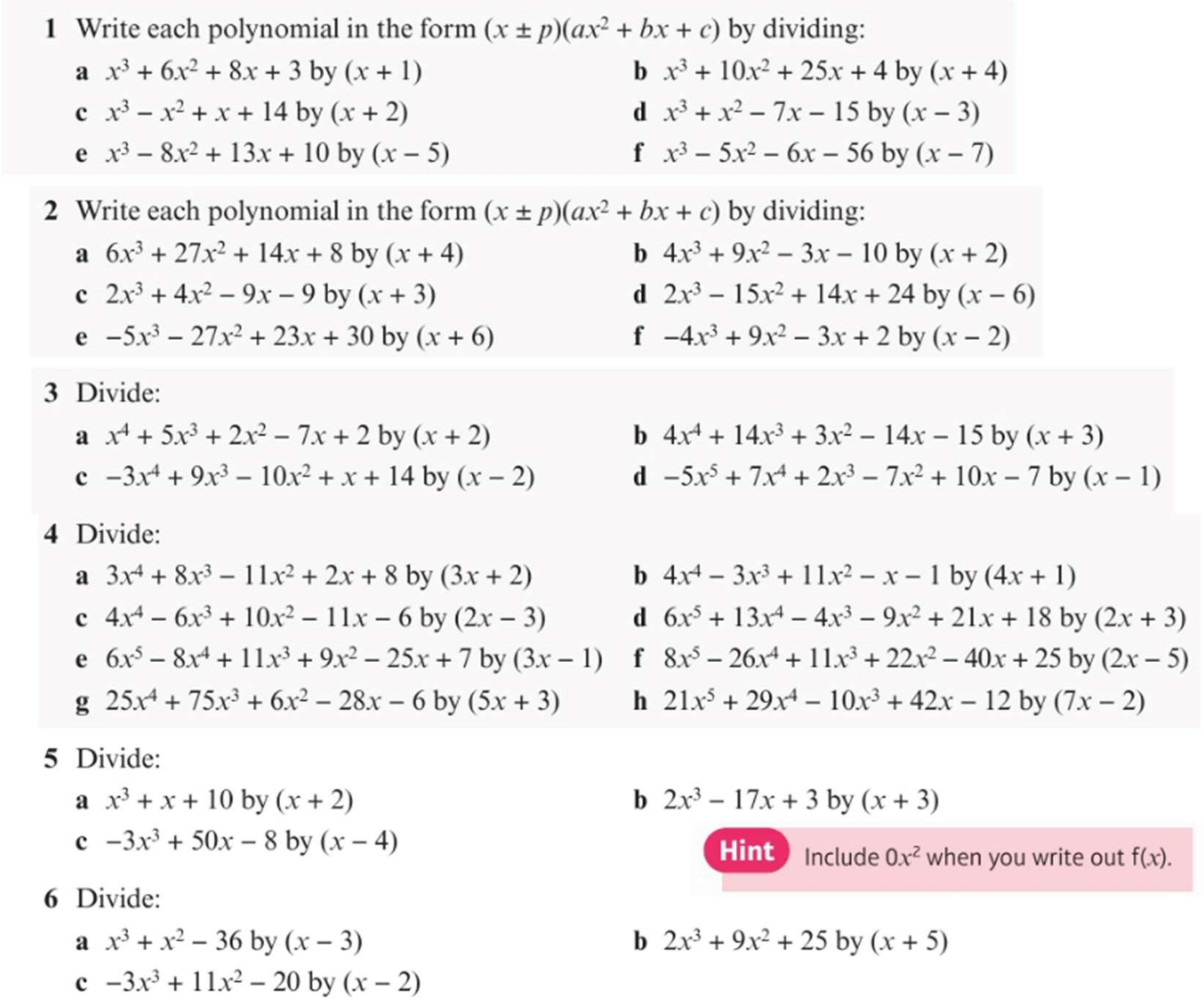
$Ax^2 + Bx + C =$ ✎ ☐

Worked Example

$f(x) = 4x^4 - 17x^2 + 4$

Divide $f(x)$ by $(2x + 1)$, giving your answer in the form $f(x) = (2x + 1)(ax^3 + bx^2 + cx + d)$.

Exercise 7B



1 Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $x^3 + 6x^2 + 8x + 3$ by $(x + 1)$

b $x^3 + 10x^2 + 25x + 4$ by $(x + 4)$

c $x^3 - x^2 + x + 14$ by $(x + 2)$

d $x^3 + x^2 - 7x - 15$ by $(x - 3)$

e $x^3 - 8x^2 + 13x + 10$ by $(x - 5)$

f $x^3 - 5x^2 - 6x - 56$ by $(x - 7)$

2 Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $6x^3 + 27x^2 + 14x + 8$ by $(x + 4)$

