Paper 2 Option A

Further Pure Mathematics 1 Mark Scheme (Section A)

Question	Scheme	Marks	AOs
1(a)	$\sec x - \tan x = \dfrac{1}{\dfrac{1-t^2}{1+t^2}} - \dfrac{2t}{1-t^2}$	M1	2.1
	$= \dfrac{1+t^2}{1-t^2} - \dfrac{2t}{1-t^2} = \dfrac{1-2t+t^2}{1-t^2}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1-t)(1+t)} = \dfrac{1-t}{1+t}$ *	A1*	2.1
		(3)	
(b)	$\dfrac{1-\sin x}{1+\sin x} = \dfrac{1-\dfrac{2t}{1+t^2}}{1+\dfrac{2t}{1+t^2}}$	M1	1.1a
	$= \dfrac{1+t^2-2t}{1+t^2+2t}$	M1	1.1b
	$= \dfrac{(1-t)^2}{(1+t)^2} = \left(\dfrac{1-t}{1+t}\right)^2 = (\sec x - \tan x)^2$ *	A1*	2.1
		(3)	
		(6 marks)	

Notes:

(a)

M1: Uses $\sec x = \dfrac{1}{\cos x}$ and the t-substitutions for both cos x and tan x to obtain an expression in terms of t

M1: Sorts out the sec x term, and puts over a common denominator of $1-t^2$

A1*: Factorises both numerator and denominator (must be seen) and cancels the $(1+t)$ term to achieve the answer

(b)

M1: Uses the t-substitution for sin x in both numerator and denominator

M1: Multiples through by $1+t^2$ in numerator and denominator

A1*: Factorises both numerator and denominator and makes the connection with part (a) to achieve the given result

Question	Scheme	Marks	AOs
2	£300 purchased one hour after opening $\Rightarrow V_0 = 3$ and $t_0 = 1$; half an hour after purchase $\Rightarrow t_2 = 1.5$, so step h required is 0.25	B1	3.3
	$t_0 = 1,\ V_0 = 3,\ \left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 \approx \frac{3^2 - 1}{1^2 + 3} = 2$	M1	3.4
	$V_1 \approx V_0 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_0 = 3 + 0.25 \times 2 = \ldots$	M1	1.1b
	$= 3.5$	A1ft	1.1b
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 \approx \frac{3.5^2 - 1.25}{1.25^2 + 1.25 \times 3.5} \left(= \frac{176}{95}\right)$	M1	1.1b
	$V_2 \approx V_1 + h\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1 = 3.5 + 0.25 \times \frac{176}{95} = 3.963\ldots$, so £396 (nearest £)	A1	3.2a
		(6)	
			(6 marks)

Notes:

B1: Identifies the correct initial conditions and requirement for h

M1: Uses the model to evaluate $\frac{\mathrm{d}V}{\mathrm{d}t}$ at t_0, using their t_0 and V_0

M1: Applies the approximation formula with their values

A1ft: 3.5 or exact equivalent. Follow through their step value

M1: Attempt to find $\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)_1$ with their 3.5

A1: Applies the approximation and interprets the result to give £396

Question	Scheme	Marks	AOs
3	$\frac{1}{x} < \frac{x}{x+2}$		
	$\frac{(x+2)-x^2}{x(x+2)} < 0$ or $x(x+2)^2 - x^3(x+2) < 0$	M1	2.1
	$\frac{x^2-x-2}{x(x+2)} > 0 \Rightarrow \frac{(x-2)(x+1)}{x(x+2)} > 0$ or $x(x+2)(2-x)(x+1) < 0$	M1	1.1b
	At least two correct critical values from $-2, -1, 0, 2$	A1	1.1b
	All four correct critical values $-2, -1, 0, 2$	A1	1.1b
	$\{x \in \mathbb{R} : x < -2\} \cup \{x \in \mathbb{R} : -1 < x < 0\} \cup \{x \in \mathbb{R} : x > 2\}$	M1 A1	2.2a 2.5
		(6)	
			(6 marks)

