KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
11 Variable Acceleration Booklet

HGS Maths

Dr Frost Course

Name: _____

Class: _____

Contents

11.1) Functions of time

11.2) Using differentiation

11.3) Maxima and minima problems

11.4) Using integration

11.5) Constant acceleration formulae

Extract from Formulae booklet
Past Paper Practice
Summary

11.1) Functions of time

Notes

Prior knowledge check

Prior knowledge check

1 Find $\frac{dy}{dx}$ given:

a $y = 3x^2 - 5x + 6$ **b** $y = 2\sqrt{x} + \frac{6}{x^2} - 1$

← Pure Year 1, Chapter 12

2 Find the coordinates of the turning points on the curve with equation:

a $y = 3x^2 - 9x + 2$ **b** $y = x^3 - 6x^2 + 9x + 5$

← Pure Year 1, Chapter 12

3 Find $f(x)$ given:

a $f'(x) = 5x + 8$, $f(0) = 1$

b $f'(x) = 3x^2 - 2x + 5$, $f(0) = 7$

← Pure Year 1, Chapter 13

4 Find the area bounded by the x-axis and:

a the line $y = 2x - 1$, $x = 2$ and $x = 5$

b the curve $y = 6x - 2 - x^2$, $x = 1$ and $x = 3$

← Pure Year 1, Chapter 13

Worked Example

A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by

$$s = 5t^3 - 2t, t > 0$$

Find:

a) s when $t = 3$

b) The time taken for the particle to return to O

T.184: 11A Qs1+2, P.77: Q1

Your Turn

A body moves in a straight line, such that its displacement, s metres, from a point O at time t seconds, is given by

$$s = 2t^3 - 3t, t > 0$$

Find:

a) s when $t = 2$

b) The time taken for the particle to return to O

a) $10\ m$

b) $\sqrt{\frac{3}{2}}\ s = 1.2\ s$ (2 sf)

Worked Example

A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by:

$$s = 8t^2 - 5t^3, \quad 0 \leq t \leq 1.6$$

Explain the restriction $0 \leq t \leq 1.6$

Your Turn

A train travels along a straight track, leaving the start of the track at time $t = 0$. It then returns to the start of the track. The distance, s metres, from the start of the track at time t seconds is modelled by:

$$s = 4t^2 - t^3, 0 \leq t \leq 4$$

Explain the restriction $0 \leq t \leq 4$

s is the distance from the start of the track: $s \geq 0$

$$4t^2 - t^3 \geq 0$$

$$t^2(4 - t) \geq 0$$

$t^2 \geq 0$ for all t and $(4 - t) < 0$ for all $t > 4$.

So $t^2(4 - t)$ is only non-negative for $t \leq 4$

Motion begins at $t = 0$, hence $t \geq 0$

Hence $0 \leq t \leq 4$

Worked Example

A body moves in a straight line such that its velocity, $v\ ms^{-1}$, at time t seconds is given by $v = 3t^2 - 24t + 36$. Find

(a) The initial velocity

(b) The values of t when the body is instantaneously at rest.

(c) The value of t when the velocity is $63\ ms^{-1}$.

(d) The greatest speed of the body in the interval $0 \leq t \leq 7$.

T.184: 11A Qs3+, P.77: Qs2+

Your Turn

A body moves in a straight line such that its velocity, $v\ ms^{-1}$, at time t seconds is given by $v = 2t^2 - 16t + 24$. Find

(a) The initial velocity

(b) The values of t when the body is instantaneously at rest.

(c) The value of t when the velocity is $64\ ms^{-1}$.

(d) The greatest speed of the body in the interval $0 \leq t \leq 5$.

a) $24\ ms^{-1}$

b) $t = 2, t = 6$

c) $t = 10$

d) $24\ ms^{-1}$

11.2) Using differentiation

Notes

Worked Example – K558 a/d

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = 3t^4 - 96t + 7$$

Find:

(a) the velocity of P when $t = 5$
(b) The value of t when P is instantaneously at rest
(c) The acceleration of P when $t = 0.5$

T.185: 11B Qs1-3, P.78: Qs1-3

Your Turn

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = t^4 - 32t + 14$$

Find:

(a) the velocity of P when $t = 3$
(b) The value of t when P is instantaneously at rest
(c) The acceleration of P when $t = 1.5$

a) $76\ ms^{-1}$
b) $t = 2$
c) $27\ ms^{-2}$

Worked Example

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = \frac{1}{3}t^3 - \frac{7}{2}t^2 + 12t + 15$$

Find the distance between the two points at which the particle is at rest.

T.185: 11B Qs4+, P.78: Qs4+

Your Turn

A particle P is moving on the x-axis.
At time t seconds, the displacement x metres from O is given by

$$x = \frac{1}{3}t^3 - \frac{11}{2}t^2 + 30t + 5$$

Find the distance between the two points at which the particle is at rest.