b $4x^3 + 9x^2 - 3x - 10$ by $(x + 2)$

c $2x^3 + 4x^2 - 9x - 9$ by $(x + 3)$

d $2x^3 - 15x^2 + 14x + 24$ by $(x - 6)$

e $-5x^3 - 27x^2 + 23x + 30$ by $(x + 6)$

f $-4x^3 + 9x^2 - 3x + 2$ by $(x - 2)$

3 Divide:

a $x^4 + 5x^3 + 2x^2 - 7x + 2$ by $(x + 2)$

b $4x^4 + 14x^3 + 3x^2 - 14x - 15$ by $(x + 3)$

c $-3x^4 + 9x^3 - 10x^2 + x + 14$ by $(x - 2)$

d $-5x^5 + 7x^4 + 2x^3 - 7x^2 + 10x - 7$ by $(x - 1)$

4 Divide:

a $3x^4 + 8x^3 - 11x^2 + 2x + 8$ by $(3x + 2)$

b $4x^4 - 3x^3 + 11x^2 - x - 1$ by $(4x + 1)$

c $4x^4 - 6x^3 + 10x^2 - 11x - 6$ by $(2x - 3)$

d $6x^5 + 13x^4 - 4x^3 - 9x^2 + 21x + 18$ by $(2x + 3)$

e $6x^5 - 8x^4 + 11x^3 + 9x^2 - 25x + 7$ by $(3x - 1)$

f $8x^5 - 26x^4 + 11x^3 + 22x^2 - 40x + 25$ by $(2x - 5)$

g $25x^4 + 75x^3 + 6x^2 - 28x - 6$ by $(5x + 3)$

h $21x^5 + 29x^4 - 10x^3 + 42x - 12$ by $(7x - 2)$

5 Divide:

a $x^3 + x + 10$ by $(x + 2)$

b $2x^3 - 17x + 3$ by $(x + 3)$

c $-3x^3 + 50x - 8$ by $(x - 4)$

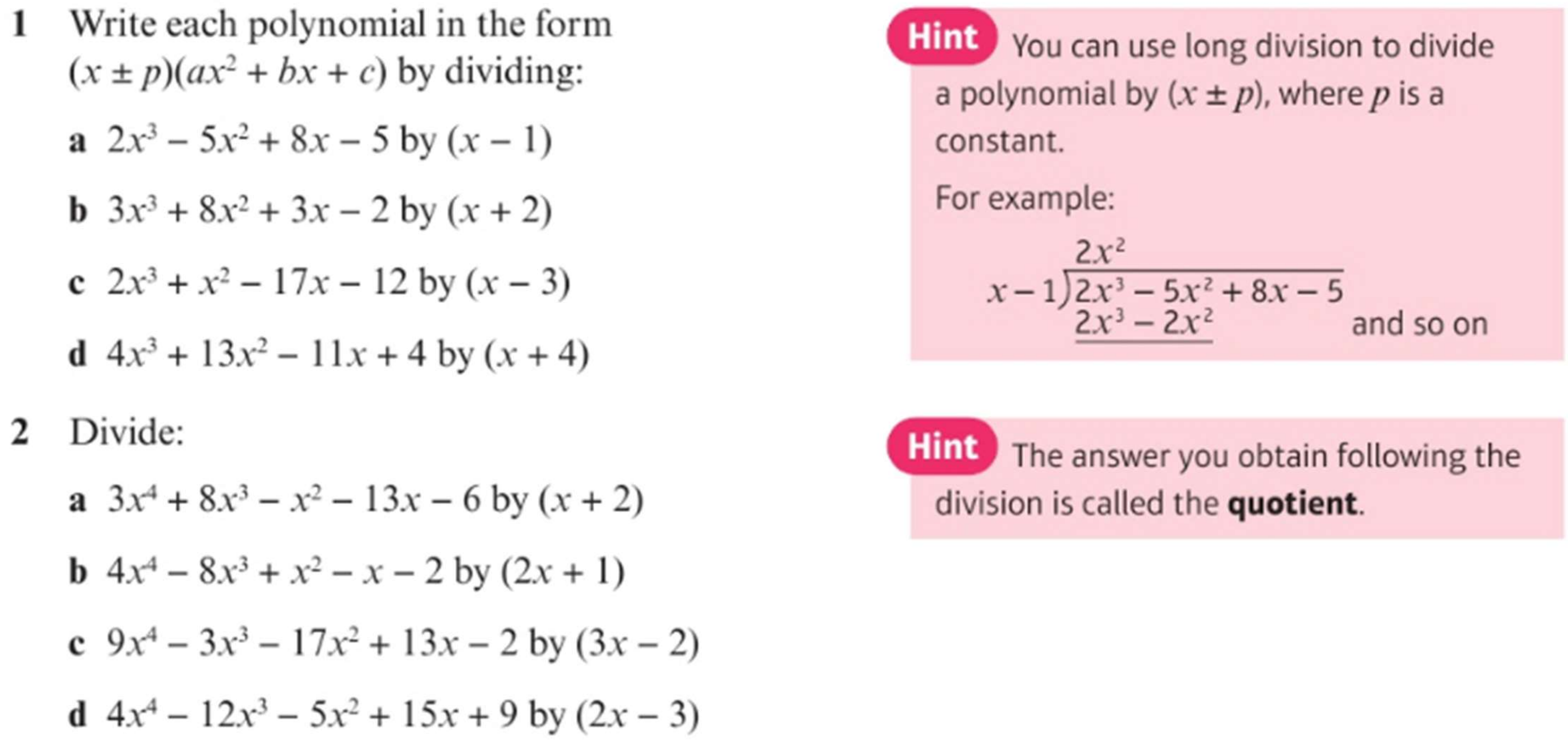
Hint Include $0x^2$ when you write out $f(x)$.

6 Divide:

a $x^3 + x^2 - 36$ by $(x - 3)$

b $2x^3 + 9x^2 + 25$ by $(x + 5)$

c $-3x^3 + 11x^2 - 20$ by $(x - 2)$



1 Write each polynomial in the form $(x \pm p)(ax^2 + bx + c)$ by dividing:

a $2x^3 - 5x^2 + 8x - 5$ by $(x - 1)$

b $3x^3 + 8x^2 + 3x - 2$ by $(x + 2)$

c $2x^3 + x^2 - 17x - 12$ by $(x - 3)$

d $4x^3 + 13x^2 - 11x + 4$ by $(x + 4)$

2 Divide:

a $3x^4 + 8x^3 - x^2 - 13x - 6$ by $(x + 2)$

b $4x^4 - 8x^3 + x^2 - x - 2$ by $(2x + 1)$

c $9x^4 - 3x^3 - 17x^2 + 13x - 2$ by $(3x - 2)$

d $4x^4 - 12x^3 - 5x^2 + 15x + 9$ by $(2x - 3)$

Hint You can use long division to divide a polynomial by $(x \pm p)$, where p is a constant.

For example:

$$\begin{array}{r|l} & 2x^2 \\ \cline{2-2} x - 1 & 2x^3 - 5x^2 + 8x - 5 \\ & \underline{2x^3 - 2x^2} \end{array}$$

and so on

Hint The answer you obtain following the division is called the **quotient**.

Worked Example

Find the remainder when $2x^3 - 5x^2 - 16x + 10$ is divided by $(x - 4)$.