Notes:

M1: Gathers terms on one side and puts over common denominator, or multiply by $x^2(x+2)^2$ and then gather terms on one side

M1: Factorise numerator or find roots of numerator or factorise resulting in equation into 4 factors

A1: At least 2 correct critical values found

A1: Exactly 4 correct critical values

M1: Deduces that the 2 "outsides" and the "middle interval" are required. May be by sketch, number line or any other means

A1: Exactly 3 correct intervals, accept equivalent set notations, but must be given as a set e.g. accept $\mathbb{R} - ([-2, -1] \cup [0, 2])$ or $\{x \in \mathbb{R} : x < -2 \text{ or } -1 < x < 0 \text{ or } x > 2\}$

Question	Scheme	Marks	AOs
4(a)	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert$	M1	3.1a
	$\frac{1}{2}\lvert\mathbf{AB}\times\mathbf{AC}\rvert = \frac{1}{2}\lvert(-2\mathbf{i}+3\mathbf{j}+\mathbf{k})\times(-\mathbf{i}+\mathbf{j}+2\mathbf{k})\rvert$	M1	1.1b
	$=\frac{1}{2}\lvert 5\mathbf{i}+3\mathbf{j}+\mathbf{k}\rvert$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\,(\text{m}^2)$	A1	1.1b
		(4)	
	Alternative		
	Identifies glued face is triangle *ABC* and attempts to find the area, e.g. evidences by use of $\frac{1}{2}\sqrt{\lvert\mathbf{AB}\rvert^2\lvert\mathbf{AC}\rvert^2-(\mathbf{AB.AC})^2}$	M1	3.1a
	$\lvert\mathbf{AB}\rvert^2=4+9+1=14,\quad \lvert\mathbf{AC}\rvert^2=1+1+4=6$ and $\mathbf{AB.AC}=2+3+2=7$	M1	1.1b
	So area of glue is = $\frac{1}{2}\sqrt{('14')('6')-('7')^2}$	M1	1.1b
	$=\frac{1}{2}\sqrt{35}\ (\text{m}^2)$	A1	1.1b
		(4)	
(b)	Volume of parallelepiped taken up by concrete is e.g. $\frac{1}{6}(\mathbf{OC}.(\mathbf{OA}\times\mathbf{OB}))$	M1	3.1a
	$=\frac{1}{6}(\mathbf{i}+\mathbf{j}+2\mathbf{k}).(2\mathbf{i}\times(3\mathbf{j}+\mathbf{k}))$	M1	1.1b
	$=\frac{10}{6}=\frac{5}{3}$	A1	1.1b
	Volume of parallelepiped is 6 $\times$ volume of tetrahedron (= 10), so volume of glass is difference between these, viz. $10-\frac{5}{3}=\ldots$	M1	3.1a
	Volume of glass $=\frac{25}{3}(\text{m}^3)$	A1	1.1b
		(5)	

Question	Scheme	Marks	AOs
	4(b) Alternative		
	$-\mathbf{j}+3\mathbf{k}$ is perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$	M1	3.1a
	Area $AOB = \frac{1}{2}\times\lvert\mathbf{OA}\rvert\times\lvert\mathbf{OB}\rvert = \frac{1}{2}\times 2\times\sqrt{10} = \sqrt{10}$	A1	1.1b
	$\mathbf{i}+\mathbf{j}+2\mathbf{k}-p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i})+\lambda(3\mathbf{j}+\mathbf{k}) \Rightarrow p = \frac{1}{2}$ and so height of tetrahedron is $h = \frac{1}{2}\lvert-\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}$	M1	3.1a
	Volume of glass is $V = 5 \times$ Volume of tetrahedron $= 5\times\frac{1}{3}\sqrt{10}\times\frac{1}{2}\sqrt{10}$	M1	1.1b
	$= \frac{25}{3}\left(\text{m}^3\right)$	A1	1.1b
		(5)	
(c)	The glued surfaces may distort the shapes / reduce the volume of concrete Measurements in m may not be accurate The surface of the concrete tetrahedron may not be smooth Pockets of air may form when the concrete is being poured	B1	3.2b
		(1)	
		(10 marks)	

