KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 12
Applied Mathematics
M1 9 Constant acceleration Booklet

HGS Maths

Dr Frost Course

Name: ________________________________

Class: ________________________________

Contents

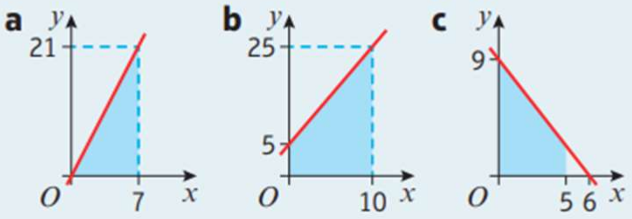
Prior knowledge check

Prior knowledge check

1 For each graph find:

i the gradient

ii the shaded area under the graph.



2 A car travels for 45 minutes at an average speed of 35 mph. Find the distance travelled.

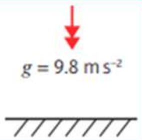
3 **a** Solve the simultaneous equations:

$$3x - 2y = 9$$

$$x + 4y + 4 = 0$$

b Solve $2x^2 + 3x - 7 = 0$. Give your answers to 3 s.f.

Modelling Assumptions

Model	Modelling assumptions
Particle – Dimensions of the object are negligible.	• mass of the object is concentrated at a single point • rotational forces and air resistance can be ignored
Rod – All dimensions but one are negligible, like a pole or a beam.	• mass is concentrated along a line • no thickness • rigid (does not bend or buckle)
Lamina – Object with area but negligible thickness, like a sheet of paper.	• mass is distributed across a flat surface
Uniform body – Mass is distributed evenly.	• mass of the object is concentrated at a single point at the geometrical centre of the body – the **centre of mass**
Light object – Mass of the object is small compared to other masses, like a string or a pulley.	• treat object as having zero mass • tension the same at both ends of a light string
Inextensible string – A string that does not stretch under load.	• acceleration is the same in objects connected by a taut inextensible string
Smooth surface	• assume that there is no friction between the surface and any object on it
Rough surface – If a surface is not smooth, it is rough.	• objects in contact with the surface experience a frictional force if they are moving or are acted on by a force
Wire – Rigid thin length of metal.	• treated as one-dimensional
Smooth and light pulley – all pulleys you consider will be smooth and light.	• pulley has no mass • tension is the same on either side of the pulley
Bead – Particle with a hole in it for threading on a wire or string.	• moves freely along a wire or string • tension is the same on either side of the bead
Peg – A support from which a body can be suspended or rested.	• dimensionless and fixed • can be rough or smooth as specified in question
Air resistance – Resistance experienced as an object moves through the air.	• usually modelled as being negligible
Gravity – Force of attraction between all objects. Acceleration due to gravity is denoted by g. 	• assume that all objects with mass are attracted towards the Earth • Earth's gravity is uniform and acts vertically downwards • g is constant and is taken as $9.8\ \text{m s}^{-2}$, unless otherwise stated in the question

Example 2

A mass is attached to a length of string which is fixed to the ceiling.
The mass is drawn to one side with the string taut and allowed to swing.
State the effect of the following assumptions on any calculations made using this model:

a The string is light and inextensible. **b** The mass is modelled as a particle.

a Ignore the mass of the string and any stretching effect caused by the mass.
b Ignore the rotational effect of any external forces that are acting on it, and the effects of air resistance.

Quantities and units

Quantity	Unit	Symbol
Mass	kilogram	kg
Length/displacement	metre	m
Time	seconds	s

Watch out

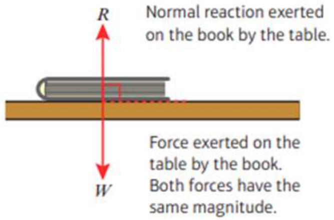
A common misconception is that kilograms measure weight not mass. However, **weight** is a **force** which is measured in **newtons (N)**.

These **derived** units are compound units built from the base units.

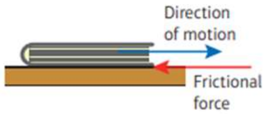
Quantity	Unit	Symbol
Speed/velocity	metres per second	$m\ s^{-1}$
Acceleration	metres per second per second	$m\ s^{-2}$
Weight/force	newton	N (= $kg\ m\ s^{-2}$)

Types of Forces

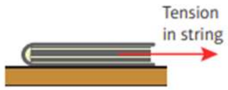
You will encounter a variety of forces in mechanics. These **force diagrams** show some of the most common forces.



