Pearson Edexcel

Mark Scheme

Summer 2023

Pearson Edexcel GCE

Advanced Subsidiary Level

Further Mathematics (8FM0)

Paper 01 Core Pure Mathematics

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 80.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎾ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

Question	Scheme	Marks	AOs
1	$y = 3$	B1	2.2a
	$z = \frac{\text{their } y}{3} = \ldots\{1\}$	B1ft	1.1b
	Uses $z - 3y = k \Rightarrow k = -8$ and $x - 3z = k \Rightarrow x = k + 3z = \text{their } k + 3 \times \text{their } z$ leading to a value for x **Alternatively** uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x.	M1	3.1a
	$x = -5$	A1	1.1b
		(4)	
		(4 marks)	

Notes:

B1: $y = 3$

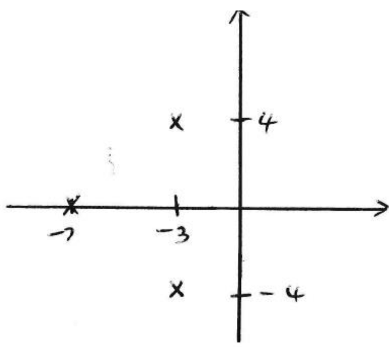
B1ft: Follow through on the value of z which comes from their y divided by 3

M1: A complete method to find the value of x. Uses $z - 3y = k$ to find a value for k then finds a value for x using $x - 3z = k$ and their values for z and k. Condone a slip with the coefficients if the intention is clear but must have the correct letters.

Alternatively uses $x - 3z = k = z - 3y$ with values for y and z to find a value for x.

A1: $x = -5$

Correct answers only scores full marks.

Question	Scheme	Marks	AOs
2(a)	$z^* = -3-4\mathrm{i}$ $\left(z-(-3+4\mathrm{i})\right)\left(z-(-3-4\mathrm{i})\right) = z^2+pz+q$ $\{\mathrm{f}(z)=\}\left(z^2+pz+q\right)(z+r)$	M1	3.1a
	$\left(z^2+6z+25\right)(z+7)$	A1	1.1b
	Multiplies out $\left(z^2+6z+25\right)(z+7) = ...\alpha z^2+\beta z...$	M1	1.1b
	$z^3+13z^2+67z+175$ or $a=13, b=67$	A1	1.1b
		(4)	
	<u>Alternative 1</u> $z^* = -3-4\mathrm{i}$ and uses product of roots = – 175 to find the third root	M1	3.1a
	Third root = –7	A1	1.1b
	Either Uses sum roots = – *a* to find a value for *a* **or** uses pair sum = *b* to find a value for *b* **Or** $\left(z-(-3+4\mathrm{i})\right)\left(z-(-3-4\mathrm{i})\right)\left(z-\text{their third root}\right)=...$	M1	1.1b
	$a=13, b=67$	A1	1.1b
		(4)	
	<u>Alternative 2</u> $(-3+4\mathrm{i})^3+a(-3+4\mathrm{i})^2+b(-3+4\mathrm{i})+175=0$ $\Rightarrow 117+44\mathrm{i}+a(-7-24\mathrm{i})+b(-3+4\mathrm{i})+175=0$ Equates real and imaginary to form two linear simultaneous equations	M1	3.1a
	$117-7a-3b+175=0 \Rightarrow -7a-3b=-292$ $44-24a+4b=0 \Rightarrow -24a+4b=-44$	A1	1.1b
	Solves simultaneously to find values for *a* or *b*	M1	1.1b
	$a=13, b=67$	A1	1.1b
		(4)	
(b)	 $-3+4\mathrm{i},\ -3-4\mathrm{i}$ -7	B1 B1	1.1b 2.2a
		(2)	

(c)	$-5+4\text{i},\ -5-4\text{i},\ -9$	B1ft	2.2a
		(1)	
			(7 marks)

Notes:

(a)
M1: Uses the given root and its complex conjugate to form a quadratic equation. Uses the quadratic equation to write $\text{f}(z)$ in the form $(z^2+pz+q)(z+r)$ where *p*, *q* and *r* are real values

A1: Correct expression for $\text{f}(z)=(z^2+6z+25)(z+7)$

M1: Multiplies out and simplifies to find the z^2 or z term.
A1: Correct values for *a* and *b* or cubic

Alternative 1
M1: Uses the complex conjugate and product of roots = – 175 to find the third root.
A1: Correct third root
M1: A complete method to find the values of *a* or *b*. Either uses the sum and pairs sum or multiplies out three brackets $(z-(-3+4\text{i}))(z-(-3-4\text{i}))(z-\text{their third root})$ to find the z^2 or z term.

A1: Correct values for *a* and *b* or cubic

Alternative 2
M1: Substitutes $-3+4\text{i}$ or $-3-4\text{i}$ into $\text{f}(z)$, sets the real and imaginary parts = 0 to form two simultaneous equations in *a* and *b*.
A1: Correct, unsimplified equations.
M1: Solves simultaneous equations to find values for *a* or *b* following an attempt at $\text{f}(-3+4\text{i})=0$ or $\text{f}(-3-4\text{i})=0$. Allow this mark for seeing a value for *a* or *b* following simultaneous equation, you do not need to check.
A1: Correct values for *a* and *b*.

