Question	Scheme	Marks	AOs
1(a)	$\mathbf{M} = \mathbf{QP} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \times \frac{1}{2}\begin{pmatrix} 1 & \sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$	M1	1.1a
	$= \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ or $\frac{1}{2}\begin{pmatrix} -1 & -\sqrt{3} \\ -\sqrt{3} & 1 \end{pmatrix}$	A1	1.1b
		(2)	
(b)	$\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} \Rightarrow -\frac{1}{2}x - \frac{\sqrt{3}}{2}y = x$ and $-\frac{\sqrt{3}}{2}x + \frac{1}{2}y = y$	M1	3.1a
	$\Rightarrow -y\sqrt{3} = 3x$ and $y = -x\sqrt{3}$	M1	1.1b
	First equation gives $y = -\frac{3x}{\sqrt{3}} = -x\sqrt{3}$, so equations are the same, hence M fixes all points on the line $y = -x\sqrt{3}$.	A1ft	2.1
		(3)	
		(5 marks)	

Notes:

(a)

M1: Attempts the multiplication of the matrices the correct way round. If no working shown, needs correct answer to imply the method.

A1: Correct matrix.

(b)

M1: Extracts simultaneous equations using their matrix $\mathbf{M}$, or using $\mathbf{M}^{-1}$ from $\mathbf{x} = \mathbf{M}^{-1}\mathbf{x}$.

M1: Gathers terms from their two equations.

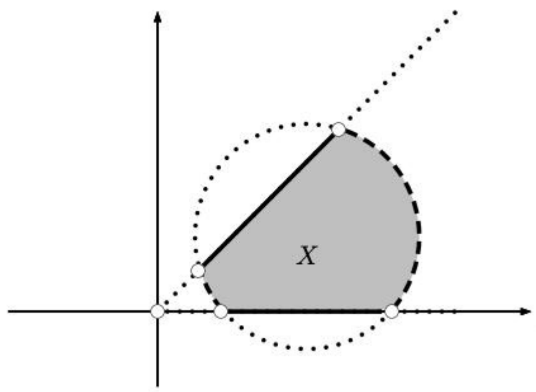
A1ft: Shows the equations are consistent and deduces the correct line. Accept equivalent equations as long as both have been shown to be the same. Follow through only on an incorrect order of matrices from part (a). If wrong order of matrices is used in (a) the equation will end up being $y = x\sqrt{3}$.

ALT (b)

M1: Identifies P is a rotation (through $\frac{\pi}{3}$ clockwise) AND Q is a reflection (in the y-axis).

M1: Hence deduce s M is a reflection (through line at angle $-\frac{\pi}{3}$ to the x-axis) and so has a line of fixed points.

A1ft: All reasoning correct (including correct reflections/rotations if stated) and identifies the equation of the line. Follow through as above.

Question	Scheme	Marks	AOs
	$A=\{z\in\mathbb{C}:\lvert z-4-2\mathrm{i}\rvert<3\}$, $B=\left\{z\in\mathbb{C}:0\text{ Ñ }\arg(z)\text{ Ñ }\frac{\pi}{4}\right\}$ and $X=A\cap B$.		
2(a)			
	Circle	B1	1.1b
	Sector	B1	1.1b
	Set X	B1ft	1.1b
		(3)	
(b)	$\lvert 5+4\mathrm{i}-4-2\mathrm{i}\rvert^2=\lvert 1+2\mathrm{i}\rvert^2=1^2+2^2=5<9$ so $5+4\mathrm{i}\in A$	M1	1.1b
	$\mathrm{Re}(5+4\mathrm{i})=5\text{Ö}\mathrm{Im}(5+4\mathrm{i})=4$ and $\mathrm{Im}(5+4\mathrm{i})=4\text{Ö}0$, so $5+4\mathrm{i}\in B$ OR $\arg(5+4\mathrm{i})=\tan^{-1}\left(\frac{4}{5}\right)=0.6747\ldots$ and $0\text{Ñ}0.6747\ldots\text{Ñ}\frac{\pi}{4}$, so $5+4\mathrm{i}\in B$	M1	2.2a
	As $5+4\mathrm{i}$ is in both A and B, so $5+4\mathrm{i}\in X=A\cap B$	A1	2.1
		(3)	
			(6 marks)

Notes:

(a)

B1: A circle/relevant arcs of circle in the first (and fourth) quadrant(s) only with centre above the real axis (shown dotted above). If only arcs shown, they should be only in the first quadrant, as shown dashed above.