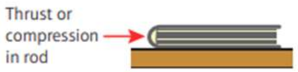
- The **weight** (or gravitational force) of an object acts vertically downwards.
- The **normal reaction** is the force which acts perpendicular to a surface when an object is in contact with the surface. In this example the normal reaction is due to the weight of the book resting on the surface of the table.
- The **friction** is a force which opposes the motion between two rough surfaces.



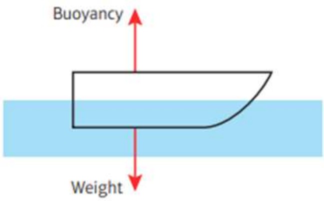
- If an object is being pulled along by a string, the force acting on the object is called the **tension** in the string.



- If an object is being pushed along using a light rod, the force acting on the object is called the **thrust** or **compression** in the rod.



- **Buoyancy** is the upward force on a body that allows it to float or rise when submerged in a liquid. In this example buoyancy acts to keep the boat afloat in the water.



- **Air resistance** opposes motion. In this example the weight of the parachutist acts vertically downwards and the air resistance acts vertically upwards.

Complete this quick exercise here:

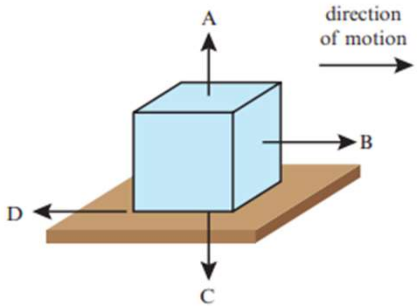
Exercise 8C

1 Convert to SI units:

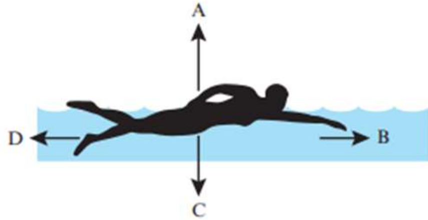
a $65\,km\,h^{-1}$ b $15\,g\,cm^{-2}$ c 30 cm per minute
d $24\,g\,m^{-3}$ e $4.5 \times 10^{-2}\,g\,cm^{-3}$ f $6.3 \times 10^{-3}\,kg\,cm^{-2}$

2 Write down the names of the forces shown in each of these diagrams.

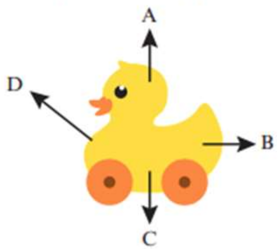
a A box being pushed along rough ground



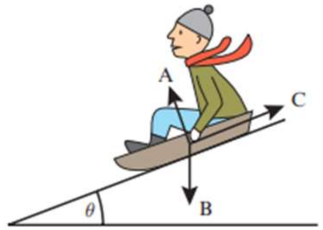
b A man swimming through the water



c A toy duck being pulled along by a string



d A man sliding down a hill on a sledge



Scalar and Vector quantities

- **A scalar quantity has magnitude only.**

These are examples of **scalar** quantities.

Quantity	Description	Unit
Distance	measure of length	metre (m)
Speed	measure of how quickly a body moves	metres per second ($m\ s^{-1}$)
Time	measure of ongoing events taking place	second (s)
Mass	measure of the quantity of matter contained in an object	kilogram (kg)

Scalar quantities are always **positive**. When considering motion in a straight line (1-dimensional motion), **vector** quantities can be **positive** or **negative**.

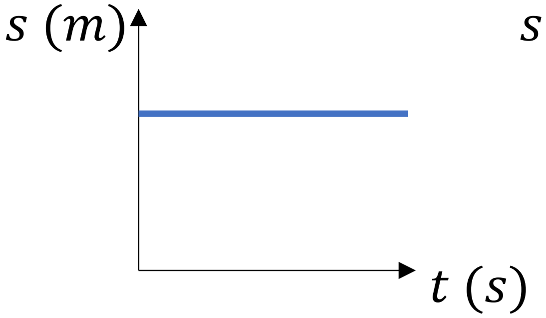
- **A vector is a quantity which has both magnitude and direction.**

These are examples of **vector** quantities.