Question 4 notes:

Accept use of column vectors throughout

(a)

M1: Shows an understanding of what is required via an attempt at finding the area of triangle *ABC*

M1: Any correct method for the triangle area is fine

M1: Finds $\mathbf{AB}$ and $\mathbf{AC}$ or any other appropriate pair of vectors to use in the vector product and attempts to use them

A1: Correct procedure for the vector product with at least 1 correct term $\frac{1}{2}\sqrt{35}$ or exact equivalent

(a) Alternative

M1: Finds two appropriate sides and attempts the scalar product and magnitudes of two of the sides

M1: May use different sides to those shown

M1: Correct full method to find the area of the triangle using their two sides

A1: $\frac{1}{2}\sqrt{35}$ or exact equivalent

Question 4 notes continued:
(b) **M1:** Attempts volume of concrete by finding volume of tetrahedron with appropriate method **M1:** Uses the formula with correct set of vectors substituted (may not be the ones shown) and vector product attempted **A1:** Correct value for the volume of concrete **M1:** Attempt to find total volume of glass by multiplying their volume of concrete by 6 and subtracting their volume of concrete. May restart to find the volume of parallelepiped **A1:** $\frac{25}{3}$ only, ignore reference to units
(b) **Alternative** **M1:** Notes (or works out using scalar products) that $-\mathbf{j}+3\mathbf{k}$ is a vector perpendicular to both $\mathbf{OA} = 2\mathbf{i}$ and $\mathbf{OB} = 3\mathbf{j}+\mathbf{k}$ **A1:** Finds (using that **OA** and **OB** are perpendicular), area of $AOB = \sqrt{10}$ **M1:** Solves $\mathbf{i}+\mathbf{j}+2\mathbf{k} - p(-\mathbf{j}+3\mathbf{k}) = \mu(2\mathbf{i}) + \lambda(3\mathbf{j}+\mathbf{k})$ to get the height of the tetrahedron $\left[(\mu=\lambda=)\ p=\frac{1}{2},\ \text{so } h=\frac{1}{2}\lvert -\mathbf{j}+3\mathbf{k}\rvert = \frac{1}{2}\sqrt{10}\right]$ **M1:** Identifies the correct area as 5 times the volume of the tetrahedron (may be done as in main scheme via the difference) **A1:** $\frac{25}{3}$ only, ignore reference to units
(c) **B1:** Any acceptable reason in context

Question	Scheme	Marks	AOs
5(a)	$y^2 = (8p)^2 = 64p^2$ and $16x = 16(4p^2) = 64p^2$ $\Rightarrow\ P(4p^2, 8p)$ is a general point on C	B1	2.2a
		(1)	
(b)	$y^2 = 16x$ gives $a = 4$, or $2y\frac{\mathrm{d}y}{\mathrm{d}x} = 16$ so $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{y}$	M1	2.2a
	$l:\ y - 8p = \left(\frac{8}{8p}\right)\left(x - 4p^2\right)$	M1	1.1b
	leading to $py = x + 4p^2$ *	A1*	2.1
		(3)	
(c)	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = \ldots$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts x-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = \ldots$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \frac{1}{2}(9 - -9)(12) - \int_0^9 4x^{\frac{1}{2}}\mathrm{d}x$	M1	2.1
	$\int 4x^{\frac{1}{2}}\mathrm{d}x = \frac{4x^{\frac{3}{2}}}{\left(\frac{3}{2}\right)}\ (+c)$ or $\frac{8}{3}x^{\frac{3}{2}}\ (+c)$	M1 A1	1.1b 1.1b
	$\text{Area}(R) = \frac{1}{2}(18)(12) - \frac{8}{3}\left(9^{\frac{3}{2}} - 0\right) = 108 - 72 = 36$ *	A1*	1.1b
		(8)	