Worked Example

Divide $8x^3 - 1$ by $(2x - 1)$.

Worked Example

$f(x) = 12x^3 - 4x^2 - 61x + 60$

Show that $(2x - 3)$ is a factor of $f(x)$ and hence find all the real roots of the equation $f(x) = 0$

Exercise 7B

7 Show that $x^3 + 2x^2 - 5x - 10 = (x + 2)(x^2 - 5)$

8 Find the remainder when:

a $x^3 + 4x^2 - 3x + 2$ is divided by $(x + 5)$

b $3x^3 - 20x^2 + 10x + 5$ is divided by $(x - 6)$

c $-2x^3 + 3x^2 + 12x + 20$ is divided by $(x - 4)$

9 Show that when $3x^3 - 2x^2 + 4$ is divided by $(x - 1)$ the remainder is 5.

10 Show that when $3x^4 - 8x^3 + 10x^2 - 3x - 25$ is divided by $(x + 1)$ the remainder is -1.

11 Show that $(x + 4)$ is a factor of $5x^3 - 73x + 28$.

12 Simplify $\dfrac{3x^3 - 8x - 8}{x - 2}$

Hint Divide $3x^3 - 8x - 8$ by $(x - 2)$.

13 Divide $x^3 - 1$ by $(x - 1)$.

Hint Write $x^3 - 1$ as $x^3 + 0x^2 + 0x - 1$.

14 Divide $x^4 - 16$ by $(x + 2)$.

E **15** $f(x) = 10x^3 + 43x^2 - 2x - 10$

Find the remainder when $f(x)$ is divided by $(5x + 4)$. **(2 marks)**

E/P **16** $f(x) = 3x^3 - 14x^2 - 47x - 14$

a Find the remainder when $f(x)$ is divided by $(x - 3)$. **(2 marks)**

b Given that $(x + 2)$ is a factor of $f(x)$, factorise $f(x)$ completely. **(4 marks)**

Problem-solving

Write $f(x)$ in the form $(x + 2)(ax^2 + bx + c)$ then factorise the quadratic factor.

E/P **17** **a** Find the remainder when $x^3 + 6x^2 + 5x - 12$ is divided by

i $x - 2$,

ii $x + 3$. **(3 marks)**

b Hence, or otherwise, find all the solutions to the equation $x^3 + 6x^2 + 5x - 12 = 0$. **(4 marks)**

E/P **18** $f(x) = 2x^3 + 3x^2 - 8x + 3$

a Show that $f(x) = (2x - 1)(ax^2 + bx + c)$ where a, b and c are constants to be found. **(2 marks)**

b Hence factorise $f(x)$ completely. **(4 marks)**

c Write down all the real roots of the equation $f(x) = 0$. **(2 marks)**

E/P **19** $f(x) = 12x^3 + 5x^2 + 2x - 1$

a Show that $(4x - 1)$ is a factor of $f(x)$ and write $f(x)$ in the form $(4x - 1)(ax^2 + bx + c)$. **(6 marks)**

b Hence, show that the equation $12x^3 + 5x^2 + 2x - 1 = 0$ has exactly 1 real solution. **(2 marks)**

3 Divide:

a $2x^3 + 6x^2 - 4$ by $(x + 1)$

b $3x^3 + 7x^2 + 18$ by $(x + 3)$

c $4x^3 - 11x - 10$ by $(x - 2)$

d $2x^3 + 7x^2 + 75$ by $(x + 5)$

Hint Write the polynomial in part **a** as $2x^3 + 6x^2 + 0x - 4$ before dividing.

4 Find the remainder when:

a $x^3 + 3x^2 + 5x - 8$ is divided by $(x + 4)$

b $2x^3 - 5x^2 + 12x - 20$ is divided by $(x - 3)$

c $3x^3 + 2x^2 - 40x + 45$ is divided by $(x + 5)$

Hint If there is a remainder, then the linear expression $(x \pm p)$ is not a factor.

The polynomial can be written as $(x \pm p)(ax^2 + bx + c) + r$ where r is the remainder.

E **5** Find the remainder when $-15x^3 + 26x^2 - 13x + 5$ is divided by $(5x - 2)$. **(2 marks)**

E/P **6** $f(x) = 6x^3 - 13x^2 - 13x + 30$

a Find the remainder when $f(x)$ is divided by $(x + 3)$. **(2 marks)**

b Given that $(x - 2)$ is a factor of $f(x)$, factorise $f(x)$ completely. **(4 marks)**

E/P **7** $f(x) = 2x^3 + 3x^2 - 4x + k$ where k is a constant.

Given that $(x + 3)$ is a factor of $f(x)$:

a find the value of k **(2 marks)**

b express $f(x)$ in the form $(x + 3)(ax^2 + bx + c)$ where a, b and c are constants **(2 marks)**

c show that $f(x) = 0$ has exactly one real solution. **(2 marks)**

E/P **8** $f(x) = 3x^3 + 10x^2 - 8x - 5$

a Find the remainder, r, when $f(x)$ is divided by $(x - 2)$. **(2 marks)**

b Express $f(x)$ in the form $(x - 2)(ax^2 + bx + c) + r$ where a, b, c and r are constants. **(4 marks)**

E/P **9** $f(x) = 10x^3 - 29x^2 + 4x + 15$

a Given that $(x - 1)$ is a factor of $f(x)$, express $f(x)$ in the form $(x - 1)(ax^2 + bx + c)$, where a, b and c are constants. **(2 marks)**

b Hence factorise $f(x)$ completely. **(4 marks)**

c Write down all the solutions to the equation $f(x) = 0$. **(2 marks)**

7.3 The Factor Theorem

$$x^3 + x^2 - 4x - 4 = (x - 2)(x^2 + 3x + 2)$$

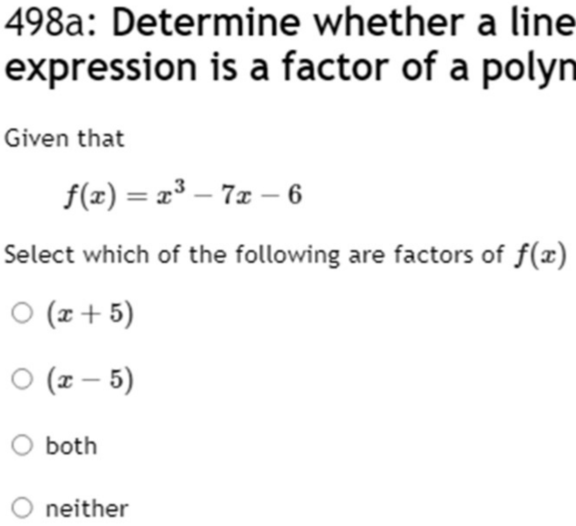
We can see that $(x - 2)$ is a factor of $x^3 + x^2 - 4x - 4$.
What would happen if x is 2?