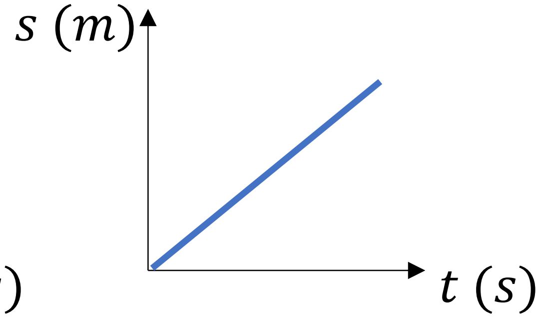
Quantity	Description	Unit
Displacement	distance in a particular direction	metre (m)
Velocity	rate of change of displacement	metres per second ($m\ s^{-1}$)
Acceleration	rate of change of velocity	metres per second per second ($m\ s^{-2}$)
Force/weight	described by magnitude, direction and point of application	newton (N)

9.1) Displacement-time graphs

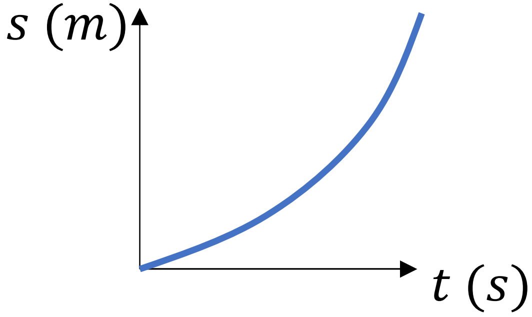
Describe the motion of each object:



Object is stationary.



Object is moving with constant velocity.



Object is accelerating.

The gradient of a displacement (s) time graph is the velocity

Velocity is the rate of change of displacement (i.e. gradient of displacement-time graph)

$$\textbf{Average Velocity} = \frac{\textbf{Displacement from starting point}}{\textbf{Time taken}}$$

$$\textbf{Average Speed} = \frac{\textbf{Total distance travelled}}{\textbf{Time taken}}$$

The distinction is important. If you went out then some time later travelled back home, your average velocity is 0 because your eventual displacement is 0!

Notes

Notes

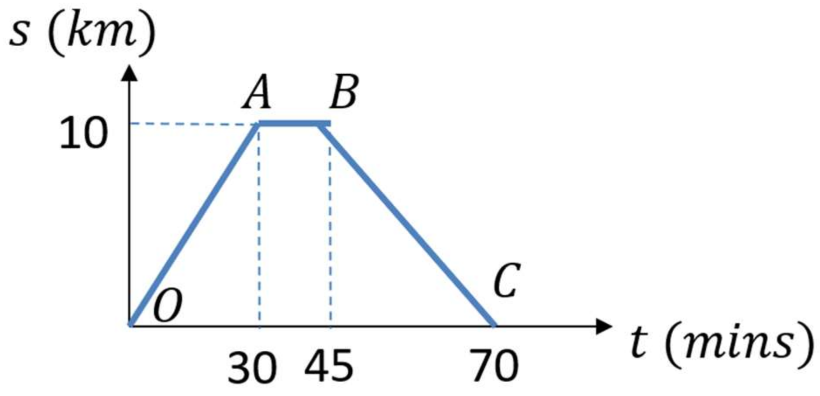
Worked Example

A cyclist rides in a straight line for 30 minutes. She waits for a quarter of an hour, then returns in a straight line to her starting point in 25 minutes.

a) Work out the average velocity for each stage of the journey in km h^{-1}.

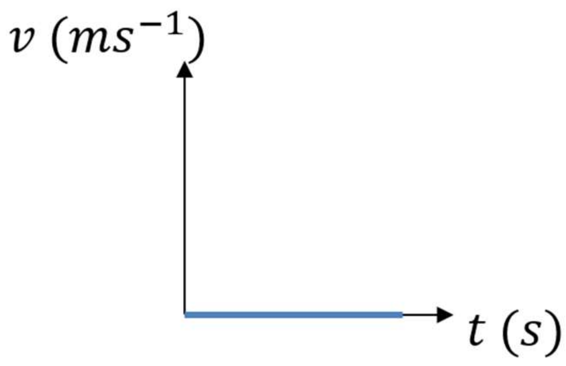
b) Write down the average velocity for the whole journey.

c) Work out average speed for the whole journey.