Question	Scheme	Marks	AOs
	5(c) Alternative 1		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ into l gives $\frac{3}{2}y = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = \frac{3}{2}y - 9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12) \Rightarrow \text{Area}(R) = \int_0^{12} \left(\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)\right) dy$	M1	2.1
	$\int \left(\frac{1}{16}y^2 - \frac{3}{2}y + 9\right) dy = \frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = \left(\frac{1}{48}(12)^3 - \frac{3}{4}(12)^2 + 9(12)\right) - (0)$ $= 36 - 108 + 108 = 36$ *	A1*	1.1b
		(8)	
	5(c) Alternative 2		
	$B\left(-4, \frac{10}{3}\right)$ into $l \Rightarrow \frac{10p}{3} = -4 + 4p^2$	M1	3.1a
	$6p^2 - 5p - 6 = 0 \Rightarrow (2p-3)(3p+2) = 0 \Rightarrow p = ...$	M1	1.1b
	$p = \frac{3}{2}$ and l cuts px-axis when $\frac{3}{2}(0) = x + 4\left(\frac{3}{2}\right)^2 \Rightarrow x = ...$	M1	2.1
	$x = -9$	A1	1.1b
	$p = \frac{3}{2} \Rightarrow P(9, 12)$ and $x = 0$ in $l: y = \frac{2}{3}x + 6$ gives $y = 6$ $\Rightarrow \text{Area}(R) = \frac{1}{2}(9)(6) + \int_0^9 \left(\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right) dx$	M1	2.1
	$\int \left(\frac{2}{3}x + 6 - 4x^{\frac{1}{2}}\right) dx = \frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}} \ (+c)$	M1	1.1b
		A1	1.1b
	$\text{Area}(R) = 27 + \left(\left(\frac{1}{3}(9)^2 + 6(9) - \frac{8}{3}\left(9^{\frac{3}{2}}\right)\right) - (0)\right)$ $= 27 + (27 + 54 - 72) = 27 + 9 = 36$ *	A1*	1.1b
		(8)	
		(12 marks)	

Question 5 notes:
(a) **B1:** Substitutes $y_P = 8p$ into y^2 to obtain $64p^2$ and substitutes $x_P = 4p^2$ into $16x$ to obtain $64p^2$ and concludes that P lies on C
(b) **M1:** Uses the given formula to deduce the derivative. Alternatively, may differentiate using chain rule to deduce it **M1:** Applies $y - 8p = m(x - 4p^2)$, with their tangent gradient *m*, which is in terms of *p*. Accept use of $8p = m(4p^2) + c$ with a clear attempt to find *c* **A1*:** Obtains $py = x + 4p^2$ by **cso**
(c) **M1:** Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l* **M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$ **M1:** Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$ **A1:** Finds that *l* cuts the *x*-axis at $x = -9$ **M1:** Fully correct method for finding the area of *R* i.e. $\frac{1}{2}(\text{their } x_P - "-9")(\text{their } y_P) - \int_0^{\text{their } x_P} 4x^{\frac{1}{2}}\,dx$ **M1:** Integrates $\pm\lambda x^{\frac{1}{2}}$ to give $\pm\mu x^{\frac{3}{2}}$, where $\lambda, \mu \neq 0$ **A1:** Integrates $4x^{\frac{1}{2}}$ to give $\frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified **A1*:** Fully correct proof leading to a correct answer of 36
(c) Alternative 1 **M1:** Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l* **M1:** Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$ Substitutes their p (which must be positive) into *l* and rearranges to give $x = ...$ **M1:** Finds *l* as $x = \frac{3}{2}y - 9$ **A1:** Fully correct method for finding the area of *R* **M1:** i.e. $\int_0^{\text{their } y_P} \left(\frac{1}{16}y^2 - \text{their}\left(\frac{3}{2}y - 9\right)\right)dy$ **M1:** Integrates $\pm\lambda y^2 \pm \mu y \pm \nu$ to give $\pm\alpha y^3 \pm \beta y^2 \pm \nu y$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$ **A1:** Integrates $\frac{1}{16}y^2 - \left(\frac{3}{2}y - 9\right)$ to give $\frac{1}{48}y^3 - \frac{3}{4}y^2 + 9y$, simplified or un-simplified **A1*:** Fully correct proof leading to a correct answer of 36

