Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 22 Further Pure Mathematics 2

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌐ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1(a)	<table><tr><th>*</th><th>0</th><th>1</th><th>2</th><th>3</th></tr><tr><th>0</th><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><th>1</th><td>1</td><td>0</td><td>3</td><td>2</td></tr><tr><th>2</th><td>2</td><td>3</td><td>0</td><td>1</td></tr><tr><th>3</th><td>3</td><td>2</td><td>1</td><td>0</td></tr></table>	M1 A1	1.1b 1.1b
		(2)	
(b)	Identity element is 0 and there is closure	M1	2.1
	0 is the identity so is self-inverse 1, 2 and 3 are self-inverse	M1	2.5
	Associative law is assumed so G forms a group	A1	1.1b
		(3)	
(c)	0 has order 1 and 1, 2 and 3 have order 2	M1 A1	1.1b 1.1b
		(2)	
(d)	There is no element with order 4 therefore G is not a cyclic group Every element is its own inverse therefore no group generator therefore G is not a cyclic group	B1	2.4
		(1)	
			(8 marks)

Notes:

(a)
M1: Finds at least 6 correct entries
A1: All entries correct

(b)
M1: States closure and identifies 0 as the identity element.
M1: Finds inverses for each element.
A1: Scores both previous method marks and **s**tates that the associative law is satisfied therefore *G* is a group

(c)
M1: States correctly the order of at least two elements
A1: States correctly the order of all four elements

(d)
B1: See scheme

Question	Scheme	Marks	AOs
2(a)(i)	$\begin{vmatrix} 5-\lambda & 1 \\ k & -3-\lambda \end{vmatrix} = (5-\lambda)(-3-\lambda) - k = 0$	M1	2.1
	$\lambda^2 - 2\lambda - k - 15 = 0$	A1	1.1b
	$b^2 - 4ac = (-2)^2 - 4(1)(-k-15) = 0$ $\Rightarrow k = \ldots$ | $-k - 15 = 1 \Rightarrow k = \ldots$	M1	1.1b
	$k = -16$	A1	1.1b
		(4)	
(a)(ii)	$\lambda^2 - 2\lambda + 1 = 0 \Rightarrow \lambda = \ldots$	M1	1.1b
	$\lambda = 1$	A1	1.1b
		(2)	
(b)	$\begin{pmatrix} 5 & 1 \\ \text{their } k & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ leading to $ax + by = 0$ or $ax = by$ $\begin{pmatrix} 5 & 1 \\ \text{their } k & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \text{their } \lambda \begin{pmatrix} x \\ y \end{pmatrix}$ leading to $ax + by = 0$ or $ax = by$ **Alternative approaches** $\begin{pmatrix} 5 & 1 \\ -16 & -3 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x \\ mx \end{pmatrix}$ leading to $\left.\begin{matrix} 5x + mx = x \\ -16x - 3mx = mx \end{matrix}\right\} -16x - 3mx = m(5x + mx)$ Leading to a value for m $\{m^2 + 8m + 16 = 0\}$ $\begin{pmatrix} 5 & 1 \\ -16 & -3 \end{pmatrix}\begin{pmatrix} x \\ mx + c \end{pmatrix} = \begin{pmatrix} x \\ mx + c \end{pmatrix}$ leading to $\left.\begin{matrix} 5x + mx + c = x \\ -16x - 3mx - 3c = mx + c \end{matrix}\right\} -16x - 3mx - 3c = m(5x + mx + c) + c$ Leading to a value for m $\{-3c = mc + c$ or $m^2 + 8m + 16 = 0\}$	M1	2.1
	$y = -4x$ o.e.	A1	1.1b
		(2)	
	(8 marks)		

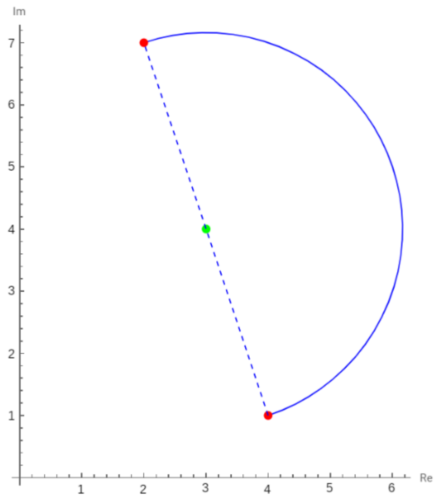
Notes:

(a)(i)

M1: Starts the process by finding the determinant of $\mathbf{M} - \lambda\mathbf{I}$ and sets = 0.