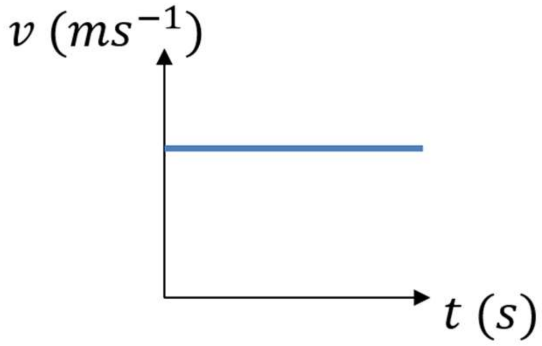


T.132 9A: Qs all, P.58 9.1: Qs all

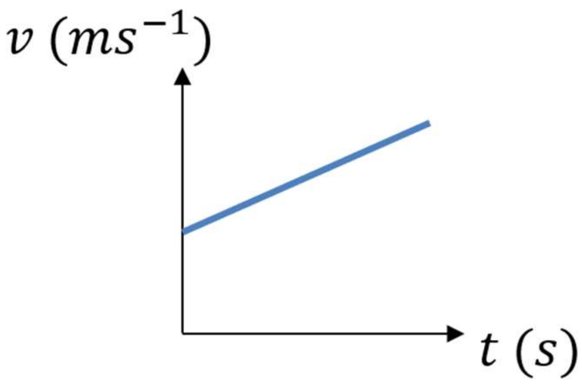
9.2) Velocity-time graphs



Object has zero velocity, i.e. is at rest.



Object has constant velocity, therefore zero acceleration.



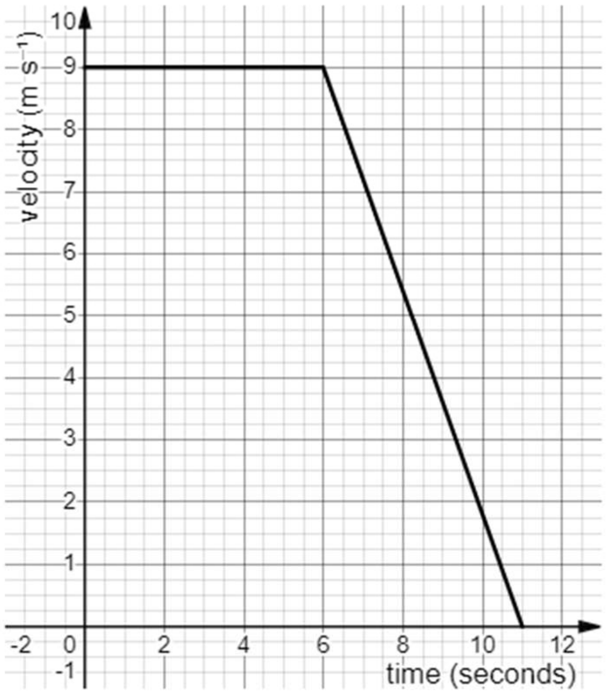
Object has constant acceleration. Velocity is increasing at a constant rate.

The **gradient** of a velocity (v) time graph is the **acceleration**
The **area under** the graph is the **displacement**

Worked Example

459h: **Calculate the total distance from a speed-time graph.**

The velocity-time graph illustrate the motion of a cyclist moving in a straight line.



Find the total distance travelled by the cyclist.

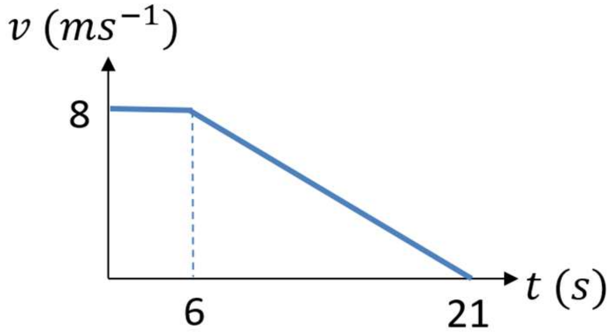
m

Worked Example

A cyclist is moving along a straight road for a period of 21 seconds. For the first 6 seconds, she moves at a constant speed of 8 ms^{-1}. She then decelerates at a constant rate, stopping after a further 15 seconds.