B1: Correct sector/portion of sector intersecting the circle, above the real axis and below the line $y=x$.

B1: Correct region shaded, inside their circle and sector (as long as they intersect), with indication that the arcs of the circle are excluded, but the rays of the sector are included - dotted/dashed lines for excluded and solid for included. Ignore lines that are not part of the boundary. If the circle was initially drawn solid, accept if it is clearly indicated the arcs are not included.

(b)

M1: Attempts to show that $5+4\mathrm{i}$ is inside the circle, ie considers $\lvert 5+4\mathrm{i}-4-2\mathrm{i}\rvert$ (or its square) and compares with 3 (or 9).

M1: Attempts to show that $5+4\mathrm{i}$ is inside the sector, ie finds the argument and checks it is in the range required.

A1: Both attempts correct with a conclusion that $5+4\mathrm{i}$ is inside the set X – must be clear it has been checked to be in both sets.

Question	Scheme	Marks	AOs
3(a)	$\begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{vmatrix} = 3(3\times 2 - k\times -1) - 2(2\times 2 - 1\times -1) + 1(2\times k - 1\times 3)$	M1	1.1b
	$= 5k + 5$	A1	1.1b
		(2)	
(b)	(i) $3x + 2y + z = 4$	B1	1.1b
	(ii) EITHER $y = 2 - \lambda \Rightarrow \lambda = 2 - y$ OR $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} = 0$ and $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 0 = 0 \Rightarrow \mathbf{n} = A\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$	M1	1.1b
	EITHER $x = 1 + (2 - y) + \mu \Rightarrow z = 3 - (2 - y) + 2(x - (2 - y) - 1)$ OR $d = A\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 5A$	M1	1.1b
	$\Rightarrow 2x + 3y - z = 5$	A1	1.1b
		(4)	
(c)	The planes meet when all three equations are satisfied, so we can find where they meet by solving $\begin{pmatrix} 3 & 2 & 1 \\ 2 & 3 & -1 \\ 1 & k & 2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 4 \\ 5 \\ -1 \end{pmatrix}$	B1	3.1a
	If the planes form a sheaf, then they must share a common line. But if $k \neq -1$ the determinant of the matrix is non-zero, so the equation has unique solution and hence the planes would meet in a single point. Therefore, we must have $k = -1$.	B1	2.3
		(4)	
			(8 marks)

Notes:

(a)

M1: Attempts determinant. Correct $()-()+()$ structure, but allow up to two slips in entries.

A1: Determinant is $5k + 5$.

(b)(i)

B1: Correct equation. (Accept multiples.)

(ii)

M1: Eliminates λ or μ from Cartesian equations OR attempts to find a vector normal to both direction vectors using scalar product (or cross product may be used).

M1: Eliminates the other variable from their equations OR uses the scalar product with $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and their normal to find *d*.

A1: Correct equation. (Accept multiples.)

(c)
B1: Makes the link between the Cartesian equations and the matrix in (a).
B1: Identifies a sheaf cannot be formed if the solution is unique and so the matrix must be singular to form a sheaf, and hence $k=-1$.

Question	Scheme	Marks	AOs
4(a)	$P_n=(K+50n)\left(20+\dfrac{n^2}{45}\right)-(1000+10n^2)$ where $K=450$ or 500	M1	2.1
	$P_n=\dfrac{10}{9}\left(n^3+900n+7200\right)$ *	A1*	1.1b
		(2)	
(b)	$T_N=\sum_{n=1}^{N}P_n=\dfrac{10}{9}\left(\sum_{n=1}^{N}n^3+900\sum_{n=1}^{N}n+7200\sum_{n=1}^{N}1\right)$		
	$=\dfrac{10}{9}\left(\underline{\dfrac{1}{4}N^2(N+1)^2+900\times\dfrac{1}{2}N(N+1)}+\underline{\underline{7200N}}\right)$	M1A1 B1	1.1a 1.1b 1.1b
	$=\dfrac{10N}{36}\left(N^3+2N^2+N+1800N+1800+28800\right)$	M1	2.1
	$T_N=\dfrac{5N}{18}\left(N^3+2N^2+1801N+30600\right)$	A1	1.1b
		(5)	
(c)	Total profit is £ $T_{20}=$ £419000	M1	3.4
	E.g. The predicted value is much less than the actual value, and so the model seems to be underestimating the profit by some way, so not a good model.	A1ft	3.5a
		(2)	
			(9 marks)

