AS Further Mathematics 8FM0

Specimen Paper - Further Pure Mathematics 1 Mark Scheme

Question	Scheme	Marks	AOs
1.	$\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0 \approx \dfrac{y_1 - y_{-1}}{2h} \Rightarrow 2 \approx \dfrac{y_1 - y_{-1}}{0.2} \Rightarrow y_1 - y_{-1} \approx 0.4$	B1	1.1b
	$\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \left(= 3 + 2\times1.5^2\times2\right) = 12$	B1	1.1b
	$\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0 \approx \dfrac{y_1 - 2y_0 + y_{-1}}{h^2} \Rightarrow 12 \approx \dfrac{y_1 - 2y_0 + y_{-1}}{0.01} \Rightarrow y_1 + y_{-1} \approx 3.12$	M1	1.1b
	$\Rightarrow y_{-1} \approx \dfrac{1}{2}(3.12 - 0.4)$	M1	2.1
	$= 1.36$	A1	1.1b
		(5)	
			(5 marks)

Notes:

B1: Uses the approximation for $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ to find the approximate value for $y_1 - y_{-1}$

B1: Correct value for $\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0$ seen or implied. Need not be simplified.

M1: Uses approximation for $\left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0$ to form second equation relating y_1 and y_{-1}, ie $y_1 + y_{-1} = 3.12$

M1: Brings together the information and solves to find y_{-1} from their equations.

M1: cao 1.36

Question	Scheme	Marks	AOs
2.	$\frac{3x}{2-x} \leqslant \frac{2}{x^2-4}$		
	$x^2-4=(x-2)(x+2)$ so $-2,\ 2$ are CVs	B1	1.1b
	$\frac{3x(2+x)+2}{(2-x)(2+x)} \leqslant 0$ or $2(x-2)(x+2)+3x(x-2)(x+2)^2 \geqslant 0$ or $3x^2+6x+2=0$	M1	2.1
	$3x^2+6x+2=0 \Rightarrow x=\frac{-6\pm\sqrt{36-4\times3\times2}}{6}$ from their quadratic in numerator/after factorisation of $(x-2)(x+2)$.	M1	1.1b
	Correct remaining critical values, $-1\pm\frac{1}{3}\sqrt{3}$	A1	1.1b
	Solution set of form $x<\alpha$, $\beta \leqslant x \leqslant \gamma$, $\delta<x$ with $\leqslant$ or < throughout, where $\alpha<\beta<\gamma<\delta$ are their CVs	M1	2.2a
	Correct assignment of strict inequalities (<) to -2 and 2 in the above and loose inequalities ($\leqslant$) with the other CVs	A1	2.3
	Solution set is $\{x\in\mathbb{R}:-2<x\}\cup\left\{x\in\mathbb{R}:-1-\frac{\sqrt{3}}{3}\leqslant x\leqslant -1+\frac{\sqrt{3}}{3}\right\}\cup\{x\in\mathbb{R}:2<x\}$ oe in set notation **from fully correct work**.	A1	2.5
		(7)	
			(7 marks)

Notes:

B1: Recognises that 2 and -2 are critical values at any point in the solution.

M1: Gathers terms on one side and puts over common denominator, or multiply by $(2-x)^2(2+x)^2$, or sets sides **equal** and proceeds to form a quadratic or cubic. Allow with any inequality sign.

M1: Attempts to solve for the remaining critical values using formula or completing the square. Only allow factorisation if their quadratic does factorise. This mark may be implied by the correct answers (from calculator).

A1: Correct remaining critical values.

M1: Deduces the two outsides and middle interval are required. May be from sketch, number line or any other means. Must have four critical values for this mark.

A1: Correct inequalities assigned to each critical value - strict for 2 and -2 and loose for the other two.

A1: cso. Fully correct solution set from fully correct work. Withhold this mark if incorrect inequalities have been `recovered' (incorrect notation has been used). Accept equivalent set notations but must be given as a set.

E.g. accept $\left\{x\in\mathbb{R}:-2<x \text{ or } -1-\frac{\sqrt{3}}{3}\leqslant x\leqslant -1+\frac{\sqrt{3}}{3} \text{ or } 2>x\right\}$ or

$\mathbb{R}-\left(\left[-2,-1-\frac{\sqrt{3}}{3}\right]\cup\left[-1+\frac{\sqrt{3}}{3},2\right]\right)$.