(a) Find the displacement from the starting point of the cyclist after this 21 second period.

(b) Work out the rate at which the cyclist decelerates.

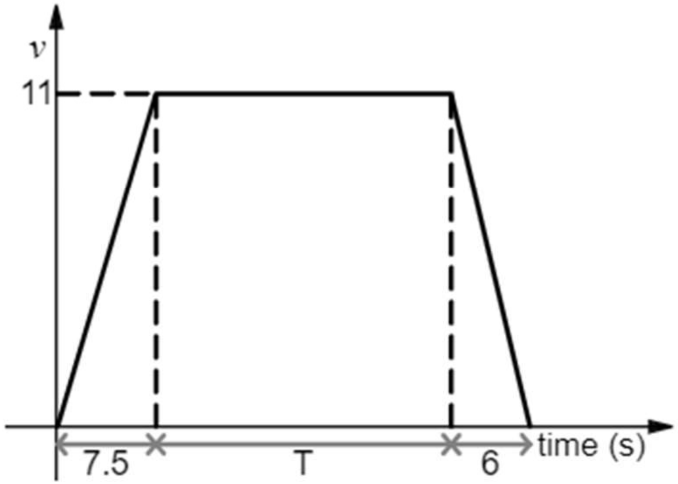


T.135 9B: Qs 1-4, P.59 9.2: Qs1-3

Worked Example

459i: **Determine the time on a speed-time graph given the total displacement.**

The velocity-time graph illustrate the motion of a particle moving in a straight line.



Given that the total displacement of the particle is 321.75 m, find the value of T.

$T =$ ☐ seconds

Worked Example

A particle moves along a straight line. The particle accelerates uniformly from rest to a velocity of 16 ms^{-1} in T seconds. The particle then travels at a constant velocity of 16 ms^{-1} for $3T$ seconds. The particle then decelerates uniformly to rest in a further 4 s.

(a) Sketch a velocity-time graph to illustrate the motion of the particle.

Give then the total displacement of the particle is 592m.

(b) find the value of T.

Worked Example

A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart.
The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.

a) Sketch a speed-time graph for the motion of the car between the two sets of traffic lights

b) Find the value of T

Worked Example

A car is travelling along a straight horizontal road. The car takes $60\ s$ to travel between two sets of traffic lights which are $1072.5\ m$ apart. The car starts from rest at the first set of traffic lights and moves with constant acceleration for $15\ s$ until its speed is $11\ m\ s^{-1}$. The car maintains this speed for T seconds. The car then moves with constant deceleration, coming to rest at the second set of traffic lights.
A motorcycle leaves the first set of traffic lights 15 s after the car has left the first set of traffic lights. The motorcycle moves from rest with constant acceleration, and passes the car at the point A which is $495\ m$ from the first set of traffic lights. When the motorcycle passes the car, the car is moving with speed $11\ ms^{-1}$

c) Find the time it takes for the motorcycle to move from the first set of traffic lights to the point A

Worked Example

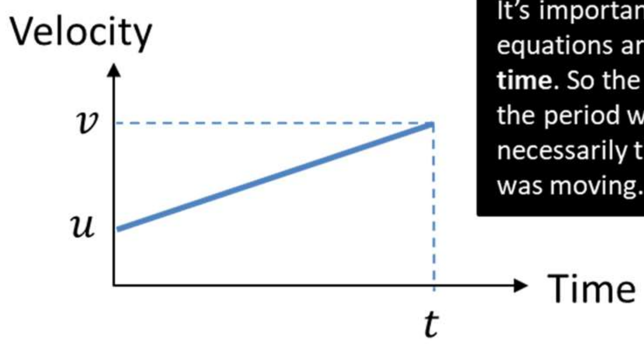
A car is moving along a straight horizontal road.
At time $t = 0$, the car is moving with speed $10\ ms^{-1}$ and is at the point A. The car maintains this speed for $50\ s$.
The car then moves with constant deceleration $0.6\ ms^{-2}$, reducing its speed from $10\ ms^{-1}$ to $4\ ms^{-1}$.
The car then moves with constant speed $4\ ms^{-1}$ for $30\ s$. The car then moves with constant acceleration until it is moving with speed $10\ ms^{-1}$ at the point B.
Given that the distance from A to B is $980\ m$, find the time taken for the car to move from A to B

