



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2026 Mathematics 2027

Unit 24 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: _____

Class: _____



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Name: _____

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1 Equations of Circles and Tangents

Worked Example

Determine whether the point with coordinates $(-5, 7)$ lies on the circle with the equation $x^2 + y^2 = 85$.

Your Turn

Determine whether the point with coordinates $(6, -8)$ lies on the circle with the equation $x^2 + y^2 = 100$.

Worked Example

Find the radius of the circle with equation:

a) $x^2 + y^2 = 196$

b) $x^2 + y^2 = 326$

Your Turn

Find the radius of the circle with equation:

a) $x^2 + y^2 = 169$

b) $x^2 + y^2 = 362$

Worked Example

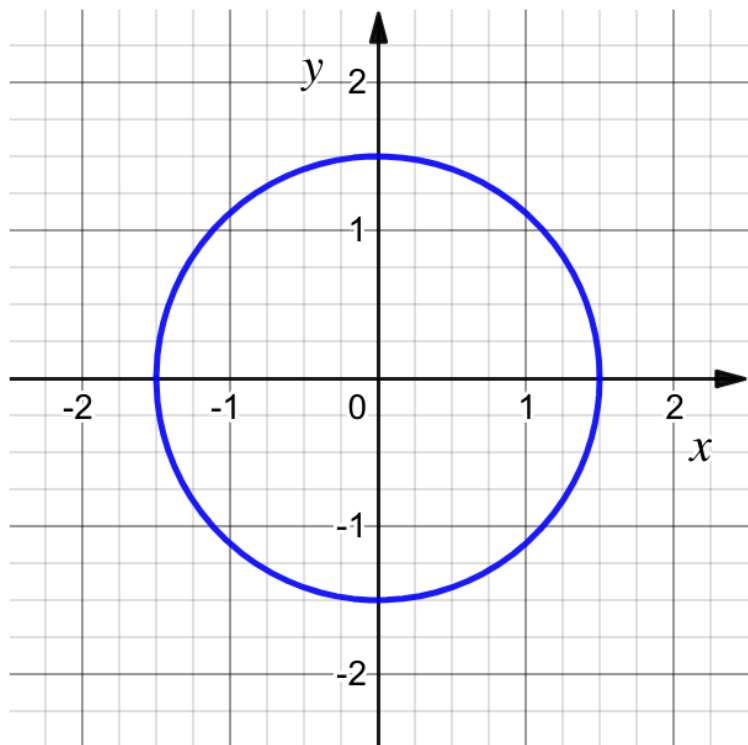
Find an equation of the circle with radius $3\sqrt{5}$ and centre $(0, 0)$.

Your Turn

Find an equation of the circle with radius $5\sqrt{2}$ and centre $(0, 0)$.

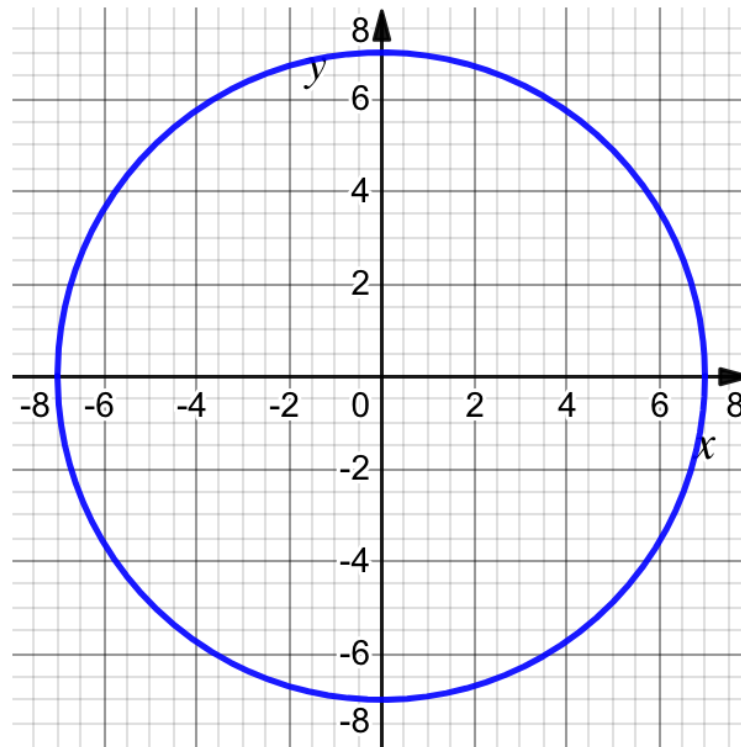
Worked Example

Find an equation of the circle drawn below.