$\mathbf{2 - 2 = 0}$ so the RHS, and hence LHS would be 0.
The converse is also true: if we could find a value $\boldsymbol{a}$ such that the LHS is 0 when we substitute in $\boldsymbol{a}$ for $\boldsymbol{x}$, then $(\boldsymbol{x} - \boldsymbol{a})$ would be a factor.

The Factor Theorem states that if $f(x)$ is a polynomial then:
- If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$.
- Conversely, if $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$.

Notes

Worked Example



498a: Determine whether a linear expression is a factor of a polynomial.

Given that

$$f(x) = x^3 - 7x - 6$$

Select which of the following are factors of $f(x)$

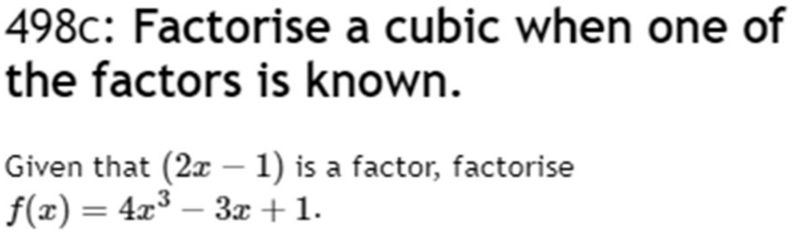
○ $(x+5)$

○ $(x-5)$

○ both

○ neither

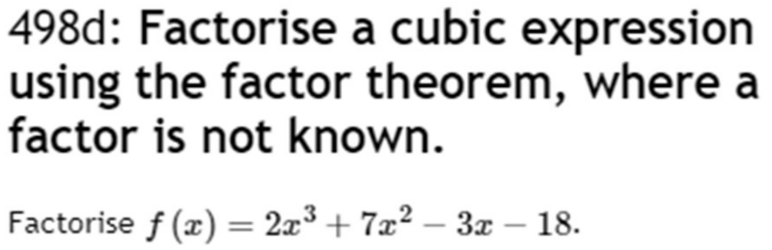
Worked Example



498c: Factorise a cubic when one of the factors is known.

Given that $(2x-1)$ is a factor, factorise
$f(x) = 4x^3 - 3x + 1$.

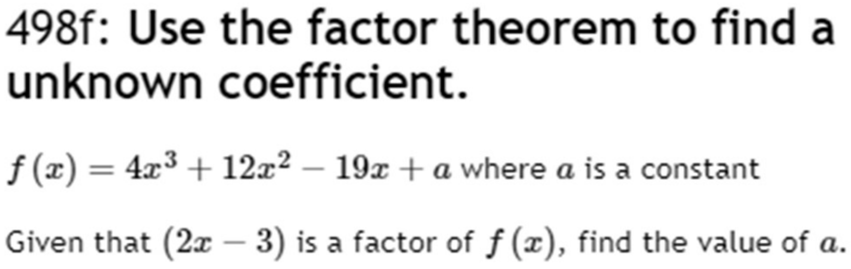
Worked Example



498d: Factorise a cubic expression using the factor theorem, where a factor is not known.

Factorise $f(x) = 2x^3 + 7x^2 - 3x - 18$.

Worked Example

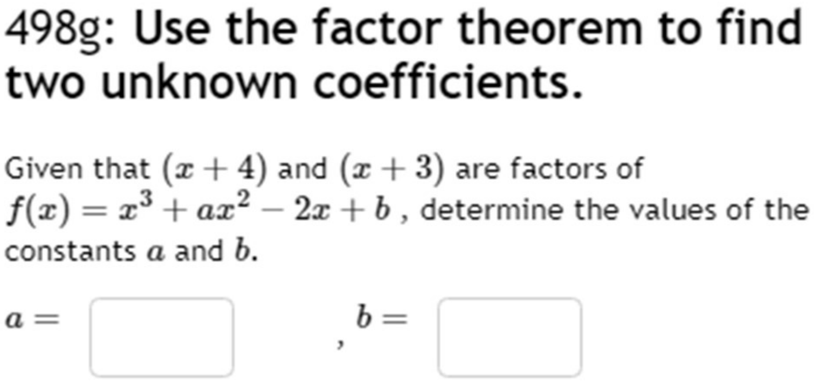


498f: Use the factor theorem to find a unknown coefficient.

$f(x) = 4x^3 + 12x^2 - 19x + a$ where a is a constant

Given that $(2x - 3)$ is a factor of $f(x)$, find the value of a.

Worked Example

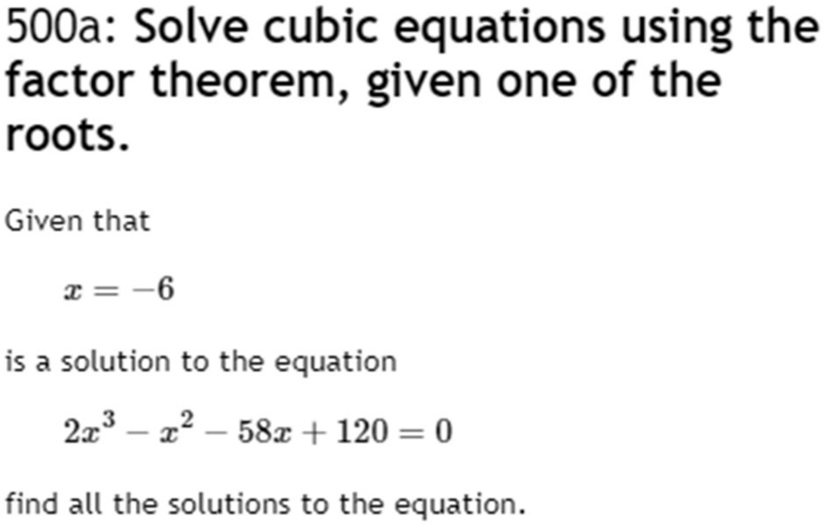


498g: Use the factor theorem to find two unknown coefficients.