T.135 9B: Qs 5+, P.59 9.2: Qs 4+

9.3) Constant acceleration formulae 1

When there is **constant acceleration**, there are a variety of formulae which relate the following 5 quantities:

s: displacement
u: initial velocity
v: final velocity
a: acceleration
t: time



It's important you recognise these equations are for **a specific interval of time.** So the time t is the duration of the period we're considering, not necessarily the time since the object was moving.

Each "suvat" equation we will see involves 4 of the 5 quantities. Typically in a problem we'll know 3 of the quantities and we wish to find an unknown 4th quantitiy. We therefore select the appropriate equation.

Using the gradient of the graph (which we know is acceleration):
$a = \frac{v-u}{t} \quad \rightarrow \quad v = u + at$

You are expected to be able to **prove** each "suvat" question using the above graph.

Using the area under the graph (which we know gives distance):
$s = \left(\frac{u+v}{2}\right)t$

Memorisation Tip: This formula is effectively "distance = average speed × time" which you knew from GCSE.

Notes

Worked Example

551a: Use the constant acceleration formula $v = u + at$

A car is decelerating at a constant rate of 1 m s^{-2} from A to B. The car has an initial velocity of 10.5 m s^{-1}. After t seconds, the car has a final velocity of 2 m s^{-1}.

Find the value of t.

Worked Example

551b: **Use the constant acceleration formula** $s = \left(\frac{u+v}{2}\right)t$

A car is moving with constant acceleration from A to B. The car has an initial velocity of 5 m s^{-1}. After 9 seconds, the car has a final velocity of 45.5 m s^{-1}.

Work out the distance travelled by the car.

$s =$ ______ m

Worked Example

A particle moves in a straight line from a point A to a point B with a constant deceleration $3\ ms^{-2}$. The velocity of the particle at A is $16\ ms^{-1}$ and the velocity of the particle at B is $4\ ms^{-1}$. Find:

(a) the time taken for the particle to move from A to B.

(b) the distance from A to B.

After reaching B the particle continues to move along the straight line with constant deceleration $3\ ms^{-2}$.

The particle is at the point C 12 seconds after passing through the point A. Find:

(c) the velocity of the particle at C.

(d) The distance from A to C.

Worked Example

A car moves from traffic lights along a straight road with constant acceleration.
The car starts from rest at the traffic lights and 20 seconds later the car passes a speed-trap where it is registered as travelling at $54\ km\ h^{-1}$. Find:

(a) the acceleration of the car

(b) the distance between the traffic lights and the speed-trap.

T.140 9C: Qs all, P.60 9.3: Qs all

9.4) Constant acceleration formulae 2

The other "suvat" equations can be derived using $v = u + at$ and $s = \left(\frac{u+v}{2}\right)t$.

Eliminating t:

$$t = \frac{v-u}{a}$$

$$s = \left(\frac{u+v}{2}\right)\left(\frac{v-u}{a}\right)$$

$$= \cdots$$

$$v^2 = u^2 + 2as$$

Eliminating v:

$$s = \left(\frac{u+u+at}{2}\right)t$$

$$= \left(\frac{2u+at}{2}\right)t$$

$$s = ut + \frac{1}{2}at^2$$

Fro Note: Because this is quadratic in t, we typically end up with two different possible times.

Eliminating u:

$$u = v - at$$

$$s = \left(\frac{v-at+v}{2}\right)t$$

$$= \left(\frac{2v-at}{2}\right)t$$

$$s = vt - \frac{1}{2}at^2$$

Fro Note: I have never seen an exam question that uses this $suvat$ formula.

Notes

Worked Example

551c: **Use the constant acceleration formula** $v^2 = u^2 + 2as$

A particle is is decelerating at a constant rate of 1.5 m s^{-2} from A to B. When it reaches B, the particle has travelled 8 metres and has a final velocity of 2.5 m s^{-1}.