Notes:

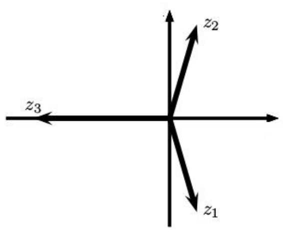
(a)
M1: Attempts forming (number of units sold) × (profit per unit) − production cost. Allow $450+50n$ or $500+50n$.
A1*: cso. All work must be correct with the clear statement of how the formula arises.
(b)
M1: Substitutes for one of the standard formulae for $\sum r^3$ or $\sum r$ into the expression.
A1: Both formulae correctly applied.
B1: For $\sum_{r=1}^{n}1=n$ used.
M1: Expanding inside the bracket and factorising out the $\dfrac{n}{4}$.

A1: Correct expression found.

(c)

M1: Evaluates the total profit after 20 years - through use of their formula or via summing 20 terms from (a).

A1ft: Makes a comparison with the known profit and draws appropriate conclusion. Accept any well reasoned response for either accepting or rejecting the model, but it must include some comparison of values. Follow through their result of sum, so, for example, if evaluated close to £500000 they should conclude the model is appropriate.

Question	Scheme	Marks	AOs
5	$\mathrm{f}(z) = 8z^3 + 12z^2 + 6z + 65$; root $\alpha = \frac{1}{2} - i\sqrt{3}$		
(a)	$\frac{1}{2} + \mathrm{i}\sqrt{3}$	B1	1.2
		(1)	
(b)	Attempts quadratic factor: $z^2 - z + \frac{13}{4}$ or $4z^2 - 4z + 13$	M1	1.1b
	So $\mathrm{f}(z) = (4z^2 - 4z + 13)(2z + 5)$ (oe)	M1	1.1b
	So roots are $z_1 = \frac{1}{2} - i\sqrt{3}$, $z_2 = \frac{1}{2} + i\sqrt{3}$ and $z_3 = -\frac{5}{2}$	A1	1.1b
		(3)	
(c)	 Correct complex roots	B1	1.1b
	Correct real root	B1ft	1.1b
		(2)	
(d)	E.g. $\lvert z_1 - z_2 \rvert = 2\sqrt{3}$, $\lvert z_1 - z_3 \rvert = \sqrt{\left(\frac{1}{2} + \frac{5}{2}\right)^2 + 3} = \sqrt{12} = 2\sqrt{3}$ and $\lvert z_2 - z_3 \rvert = 2\sqrt{3}$ by symmetry. OR $\arg(z_2 - z_3) = \arg\left(3 + \mathrm{i}\sqrt{3}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$, so (by symmetry) angle at z_3 is $2 \times \frac{\pi}{6} = \frac{\pi}{3}$, and since by symmetry the angles at z_1 and z_2 are equal, they must also each be $\frac{\pi}{3}$ (so all add to π).	M1	3.1a
	All three sides of the triangle are the same length, and so the vertices form an equilateral triangle. OR All three angles are $\frac{\pi}{3}$ and so the triangle formed by the vertices	A1	2.1

	is equilateral.		
		(2)	
			(8 marks)

Notes:

(a)

B1: Correct conjugate root.

(b)

M1: Attempts quadratic factor $z - 2\mathrm{Re}(\alpha)z + |\alpha|^2$ or $(z-\alpha)(z-\alpha^*)$. As a minimum accept an attempt at the product of roots.

M1: Attempts to find the linear term, e.g. by factorisation or dividing by quadratic term or use of product of roots being 65.

A1: Correct solutions. All three must be stated. Ignore labelling. Answers only score zero marks in (b). Algebra must be used.