Given that $(x+4)$ and $(x+3)$ are factors of $f(x) = x^3 + ax^2 - 2x + b$, determine the values of the constants a and b.

$a =$, $b =$

Worked Example



500a: Solve cubic equations using the factor theorem, given one of the roots.

Given that

$$x = -6$$

is a solution to the equation

$$2x^3 - x^2 - 58x + 120 = 0$$

find all the solutions to the equation.

Worked Example

a) Fully factorise $2x^3 + x^2 - 18x - 9$

b) Hence sketch the graph of $y = 2x^3 + x^2 - 18x - 9$

Exercise 7C

1 Use the factor theorem to show that:

a $(x - 1)$ is a factor of $4x^3 - 3x^2 - 1$

b $(x + 3)$ is a factor of $5x^4 - 45x^2 - 6x - 18$

c $(x - 4)$ is a factor of $-3x^3 + 13x^2 - 6x + 8$.

2 Show that $(x - 1)$ is a factor of $x^3 + 6x^2 + 5x - 12$ and hence factorise the expression completely.

3 Show that $(x + 1)$ is a factor of $x^3 + 3x^2 - 33x - 35$ and hence factorise the expression completely.

4 Show that $(x - 5)$ is a factor of $x^3 - 7x^2 + 2x + 40$ and hence factorise the expression completely.

5 Show that $(x - 2)$ is a factor of $2x^3 + 3x^2 - 18x + 8$ and hence factorise the expression completely.

6 Each of these expressions has a factor $(x \pm p)$. Find a value of p and hence factorise the expression completely.

a $x^3 - 10x^2 + 19x + 30$

b $x^3 + x^2 - 4x - 4$

c $x^3 - 4x^2 - 11x + 30$

7 i Fully factorise the right-hand side of each equation.

ii Sketch the graph of each equation.

a $y = 2x^3 + 5x^2 - 4x - 3$

b $y = 2x^3 - 17x^2 + 38x - 15$

c $y = 3x^3 + 8x^2 + 3x - 2$

d $y = 6x^3 + 11x^2 - 3x - 2$

e $y = 4x^3 - 12x^2 - 7x + 30$

Ⓟ **8** Given that $(x - 1)$ is a factor of $5x^3 - 9x^2 + 2x + a$, find the value of a.

Ⓟ **9** Given that $(x + 3)$ is a factor of $6x^3 - bx^2 + 18$, find the value of b.

Ⓟ **10** Given that $(x - 1)$ and $(x + 1)$ are factors of $px^3 + qx^2 - 3x - 7$, find the values of p and q.

Ⓟ **11** Given that $(x + 1)$ and $(x - 2)$ are factors of $cx^3 + dx^2 - 9x - 10$, find the values of c and d.

Problem-solving

Use the factor theorem to form simultaneous equations.

Ⓟ **12** Given that $(x + 2)$ and $(x - 3)$ are factors of $gx^3 + hx^2 - 14x + 24$, find the values of g and h.

Ⓔ **13** $f(x) = 3x^3 - 12x^2 + 6x - 24$

a Use the factor theorem to show that $(x - 4)$ is a factor of $f(x)$. **(2 marks)**

b Hence, show that 4 is the only real root of the equation $f(x) = 0$. **(4 marks)**

Ⓔ **14** $f(x) = 4x^3 + 4x^2 - 11x - 6$

a Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$. **(2 marks)**

b Factorise $f(x)$ completely. **(4 marks)**

c Write down all the solutions of the equation $4x^3 + 4x^2 - 11x - 6 = 0$. **(1 mark)**

Ⓔ **15 a** Show that $(x - 2)$ is a factor of $9x^4 - 18x^3 - x^2 + 2x$. **(2 marks)**

b Hence, find four real solutions to the equation $9x^4 - 18x^3 - x^2 + 2x = 0$. **(5 marks)**

Challenge

$f(x) = 2x^4 - 5x^3 - 42x^2 - 9x + 54$

a Show that $f(1) = 0$ and $f(-3) = 0$.

b Hence, solve $f(x) = 0$.

1 Use the factor theorem to show that:

- **a** $(x + 1)$ is a factor of $2x^3 + 7x^2 - 5$
- **b** $(x + 2)$ is a factor of $x^3 + 4x^2 + 3x - 2$
- **c** $(x - 3)$ is a factor of $2x^3 - 3x^2 - 7x - 6$
- **d** $(x - 4)$ is a factor of $x^4 - 3x^3 - 15x - 4$

Hint The **factor theorem** states that if f(x) is a polynomial, then:

- if f$(p) = 0$ then $(x - p)$ is a factor of f(x)
- if $(x - p)$ is a factor of f(x) then f$(p) = 0$

2 Use the factor theorem to show that the linear expression is a factor of the polynomial f(x) and factorise f(x) completely:

- **a** $(x - 2)$, $2x^3 + x^2 - 13x + 6$
- **b** $(x + 3)$, $2x^3 + 17x^2 + 38x + 15$
- **c** $(x - 1)$, $6x^3 - x^2 - 11x + 6$
- **d** $(x + 4)$, $15x^3 + 61x^2 - 2x - 24$

Hint When you have used the factor theorem to show the linear expression is a factor, you can use long division to find the quadratic factor. Factorise the quadratic factor to write the polynomial as a product of three linear factors.