$0.17\ m$ (2s f)

11.3) Maxima and minima problems

Notes

Worked Example

A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight line before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 2.4t - 0.4t^2 - 0.4t^3, \qquad 0 \leq t \leq 2$$

(a) Justify the restriction $0 \leq t \leq 2$

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

T.187: 11C Qs1-2, P.79: Q 1

Your Turn

A child is playing with a yo-yo. The yo-yo leaves the child's hand at time $t = 0$ and travels vertically in a straight line before returning to the child's hand. The distance, s m, of the yo-yo from the child's hand after time t seconds is given by:

$$s = 0.6t + 0.4t^2 - 0.2t^3, \qquad 0 \leq t \leq 3$$

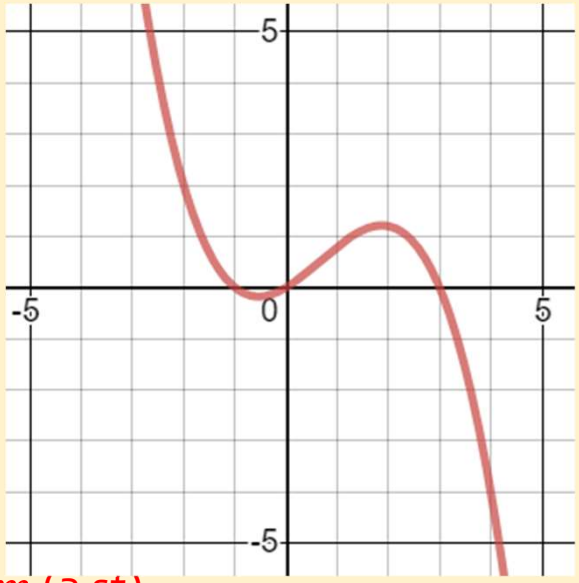
(a) Justify the restriction $0 \leq t \leq 3$

(b) Find the maximum distance of the yo-yo from the child's hand, correct to 3sf.

a) $s = 0.2t(3 + 2t - t^2) = 0.2t(3 - t)(1 + t)$

$t \geq 0$ as time cannot be negative.

If $t > 3, s < 0$ (but distance cannot be negative)



b) 1.21 m (3 sf)

Worked Example

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = \frac{5}{3}t^3 - 18t^2 + 36t$$

Find the maximum velocity of the particle

T.187: 11C Q 2, P.79: Q 2-3

Your Turn

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = t^3 - 16t^2 + 64t$$

Find the maximum velocity of the particle

$75.9\ ms^{-1}$ (3 sf)

Worked Example

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = 3t^2 - 21t + 30, t \geq 0$$

Find the maximum speed of the particle

T.187: 11C Q 3+, P.79: Q 4+

Your Turn

A particle P is moving along the x-axis. At time t seconds, the velocity of P in the direction of x increasing, is:

$$v = 2t^2 - 14t + 20, t \geq 0$$

Find the maximum speed of the particle

$20\ ms^{-1}$

11.4) Using integration

Notes

Worked Example k558f/h

A particle is moving on the x-axis.
At time $t = 0$, the particle is at the point where $x = 7$.
The velocity of the particle at time t seconds (where $t \geq 0$) is $(8t - 3t^2)$ ms^{-1}. Find:
(a) An expression for the displacement of the particle from O at time t seconds.
(b) The distance of the particle from its starting point when $t = 4$.

T.189 11D Q 1+3, P.81: Q 2

Your Turn

A particle is moving on the x-axis.
At time $t = 0$, the particle is at the point where $x = 5$.
The velocity of the particle at time t seconds (where $t \geq 0$) is $(6t - t^2)$ ms^{-1}. Find:
(a) An expression for the displacement of the particle from O at time t seconds.
(b) The distance of the particle from its starting point when $t = 6$.

a) $x = 3t^2 - \frac{1}{3}t^3 + 5$
b) $36\ m$

Worked Example

A particle travels in a straight line.
After t seconds its velocity, v ms^{-1}, is given by $v = 7 - 6t^2$, $t \geq 0$.
Find the distance travelled by the particle in the fifth second of its motion.

T.189 11D Q 2-3 P.81: Q 1,3

Your Turn

A particle travels in a straight line.
After t seconds its velocity, v ms^{-1}, is given by $v = 5 - 3t^2$, $t \geq 0$.
Find the distance travelled by the particle in the third second of its motion.