(c)

B1: Correct placement of complex roots, symmetric about real axis, in first and fourth quadrants, closer to imaginary axis than real axis. Lines/arrows not needed, just points.

B1ft: Correct placement for real root. If root is correct then on real axis further from origin than other roots, but follow through if a positive root found in (b).

(d)

M1: A complete method to find either all three sides or all three angles of the triangle.

A1: Sides/angles all correct from correct work/reasoning and conclusion made to draw the argument together.

Question	Scheme	Marks	AOs
6(a)	For $n=1$: $\mathrm{f}(1)=1^5+4(1)=5$. So the statement is true for $n=1$	B1	2.2a
	Assume true for $n=k$Ö1, so k^5+4k is divisible by 5	M1	2.4
	$\mathrm{f}(k+1)=(k+1)^5+4(k+1)=k^5+5k^4+\ldots$	M1	2.1
	$=k^5+5k^4+10k^3+10k^2+5k+1+4k+4$	A1	1.1b
	$=k^5+4k+5\left(k^4+2k^3+2k^2+k+1\right)$ $=\mathrm{f}(k)+5\left(k^4+2k^3+2k^2+k+1\right)$	A1	1.1b
	Hence the result is true for $n=k+1$. Since it is true for $n=1$, and if true for $n=k$ then true for $n=k+1$, thus by mathematical induction the result holds for all $n \in \mathbb{Z}^+$	A1cso	2.4
		(6)	
(b)	For any x, $\mathrm{f}(-x)=\left((-x)^5+4(-x)=\right)-x^5-4x=-(x^5+4x)=-\mathrm{f}(x)$ *	B1*	2.5
		(1)	
(c)	We know from (a) that $\mathrm{f}(n)$ is divisible 5 for all positive integers, and since by (b) for negative n we have $\mathrm{f}(n)=-\mathrm{f}(-n)$ is divisible by 5 (as $-n$ is a positive integer).	M1	2.1
	Since $\mathrm{f}(0)=0$ is also divisible by 5, so $\mathrm{f}(n)$ is divisible by 5 for all integers n.	A1	2.4
		(2)	
		(9 marks)	

Notes:

(a)

B1: Shows the statement is true for $n=1$.

M1: Makes the inductive assumption, assume true for $n=k$.

M1: Attempts to expand $\mathrm{f}(k+1)$ or $\mathrm{f}(k+1)-\mathrm{f}(k)$ using the binomial theorem.

A1: Correct expansion.

A1: Correct expression for $\mathrm{f}(k+1)$ with common factor 5 made clear.

A1: Completes the inductive argument conveying **all** three underlined points or equivalent at some point in their argument.

(b)

B1*: Correct proof with each of the un-bracketed expressions shown in the scheme.

(c)

M1: Reasons that result holds for negative integers by results of (a) and (b).

A1t: Considers the $n=0$ case and concludes true for all integers. Note: if $n=0$ case was used as the inductive base in (a), the reasoning here must clearly refer to all integers for the mark to be awarded.