3 Fully factorise each expression:

- **a** $x^3 + 2x^2 - 21x + 18$
- **b** $2x^3 + 13x^2 + 13x - 10$
- **c** $3x^3 + 2x^2 - 41x - 60$

Hint Try values of p in each expression for f(x), e.g. $p = -1, 1, 2, 3, \dots$ until you find f$(p) = 0$. Then use the factor theorem to deduce that $(x - p)$ is a factor of f(x).

4 For each of the following polynomials,

- **i** fully factorise each polynomial f(x).
- **ii** Hence sketch the graph of $y = $ f(x).

- **a** $2x^3 - 11x^2 + 5x + 18$
- **b** $2x^3 - 3x^2 - 39x + 20$
- **c** $6x^3 + 37x^2 + 50x - 21$

Hint To sketch the graph, you need to identify the points where the curve crosses the axes.

Set $x = 0$ to find the y-intercept and $y = 0$ to find the x-intercepts.

The general shapes of cubic graphs are:

if the coefficient of x^3 is positive

if the coefficient of x^3 is negative

← Section 4.1

E/P 5 f$(x) = 6x^3 - 17x^2 - 15x + 36$
Given that $(x - 3)$ is a factor of f(x), find all the solutions to f$(x) = 0$. **(5 marks)**

E/P 6 f$(x) = 9x^3 + 24x^2 - 44x + 16$

- **a** Use the factor theorem to show that $(x + 4)$ is a factor of f(x). **(2 marks)**
- **b** Hence show that f(x) can be written in the form f$(x) = (x + 4)(px + q)^2$, where p and q are integers to be found. **(4 marks)**

E 7 f$(x) = 2x^3 - 3x^2 - 5x + 6$. Factorise f$(x)$ completely. **(5 marks)**

E/P 8 g$(x) = x^3 + 2x^2 - 19x + k$

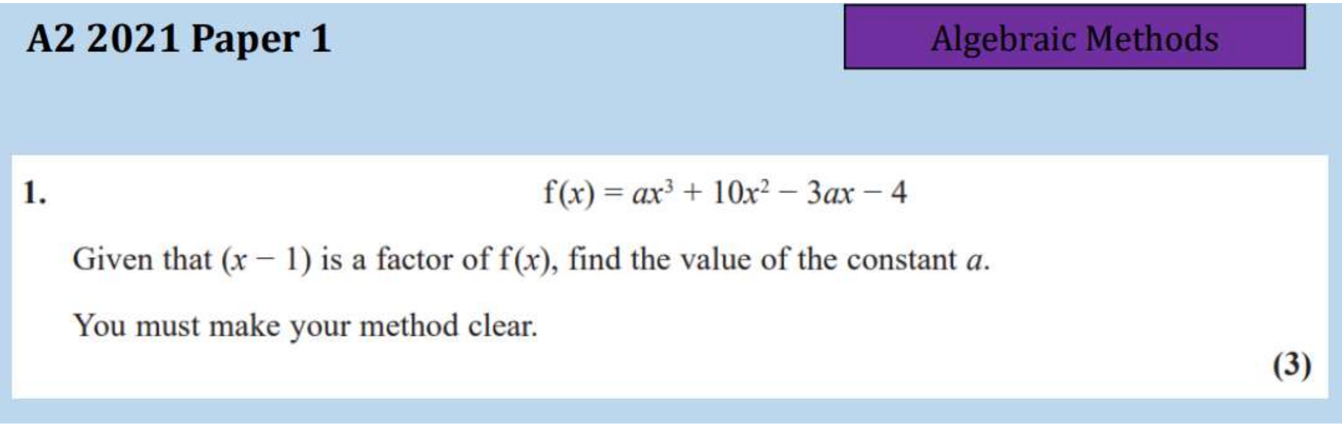
Given that $(x + 1)$ is a factor of g(x),

- **a** show that $k = -20$ **(2 marks)**
- **b** express g(x) as a product of three linear factors. **(3 marks)**
- **c** Sketch the curve with equation $y = x^3 + 2x^2 - 19x - 20$, indicating the values where the curve crosses the x-axis and the y-axis. **(4 marks)**

E/P 9 p$(x) = 25x^3 + 55x^2 - 56x + 12$

- **a** Use the factor theorem to show that $(x + 3)$ is a factor of p(x). **(2 marks)**
- **b** Fully factorise p(x). **(3 marks)**
- **c** Hence show that there are exactly two real roots of the equation p$(x) = 0$. **(2 marks)**

Past Paper Questions



A2 2021 Paper 1

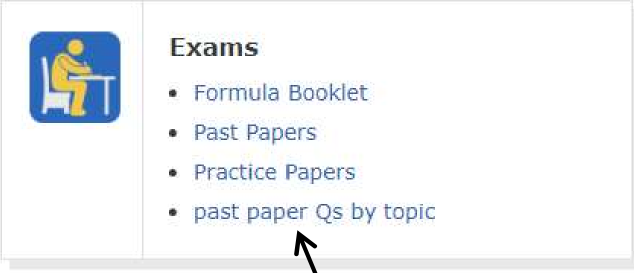
Algebraic Methods

1. $$f(x) = ax^3 + 10x^2 - 3ax - 4$$

Given that $(x - 1)$ is a factor of $f(x)$, find the value of the constant a.

You must make your method clear.

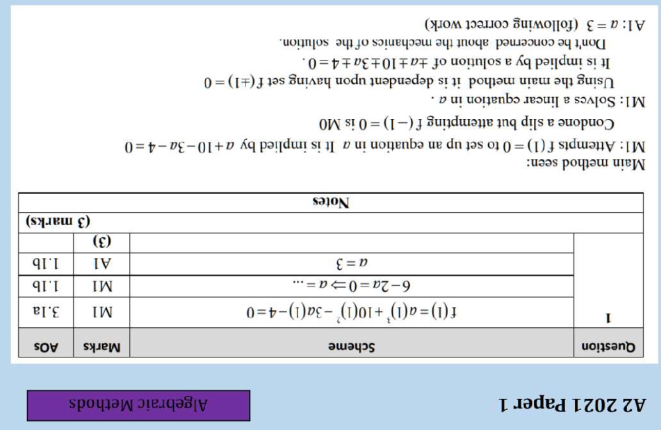
(3)



Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



A2 2021 Paper 1

Algebraic Methods

Question	Scheme	Marks	AOs
1	$f(1) = a(1)^3 + 10(1)^2 - 3a(1) - 4 = 0$	M1	3.1a
	$6 - 2a = 0 \Rightarrow a = \ldots$	M1	1.1b
	$a = 3$	A1	1.1b
		(3)	
			(3 marks)

Notes

Main method seen:

M1: Attempts $f(1) = 0$ to set up an equation in a It is implied by $a + 10 - 3a - 4 = 0$

Condone a slip but attempting $f(-1) = 0$ is M0

M1: Solves a linear equation in a .