Your Turn

Find an equation of the circle drawn below.



Worked Example

The point $(-5, 3)$ lies on a circle centered on the origin.
Find an equation for this circle.

Your Turn

The point $(-7, -2)$ lies on a circle centered on the origin.
Find an equation for this circle.

Worked Example

The circle below is given by the equation $x^2 + y^2 = 16$.

- a) Calculate its circumference, C .
- b) Calculate the shaded area, A .

Give your answers correct to 2 decimal places.

Your Turn

The circle below is given by the equation $x^2 + y^2 = 64$.

- a) Calculate its circumference, C .
- b) Calculate the shaded area, A .

Give your answers correct to 2 decimal places.

Worked Example

- a) A circle has a circumference of 6π . Find an equation for the circle.
- b) A circle has an area of 49π . Find an equation for the circle.

Your Turn

- a) A circle has a circumference of 12π . Find an equation for the circle.
- b) A circle has an area of 25π . Find an equation for the circle.

Fill in the Gaps

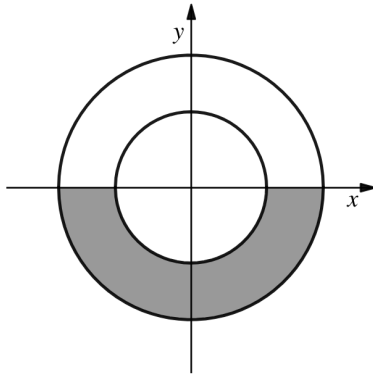
Equation	Radius	Area	Point 1	Point 2	Where is (3, 7)?
$x^2 + y^2 = 25$			(3, ____)	(____, 0)	Outside
$x^2 + y^2 = 50$			(-5, ____)	(____, 7)	
$x^2 + y^2 = 65$			(1, ____)	(____, 7)	
	15		(9, ____)	(____, 0)	
	$5\sqrt{5}$		(-5, ____)	(____, 11)	
		130π	(-7, ____)	(____, 11)	
		2042	(19, ____)	(____, 11)	
			(-4, ____)	(8, 11)	
			(1, ____)	(-7, 11)	
			(-7, ____)	(____, $\sqrt{22}$)	On the circle

Worked Example

The annulus below is formed of two circles centred on the origin. The equations of the circles are:

$$x^2 + y^2 = 49$$

$$x^2 + y^2 = 16$$



- a) Calculate the perimeter of the shaded shape.
- b) Calculate the area of the shaded shape.

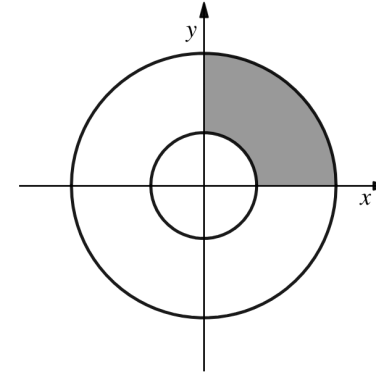
Give your answers correct to 2 decimal places.

Your Turn

The annulus below is formed of two circles centred on the origin. The equations of the circles are:

$$x^2 + y^2 = 25$$

$$x^2 + y^2 = 4$$

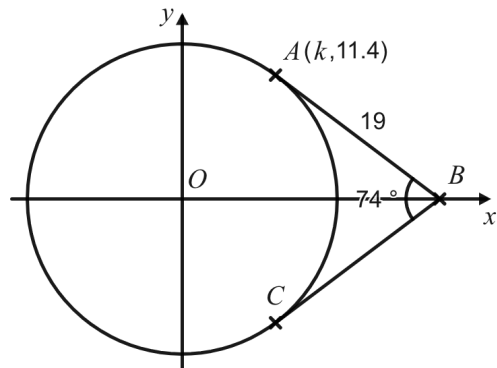


- a) Calculate the perimeter of the shaded shape.
- b) Calculate the area of the shaded shape.

Give your answers correct to 2 decimal places.

Worked Example

The diagram shows a circle, centre O



AB and BC are tangents to the circle.