Question	Scheme	Marks	AOs

3(a)	$y=15x^{-1} \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x}=-15x^{-2}$ May use product rule or implicit - must be correct expression	B1	1.1b
	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-15(5p)^{-2}=-\dfrac{3}{5p^2}$ oe	M1	1.1b
	$y-\dfrac{3}{p}=m(x-5p)$ with m their tangent gradient in terms of p	M1	1.1b
	$y-\dfrac{3}{p}=-\dfrac{3}{5p^2}(x-5p) \Rightarrow 5p^2y-15p=-3x+15p$ $\Rightarrow 3x+5p^2y=30p$ *	A1*	2.1
		(4)	
(b)	$3x+5q^2y=30q$	B1	2.2a
		(1)	
(c)	$30p-5p^2y=30q-5q^2y$ or $3x+5p^2\left(\dfrac{30q-3x}{5q^2}\right)=30p$ oe	M1	3.1a
	$\Rightarrow y=\dfrac{30p-30q}{5p^2-5q^2}$ or $x=\dfrac{30p-30\frac{p^2}{q}}{3-3\frac{p^2}{q^2}}$	M1	1.1b
	One of $x=\dfrac{10pq}{p+q}$ or $y=\dfrac{6}{p+q}$	A1	1.1b
	Both of $x=\dfrac{10pq}{p+q}$ and $y=\dfrac{6}{p+q}$	A1	1.1b
		(4)	
			(9 marks)

Question 3 notes:
(a) **B1:** A correct expression involving $\frac{\mathrm{d}y}{\mathrm{d}x}$. **M1:** Substitutes coordinates into their derivative to find tangent gradient in terms of p. **M1:** Applies line equation with their tangent gradient in terms of p, and with correct x and y coordinates. Use of $-1/m_t$ is M0. **A1*:** Expands and gathers term - must be an intermediate step before answer given. This is a given answer, so all aspects must be correct throughout.
(b) **B1:** Deduces correct tangent at Q. Allow if reworked completely.
(c) **M1:** Attempts to obtain an equation in one variable, either x or y. **M1:** Makes x or y the subject – must reach x or $y = \mathrm{f}(p, q)$ or $\mathrm{f}(p)$ or $\mathrm{f}(q)$. **A1:** One correct **simplified** coordinate. **A1**: Both coordinates correct and **simplified**.

Question	Scheme	Marks	AOs
4	$A(3,6,9)$, $B(1,5,7)$, $C(2,3,8)$, $D(3+k,6,9-k)$		
(a)	Finds $\overrightarrow{AB}=\begin{pmatrix}-2\\-1\\-2\end{pmatrix}$, $\overrightarrow{AC}=\begin{pmatrix}-1\\-3\\-1\end{pmatrix}$ and $\overrightarrow{AD}=\begin{pmatrix}k\\0\\-k\end{pmatrix}$ and attempts scalar products OR attempts $\overrightarrow{AB}\times\overrightarrow{AC}$	M1	1.1b
	$\overrightarrow{AB}\bullet\overrightarrow{AD}=0$ and $\overrightarrow{AC}\bullet\overrightarrow{AD}=0$ hence $\overrightarrow{AD}$ perpendicular to ABC OR $\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}-5\\0\\5\end{pmatrix}=\frac{-5}{k}\left(\overrightarrow{AD}\right)$ hence $\overrightarrow{AD}$ perpendicular to ABC.	A1	2.1
		(2)	
(b)	Area $ABC=\frac{1}{2}\left\|\overrightarrow{AB}\times\overrightarrow{AC}\right\|=\frac{1}{2}\left\|\begin{pmatrix}-5\\0\\5\end{pmatrix}\right\|=\frac{5\sqrt{2}}{2}$ OR applies $\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\bullet\overrightarrow{AD}=\begin{pmatrix}-5\\0\\5\end{pmatrix}\bullet\begin{pmatrix}k\\0\\-k\end{pmatrix}=-10k$	M1	3.1a
	So Volume of prism $=\frac{5\sqrt{2}}{2}\times\|\overrightarrow{AD}\|=\frac{5\sqrt{2}}{2}\times\|k\|\sqrt{2}=\ldots$ OR $=\frac{1}{2}\|-10k\|=\ldots$	M1	1.1b
	$\ldots=5\|k\|$ or accept $5k$	A1	1.1b
	$\left\{\frac{1}{6}\right\}\left(\overrightarrow{OB}\times\overrightarrow{OC}\right)\bullet\overrightarrow{OD}=\left\{\frac{1}{6}\right\}\begin{pmatrix}19\\6\\-7\end{pmatrix}\bullet\begin{pmatrix}3+k\\6\\9-k\end{pmatrix}=\ldots$	M1	2.1
	So Volume parallelepiped $OBCD=\|30+26k\|$ or volume tetrahedron $OBCD=\frac{1}{6}\|30+26k\|$	A1	1.1b
	So $5\|k\|=\frac{2}{6}\|30+26k\|\Rightarrow 15k=30+26k$ or $-15k=30+26k$ (need both)	M1	3.1a
	$k=-\frac{30}{11}$ or $-\frac{30}{41}$ (need both)	A1	1.1b
		(7)	
		(6 marks)	

Question 4 notes:

(a)

M1: Either finds two appropriate vectors in the plane ABC and takes scalar products with $\overrightarrow{AD}$ OR attempts to find the normal to plane ABC with suitable cross product.

A1: Shows their two directions are each perpendicular to $\overrightarrow{AD}$ and concludes, OR shows the normal vector is a scalar multiple of $\overrightarrow{AD}$ and concludes.

(b)

M1: Correct method to find area of triangle ABC OR attempts $(\overrightarrow{AB}\times\overrightarrow{AC})\bullet\overrightarrow{AD}$ oe. The attempt at the area of the triangle may be more lengthy, but must be complete. E.g. use scalar product to find $\angle BAC$ then use $\frac{1}{2}\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|\sin\angle BAC$.