Question	Scheme	Marks	AOs
7(a)	Explains that there are $0.23A_n$ new juveniles from births AND $\frac{2}{3}(0.87)J_n$ surviving juveniles staying juveniles	B1	2.4
	$J_{n+1} = 0.23A_n + \frac{2}{3}(0.87)J_n \ = 0.23A_n + 0.58J_n$	B1	1.1b
		(2)	
(b)	$p = 0.30$	B1	3.3
	$\mathbf{M} = \begin{pmatrix} 0.97 & 0.29 \\ 0.23 & 0.58 \end{pmatrix}$	B1ft B1	3.3 3.3
		(3)	
(c)(i)	$\begin{pmatrix} A_{-1} \\ J_{-1} \end{pmatrix} = \begin{pmatrix} 0.97 & 0.29 \\ 0.23 & 0.58 \end{pmatrix}^{-1} \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} = \begin{pmatrix} 1.228\ldots \\ 0.0302\ldots \end{pmatrix}$ $\left(\text{NB } \mathbf{M}^{-1} = \begin{pmatrix} 1.1695\ldots & -0.58479\ldots \\ -0.46380\ldots & 1.9560\ldots \end{pmatrix} \right)$	M1	3.4
	Hence total population is $1.228\ldots + 0.0302\ldots = \ldots$	M1	1.1b
	$= 1.26$ million	A1	2.2b
		(3)	
(c)(ii)	$\begin{pmatrix} A_7 \\ J_7 \end{pmatrix} = \mathbf{M}^7 \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} \left(= \begin{pmatrix} 2.117\ldots \\ 0.938\ldots \end{pmatrix} \right)$	M1	3.4
	So the juvenile population on 1st January 2025 is expected to be 0.938 million (or 938000)	A1	1.1b
		(2)	
(d)	For attempting to include 15000 juvenile being exported by adding (or subtracting) a suitable vector, ie. $\begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_n \\ J_n \end{pmatrix} - \begin{pmatrix} 0 \\ \text{their value} \end{pmatrix}$	M1	3.5c
	$\begin{pmatrix} A_1 \\ J_1 \end{pmatrix} = \begin{pmatrix} 1.2 \\ 0.3 \end{pmatrix} \quad \begin{pmatrix} A_{n+1} \\ J_{n+1} \end{pmatrix} = \mathbf{M} \begin{pmatrix} A_n \\ J_n \end{pmatrix} - \begin{pmatrix} 0 \\ 0.015 \end{pmatrix} \quad$ nÖ1	A1ft	3.3
		(2)	
(e)	E.g. The exportation may have an effect on the proportion of juveniles becoming adults/proportion who become adults each year may fluctuate/birth rates and death rates may change over time/two sub-populations may not sufficiently reflect the population.	B1	3.5b
		(1)	
			(13 marks)

Notes:

(a)

B1: Explains the two components of the equation with reference to surviving juveniles remaining juveniles and that new juveniles arise through birth.

B1: Correct equation.

(b)

B1: Correct value for p.

B1ft: One row or column correct, follow through their equation in (a) on second row.

B1: Completely correct matrix.

(c)(i)

M1: Uses the model to find the population for 2017, ie $n = -1$, using their $\mathbf{M}$ and initial vector. May see $\mathbf{M}^{-1}$ used, or the correct answer can imply the method

M1: Adds the two components of their vector to give the total population.

A1: awrt 1.26 million.

(c)(ii)

M1: Uses a calculator to evaluate $\mathbf{M}^7(1.2, 0.3)^{\mathrm{T}}$, or multiplies by $\mathbf{M}$ seven times oe. Awrt $(2.12, 0.938)^{\mathrm{T}}$, or just awrt 0.938 is sufficient for this mark.

A1: Concludes juvenile population is 0.938 million, or 938000.

(d)

M1: For incorporating the exportation into the model by subtracting (or adding if a negative entry is used) a vector with top entry zero and bottom entry an attempt at the 15 000 exported. So allow if an incorrect value of e.g. 15 000 or 0.15 is used as the value for this mark.

A1ft: Sets up the new system in full with **correct** vector subtracted (or added), but follow through on their p and $\mathbf{M}$. Allow if the range on n is omitted**.**

(e)

B1: Any valid limitation of the model - allow for limitations of the new model or original model. Some examples are given above but accept any sensible limitation.

Question	Scheme	Marks	AOs
8(a)	Vol $V_1 = (\pi)\int_{(0)}^{(8)} y^2 \mathrm{d}x = (\pi)\int_{(0)}^{(8)} x^{\frac{4}{3}} \mathrm{d}x$	B1	1.1b
	$= (\pi)\left[\dfrac{x^{\frac{7}{3}}}{\frac{7}{3}}\right]_{(0)}^{(8)}$	M1	1.1b
	$= (\pi)\left[\dfrac{3\times 8^{\frac{7}{3}}}{7} - 0\right]$	M1	1.1b
	$= \pi \times \dfrac{3\times 2^7}{7} = \dfrac{384\pi}{7}$ *	A1*	2.1
		(4)	
(b)	Attempts to find the ratio of volume of V_2 to total volume, using $\pi\int x^2 \mathrm{d}y$ to get V_2.	M1	3.1a
	Vol $V_2 = (\pi)\int_{(0)}^{(4)} x^2 \mathrm{d}y = (\pi)\int_{(0)}^{(4)} y^3 \mathrm{d}y$	B1	1.1b
	So Vol $V_2 = (\pi)\left[\dfrac{y^4}{4}\right]_0^4 = (\pi)\left(\dfrac{4^4}{4} - 0\right)$	M1	1.1b
	$= 64\pi$	A1	1.1b
	So probability V_2 selected in a single trial is $p = \dfrac{64\pi}{64\pi + \frac{384\pi}{7}}\left(= \dfrac{7}{13}\right)$	M1	1.1b
	Identifies binomial distribution needed, $X \sim \mathrm{B}(10, \text{their } p)$.	M1	3.1a
	$P(X = 8) = {}^{10}C_8\left(\dfrac{7}{13}\right)^8\left(\dfrac{6}{13}\right)^2 = 0.0677$ (4 d.p.)	A1	1.1b
		(7)	
			(11 marks)