Angle $ABC = 74^\circ$

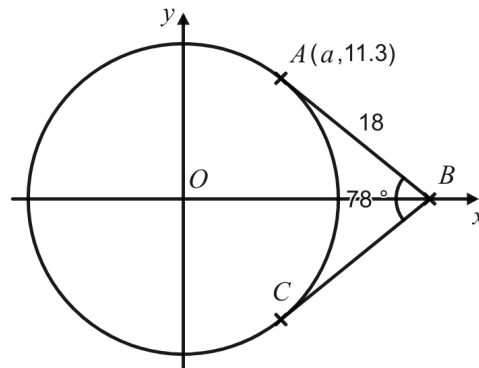
AB has a length of 19 units.

Calculate the value of k

Give your answer correct to 1 decimal place.

Your Turn

The diagram shows a circle, centre O



AB and BC are tangents to the circle.

Angle $ABC = 78^\circ$

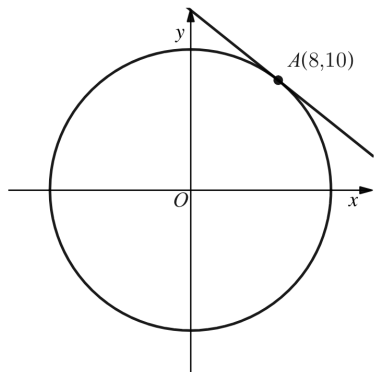
AB has a length of 18 units.

Calculate the value of a

Give your answer correct to 1 decimal place.

Worked Example

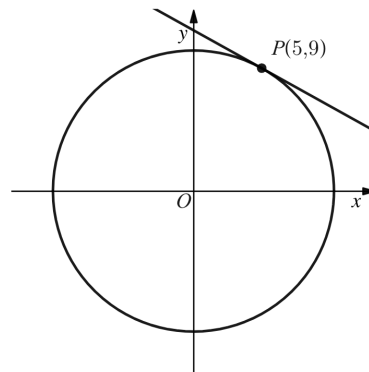
The diagram shows the circle with equation $x^2 + y^2 = 164$



A tangent to the circle is drawn at point A with coordinates $(8, 10)$. Find an equation of the tangent at A .

Your Turn

The diagram shows the circle with equation $x^2 + y^2 = 106$



A tangent to the circle is drawn at point P with coordinates $(5, 9)$. Find an equation of the tangent at P .

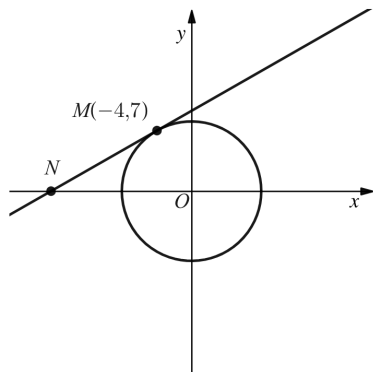
Fill in the Gaps

Equation of Circle	Point on Circle	Gradient of Radius	Gradient of Tangent	Equation of Tangent
$x^2 + y^2 = 45$	$(3, 6)$	2	$-\frac{1}{2}$	
$x^2 + y^2 = 10$	$(3, -1)$	$m = -\frac{1}{3}$		
$x^2 + y^2 = 68$	$(-2, -8)$			
$x^2 + y^2 = 25$	$(-4, 3)$			
$x^2 + y^2 = 73$	$(8, 3)$			
$x^2 + y^2 = \frac{53}{2}$	$\left(\frac{5}{2}, -\frac{9}{2}\right)$			
$x^2 + y^2 = 6$	$(-2, \sqrt{2})$			
$x^2 + y^2 = 100$				$y = \frac{3}{4}x - \frac{25}{2}$

Worked Example

A circle has equation $x^2 + y^2 = 65$

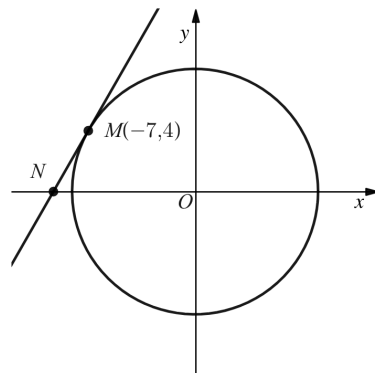
M is the point on the circle with coordinates $(-4, 7)$



The tangent to the circle at M intersects the x-axis at point N .
Work out the x -coordinate of N .