Work out the initial velocity of the particle.

Worked Example

551d: **Use the constant acceleration formula** $s = ut + \frac{1}{2}at^2$

A car is moving with constant acceleration from A to B. The car has an initial velocity of 4.5 m s^{-1}. When it reaches B, the car has travelled 5.5 metres over a period of 1 seconds.

Find the acceleration of the car.

Worked Example

551e: Use the constant acceleration formula $s = vt - \frac{1}{2}at^2$

A particle is accelerating at a constant rate of 2.5 m s^{-2} from A to B. The particle travels for 3 seconds and has a final velocity of 9 m s^{-1}.

Work out the distance travelled by the particle.

Worked Example

A particle is moving along a straight line from A to B with constant acceleration 3 ms^{-2}.
The velocity of the particle is 5 ms^{-1} in the direction $\overrightarrow{AB}$.
The velocity of the particle at B is 81 ms^{-1} in the same direction.
Find the distance from A to B.

Worked Example

A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2}.

At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1} travelling towards a point A, where $OA = 40$ m. Find:

(a) the times when the particle passes through A

(b) the value of t when the particle returns to O.

Worked Example

551f: Be able to use all constant acceleration formulae.

A car is moving with constant acceleration from A to B. When it reaches B, the car has travelled 8 metres over a period of 2 seconds, and has a final velocity of 5 m s^{-1}.

Find the acceleration of the car.

Worked Example

A particle is moving in a straight horizontal line with constant deceleration 6 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 23 ms^{-1}.
Find the total distance travelled by the particle between when it first passes O and returns to O

Your Turn

A particle is moving in a straight horizontal line with constant deceleration 4 ms^{-2}.
At time $t = 0$ the particle passes through a point O with speed 13 ms^{-1}.
Find the total distance travelled by the particle between when it first passes O and returns to O

$42.25\ m$

Worked Example

551i: Use the constant acceleration formulae with two linked objects.

A pair of bikes A and B are moving in the same direction along a straight track.

Bike A is accelerating at 8.5 m s^{-2}. When its velocity is 4 m s^{-1}, it is passed by bike B which is moving at 32 m s^{-1} and accelerating at 4.5 m s^{-2}.

What distance do the bikes travel before bike A overtakes bike B?

T.145 9D: Qs 1-10, P.62 9.4: Qs 1-8

Worked Example

Two particles P and Q are moving along the same straight horizontal line with constant accelerations 2 and 4 ms^{-2} respectively. At time $t = 0$, P passes through a point A with speed 12 ms^{-1}. One second later Q passes through A with speed 6 ms^{-1}, moving in the same direction as P.

a) Find the value of t where the particles meet.

b) Find the distance of A from the point where the particles meet.

Worked Example

A particle moves in a straight horizontal line with constant acceleration from A to B, then B to C.
AB = 3 km and BC = 12 km.
It takes 2 hour from A to B and 4 hours from B to C.
Find:

a) The acceleration of the particle
b) The particle's speed as it passes A

T.145 9D: Qs 11+, P.62 9.4: Qs 9+

9.5) Vertical motion under gravity

Notes

Worked Example

552c: **Determine the greatest height or total distance travelled when a particle is projected vertically upwards.**

A ball is projected vertically upwards from a point 4 metres above the ground, and initial velocity 9.5 m s^{-1}.

Find the greatest height above the ground reached by the ball.

Worked Example

552d: **Determine the time of flight of a particle projected vertically.**

A particle is projected vertically downwards from a point 3 metres above the ground, with initial velocity of 10 m s^{-1}.

Find the time of flight of the particle.

Worked Example

A book falls off the top shelf of a bookcase.
The shelf is 2.8 m above a wooden floor. Find:

(a) the time the book takes to reach the floor,

(b) the speed with which the book strikes the floor.

Worked Example

A ball is projected vertically upwards, from a point X which is 5m above the ground, with speed 15 ms^{-1}. Find

(a) the greatest height above the ground reached by the ball,

(b) the time of flight of the ball

Worked Example

A ball is projected vertically upwards from ground level at a speed of 40 ms^{-1}.
Determine the amount of time the ball is at least 20m above ground level.