(b)
B1: Correctly plotting $-3+4\text{i},\ -3-4\text{i}$

B1: Correctly plotting – 7

(c)
B1ft: $-5+4\text{i},\ -5-4\text{i}$ and subtracts 2 from their real root shown on their Argand diagram

Question	Scheme	Marks	AOs
3 (a)	Rotation	B1	1.1b
	30 degrees or $\frac{\pi}{6}$ about the x – axis Ignore any reference to direction	B1	1.1b
		(2)	
(b)	They have found **AB** when they should find **BA** Multiplication is the wrong way round It should be **BA** Matrix B should be on the left instead of the right Student has done transformation B followed by transformation A It should be $\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	B1	2.3
		(1)	
(c)	$\left\{\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\right\} = \left\{\begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -\frac{3}{2} \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & \frac{15}{2} \\ 1 & \sqrt{3} & -1 \end{pmatrix}\right\}$ $\left\{\begin{pmatrix} 1 & 3 & 0 \\ \sqrt{3} & 0 & 5\sqrt{3} \\ 1 & 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\right\} = \left\{\begin{pmatrix} 1 & \frac{3\sqrt{3}}{2} & -1.5 \\ \sqrt{3} & \frac{5\sqrt{3}}{2} & 7.5 \\ 1 & \sqrt{3} & -1 \end{pmatrix}\right\}$	B1	1.1b
		(1)	
		(4 marks)	

Notes:

(a)
B1: Identifies the single transformation as a rotation only
B1: Correct angle and axis. Ignore any reference to direction.
Note x-plane, zy-plane and $x = 0$ are 2nd B0
Any additional incorrect statements is 2nd B0

(b) **B1:** Explains that they should be multiplied the other way around
(c) **B1:** Correct exact matrix **Note:** $5\sqrt{3}\times\frac{\sqrt{3}}{2}$ must be simplified to $\frac{15}{2}$ Condone $\frac{2\sqrt{3}}{2}$ not simplified

Question	Scheme		Marks	AOs
4(i) (a)	$\frac{2+3i}{5+i}\times\frac{5-i}{5-i}$	$2+3i=k(1+i)(5+i)=...$	M1	1.1a
	$\frac{10-2i+15i+3}{25+1}$ or $\frac{13+13i}{26}$	$2+3i=k(5+i+5i-1)=...$	dM1	1.1b
	$\frac{1}{2}(1+i)$ cso	$2+3i=k(4+6i)$ therefore $\frac{2+3i}{5+i}=k(1+i)$ where $k=\frac{1}{2}$ cso	A1	2.1
			(3)	
(i)(b)	$n=4$		B1	2.2a
			(1)	
(ii)	$\lvert z\rvert=3$		B1	1.2
	$\arg(z^{10})=10\arg(z)=-\frac{5\pi}{3}\Rightarrow\arg(z)=...\left\{-\frac{\pi}{6}\right\}$ $\arg(z^{10})=10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	$z=3\left(\cos\left(-\frac{\pi}{6}\right)+i\sin\left(-\frac{\pi}{6}\right)\right)=...$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
	Alternative $a^2+b^2=9$		B1	1.2
	$10\arg z=-\frac{5\pi}{3}\Rightarrow\arg z=-\frac{5\pi}{3}\div 10$ Or e.g $10\arg(z)=\frac{\pi}{3}\Rightarrow\arg(z)=...\left\{\frac{\pi}{30}\right\}$		M1	1.1b
	Forming and solving simultaneous equations to find a value for *a* or *b* $\frac{b}{a}=\arctan\left(-\frac{\pi}{6}\right)\Rightarrow\frac{b}{a}-\frac{\sqrt{3}}{3}\Rightarrow b=-a\frac{\sqrt{3}}{3}$ or $\frac{b}{a}=\arctan\frac{\pi}{30}\Rightarrow b=0.104...a$		M1	2.1
	$z=\frac{3\sqrt{3}}{2}-\frac{3}{2}i$ or $a=\frac{3\sqrt{3}}{2}$ and $b=-\frac{3}{2}$		A1	1.1b
			(4)	
				(8 marks)

Notes:

(i) (a)

M1: Selects the process $\frac{2+3\text{i}}{5+\text{i}}\times\frac{5-\text{i}}{5-\text{i}}$

dM1: Evidence of multiplying out brackets

A1: Achieves $\frac{1}{2}(1+\text{i})$ or $\frac{13}{26}(1+\text{i})$ with no errors cso, isw.

Note: Correct answer from no working score no marks

Note: Going from $\frac{13+13\text{i}}{26}$ and then stating $k=\frac{1}{2}$ is A0, they have not shown the form asked for

Alternative

M1: Multiplies across by $(5+\text{i})$ and expands the brackets

dM1: Collects terms

A1: Achieves $2+3\text{i}=k(4+6\text{i})$ and draws the conclusion that therefore $\frac{2+3\text{i}}{5+\text{i}}=k(1+\text{i})$ where $k=\frac{1}{2}$

(i) (b)

B1: Deduces $n=4$ only

(ii)

Note: Send to review any attempts where they are finding additional solutions such as arguments of *z* is $\frac{(6k-5)\pi}{30}$ For example correctly uses $\arg(z)=\frac{\pi}{30}$

B1 (M1 on ePen): $|z|=3$ can be implied by $a^2+b^2=9$ isw

M1: Uses $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ to find $\arg(z)=-\frac{5\pi}{3}\div 10$ or $\arg(z)=\frac{\pi}{3}\div 10$

M1: Uses $z=\text{their}|z|(\cos(\text{their arg})+\text{i}\sin(\text{their arg}))$ to find the complex number *z* or values for *a* or *b*.

As long as the modulus has changed.

A1: Correct complex number or values for *a* and *b*.

Alternative

B1: $a^2+b^2=9$ isw

M1: Uses $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ to find $\arg(z)=-\frac{5\pi}{3}\div 10$

M1: Uses the argument of *z* find an equation in *a* and *b* . Then solve simultaneously to find a value for *a* or *b*.

As long as $\sqrt{a^2+b^2}\neq 59049$

A1: Correct complex number or values for *a* and *b*.

Note there are other correct answers

$z_1=\frac{3\sqrt{3}}{2}-\frac{3}{2}\text{i}$

$z_2=\text{awrt } 2.98+\text{awrt } 0.314\text{i}$

$z_3=\text{awrt } 2.23+\text{awrt } 2.01\text{i}$

$z_4=\text{awrt } 0.624+\text{awrt } 2.93\text{i}$

$z_5=\text{awrt } -1.22+\text{awrt } 2.74\text{i}$

$z_6=-\frac{3\sqrt{3}}{2}+\frac{3}{2}\text{i}$

$z_7=\text{awrt } -2.98+\text{awrt } -0.314\text{i}$

$z_8=\text{awrt } -2.23+\text{awrt } -2.01\text{i}$

$z_9=\text{awrt } -0.624+\text{awrt } -2.93\text{i}$

$z_{10}=\text{awrt } 1.22+\text{awrt } -2.74\text{i}$

Question	Scheme	Marks	AOs
5(a)	$\{V=\}\pi\int_0^2\left[(2-y)^{\frac{1}{2}}\right]^2 \mathrm{d}y$ or $\{V=\}\pi\int_0^2(2-y)\,\mathrm{d}y$	B1	3.3
	Integrates to the form $\alpha y \pm \beta y^2$	M1	1.1b
	Correct integration $2y-\frac{1}{2}y^2$	A1	1.1b
	Uses their *y* limits correctly in a changed expression $\pi\left[2y-\frac{1}{2}y^2\right]_0^2=\pi\left(2(2)-\frac{1}{2}(2^2)\right)-0=...\{2\pi \text{ or } 6.28...\}$	M1	3.4
	mass = 'their volume' $\times 900$	M1	3.1b
	Mass = 5700 (kg) 2 s.f. cao	A1	2.2b
		(6)	
(b)	eg The surface will not be smooth The pile will not follow the shape of the curve The pile will not be solid Equation of the curves may not be a suitable model Concrete is likely to be uneven/may have bumps The pile is unlikely to be symmetrical	B1	3.5b
		(1)	
(c)	Makes a comparison about the difference between their mass and 5500 and draws a conclusion e.g. 200 difference which is a lot of concrete therefore not a good model e.g. the mass of 5700 is very close to 5500 kg and draws a conclusion about the model – e.g. therefore a good model e.g. Finds the percentage error and draw a conclusion about the model e.g. The masses are very close/significantly different and draws an appropriate conclusion Not sufficient to say 5700 > 5500 B0	B1ft	3.5a
		(1)	
			(8 marks)

Notes:

(a)

B1: Sets up the model to find a correct expression for the volume, including limits, d*y* may be implied. The limits may be seen later.

M1: Integrates to the form $\alpha y \pm \beta y^2$

A1: Correct integration

M1: Substitutes their *y* limits the correct way round and subtracts, must be a changed expression

M1: Multiplies their volume by 900 to find the mass

A1: 5700 cao

Note incorrect upper limit of $\sqrt{2}$ leads to 5200kg Scores B0 M1 A1 M1 M1 A0

Note: Finding the volume around the x-axis can score B0 M0 A0 M0 M1 A0 only

Note If they use their calculator to find the value of the definite integration and achieve the correct answer the maximum they can score is B1M0A0M1M1A1

(b)

B1: See scheme, must be referring to the model and not the value of the density etc

(c)

B1ft: See scheme, follow through on their answer to (a). If using a calculation, it must be correct. Ignore any contradictory comments e.g. 3.6% out so it’s fairly close so it’s a good model but it’s an overestimate which isn’t good. You may need to use your own judgement but any sensible comment comparing their value to 5500 is acceptable.

Question	Scheme	Marks	AOs
6(a)	$\begin{pmatrix}3\\0\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=3\{+0\}-3$	M1	1.1b
	= **0** therefore the lines are **perpendicular.**	A1	2.4
		(2)	
(b)	$\mathbf{r}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}-2\\2\\0\end{pmatrix}\bullet\begin{pmatrix}1\\2\\-3\end{pmatrix}=...\{2\}$	M1	1.1b
	$x+2y-3z=2$ o.e.	A1	2.5
		(2)	
(c)	$3+2(1)-3(1)=2$ (therefore lies on the plane)	B1	1.1b
		(1)	
(d)	$\begin{pmatrix}p\\q\\r\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}$ **or** $\begin{pmatrix}p\\q\\r\end{pmatrix}+\mu\begin{pmatrix}1\\2\\-3\end{pmatrix}=\begin{pmatrix}2\\3\\2\end{pmatrix}$ leading to $p=2-\mu$, $q=3-2\mu$, $r=2+3\mu$	M1	3.1a
	$(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2+\mu)-3)^2+((3+2\mu)-1)^2+((2-3\mu)-1)^2=(2\sqrt{5})^2$ $(-1+\mu)^2+(2+2\mu)^2+(1-3\mu)^2=20$ **or** $(p-3)^2+(q-1)^2+(r-1)^2=(2\sqrt{5})^2$ $((2-\mu)-3)^2+((3-2\mu)-1)^2+((2+3\mu)-1)^2=(2\sqrt{5})^2$ $(-1-\mu)^2+(2-2\mu)^2+(1+3\mu)^2=20$	M1	3.1a
	$14\mu^2-14=0$ o.e	A1	1.1b
	Solves their quadratic $\{\mu=-1 \text{ or } \mu=1\}$	M1	1.1b
	Uses $\mu=-1$ Using $\mu=1$	ddM1	1.1b

	$p = 2 + (-1) = \ldots$ $q = 3 + 2(-1) = \ldots$ or $p = 2 - (1) = \ldots$ $q = 3 - 2(1) = \ldots$ $r = 2 - 3(-1) = \ldots$ $r = 2 + 3(1) = \ldots$		
	$(1, 1, 5)$ only	A1	3.2a
		(6)	
	Alternative $\lvert AX \rvert = \sqrt{(3-2)^2 + (1-3)^2 + (1-2)^2} = \sqrt{6}$	M1	3.1a
	Correctly uses Pythagoras to find the length of *XB* $\lvert XB \rvert = \sqrt{(2\sqrt{5})^2 - 6} = \sqrt{14}$	M1	3.1a
	Find the magnitude of the direction vector and compares to the length of *XB* to find a value for μ	M1	1.1b
	$\mu = -1$ or $\mu = 1$	A1	1.1b
	Uses $\mu = -1$ Using $\mu = 1$ $p = 2 + (-1) = \ldots$ $p = 2 - (1) = \ldots$ $q = 3 + 2(-1) = \ldots$ or $q = 3 - 2(1) = \ldots$ $r = 2 - 3(-1) = \ldots$ $r = 2 + 3(1) = \ldots$	ddM1	1.1b
	$(1, 1, 5)$ only	A1	3.2a
		(6)	
		(11 marks)	

Notes:

(a)
M1: Applies the dot product to the direction vectors. Minimum requirement is 3 – 3
A1: Shows that the dot product **= 0** and concludes that the lines are **perpendicular**.

(b)
M1: Applies $\mathbf{r} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \ldots$ or $\begin{pmatrix} x \\ y \\ z \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} = \ldots$
A1: Correct Cartesian equation $x + 2y - 3z = 2$ o.e.
Note: $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} = 2$ is M1A0

(c)
B1: **See** scheme, no conclusion required

(d)
M1: Uses the point of intersection to find the coordinates of *B* as functions of a parameter
M1: Uses the distance between the point *A* and the point *B* to form an equation for their parameter only.
A1: Correct simplified quadratic equation

M1: Solves their quadratic equation to find a value for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of *B*, it need not be the correct one.

A1: Correct coordinates for *B*, condone as a vector, if seen (3, 5, -1) must be disregarded

Alternative

M1: Finds the length *AX*

M1: Uses Pythagoras to find the length of *XB*

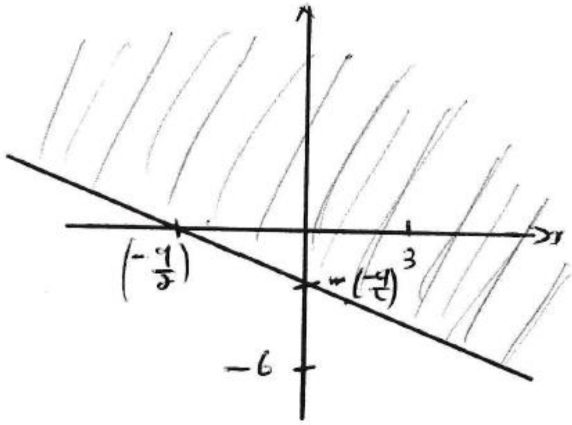
NOTE the change in order of the M1 and A1

M1: Find the length of the direction vector and compares to find a value for μ

A1: A correct values for μ

ddM1: Dependent on the first two method marks. Uses any one of their values for their parameter to find the coordinates of *B*, it need not be the correct one.

A1: Correct coordinates for *B*, condone as a vector, if seen (3, 5, -1) must be disregarded

Question	Scheme	Marks	AOs
7(i)		M1 A1 B1	3.1a 1.1b 1.1b
		(3)	
(ii)	$m=\tan\left(\frac{\pi}{3}\right)\left\{=\sqrt{3}\right\}$ and $y-0=m(x-2)$ leads to $y-0=\sqrt{3}(x-2)$ or $y=\sqrt{3}x-2\sqrt{3}$ $m=\tan\left(\frac{\pi}{6}\right)\left\{=\frac{\sqrt{3}}{3}\right\}$ and $y-0=m(x-(-1))$ leads to $y-0=\frac{\sqrt{3}}{3}(x-(-1))$ or $y=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}$	M1 A1 A1	3.1a 1.1b 1.1b
	$\sqrt{3}x-2\sqrt{3}=\frac{\sqrt{3}}{3}x+\frac{\sqrt{3}}{3}\Rightarrow x=\ldots$	M1	1.1b
	$y=\sqrt{3}\left(\frac{7}{2}\right)-2\sqrt{3}=\ldots$	M1	1.1b
	$\{w=\}\frac{7}{2}+\frac{3\sqrt{3}}{2}\mathrm{i}$	A1	2.1
		(6)	
	Alternative $\tan\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{3}=\frac{y}{x_{-1}}$ and $\tan\left(\frac{\pi}{3}\right)=\sqrt{3}=\frac{y}{x_2}$	M1	1.1b
	 $x_{-1}=\sqrt{3}y$ $\quad$ $x_2=\frac{\sqrt{3}}{3}y$	A1 A1	1.1b 1.1b

	$y\sqrt{3} = y\frac{\sqrt{3}}{3} + 3 \Rightarrow y = ...$	M1	3.1a
	Uses $x = y\sqrt{3} - 1$ or $x = \frac{\sqrt{3}}{3}y + 2$ with their value of *y* leading to a value for *x*	M1	1.1b
	$(w =)\frac{7}{2} + \frac{3\sqrt{3}}{2}\text{i}$	A1	2.1
		(6)	
			(9 marks)

Notes:

(i)

M1: Draws a **single** straight line through **both axes** with a negative gradient. Ignore any line joining (3, 0) and (0, -6)

A1: Draws a **single** straight line through **both axes** with a negative gradient which has a negative *y* intercept**.** Ignore any intercept marked on the axes. Ignore any line joining (3, 0) and (0, -6)

B1: Shades the area above their straight line (not a bounded region such as a triangle bounded by the axes and the line)

(ii)

M1: Finds the Cartesian equations for both loci by using the gradient as tan(argument) and correct coordinate. Must be an attempt at both equations but one correct equation scores this mark

A1: One equation correct, need not be simplified

A1: Both equations correct, need not be simplified

M1: Solve simultaneously to find either the real or imaginary component.

M1: Finds the other component to complete the process of finding *w*.

A1: Correct exact answer

Note: If leaves the answer as a coordinate this is A0. If defines $w = a + b\text{i}$ and then states $a = \frac{7}{2}$ and $b = \frac{3\sqrt{3}}{2}$ this is A1

Alternative

M1: Use both arguments to form equations involving *x* and *y*

A1: (One correct triangle) value for *x* in terms of *y*

A1: (Two correct triangles), values for *x* in terms of *y*

M1: Forms and solves an equation $y\sqrt{3} = y\frac{\sqrt{3}}{3} + 3 \Rightarrow y = ...$ must be come from $x_2 = x_{-1} + 3$

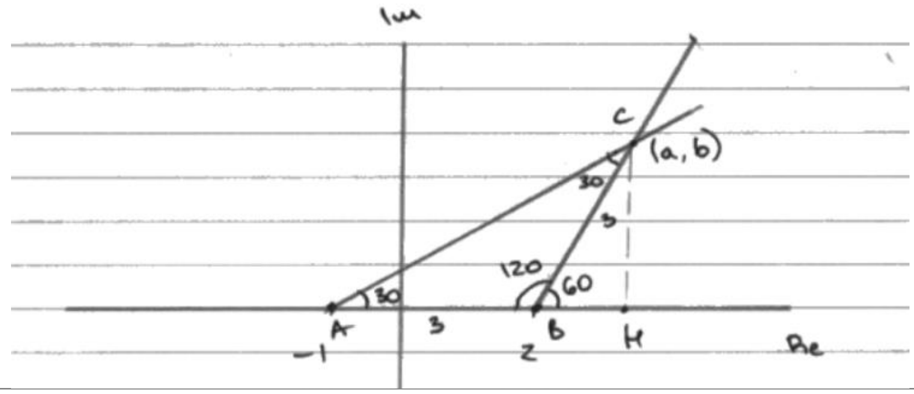
M1: Uses their *y* value and $x = y\sqrt{3} - 1$ or $x = \frac{\sqrt{3}}{3}y + 2$ to find a value for *x*

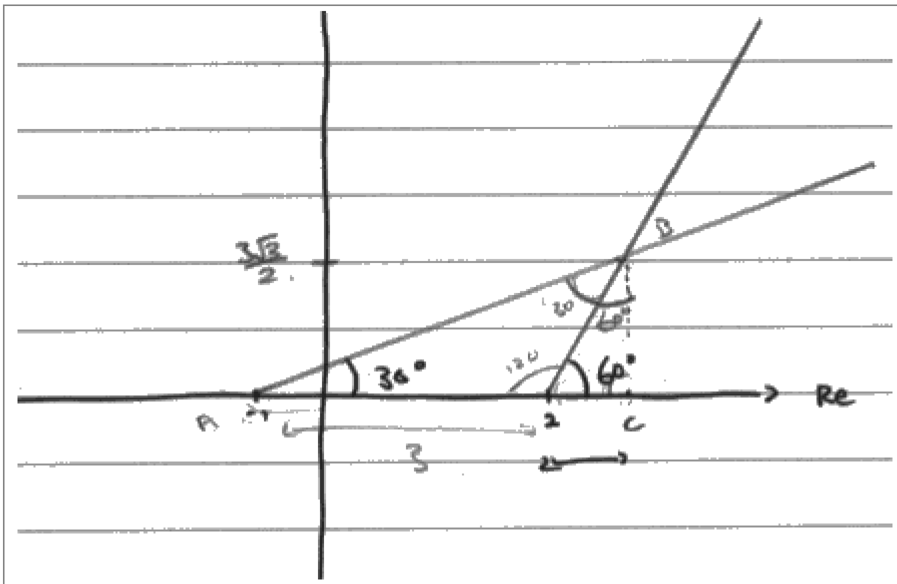
A1: Correct exact answer

Note: If candidates use decimal instead of exact values throughout allow the method marks

$y = 1.73x - 3.46$ and $y = 0.58x + 0.58$

Q7(ii) Two alternatives seen

		
Alternative 3 $b=3\sin\left(\frac{\pi}{3}\right)$ and $c=3\cos\left(\frac{\pi}{3}\right)$	M1 A1 A1	3.1a 1.1b 1.1b
$b=3\sin\left(\frac{\pi}{3}\right)=\ldots$	M1	1.1b
$a=2+3\cos\left(\frac{\pi}{3}\right)=\ldots$	M1	1.1b
$(w=)\frac{7}{2}+\frac{3\sqrt{3}}{2}\text{i}$	A1	2.1
	(6)	
Alternative 3 **M1**: Uses correct geometry to form equations involving *a* and *c* **A1**: One correct equation **A1:** Two correct equations **M1:** Finds the imaginary component **M1:** Uses 2 + their *c* to find the real component **A1:** Correct exact answer		

Alternative 4 $\frac{3}{\sin 30} = \frac{AB}{\sin 120}$ $AB = 3\sqrt{3}$		M1 A1	3.1a 1.1b
$\sin 30 = \frac{BC}{3\sqrt{3}}$ $BC = \frac{3}{2}\sqrt{3}$	$\sin 60 = \frac{AC}{3\sqrt{3}}$ $AC = \frac{7}{2}$	M1 A1	1.1b 1.1b
Uses trigonometry to find the other component		M1	1.1b
$(w =)\frac{7}{2} + \frac{3\sqrt{3}}{2}\text{i}$		A1	2.1
		(6)	
Alternative 4 **M1**: Uses the sine rule to find the length *AB* **A1**: Correct length *AB* **M1:** Uses trigonometry to find either the real or imaginary component A1: Correct real or imaginary component **M1:** Uses trigonometry to find the other component **A1:** Correct exact answer			

Question	Scheme	Marks	AOs
8(a)	$(2r-1)^2 = 4r^2 - 4r + 1$	B1	1.1b
	$\sum_{r=1}^{n}(2r-1)^2 = 4\sum_{r=1}^{n} r^2 - 4\sum_{r=1}^{n} r + \sum_{r=1}^{n} 1$ $= 4\frac{n}{6}(n+1)(2n+1) - 4\frac{n}{2}(n+1) + n$	M1 A1	1.1b 1.1b
	$= \frac{n}{3}\left[2(n+1)(2n+1) - 6(n+1) + 3\right]$ Or $= n\left[\frac{2}{3}(n+1)(2n+1) - 2(n+1) + 1\right]$	dM1	1.1b
	$\left\{\frac{n}{3}\left(4n^2 + 6n + 2 - 6n - 6 + 3\right)\right\}$ $= \frac{n}{3}\left(4n^2 - 1\right)$ cso	A1	2.1
		(5)	
(b)	$\sum_{r=51}^{500}(2r-1)^2$	B1	3.1a
	$\sum_{r=51}^{500}(2r-1)^2 = \sum_{r=1}^{500}(2r-1)^2 - \sum_{r=1}^{50}(2r-1)^2$ $= \frac{500}{3}\left(4(500)^2 - 1\right) - \frac{50}{3}\left(4(50)^2 - 1\right)$ $\{ = 166666500 - 166650\}$	M1	1.1b
	166 499 850	A1	1.1b
		(3)	
		(8 marks)	

Notes:

(a)

B1: Correct expanded expression.

M1: Substitutes at least one of the standard formulae into their expanded expression.

A1: Fully correct unsimplified expression.

dM1: Dependent on previous method. Attempts to factorises out *n*. Must have a *n* in every term. Condone a slip with one term as long as the intention is clear.

A1: Achieves the correct answer, with a correct intermediate line of working. cso

Note If uses $\sum 1 = 1$ scores B1 M1 A0 M0 A0

An attempt at proof by induction may score B1 only

(b)

B1: Correct summation formula for the sum of the squares of all positive odd three-digit integers including limits. This can be implied by later work.

M1: Uses the answer to part (a) and $\sum_{r=p}^{q}(2r-1)^2=\sum_{r=1}^{q}(2r-1)^2-\sum_{r=1}^{p-1}(2r-1)^2$ where p, q are numerical and $q > p$, to find a value. There must be some indication of the sum that they are finding or the correct values for p and q.

States $\sum_{r=1}^{500}(2r-1)^2-\sum_{r=1}^{50}(2r-1)^2$ implies B1

States $\frac{500}{3}\left(4(500)^2-1\right)-\frac{50}{3}\left(4(50)^2-1\right)$ this scores B1 (implied) and M1

A1: Correct value

Note $\sum_{r=51}^{500}(2r-1)^2=166499850$ or correct answer only scores B1 M0 A0, must be evidence of using the answer to (a)

Question	Scheme	Marks	AOs
9(i)	$\begin{vmatrix} k & -2 & 7 \\ -3 & -5 & 2 \\ k & k & 4 \end{vmatrix} = k(-20-2k)+2(-12-2k)+7(-3k+5k)$ or $\begin{vmatrix} k & -2 & 7 & k & -2 \\ -3 & -5 & 2 & -3 & -5 \\ k & k & 4 & k & k \end{vmatrix} = k(-5)(4)-2(2)(k)+7(-3)(k)$ $-7(-5)(k)-k(2)(k)-(-2)(-3)(4)$	M1	1.1b
	$-2k^2-10k-24(=0)$ isw	A1	1.2
	$b^2-4ac=(10)^2-4(-2)(-24)=\ldots$ $b^2-4ac=(5)^2-4(-1)(-12)=\ldots$ Or $k^2+5k+12=0 \Rightarrow (k+2.5)^2+5.75=0 \Rightarrow (k+2.5)^2=-5.75$ $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow (k+2.5)^2=-5.75$ Or $k^2+5k+12 \Rightarrow (k+2.5)^2+5.75 \Rightarrow (k+2.5)^2 \geqslant 0$ or $-2k^2-10k-25=0 \Rightarrow -2(k+2.5)^2-12.5=0 \Rightarrow -2(k+2.5)^2 \leqslant 0$ Or $\frac{\mathrm{d}(-2k^2-10k-24)}{\mathrm{d}k}=-4k-10=0 \Rightarrow k=-2.5 \Rightarrow$ determinant $=-5.75$ Or $k=\frac{10\pm\sqrt{(-10)^2-4(-2)(-25)}}{2(-2)}=\frac{-5\pm\sqrt{23}\mathrm{i}}{2}$	M1	1.1b
	$b^2-4ac=-92<0$ therefore **no real roots** so **non-singular** $b^2-4ac=-23<0$ therefore **no real roots** so **non-singular** Or Square of negative is not real therefore **non-singular** Or $(k+2.5)^2+5.75>0$ therefore **no real roots** so **non-singular** $-2(k+2.5)^2-12.5<0$ therefore **no real roots** so **non-singular** Or As negative quadratic maximum value of determinant = – 5.25 therefore **no real roots** so **non-singular** Or Imaginary roots therefore **no real roots** so **non-singular**	A1	2.4
		(4)	

(ii)	$\begin{pmatrix} 2 & -1 \\ -3 & 0 \end{pmatrix}\begin{pmatrix} a & 4 \\ 2 & -a \end{pmatrix} = \begin{pmatrix} \dots & \dots \\ \dots & \dots \end{pmatrix}$ can be done separately for each point	M1	3.1a
	$\begin{pmatrix} 2a-2 & 8+a \\ -3a & -12 \end{pmatrix}$ or $(2a-2, -3a)$ and $(8+a, -12)$	A1	1.1b
	$\sqrt{[(2a-2)-(8+a)]^2+[-3a-(-12)]^2}=\sqrt{58}$ or $\overrightarrow{AB}=\begin{pmatrix} 8+a \\ -12 \end{pmatrix}-\begin{pmatrix} 2a-2 \\ -3a \end{pmatrix}=\begin{pmatrix} 10-a \\ -12+3a \end{pmatrix}$ or $\overrightarrow{BA}=\begin{pmatrix} 2a-2 \\ -3a \end{pmatrix}-\begin{pmatrix} 8+a \\ -12 \end{pmatrix}=\begin{pmatrix} a-10 \\ 12-3a \end{pmatrix}$ $(a-10)^2+(12-3a)^2=58$ or $(10-a)^2+(3a-12)^2=58$ leading to a 3TQ	M1	3.1a
	$10a^2-92a+186=0$	A1	1.1b
	$a=3, \frac{31}{5}$ o.e. cso	A1	1.1b
		(5)	

(9 marks)

Notes:

(i)

M1: Correct method to find the determinant**,** condone a single sign slip but not on second term must be +2 (…)

Note: May expand along any row or column.

A1: Correct simplified determinant

M1: Either

- Finds the value of the discriminant or sufficient working seen to identify the sign e.g. 100 – 192
- Completes the square an rearranges so that $(k \pm a)^2 = -b$
- Completes the square and states that $(k \pm a)^2 \geqslant 0$
- Completes the square and states that $-\alpha(k \pm a)^2 \leqslant 0$
- Differentiates the determinant to find the coordinates of the vertex
- Use the quadratic formula to find the imaginary roots

A1: Correct solution only

Either

- Correct value for the discriminant (may be implied), concludes less than 0, therefore no real roots and non singular.
- Correct completing the square and conclude no real roots as square root of negative therefore non singular
- Correct completing the square and shows > 0 therefore no real roots and non singular.
- Correct completing the square and shows < 0 therefore no real roots and non singular.

• Correct coordinates of the vertex and negative quadratic therefore no real roots and non singular. • Use the quadratics formula to find the correct imaginary roots therefore no real roots/value for k and non singular. **Note** $k=\frac{-5\pm\sqrt{23}\mathrm{i}}{2}$ which is not real is M0 A0 unless uses the quadratic formula or completing the square to show where this has come from
(ii) **M1:** Uses matrix **Q** to find the coordinates of the points A' and B'. Condone a sign slip. **A1:** Correct coordinates for the points A' and B', they do not need to be labelled **M1:** Finds the distance between their points A' and B' which must not be equal to A and B, sets equal to $\sqrt{58}$, forms a 3TQ. **A1:** Correct 3TQ form correct coordinates **A1:** Correct values cso **Misread: A common misread is 3 instead of – 3, the first 3 mark only can be scored using the misread rule** **M1:** $\begin{pmatrix}2 & -1\\ 3 & 0\end{pmatrix}\begin{pmatrix}a & 4\\ 2 & -a\end{pmatrix}=\begin{pmatrix}\ldots & \ldots\\ \ldots & \ldots\end{pmatrix}$ **A1:** $\begin{pmatrix}2a-2 & 8+a\\ 3a & 12\end{pmatrix}$ or $(2a-2,\ 3a)$ and $(8+a, 12)$ **M1:** $\sqrt{\left[(2a-2)-(8+a)\right]^2+\left[3a-12\right]^2}=\sqrt{58}$ **A0, A0 This does lead to the correct answer but can score the first three marks only.**

Question	Scheme	Marks	AOs
10(i)	$w = 3z - 1 \Rightarrow z = \frac{w+1}{3}$	B1	3.1a
	$\left(\frac{w+1}{3}\right)^4 + 5\left(\frac{w+1}{3}\right)^2 - 30 = 0$	M1	3.1a
	$\frac{1}{81}\left(w^4 + 4w^3 + 6w^2 + 4w + 1\right) + \frac{5}{9}\left(w^2 + 2w + 1\right) - 30 = 0$ leading to $w^4 + aw^3 + bw^2 + cw + d\ \{= 0\}$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
	Alternative $p + q + r + s = 0, \quad pq + pr + ps + qr + qs + rs = 5$ $pqr + pqs + prs + qrs = 0, \quad pqrs = -30$	B1	3.1a
	New sum $= 3(p + q + r + s) - 4 = \ldots\{-4\}$ New pair sum $= 9(pq + pr + ps + qr + qs + rs) - 9(p + q + r + s) + 6 = \ldots\{51\}$ New triple sum $= 27(pqr + pqs + prs + qrs) - 18(pq + pr + ps + qr + qs + rs)$ $+6(p + q + r + s) - 4 = \ldots\{-94\}$ New product $= 81(pqrs) - 27(pqr + pqs + prs + qrs)$ $+9(pq + pr + ps + qr + qs + rs) - 3(p + q + r + s) + 1$ $= \ldots\{-2384\}$	M1	3.1a
	Applies $w^4 - (\text{new sum})w^3 + (\text{new pair sum})w^2 - (\text{new triple sum})w$ $+ (\text{new product}) = 0$	M1	1.1b
	$w^4 + 4w^3 + 51w^2 + 94w - 2384 = 0$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	$\alpha + 2\alpha + \alpha - \beta = 0$ and $\alpha \times 2\alpha \times (\alpha - \beta) = -\frac{81}{4}$	M1 A1	3.1a 1.1b
	Solves simultaneously e.g. $4\alpha - \beta = 0 \Rightarrow \beta = 4\alpha$ $2\alpha^2(\alpha - 4\alpha) = -\frac{81}{4} \Rightarrow \alpha^3 = \frac{27}{8} \Rightarrow \alpha = \ldots$	M1	3.1a
	Uses their values $\alpha = \frac{3}{2}\ \beta = 6$ to find the roots $\alpha, 2\alpha, \alpha - \beta$	M1	1.1b
	Roots 1.5, 3, – 4.5	A1	1.1b

		(5)	
(ii) (b)	$n=\left[(1.5\times3)+(1.5\times-4.5)+(3\times-4.5)\right]\times4$ Or Multiplies out $(x-3)\left(x-\frac{3}{2}\right)\left(x+\frac{9}{2}\right)$ or $(x-3)(2x-3)(2x+9)$ to achieve the form $4x^3+...$	M1	1.1b
	$n=-63$ cso (must have correct roots in (a)	A1	1.1b
		(2)	
			(12 marks)

Notes:

(i)

B1: Selects the method of making a connection between *z* and *w* by writing $z=\frac{w+1}{3}$. Other variables may be used

M1: Applies the process of substituting their $z=\frac{w+1}{3}$ into $z^4+5z^2-30=0$

M1: Manipulates their equation into the form $w^4+aw^3+bw^2+cw+d=0$ having substituted their *z* in terms of *w*. Note that the "= 0" can be missing for this mark.

A1: At least two of *a*, *b*, *c*, *d* correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of *w*

(i) Alternative

B1: Selects the method of giving four correct equations containing *p*, *q*, *r* and *s*

M1: Applies the process of finding **at least three** of the new sum, new pair sum, new triple sum and new product. Condone slips but the intention is clear and uses their values.

M1: Applies $w^4-(\text{new sum})w^3+(\text{new pair sum})w^2-(\text{new triple sum})w+(\text{new product})=0$.

Condone use of any variable for this mark.

Note that the "= 0" can be missing for this mark.

A1: At least two of *a*, *b*, *c*, *d* correct. Note that the "= 0" can be missing for this mark.

A1: Fully correct equation including "= 0" Must be in terms of *w*

(ii) (a)

M1: Uses the sum and product to form two equations in α and β. Condone product = $\frac{81}{4}$ for this mark

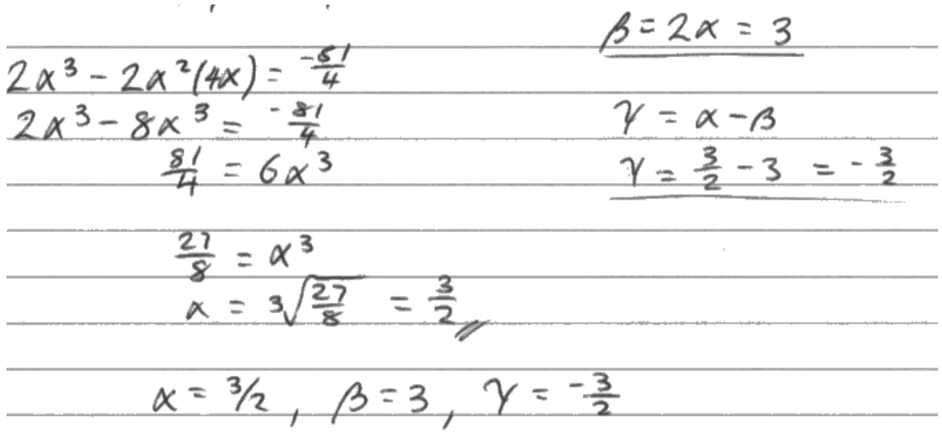
Note: $4\alpha-\beta=-\frac{n}{4}$ or $4\alpha-\beta=81$ is M0

A1: Correct equations need not be simplied

M1: Solves simultaneous equations to find a value for α or β

M1: Uses their values for α and β to find the roots using $\alpha,\ 2\alpha,\ \alpha-\beta$. Condone third root as β

A1: Correct roots

Candidates may use their own notation for the roots Condone confusion with β for M1 but A0 
(ii) (b) **M1:** Finds the pair sum for their numerical roots and multiplies by 4 Alternative multiplies out three brackets $(x - \text{their } \alpha)(x - \text{their } 2\alpha)(x - (\alpha - \beta))$ to achieve the form $4x^3 + \ldots$ **A1:** Correct value **from correct roots only**

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