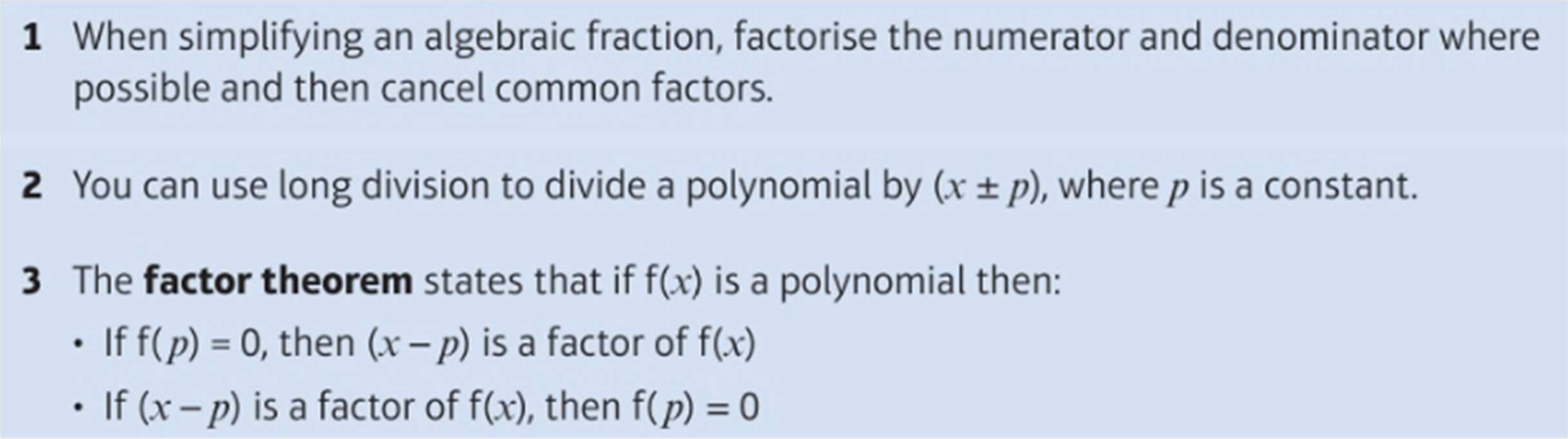
Using the main method it is dependent upon having set $f(\pm 1) = 0$

It is implied by a solution of $\pm a \pm 10 \pm 3a \pm 4 = 0$.

Don't be concerned about the mechanics of the solution.

A1: $a = 3$ (following correct work)

Summary of Key Points



1 When simplifying an algebraic fraction, factorise the numerator and denominator where possible and then cancel common factors.

2 You can use long division to divide a polynomial by $(x \pm p)$, where p is a constant.

3 The **factor theorem** states that if $f(x)$ is a polynomial then:

- If $f(p) = 0$, then $(x - p)$ is a factor of $f(x)$
- If $(x - p)$ is a factor of $f(x)$, then $f(p) = 0$

Mixed Exercise

1 Simplify these fractions as far as possible:

a $\frac{3x^4 - 21x}{3x}$

b $\frac{x^2 - 2x - 24}{x^2 - 7x + 6}$

c $\frac{2x^2 + 7x - 4}{2x^2 + 9x + 4}$

2 Divide $3x^3 + 12x^2 + 5x + 20$ by $(x + 4)$.

3 Simplify $\frac{2x^3 + 3x + 5}{x + 1}$

Ⓔ 4 a Show that $(x - 3)$ is a factor of $2x^3 - 2x^2 - 17x + 15$. **(2 marks)**

b Hence express $2x^3 - 2x^2 - 17x + 15$ in the form $(x - 3)(Ax^2 + Bx + C)$, where the values A, B and C are to be found. **(3 marks)**

Ⓔ 5 a Show that $(x - 2)$ is a factor of $x^3 + 4x^2 - 3x - 18$. **(2 marks)**

b Hence express $x^3 + 4x^2 - 3x - 18$ in the form $(x - 2)(px + q)^2$, where the values p and q are to be found. **(4 marks)**

Ⓔ 6 Factorise completely $2x^3 + 3x^2 - 18x + 8$. **(6 marks)**

E/P 7 Find the value of k if $(x - 2)$ is a factor of $x^3 - 3x^2 + kx - 10$. **(4 marks)**

E/P 8 $f(x) = 2x^2 + px + q$. Given that $f(-3) = 0$, and $f(4) = 21$:

a find the value of p and q **(6 marks)**

b factorise $f(x)$. **(3 marks)**

E/P 9 $h(x) = x^3 + 4x^2 + rx + s$. Given $h(-1) = 0$, and $h(2) = 30$:

a find the values of r and s **(6 marks)**

b factorise $h(x)$. **(3 marks)**

Ⓔ 10 $g(x) = 2x^3 + 9x^2 - 6x - 5$.

a Factorise $g(x)$. **(6 marks)**

b Solve $g(x) = 0$. **(2 marks)**

Mixed Exercise

Ⓔ **11** **a** Show that $(x - 2)$ is a factor of $f(x) = x^3 + x^2 - 5x - 2$. **(2 marks)**

b Hence, or otherwise, find the exact solutions of the equation $f(x) = 0$. **(4 marks)**

Ⓔ **12** Given that -1 is a root of the equation $2x^3 - 5x^2 - 4x + 3$, find the two positive roots. **(4 marks)**

