KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
M2 5 Forces Booklet

HGS Maths

Dr Frost Course

Name: _____

Class: _____

Contents

5.1) Resolving forces

5.2) Inclined planes

5.3) Friction

Extract from Formulae booklet
Past Paper Practice
Summary

Prior knowledge check

Prior knowledge check

1 A particle of mass 5 kg is acted on by two forces:

$\mathbf{F}_1 = (8\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + 8\mathbf{j})$ N.

Find the acceleration of the particle in the form $(p\mathbf{i} + q\mathbf{j})$ m s^{-2}.

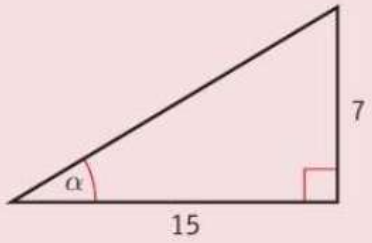
← Year 1, Chapter 10

2 In the diagram below, calculate

a the length of the hypotenuse

b the size of α.

Give your answers correct to 2 d.p.



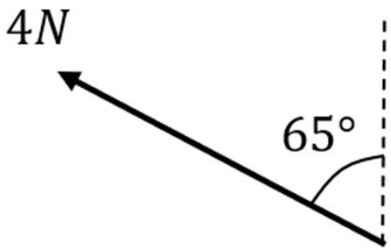
← GCSE Mathematics

5.1) Resolving forces

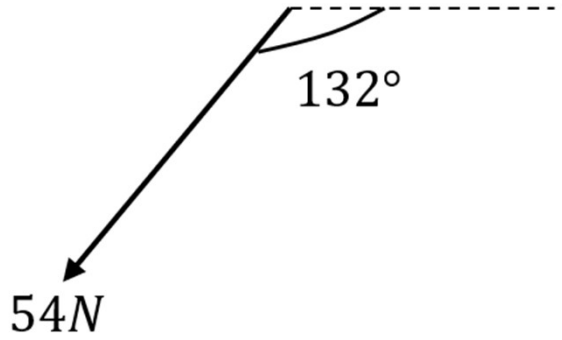
Notes

Worked Example

Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Convert each force to the form $a\boldsymbol{i} + b\boldsymbol{j}$, where $\boldsymbol{i}$ and $\boldsymbol{j}$ are the positive x and y directions respectively.



Worked Example

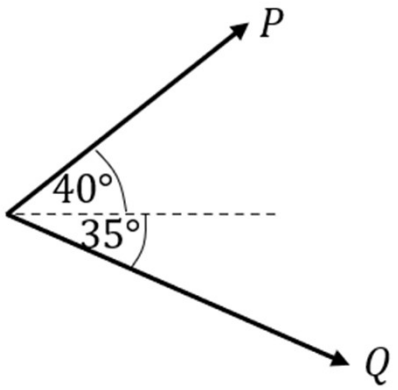
A box of mass 10kg lies on a smooth horizontal floor.
A force of 8N is applied at an angle of $50°$ causing the box to accelerate horizontally along the floor.

(a) Work out the acceleration of the box.

(b) Calculate the normal reaction between the box and the floor.

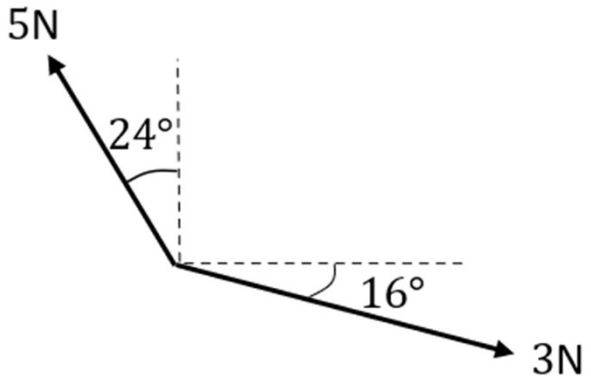
Worked Example

Two forces P and Q act on a particle as shown.
P has a magnitude of 5N and Q has a magnitude of 4N. Work out the magnitude and direction of the resultant force.