Notes:

(a)

B1: Correct integral with $y^2 = x^{\frac{4}{3}}$. No need for π or limits for this mark.

M1: Attempts the integration ($x^n \to x^{n+1}$).

M1: Applies limits 0 and 8, subtracts correct way. The lower limit of zero may be missing for this mark.

A1*: Simplifies to correct answer, no errors. Evidence of the lower limit being correctly applied should be seen for this mark.

(b)

M1: A full method to find the ratio V_2 / Total Volume to establish the probability V_2 is drawn in a single trial.

B1: Correct integral, need not have π or limits at this stage.

M1: Attempts the integration and applies limits of 4 and 0.

A1: Correct volume for V_2

M1: Combines result of (a) and their second integral to find the probability V_2 is drawn in a single trial, ie $p = \dfrac{\text{their } V_2 \text{ volume}}{\text{Sum of both volumes}}$ or ratio $V_1 : V_2 = \dfrac{384\pi}{7} : 64\pi = 6:7$, so $p = \dfrac{7}{13}$

M1: Demonstrates awareness of the binomial distribution being needed. This may be implied by a correct value (from calculator) if a correct p has been seen, or may be evidenced by writing out the distribution as shown in scheme.

A1: Correct answer to 4 decimal places.

Question	Scheme	Marks	AOs
9(a)	Forms a correct strategy to find the minimum distance between the comet and O. Using $\overrightarrow{OC} = 146\mathbf{i} + 234\mathbf{j} - 85\mathbf{k} + \lambda(-21\mathbf{i} - 33\mathbf{j} + 13\mathbf{k})$ **Way 1** Attempts dot product of $\overrightarrow{OC}$ and the direction $\mathbf{d}$ to form an equation in λ, then uses λ to find min distance or its square. **Way 2** Attempts distance formula for $\overrightarrow{OC}$ in terms of λ, then completes the square in λ to find min distance or its square. **Way 3** Attempts dot product to find angle, θ between the line and $\overrightarrow{OX} = 146\mathbf{i} + 234\mathbf{j} - 85\mathbf{k}$ and use a trigonometric approach to find the minimum distance within a right angle triangle.	M1	3.1a
	Way 1 $\overrightarrow{OC} \bullet \mathbf{d} = 0 \Rightarrow \begin{pmatrix} 146 - 21\lambda \\ 234 - 33\lambda \\ -85 + 13\lambda \end{pmatrix} \bullet \begin{pmatrix} -21 \\ -33 \\ 13 \end{pmatrix} = 0 \Rightarrow -3066 + 441\lambda -$ $7722 + 1089\lambda - 1105 + 169\lambda = 0 \Rightarrow 1699\lambda = 11893 \Rightarrow \lambda = \ldots$ **Way 2** Min distance, d, given by $d^2 = (146 - 21\lambda)^2 + (234 - 33\lambda)^2 + (-85 + 113\lambda)^2 = \ldots$ **Way 3** $\cos\theta = \left(\overrightarrow{OX} \bullet \mathbf{d}\right) / \left(\lvert\overrightarrow{OX}\rvert \lvert\mathbf{d}\rvert\right)$ $= \dfrac{\pm(146 \times (-21) + 234 \times (-33) + (-85) \times 13)}{\sqrt{146^2 + 234^2 + (-85)^2}\sqrt{(-21)^2 + (-33)^2 + 13^2}} = -0.9997\ldots$	M1	1.1b
	Way 1 $\lambda = 7$ **Way 2** So $d^2 = 1699\lambda^2 - 23786\lambda + 83297$ **Way 3** $\theta = 178.653\ldots°$ or $1.34653\ldots°$ (oe) or $\sin\theta = 0.023499\ldots$	A1	1.1b
	Way 1 So distance is $d = \sqrt{(146 - 7 \times 21)^2 + (234 - 7 \times 33)^2 + (-85 + 7 \times 13)^2} = \ldots\left(= \sqrt{46} = 6.782\ldots\right)$ **Way 2** $= 1699\left[(\lambda - 7)^2 - 49\right] + 83297 = 1699(\lambda - 7)^2 + 46$, so $d_{\min} = \sqrt{\text{their } 46}$ or $d_{\min}^2 = \text{their } 46$ **Way 3** So $d_{\min} = \sqrt{146^2 + 234^2 + (-85)^2}\sin\theta = \ldots(= 6.782\ldots)$	M1	1.1b
	Interprets situation correctly and compares their minimum distance with the radius of the planet with correct units, e.g. 6500km compared with 6782km or 6.5^2 with 46.	M1	3.1b
	The closest distance of the comet to the planet is more than a radius away from the centre, so comet (just) misses planet.	A1	3.2a
		(6)	