$14\ m$

Worked Example

A particle P moves on the positive x-axis.
The velocity of P at time t seconds is $(4t^2 - 9t + 2)ms^{-1}$.
When $t = 0, P$ is $5\ m$ from the origin O. Find:
a) The values of t when P is instantaneously at rest
b) The acceleration of P when $t = 10$
c) The total distance travelled by P in the interval $0 \leq t \leq 3$

T.189 11D Q 4-7 P.81: Q 4-6

Your Turn

A particle P moves on the positive x-axis.
The velocity of P at time t seconds is $(2t^2 - 9t + 4)ms^{-1}$.
When $t = 0, P$ is 15 m from the origin O. Find:
a) The values of t when P is instantaneously at rest
b) The acceleration of P when $t = 5$
c) The total distance travelled by P in the interval $0 \leq t \leq 5$

a) $t = \frac{1}{2}, t = 4$
b) $11\ ms^{-2}$
c) $19.4\ m$ (3 sf)

Worked Example

A particle travels in a straight line such that its acceleration, $a\ ms^{-2}$, at time t seconds, is given by $a = 18t + 6$.
When $t = 2$ seconds, the displacement, s, is 40 metres.
When $t = 3$ seconds, the displacement is 117 metres.
Find:
a) The displacement when $t = 4$ seconds.
b) The velocity when $t = 4$ seconds.

T.189 11D Q 8+ P.81: Q 7+

Your Turn

A particle travels in a straight line such that its acceleration, $a\ ms^{-2}$, at time t seconds, is given by $a = 12t + 4$.
When $t = 1$ seconds, the displacement, s, is 6 metres.
When $t = 2$ seconds, the displacement is 196 metres.
Find:
a) The displacement when $t = 3$ seconds.
b) The velocity when $t = 3$ seconds.

a) $98\ m$
b) $76\ ms^{-1}$

11.5) Constant acceleration formulae

Notes

Worked Example

A particle moves in a straight line with constant acceleration $a\ ms^{-2}$.
Given that its initial velocity is $u\ ms^{-1}$ and its initial displacement is $0\ m$, prove that:

Its velocity, $v\ ms^{-1}$, at time t s is given by $v = u + at$

T.192 11E , P.81

Your Turn

A particle moves in a straight line with constant acceleration $a\ ms^{-2}$.
Given that its initial velocity is $u\ ms^{-1}$ and its initial displacement is $0\ m$, prove that:

Its displacement, $s\ m$, at time t s is given by $s = ut + \frac{1}{2}at^2$

Proof

AS 2019

Variable Acceleration

3. A particle, P, moves along a straight line such that at time t seconds, $t \geqslant 0$, the velocity of P, $v\,\text{m s}^{-1}$, is modelled as

$$v = 12 + 4t - t^2$$

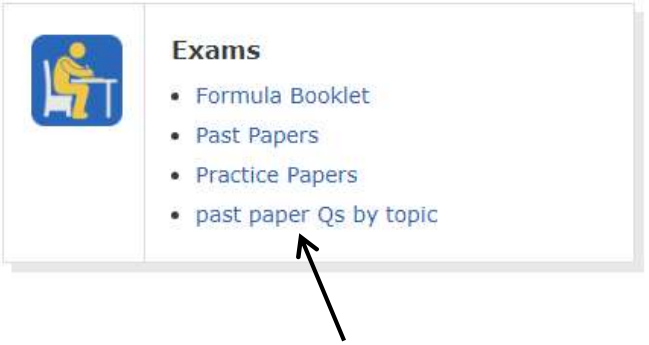
Find

(a) the magnitude of the acceleration of P when P is at instantaneous rest,

(5)

(b) the distance travelled by P in the interval $0 \leqslant t \leqslant 3$

(3)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

	Working or answer as given in mark scheme	Mark	Notes
(a)	$v = 12 + 4t - t^2 = 0$	M1	Attempt to solve for either root
	$(t+2)(t-6) = 0$	A1	Correct root
	$t = 6$		
	$a = \frac{dv}{dt}$	M1	Attempt to differentiate v
	$a = 4 - 2t$	A1	Correct expression for a
	When $t = 6$, $a = -8$	A1	Correct magnitude and direction
	The magnitude of the acceleration is 8		
(b)	$s = \int 12 + 4t - t^2 \, dt$	M1	Attempt to integrate v
	$s = 12t + 2t^2 - \frac{1}{3}t^3 (+c)$	A1	Correct integral for v
	$\left[12t + 2t^2 - \frac{1}{3}t^3\right]_0^3 = 27\ (\text{m})$	A1	Correct answer obtained

Summary of Key Points

1 If the displacement, s, is expressed as a function of t, then the velocity, v, can be expressed as

$$v = \frac{ds}{dt}$$

2 If the velocity, v, is expressed as a function of t, then the acceleration, a, can be expressed as

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

3

Differentiate ↓

displacement $= s = \int v\,dt$

$\frac{ds}{dt}$ = velocity $= v = \int a\,dt$

$\frac{dv}{dt} = \frac{d^2s}{dt^2}$ = acceleration $= a$

↑ Integrate

T.Mixed Ex pgs 193-195 , P.83+84 BSG Qs