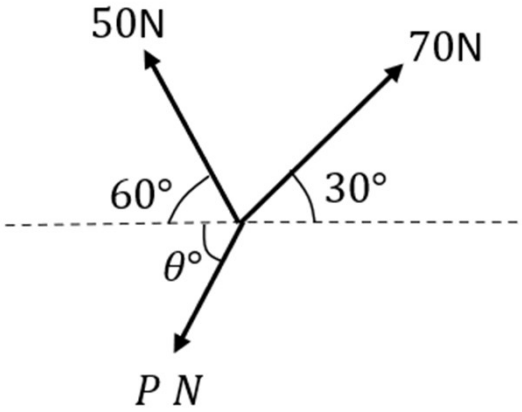
Worked Example

Two forces act on a particle as shown.
Determine the magnitude and direction (anticlockwise from the positive x direction) of the resultant force.



Worked Example

Three forces act on a particle as shown.
Given that the particle is in equilibrium, calculate the magnitude of P



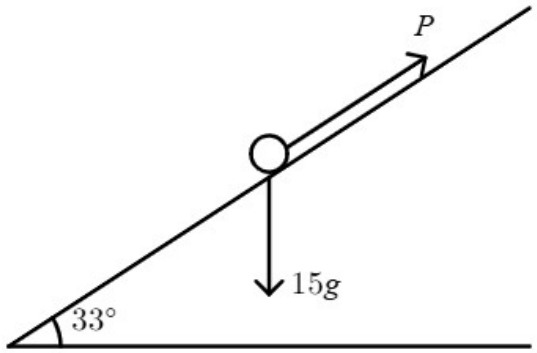
5.2) Inclined planes

Notes

Worked Example

649a: Determine the force needed to hold a particle in equilibrium on a smooth inclined plane, when the force is parallel to the plane.

A particle of mass 15 kg on a smooth inclined plane is held in equilibrium by a force P that is parallel to the plane as shown on the diagram. The plane makes an angle of $33°$ with the horizontal.

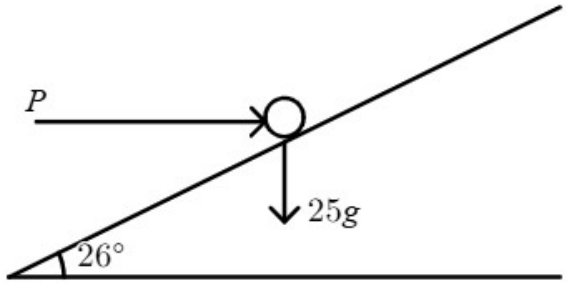


Work out the magnitude of the force P.

Worked Example

649b: **Determine the force needed to hold a particle in equilibrium on a smooth inclined plane, when the force is horizontal.**

A particle of mass 25 kg on a smooth inclined plane is held in equilibrium by a force P as shown on the diagram. The plane makes an angle of $26°$ with the horizontal.

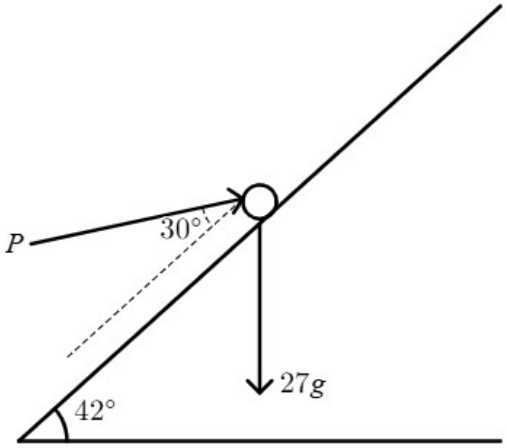


Work out the magnitude of the force P.

Worked Example

649e: **Determine the force needed to hold a particle in equilibrium on a smooth inclined plane, when the force is neither horizontal nor parallel to the plane.**

A particle of mass 27 kg on a smooth inclined plane is held in equilibrium by a force P that makes an angle of 30° with the plane as shown on the diagram. The plane makes an angle of 42° with the horizontal.