M1: Applies $\frac{1}{2}(\text{Area } ABC)\left|\overrightarrow{AD}\right|$ for volume of prism OR $\frac{1}{2}|(\overrightarrow{AB}\times\overrightarrow{AC})\bullet\overrightarrow{AD}|$ oe Must include the $\frac{1}{2}$.

A1: Volume of prism is $|5k|$ – accept just $5k$.

M1: Attempts scalar triple product formula on appropriate vectors to find volume of tetrahedron $OBCD$ or its associated parallelepiped, it should be clear which they are attempting. (May see more convoluted methods not using triple product.)

A1: Correct volume $|30+26k|$ or $\frac{1}{6}|30+26k|$ -- need not be simplified and accept without modulus.

M1: Uses their volumes to produce **two** possible values for k by equating twice volume of tetrahedron to plus/minus volume of prism.

A1: Both $k=-\frac{30}{11}$ and $k=-\frac{30}{41}$.

Question	Scheme	Marks	AOs
5.	$T=\dfrac{119+38\cos\left(\frac{x}{3}\right)+79\sin\left(\frac{x}{3}\right)}{6+4\sin\left(\frac{x}{3}\right)+2\cos\left(\frac{x}{3}\right)}$		
(a)	$T=\dfrac{119+38\left(\frac{1-t^2}{1+t^2}\right)+79\left(\frac{2t}{1+t^2}\right)}{6+4\left(\frac{2t}{1+t^2}\right)+2\left(\frac{1-t^2}{1+t^2}\right)}$	M1	1.1b
	$=\dfrac{119(1+t^2)+38(1-t^2)+79(2t)}{6(1+t^2)+4(2t)+2(1-t^2)}$	M1	1.1b
	$=\dfrac{81t^2+158t+157}{4t^2+8t+8}$ *	A1*	2.1
		(3)	
(b)	$T=20\Rightarrow 81t^2+158t+157=20(4t^2+8t+8)$	M1	3.4
	$\Rightarrow t^2-2t-3=0$	A1	1.1b
	$\Rightarrow (t-3)(t+1)=0\Rightarrow \frac{x}{6}=\tan^{-1}(3)=..$ or $\tan^{-1}(-1)=..$	M1	1.1b
	So $\left(\text{for } 0\,\text{Ñ}\,\frac{x}{3}\,\text{Ñ}\backslash\pi\right)$ solutions are $\frac{x}{6}=1.249\ldots$ and $\frac{3\pi}{4}(=2.356\ldots)$ (or $x=7.494\ldots$ and $14.137\ldots$) (accept consecutive pairs in other ranges, e.g. $=-\frac{\pi}{4}$ and $1.249\ldots$)	A1ft	3.4
	So proportion of time above 20° C is $\dfrac{2.356\ldots-1.249\ldots}{\pi}$ or $\dfrac{14.137\ldots-7.494\ldots}{6\pi}$	M1	3.1b
	$=0.352\ldots$ so 35% of the time. Accept awrt 0.35 or 35%	A1	3.2a
		(6)	
(c)	Any reasonable comments such as: The ambient temperature outside the room may increase and so the temperature may not drop below $20°\text{C}$, so not need heating/graph will drop less sharply from peaks etc.	B1	3.5b
		(1)	
			(10 marks)

Question 5 notes:
(a) **M1:** Use of $\cos\left(\frac{x}{3}\right)=\frac{1-t^2}{1+t^2}$ and $\sin\left(\frac{x}{3}\right)=\frac{2t}{1+t^2}$ at least once each in the expression. **M1:** Multiplies numerator and denominator through by $1+t^2$. **A1*:** ully correct solution, no errors seen.
(b) **M1:** Sets $T=20$ and multiplies across to form a quadratic in t (not necessarily 3 term). May use Ö instead of $=$. **A1:** Correct 3 term quadratic formed. **M1:** Solves the quadratic and applies inverse $\tan$ to find at least one solution. **A1ft:** Finds consecutive solutions for the equation using both roots and uses the period of $\tan$ is π, or finds solutions for x (ie multiplies by 6 first) and uses period 3π. Note that $-\frac{\pi}{4}$ and $1.249\ldots$ are a consecutive pair, so gain A1. If quadratic is incorrect the answers will need to be checked. **M1:** Correct method to find the proportion of time above $20°$C. If using e.g. $-\frac{\pi}{4}$ and $1.249\ldots$ they would need $1-\frac{1.240\ldots+\frac{\pi}{4}}{\pi}$ oe. If x values have been found, they need to be using period 3π, e.g $\frac{14.137\ldots-7.494\ldots}{6\pi}$. NB any other scaling with correct associated period is fine. **A1**: awrt 0.35 or 35%
(c) **B1:** Any reasonable comment. Either as per scheme, or accept reasons along the lines of changes to the state of the room may occur (e.g a window may be opened letting out heat). Also accept reasons along the lines of one hour being insufficient data to extrapolate from.