Question 5 notes continued:	
(c)	**Alternative 2**
M1:	Substitutes their $x = "-a"$ and $y = \frac{10}{3}$ into *l*
M1:	Obtains a 3 term quadratic and solves (using the usual rules) to give $p = ...$
M1:	Substitutes their p (which must be positive) and $y = 0$ into *l* and solves to give $x = ...$
A1:	Finds that *l* cuts the *x*-axis at $x = -9$
M1:	Fully correct method for finding the area of *R* i.e. $\frac{1}{2}(\text{their } 9)(\text{their } 6) + \int_0^{\text{their } x_P} \left(\text{their}\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)\right)\text{d}y$
M1:	Integrates $\pm\lambda x \pm \mu \pm \nu x^{\frac{1}{2}}$ to give $\pm\alpha x^2 \pm \mu x \pm \beta x^{\frac{3}{2}}$, where $\lambda, \mu, \nu, \alpha, \beta \neq 0$
A1:	Integrates $\left(\frac{2}{3}x + 6\right) - \left(4x^{\frac{1}{2}}\right)$ to give $\frac{1}{3}x^2 + 6x - \frac{8}{3}x^{\frac{3}{2}}$, simplified or un-simplified
A1*:	Fully correct proof leading to a correct answer of 36

Question	Scheme	Marks	AOs
6(a)	Consider $\det\begin{pmatrix}3-\lambda & 1\\ 6 & 4-\lambda\end{pmatrix}=(3-\lambda)(4-\lambda)-6$	M1	1.1b
	So $\lambda^2-7\lambda+6=0$ is characteristic equation	A1	1.1b
		(2)	
	So $\mathbf{A}^2=7\mathbf{A}-6\mathbf{I}$	B1ft	1.1b
(b)	Multiplies both sides of their equation by **A** so $\mathbf{A}^3=7\mathbf{A}^2-6\mathbf{A}$	M1	3.1a
	Uses $\mathbf{A}^3=7(7\mathbf{A}-6\mathbf{I})-6\mathbf{A}$ So $\mathbf{A}^3=43\mathbf{A}-42\mathbf{I}$ *	A1*cso	1.1b
		(3)	
			(5 marks)

Notes:

(a)

M1: Complete method to find characteristic equation

A1: Obtains a correct three term quadratic equation – may use variable other than λ

(b)

B1ft: Uses Cayley Hamilton Theorem to produce equation replacing λ with **A** and constant term with constant multiple of identity matrix, **I**

M1: Multiplies equation by **A**

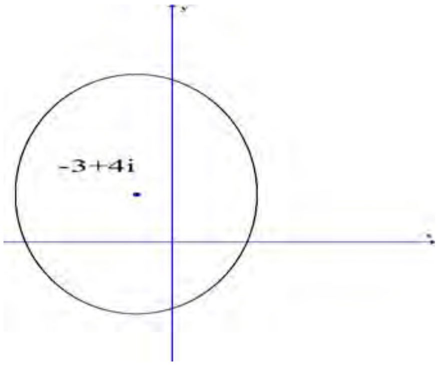
A1*: Replaces $\mathbf{A}^2$ by linear expression in **A** and achieves printed answer with no errors