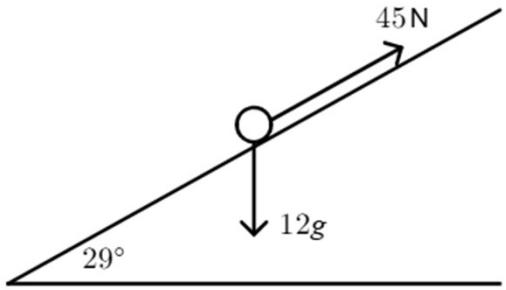
Work out the magnitude of the force P.

Worked Example

652a: Determine the acceleration of a particle pulled by a string on an inclined plane.

A particle of mass 12 kg is held at rest on a smooth plane inclined at $29°$ to the horizontal.

A light inextensible string is attached to the particle and exerts a force of 45 N on the particle parallel to the plane as shown.



The particle is released and moves down the slope.
Find the acceleration of the particle.

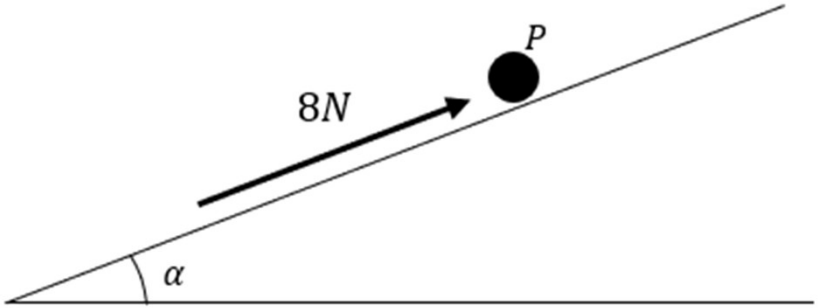
Worked Example

A particle of mass m is pushed up a smooth slope, inclined at $60°$ by a force of magnitude $10g$ N acting at angle of $30°$ to the slope, causing the particle to accelerate up the slope at 0.25 ms^{-2}.

Show that the mass of the particle is $\left(\frac{20\sqrt{3}g}{1+2\sqrt{3}g}\right)$ kg

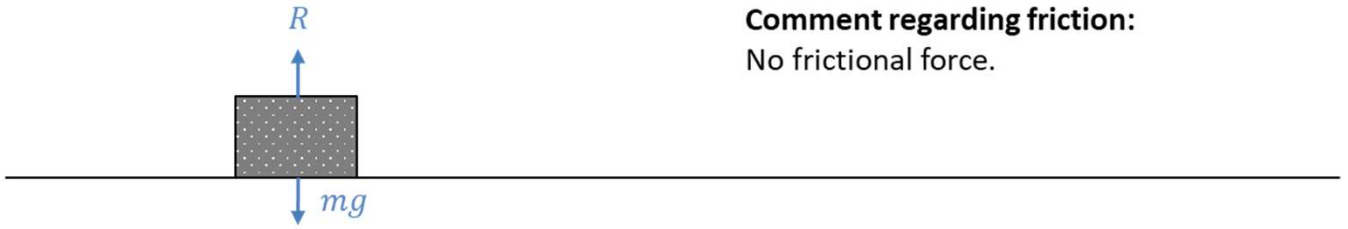
Worked Example

A particle P of mass 4kg is moving on a smooth slope and is being acted on by a force of 8N that acts parallel to the slope, as shown.
The slop is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$.
Work out the acceleration of the particle.



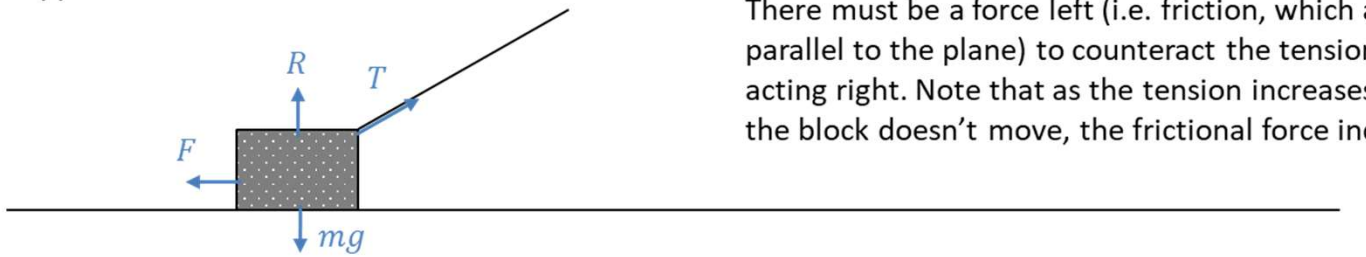
5.3) Friction

Scenario 1: A block is on a horizontal rough surface with no forces (other than gravity) acting on it.



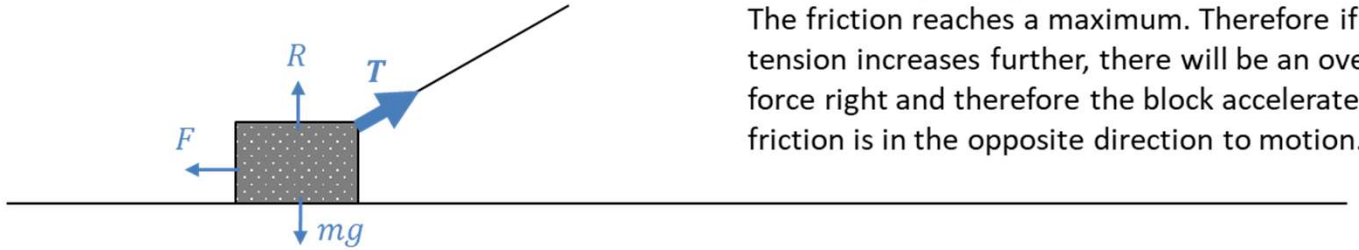
Comment regarding friction:
No frictional force.

Scenario 2: A cable is attached to the block and a force applied. The block doesn't move.



Comment regarding friction:
There must be a force left (i.e. friction, which acts parallel to the plane) to counteract the tension force acting right. Note that as the tension increases but the block doesn't move, the frictional force increases.

Scenario 3: The tension is increased until the block starts to move.



Comment regarding friction:
The friction reaches a maximum. Therefore if the tension increases further, there will be an overall force right and therefore the block accelerates. The friction is in the opposite direction to motion.

Notes

This ‘maximum friction’ depends on two things:

- How **rough** the surface is (i.e. the rougher the surface, the more force required before the block starts moving).
- How hard the block is pressing against the surface (and more formally, by application of Newton’s 3rd Law, how large the **reaction force $\boldsymbol{R}$** is).

The maximum friction between two surfaces:

$$F_{max} = \mu R$$

where μ is the coefficient of friction and R is the normal reaction between two surfaces.

Example μ: (source physlink.com)

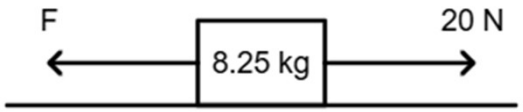
Materials	Coeff. of Static Friction μ_s
Steel on Steel	0.74
Aluminum on Steel	0.61
Copper on Steel	0.53
Rubber on Concrete	1.0
Wood on Wood	0.25-0.5
Glass on Glass	0.94
Waxed wood on Wet snow	0.14
Waxed wood on Dry snow	-
Metal on Metal (lubricated)	
Ice on Ice	0.1
Teflon on Teflon	0.04
Synovial joints in humans	0.01

Notes

Worked Example

647a: Determine the coefficient of friction where a horizontal force pulls a particle in limiting equilibrium.

A particle of mass 8.25 kg lies on a rough horizontal plane. The particle is pulled by a horizontal force of 20 N as show

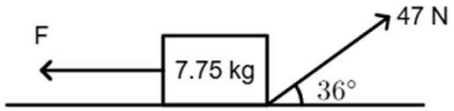


Given that the particle is on the point of sliding, find the coefficient of friction.

Worked Example

650a: Determine the coefficient of friction where an inclined force pulls a particle in limiting equilibrium.