Worked Example

A stone is thrown vertically upward from a point which is $8\ m$ above the ground with speed $5\ ms^{-1}$.

Find:

a) The time of flight of the stone

b) The total distance travelled by the stone

T.151 9E: Qs 1-10, P.64 9.5: Qs 1-6

Worked Example

Ball A falls vertically from rest from the top of a tower $48\ m$ high. At the same time as A begins to fall, another ball B is projected vertically upwards from the bottom of the tower with speed $24\ ms^{-1}$. The balls collide. Find the distance to the point where the balls collide from the bottom of the tower.

Worked Example

552f: Use constant acceleration formulae for two particles colliding in vertical motion.

An object A falls vertically from rest from the top of a tower 100 m high. At the same time as A begins to fall, another object B is projected vertically upwards from the bottom of the tower with speed 25 m s^{-1}.
The objects collide.

Find the distance of the point where the objects collide from the bottom of the tower.

Worked Example

At time $t = 0$, two balls A and B are projected vertically upwards. Ball A is projected upwards with speed $3\ ms^{-1}$ from a point $40\ m$ above the horizontal ground. Ball B is projected vertically upwards from the ground with speed $30\ ms^{-1}$. The balls are modelled as particles moving freely under gravity. Find the time and the height at which the balls are at the same vertical height.

Worked Example

A ball is released from rest at a point which is $20\ m$ above a wooden floor. Each time the ball strikes the floor, it rebounds with $\frac{2}{3}$ of the speed with which it strikes the floor.

Find the greatest height above the floor reached by the ball:

a) The first time it rebounds from the floor

b) The second time it rebounds from the floor.

T.151 9E: Qs 11+, P.64 9.5: Qs 7+

Extract from Formulae book

Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

6. A man throws a tennis ball into the air so that, at the instant when the ball leaves his hand, the ball is 2 m above the ground and is moving vertically upwards with speed $9\,\text{m}\,\text{s}^{-1}$

The motion of the ball is modelled as that of a particle moving freely under gravity and the acceleration due to gravity is modelled as being of constant magnitude $10\,\text{m}\,\text{s}^{-2}$

The ball hits the ground T seconds after leaving the man's hand.

Using the model, find the value of T.

(4)

Exams

- Formula Booklet
- Past Papers
- Practice Papers
- past paper Qs by topic

Past paper practice by topic. Both new and old specification can be found via this link on hgsmaths.com

6.	Equation in t only	M1	2.1
	$-2 = 9t - \frac{1}{2} 10t^2$	A1	1.1b
	$5t^2 - 9t - 2 = 0 = (5t+1)(t-2)$	DM1 [No Title]	1.1b
	$T = 2$ (only)	A1	1.1b
		(4)	
		(4 marks)	

Summary of Key Points

Summary of key points

1 Velocity is the **rate of change** of displacement.
On a displacement–time graph the **gradient** represents the velocity.
If the displacement–time graph is a straight line, then the velocity is constant.

2 Average velocity $= \dfrac{\text{displacement from starting point}}{\text{time taken}}$

3 Average speed $= \dfrac{\text{total distance travelled}}{\text{time taken}}$

4 Acceleration is the **rate of change** of velocity.
In a velocity–time graph the **gradient** represents the acceleration.
If the velocity–time graph is a straight line, then the acceleration is constant.

5 The area between a velocity–time graph and the horizontal axis represents the distance travelled.
For motion in a straight line with positive velocity, the area under the velocity–time graph up to a point t represents the displacement at time t.

6 You need to be able to use and to derive the five formulae for solving problems about particles moving in a straight line with constant acceleration.

- $v = u + at$
- $s = \left(\frac{u+v}{2}\right)t$
- $v^2 = u^2 + 2as$
- $s = ut + \frac{1}{2}at^2$
- $s = vt - \frac{1}{2}at^2$

7 The force of **gravity** causes all objects to accelerate towards the earth. If you ignore the effects of air resistance, this acceleration is constant. It does not depend on the mass of the object.

8 An object moving vertically in a straight line can be modelled as a particle with a constant downward acceleration of $g = 9.8\text{ m s}^{-2}$.

T.152 mixed ex, P.65 BSG