Question	Scheme	Marks	AOs
7(i)	Adding digits 8 + 1 + 8 + 4 = 21 which is divisible by 3 (or continues to add digits giving 2+1=3 which is divisible by 3) so concludes that 8184 is divisible by 3	M1	1.1b
	8184 is even, so is divisible by 2 and as divisible by both 3 and 2, so it is divisible by 6	A1	1.1b
		(2)	
(ii)	Starts Euclidean algorithm $31 = 27 \times 1 + 4$ and $27 = 4 \times 6 + 3$	M1	1.2
	$4 = 3 \times 1 + 1$ (so hcf = 1)	A1	1.1b
	So $1 = 4 - 3 \times 1 = 4 - (27 - 4 \times 6) \times 1 = 4 \times 7 - 27 \times 1$	M1	1.1b
	$(31 - 27 \times 1) \times 7 - 27 \times 1 = 31 \times 7 - 27 \times 8$ $a = -8$ and $b = 7$	A1cso	1.1b
		(4)	
			(6 marks)

Notes:

(i)
M1: Explains divisibility by 3 rule in context of this number by adding digits
A1: Explains divisibility by 2, giving last digit even as reason and makes conclusion that number is divisible by 6

(ii)
M1: Uses Euclidean algorithm showing two stages
A1: Completes the algorithm. Does not need to state that hcf = 1
M1: Starts reversal process, doing two stages and simplifying
A1cso: Correct completion, giving clear answer following complete solution

Question	Scheme	Marks	AOs
8(a)	$(x-9)^2+(y+12)^2=4[x^2+y^2]$	M1	2.1
	$3x^2+3y^2+18x-24y-225=0$ which is the equation of a circle	A1*	2.2a
	As $x^2+y^2+6x-8y-75=0$ so $(x+3)^2+(y-4)^2=10^2$	M1	1.1b
	Giving centre at (-3, 4) and radius = 10	A1ft	1.1b
		(4)	
(b)		M1	1.1b
		A1	1.1b
		(2)	
(c)	Values range from **their** – 3 –10 to their –3 + 10	M1	3.1a
	So $-13 \le \text{Re}(w) \le 7$	A1ft	1.1b
		(2)	
			(8 marks)

Notes:

(a)
M1: Obtains an equation in terms of *x* and *y* using the given information
A1: Expands and simplifies the algebra, collecting terms and obtains a circle equation correctly, deducing that this is a circle
M1: Completes the square for their equation to find centre and radius
A1ft: Both correct

(b)
M1: Draws a circle with centre and radius as given from **their** equation
A1: Correct circle drawn, as above, with centre at –3 + 4i and passing through all four quadrants

(c)
M1: Attempts to find where a line parallel to the real axis, passing through the centre of the circle, meets the circle so using " their – 3 –10" to "their –3 + 10"
A1ft: Correctly obtains the correct answer for their centre and radius

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A1</td><td>1.1b
1.1b</td></tr>
<tr><td rowspan="4">(ii)</td><td>Identity is zero and there is closure as shown above</td><td>M1</td><td>2.1</td></tr>
<tr><td>3 and 5 are inverses, 4 and 6 are inverses, 2 is self-inverse,
0 is identity so is self-inverse</td><td>M1</td><td>2.5</td></tr>
<tr><td>Associative law may be assumed so S forms a group</td><td>A1</td><td>1.1b</td></tr>
<tr><td></td><td>(6)</td><td></td></tr>
<tr><td rowspan="3">(b)</td><td>4*4*4 = 4* (4 * 4) = 4 * 6 or 4*4*4 = (4* 4) * 4 = 6* 4</td><td>M1</td><td>2.1</td></tr>
<tr><td>= 0 (the identity) so 4 has order 3</td><td>A1</td><td>2.2a</td></tr>
<tr><td></td><td>(2)</td><td></td></tr>
<tr><td rowspan="3">(c)</td><td>3 and 5 each have order 6 so either generates the group</td><td>M1</td><td>3.1a</td></tr>
<tr><td>Either $3^1 = 3,\ 3^2 = 4,\ 3^3 = 2,\ 3^4 = 6,\ 3^5 = 5,\ 3^6 = 0$
Or $5^1 = 5,\ 5^2 = 6,\ 5^3 = 2,\ 5^4 = 4,\ 5^5 = 3,\ 5^6 = 0$</td><td>A1, A1</td><td>1.1b
1.1b</td></tr>
<tr><td></td><td>(3)</td><td></td></tr>
<tr><td colspan="4" align="right">(11 marks)</td></tr>
</table>

Question 9 notes:
(a)(i) **M1:** Begins completing the table – obtaining correct first row and first column and using symmetry **M1:** Mostly correct – three rows or three columns correct (so demonstrates understanding of using * **A1:** Completely correct
(a)(ii) **M1:** States closure and identifies the identity as zero **M1:** Finds inverses for each element **A1:** States that associative law is satisfied and so all axioms satisfied and *S* is a group
(b) **M1:** Clearly begins process to find 4*4*4 reaching 6*4 or 4*6 with clear explanation **A1:** Gives answer as zero, states identity and deduces that order is 3
(c) **M1:** Finds either 3 or 5 or both **A1:** Expresses four of the six terms as powers of either generator correctly (may omit identity and generator itself) **A1:** Expresses all six terms correctly in terms of either 3 or 5 (Do not need to give both)

Question	Scheme	Marks	AOs
10(a)	P_{n-1} is the population at the end of year $n-1$ and this is increased by 10% by the end of year n, so is multiplied by 110% = 1.1 to give $1.1\times P_{n-1}$ as new population by natural causes	B1	3.3
	Q is subtracted from $1.1\times P_{n-1}$ as Q is the number of deer removed from the estate	B1	3.4
	So $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ as population at start is 5000 and $n \in \mathbb{Z}^+$	B1	1.1b
		(3)	
(b)	Let $n = 0$, then $P_0 = (5000-10Q)(1.1)^0 + 10Q = 5000$ so result is true when $n = 0$	B1	2.1
	Assume result is true for $n = k$, $P_k = (1.1)^k(5000-10Q)+10Q$, then **as** $P_{k+1} = 1.1P_k - Q$, so $P_{k+1} = \ldots$	M1	2.4
	$P_{k+1} = 1.1\times 1.1^k(5000-10Q) + 1.1\times 10Q - Q$	A1	1.1b
	So $P_{k+1} = (5000-10Q)(1.1)^{k+1} + 10Q,$	A1	1.1b
	Implies result holds for $n = k+1$ and so by induction $P_n = (5000-10Q)(1.1)^n + 10Q,$ is true for all integer n	B1	2.2a
		(5)	
(c)	For $Q < 500$ the population of deer will grow, for $Q > 500$ the population of deer will fall	B1	3.4
	For $Q = 500$ the population of deer remains steady at 5000,	B1	3.4
		(2)	
			(10 marks)

Notes:

(a)
B1: Need to see 10% increase linked to multiplication by scale factor 1.1
B1: Needs to explain that subtraction of Q indicates the removal of Q deer from population
B1: Needs complete explanation with mention of $P_n = 1.1P_{n-1} - Q, \quad P_0 = 5000$ being the initial number of deer

(b)
B1: Begins proof by induction by considering $n = 0$
M1: Assumes result is true for $n = k$ and uses iterative formula to consider $n = k + 1$
A1: Correct algebraic statement
A1: Correct statement for $k + 1$ in required form
B1: Completes the inductive argument

(c)
B1: Consideration of both possible ranges of values for Q as listed in the scheme
B1: Gives the condition for the steady state