A particle of mass 7.75 kg lies on a rough horizontal plane. The particle is pulled by a force of 47 N inclined at an angle of $36°$ to the horizontal as shown.



Given that the particle is on the point of sliding, find the coefficient of friction.

$\mu =$

Worked Example

A block of mass $10\ kg$ lies on rough horizontal ground.
The coefficient of friction between the block and the ground is 0.4.
A horizontal force P is applied to the block.
Find the magnitude of the frictional force acting on the block and the acceleration of the block when the magnitude of P is:
a) $30\ N$
b) $39.2\ N$
c) $90\ N$

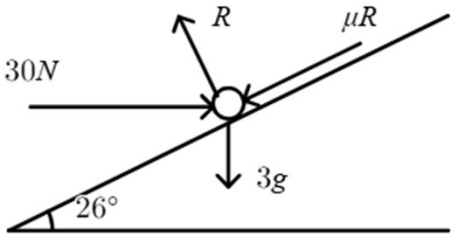
Worked Example

A particle of mass 4kg is sliding down a rough slope that is inclined at $60°$ to the horizontal.
Given that the acceleration of the particle is 2 ms^{-2}, find the coefficient of friction μ between the particle and the slope.

Worked Example

653d: Determine the coefficient of friction of a particle accelerating on an inclined plane, where additional forces are involved.

A particle of mass 3 kg is pushed up a rough plane by a horizontal force of magnitude 30 N. The plane is inclined at 26° to the horizontal and the particle accelerates at 4 m s $^{-2}$.



Calculate the coefficient of friction between the particle and the plane.

Worked Example

A box of mass $4\ kg$ is held in equilibrium on a fixed rough inclined plane by a rope.
The rope lies in a vertical plane containing a line of greatest slope of the inclined plane.
The ropes is inclined to the plane at an angle α, where $\tan\alpha = \frac{5}{12}$, and the plane is at an angle of $45°$ to the horizontal.
The coefficient of friction between the box and the inclined plane is $\frac{1}{4}$ and the box is on the point of slipping up the plane.
By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

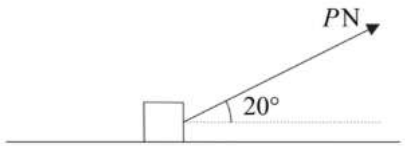
$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Past Paper Questions

6.

Figure 3



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown in Figure 3. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

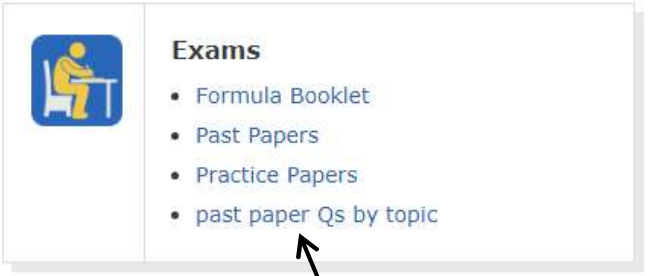
(a) Find the value of P.

(8)

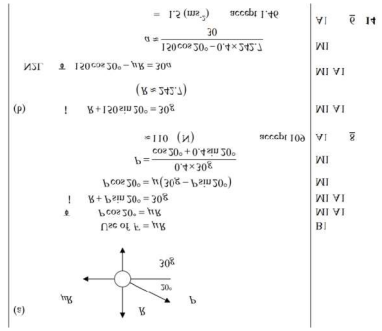
The tension in the rope is now increased to 150 N.

(b) Find the acceleration of the box.

(6)



Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com



Summary of Key Points

Summary of key points

1. If a force is applied at an angle to the direction of motion, you can resolve it to find the component of the force that acts in the direction of motion.
2. The component of a force of magnitude F in a certain direction is $F\cos\theta$, where θ is the size of the angle between the force and the direction.
3. To solve problems involving inclined planes, it is usually easier to resolve parallel to and at right angles to the plane.
4. The maximum or limiting value of the friction between two surfaces, F_{MAX}, is given by $\mathbf{F_{MAX} = \mu R}$ where μ is the coefficient of friction and R is the normal reaction between the two surfaces.