Question	Scheme	Marks	AOs
(b)	$C_{\lambda=4}=\begin{pmatrix}62\\102\\-33\end{pmatrix}$, so need $\mathbf{d}_1=\begin{pmatrix}5\\0\\12\end{pmatrix}-\begin{pmatrix}62\\102\\-33\end{pmatrix}$ and $\mathbf{d}_2=\begin{pmatrix}4\\12\\-3\end{pmatrix}-\begin{pmatrix}62\\102\\-33\end{pmatrix}$	M1	3.1a
	$\mathbf{d}_1=\begin{pmatrix}-57\\-102\\45\end{pmatrix}$ and $\mathbf{d}_2=\begin{pmatrix}-58\\-90\\30\end{pmatrix}$	A1	1.1b
	$\cos\angle ACB=\dfrac{(-57)(-58)+(-102)(-90)+(45)(30)}{\sqrt{(-57)^2+(-102)^2+45^2}\sqrt{(-58)^2+(-90)^2+30^2}}$ $\left(=\dfrac{13836}{\sqrt{15678}\sqrt{12364}}=0.9937\ldots\right)$	M1	1.1b
	$\angle ACB=6.4°$ (awrt) ($6.399\ldots°$)	A1	1.1b
		(4)	
(c)	The comet may not follow a straight line course, (as e.g. gravity when nearing the planet will affect it).	B1	3.2b
		(1)	
			(11 marks)

Notes:

(a)

M1: Demonstration of a correct overall strategy to find the minimum distance, or its square, between the comet and O. See examples in scheme. There are other variations, e.g. via differentiating an expression for d^2 which follow a similar pattern.

M1: See scheme. A correct starting method, setting up an equation to find λ, or an expression for the square of distance, or correct equation to find an angle in an appropriate triangle.

A1: Correct λ (Way 1) or quadratic in λ (Way 2) or angle or its sine (Way 3). Equivalents in radians are $33.11809\ldots$ or $0.02350\ldots$.

M1: A correct attempt to find the minimum distance or the square of the minimum distance.

M1: Translates the information about the planet into the context of the question to draw an appropriate comparison between the minimum distance, d, between comet and O with the radius, or compares d^2 with the square of the radius. Units must match (ie both in km or in thousands of km).

A1: Correct conclusion drawn, following a correct minimum distance found.

(b)

M1: Identifies the correct direction vectors required to find the angle.

A1: Correct two vectors found. Any (non-zero) multiples of these are fine.

M1: Applies dot product formula with their direction vectors.

A1: Correct angle.

(c)

B1: See scheme. Accept any other reasonable comment, e.g. comet may lose mass as it travels, and this may affect its motion, satellites may not be in exactly the same positions and so on.