Ⓔ **13** $f(x) = x^3 - 2x^2 - 19x + 20$

a Show that $(x + 4)$ is a factor of $f(x)$. **(3 marks)**

b Hence, or otherwise, find all the solutions to the equation
$x^3 - 2x^2 - 19x + 20 = 0$. **(4 marks)**

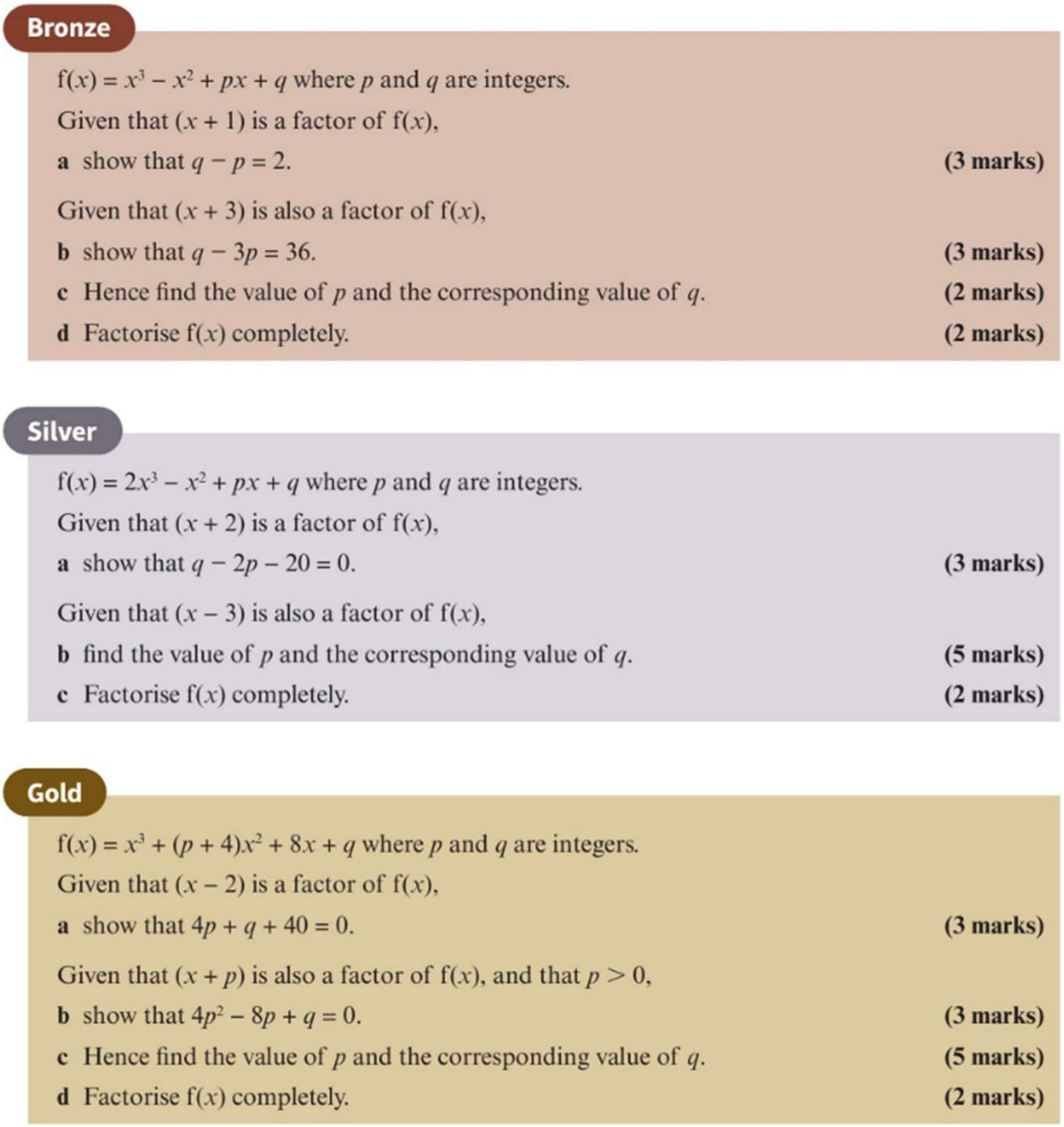
Ⓔ **14** $f(x) = 6x^3 + 17x^2 - 5x - 6$

a Show that $f(x) = (3x - 2)(ax^2 + bx + c)$, where a, b and c are constants to be found. **(2 marks)**

b Hence factorise $f(x)$ completely. **(4 marks)**

c Write down all the real roots of the equation $f(x) = 0$. **(2 marks)**

Problem Solving Set B



Bronze

$f(x) = x^3 - x^2 + px + q$ where p and q are integers.

Given that $(x + 1)$ is a factor of $f(x)$,

a show that $q - p = 2$. **(3 marks)**

Given that $(x + 3)$ is also a factor of $f(x)$,

b show that $q - 3p = 36$. **(3 marks)**

c Hence find the value of p and the corresponding value of q. **(2 marks)**

d Factorise $f(x)$ completely. **(2 marks)**

Silver

$f(x) = 2x^3 - x^2 + px + q$ where p and q are integers.

Given that $(x + 2)$ is a factor of $f(x)$,

a show that $q - 2p - 20 = 0$. **(3 marks)**

Given that $(x - 3)$ is also a factor of $f(x)$,

b find the value of p and the corresponding value of q. **(5 marks)**

c Factorise $f(x)$ completely. **(2 marks)**

Gold

$f(x) = x^3 + (p + 4)x^2 + 8x + q$ where p and q are integers.

Given that $(x - 2)$ is a factor of $f(x)$,

a show that $4p + q + 40 = 0$. **(3 marks)**

Given that $(x + p)$ is also a factor of $f(x)$, and that $p > 0$,

b show that $4p^2 - 8p + q = 0$. **(3 marks)**

c Hence find the value of p and the corresponding value of q. **(5 marks)**

d Factorise $f(x)$ completely. **(2 marks)**