A1: Correct quadratic equation.

M1: Sets the discriminant of their 3TQ = 0 to find a value for *k*. Alternatively identifies that for a perfect square the constant term must equal 1 and finds a value for *k*. **A1:** Correct value for *k*.
(a)(ii) **M1:** Uses their value of *k* to form and solve their 3TQ to find the eigenvalue. This mark can be implied by a correct eigenvalue. **A1:** Correct eigenvalue
(b) **M1:** Constructs a rigorous argument using their eigenvalue to find the Cartesian equation of the invariant line. **A1:** Correct equation.

Question	Scheme		Marks	AOs
3(a)		Draws part of a circle with end points (4,1) and (2, 7),	M1	1.1b
		A semi-circle with end points (4,1) and (2, 7) to the right	A1	1.1b
			(2)	
(b)	Centre $(3, 4)$		B1	2.2a
	Finds the radius using Pythagoras $r=\sqrt{(4-"3")^2+(1-"4")^2}$ or $r=\sqrt{(2-"3")^2+(7-"4")^2}$ or $r=\frac{1}{2}\sqrt{(4-2)^2+(1-7)^2}$		M1	1.1b
	Finds the distance from the origin to their centre using Pythagoras $d=\sqrt{("3"-0)^2+("4"-0)^2}$		M1	1.1b
	Adds this distance to their radius		dM1	3.1a
	$\lvert z\rvert=5+\sqrt{10}$		A1	1.1b
			(5)	
			(7 marks)	

Notes:

(a)

M1: See scheme

A1: See scheme

(b)

B1: Deduces the correct centre coordinates.

M1: Uses Pythagoras and their centre coordinates to find the radius

$r=\sqrt{(4-"\text{their centre } x")^2+(1-"\text{their centre } y")^2}$ or

$r=\sqrt{(2-"\text{their centre } x")^2+(7-"\text{their centre } y")^2}$ or

half of the diameter $r=\frac{1}{2}\sqrt{(4-2)^2+(1-7)^2}$

M1: Finds the distance from their centre to the origin

dM1: Dependent on previous method mark. A complete method to find the maximum value of $|z|$ Adds the distance to the centre to their radius

A1: $|z| = 5 + \sqrt{10}$

Question	Scheme	Marks	AOs
4(a)	$A = 50$	B1	3.3
	Interest rate is 0.5% so multiplied by 1.005	B1	2.4
		(2)	
(b)	A complete method to solve the recurrence relation using $U_n = \text{CF} + \text{PS} = c(1.005)^n + \lambda$	M1	3.1a
	$PS = l$ Þ $l = 1.005l$ - "50" leading to $l = ...$	M1	1.1b
	$l = 10000$	A1	1.1b
	Uses $U_0 = 1000$ and their value of l to find the value of c $1000 = c(1.005)^0 + "10000"$ Leading to $c = ...\{-9000\}$	M1	1.1b
	$U_n = 10000 - 9000(1.005)^n$	A1	1.1b
		(5)	
	Alternative A complete method to solve the recurrence relation using $U_n = \text{CF} + \text{PS} = c(1.005)^n + \lambda$	M1	3.1a
	$U_0 = c(1.005)^0 + \lambda \Rightarrow 1000 = c + \lambda$	M1	1.1b
	$U_1 = 1.005(1000) - 50 = 955$ $U_1 = c(1.005)^1 + \lambda \Rightarrow 955 = 1.005c + \lambda$	M1 A1	1.1b 1.1b
	$U_n = 10000 - 9000(1.005)^n$	A1	1.1b
		(5)	
(c)	$10000 - 9000(1.005)^n = 0 \Rightarrow n = ...$ $(1.005)^n = \frac{10000}{9000} \Rightarrow n = \frac{\log\frac{10}{9}}{\log 1.005} = ...$ $(1.005)^n = \frac{10000}{9000} \Rightarrow n = \log_{1.005}\frac{10}{9} = ...$	M1	3.4
	$n = 21.1$ therefore 22	A1	3.2a
		(2)	
		(9 marks)	
Notes:			
(a) **B1:** Uses the model to state $A = 50$			

