Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for each question are shown in brackets
    – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions.

1. (a) Using a suitable test, and without performing any division explain why 1485 is divisible by 11.

    (1)

    (b) Use the Euclidean algorithm to find integers p and q such that $1485p + 143q = 11$.

    (4)

    (c) Hence find integers a and b such that $1485a + 143b = 22$.

    (1)

    (Total for Question 1 is 6 marks)

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2. The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ forms a group under the operation of multiplication modulo 20.

    (a) Complete the Cayley table below for this group.

$\times_{20}$	1	3	7	9	11	13	17	19
1		3						
3				7				
7						11		
9					19			
11		17						
13							1	
17				13				
19					9			

    (4)

    (b) Explain why $(G, \times_{20})$ cannot contain a subgroup of order 3.

    (1)

    (c) Find three different subgroups of $(G, \times_{20})$ of order 2.

    (2)

    (Total for Question 2 is 7 marks)

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3. L is the locus of points on an Argand diagram satisfied by $|z - 3| = \sqrt{2}\,|z + 9|$

(a) Sketch clearly the locus L.

(4)

On the same Argand diagram a finite region R is bounded by L and the condition

$$0 \le \arg (z + 21) \le \frac{\pi}{6}.$$

(b) (i) Show this region on your sketch.

(ii) Find the exact area of region R.

(3)

(Total for Question 3 is 7 marks)

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4. Given $\mathbf{M} = \begin{pmatrix} 6 & 1 - p \\ -3 & 4 \end{pmatrix}$ and $p \in \mathbb{R}$, and that $\mathbf{M}$ has distinct, real eigenvalues,

(a) find the range of possible values for p.

(3)

Given further that one of the eigenvalues of $\mathbf{M}$ is 1,

(b) (i) find p

(ii) find the other eigenvalue of $\mathbf{M}$.

(3)

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{MP} = \mathbf{D}$.

(4)

(Total for Question 4 is 10 marks)

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5. Dishwasher salt should be added regularly to a dishwasher to enhance its performance. A new dishwasher is initially filled with 500 g of salt.

The dishwasher uses its supply of salt at a rate of 30% per week. Each Saturday the dishwasher owner adds a further 50 g of salt to the supply.

The amount of salt in the dishwasher s_n immediately after n Saturday additions may be defined by a recurrence relation in the form

$$s_n = as_{n-1} + b, \quad s_0 = c.$$

(a) Deduce appropriate values for a, b and c.

(2)

(b) Solve your recurrence relation and show that the closed form may be expressed by

$$s_n = \frac{500}{3}(2a^n + 1)$$

(5)

Within the dishwasher a sensor measures the salt supply to the nearest 0.1 g. This sensor causes a warning light to come on when the salt level falls below x g.

This warning light first came on during the 12th week of operation but was only noticed on the Saturday of that week just before the usual weekly top up.

(c) Find the range of possible values for x.

(3)

(Total for Question 5 is 10 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS