Write your name here

Surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary
Further Mathematics options
22: Further Pure Mathematics 2
(Part of option A only)

Thursday 17 May 2018 – Afternoon

Paper Reference **8FM0/22**

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 5 questions.
- The marks for each question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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P 6 0 1 5 3 A 0 1 1 6

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Answer ALL questions. Write your answers in the spaces provided.

1. (i) Using a suitable algorithm and without performing any division, determine whether 23 738 is divisible by 11

(2)

(ii) Use the Euclidean algorithm to find the highest common factor of 2322 and 654

(3)

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Question 1 continued

(Total for Question 1 is 5 marks)

2.

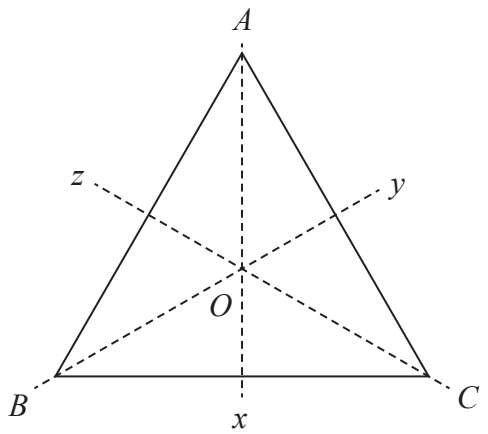

Figure 1

Figure 1 shows an equilateral triangle ABC. The lines x, y and z and their point of intersection, O, are fixed in the plane. The triangle ABC is transformed about these fixed lines and the fixed point O. The lines x, y and z each pass through a vertex of the triangle and the midpoint of the opposite side.

The transformations I, X, Y, Z, R_1 and R_2 of the plane containing triangle ABC are defined as follows:

- I: Do nothing
- X: Reflect in the line x
- Y: Reflect in the line y
- Z: Reflect in the line z
- R_1: Rotate 120° anticlockwise about O
- R_2: Rotate 240° anticlockwise about O

The operation * is defined as 'followed by' on the set $T = \{I, X, Y, Z, R_1, R_2\}$.
For example, $X * Y$ means a reflection in the line x followed by a reflection in the line y.

(a) (i) Complete the Cayley table on page 5

Given that the associative law is satisfied,

(ii) show that T is a group under the operation *

(6)

(b) Show that the element R_2 has order 3

(2)

(c) Explain why T is not a cyclic group.

(1)

(d) Write down the elements of a subgroup of T that has order 3

(1)

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Question 2 continued

		Second transformation					
	*	I	X	Y	Z	R_1	R_2
First Transformation	I						
	X		I			Z	
	Y						
	Z						
	R_1		Y				
	R_2						

Turn over for a spare table if you need to re-write your Cayley table

Question 2 continued

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Question 2 continued

Only use this grid if you need to re-write your Cayley table

		Second transformation					
	*	I	X	Y	Z	R_1	R_2
First Transformation	I						
	X		I			Z	
	Y						
	Z						
	R_1		Y				
	R_2						

(Total for Question 2 is 10 marks)

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3 A tree at the bottom of a garden needs to be reduced in height. The tree is known to increase in height by 15 centimetres each year.

On the first day of every year, the height is measured and the tree is immediately trimmed by 3% of this height.

When the tree is measured, before trimming on the first day of year 1, the height is 6 metres.

Let H_n be the height of the tree immediately before trimming on the first day of year n.

(a) Explain, in the context of the problem, why the height of the tree may be modelled by the recurrence relation

$$H_{n+1} = 0.97H_n + 0.15, \quad H_1 = 6, \quad n \in \mathbb{Z}^+$$

(3)

(b) Prove by induction that $H_n = 0.97^{n-1} + 5, \quad n \geqslant 1$

(4)

(c) Explain what will happen to the height of the tree immediately before trimming in the long term.

(1)

(d) By what fixed percentage should the tree be trimmed each year if the height of the tree immediately before trimming is to be 4 metres in the long term?

(2)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 10 marks)

4.

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$$

Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{D} = \mathbf{P}^{-1}\mathbf{AP}$

(7)

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Question 4 continued

(Total for Question 4 is 7 marks)

5. A complex number z is represented by the point P on an Argand diagram.

Given that $\arg\left(\dfrac{z - 6\mathrm{i}}{z - 3\mathrm{i}}\right) = \dfrac{\pi}{3}$

(a) sketch the locus of P as z varies,

(3)

(b) find the exact maximum possible value of $|z|$

(5)

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Question 5 continued

Question 5 continued

(Total for Question 5 is 8 marks)

TOTAL FOR FURTHER PURE MATHEMATICS 2 IS 40 MARKS

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