B1: A correct explanation

(b)

M1: A complete method to solve the recurrence relation using $U_n = \text{CF} + \text{PS} = c(1.005)^n + \lambda$

M1: Uses their value of *A* with $PS = \lambda \Rightarrow \lambda = 1.005\lambda - \text{"}50\text{"}$ to find a value for λ

A1: $\lambda = 10000$

M1: Uses $U_0 = 1000$ and their value of λ to find the value of *c*

A1: Fully correctly defined sequence $U_n = 10000 - 9000(1.005)^n$

Alternative

M1: A complete method to solve the recurrence relation using $U_n = \text{CF} + \text{PS} = c(1.005)^n + \lambda$

M1: Uses U_0 to form an equation for their *c* and their λ

M1: Uses the given formula to find U_1 and then uses their value to form another equation for their *c* and their λ

A1: Correct second equation using U_1

A1: Solve simultaneously to find $U_n = 10000 - 9000(1.005)^n$

(c)

M1: Sets their expression = 0 and solves using logarithms to find a value for *n*.

A1: 22 (months)

Note: Using trail and error must show that $U_{21} > 0$ is positive and $U_{22} < 0$

Question	Scheme	Marks	AOs
5 (i)	$214 \equiv 6(\bmod 8)$	B1	1.1b
	$(214)^2 \equiv 6^2(\bmod 8) \Rightarrow (214)^2 \equiv 4(\bmod 8)$ $(214)^6 \equiv 4^3(\bmod 8) \Rightarrow (214)^6 \equiv 64(\bmod 8)$ or $(214)^6 \equiv 6^6(\bmod 8) \equiv (6^3)^2(\bmod 8) \equiv (0)^2(\bmod 8)$ Or $(214)^3 \equiv 6^3(\bmod 8) \Rightarrow (214)^3 \equiv 0(\bmod 8)$ Or $214 \equiv -2(\bmod 8) \Rightarrow (214)^6 \equiv (-2)^6(\bmod 8) \equiv 64(\bmod 8)$	M1	2.1
	$(214)^6 \equiv 0(\bmod 8)$ therefore 214^6 is divisible by 8	A1	2.4
		(3)	
(ii)(a)	Uses $a-5+b-8+a-b+0=2a-13=11n$ to find a possible value of a.	M1	1.1b
	$a = 1$	A1	2.2a
		(2)	
(ii)(b)	$"1"+5+b+8+"1"+b+0=2b+15=3n$ to find a possible value of b.	M1	3.1a
	Any two of $b = 0,\ 3,\ 6$ or 9	A1	1.1b
	All four of $b = 0,\ 3,\ 6$ and 9	A1	2.2a
		(3)	
			(8 marks)

Notes:

(i)

B1: States $214 \equiv 6(\bmod 8)$

M1: Uses modulo arithmetic to find $(214)^6 \equiv a(\bmod 8)$

A1: Achieves $(214)^6 \equiv 0(\bmod 8)$ from correct modulo arithmetic and draws the conclusion that 214^6 is divisible by 8

(ii) (a)
M1: Uses the division rule for 11 to find a value for a.
A1: $a = 1$

(ii) (b)

M1: Uses the division rule for 3 to find a value for *b*.
A1: Any two correct values for *b*.
A1: All four correct values for *b*.

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