Your Turn

The diagram shows a circle with centre $(0, 0)$ and a tangent at the point $M(-7, 4)$

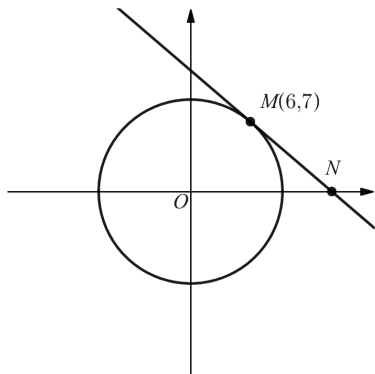


The tangent to the circle at M intersects the x-axis at point N .
Work out the x -coordinate of N .

Worked Example

A circle has equation $x^2 + y^2 = 85$

M is the point on the circle with coordinates $M(6, 7)$

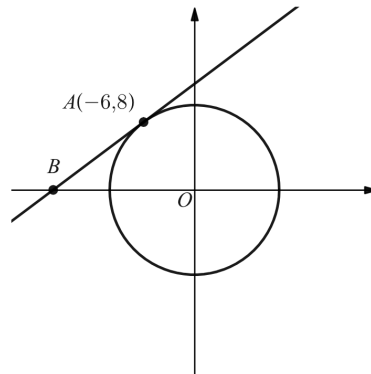


The tangent to the circle at M intersects the x -axis at point N .
Work out the area of triangle OMN .

Your Turn

A circle has equation $x^2 + y^2 = 100$

A is the point on the circle with coordinates $A(-6, 8)$



The tangent to the circle at A intersects the x -axis at point B .
Work out the area of triangle OAB .

Extra Notes

2 Advanced Sequences

Geometric Sequences

Worked Example	Your Turn
Generate the first 5 terms of the following geometric sequence: $4 \times 3^{n-1}$	Generate the first 5 terms of the following geometric sequence: $5 \times 4^{n-1}$

Worked Example

Write down the n^{th} term of the following geometric sequences:

- a) 4, 12, 36, 108
- b) 4, -12, 36, -108
- c) 108, 36, 12, 4
- d) $\sqrt{7}$, 7, $7\sqrt{7}$, 49
- e) $3p^4$, $6p^4q^4$, $12p^4q^8$

Your Turn

Write down the n^{th} term of the following geometric sequences:

- a) 5, 20, 80, 320
- b) 5, -20, 80, -320
- c) 320, 80, 20, 5
- d) $\sqrt{3}$, 3, $3\sqrt{3}$, 9
- e) $2x^4$, $\frac{8x^4}{y^4}$, $\frac{32x^4}{y^8}$

Worked Example

The second term of a geometric sequence is 78. The sixth term of the same sequence is 101,088. Calculate the value of the common ratio.

Your Turn

A geometric sequence has second and fifth terms 108 and 4, respectively. Calculate the value of the common ratio.

Worked Example

A geometric sequence has first term $(x - 3)$, second term $(x + 1)$ and third term $(4x - 2)$. Find the two possible values of x .

Your Turn

The first three terms of a geometric sequence are $4p$, $(3p + 15)$ and $(5p + 20)$ respectively, where p is a positive constant. Find the value of p .

Quadratic Sequences

Worked Example

Generate the first 5 terms of the following quadratic sequence:
 $3n^2 + 2n - 5$

Your Turn

Generate the first 5 terms of the following quadratic sequence:
 $3n^2 - 2n + 5$

Worked Example

By comparison with the first four terms of the sequence n^2 , find the n th term of the second sequence:

a)

First four terms				n th term
1	4	9	16	n^2
7	28	63	112	_____

b)

First four terms				n th term
1	4	9	16	n^2
-1	2	7	14	_____

Your Turn

By comparison with the first four terms of the sequence n^2 , find the n th term of the second sequence:

a)

First four terms				n th term
1	4	9	16	n^2
4	7	12	19	_____

b)

First four terms				n th term
1	4	9	16	n^2
-7	-28	-63	-112	_____

Worked Example	Your Turn
Find the n^{th} term of the following sequence: 0, 11, 28, 51, 80	Find the n^{th} term of the following sequence: 6, 13, 26, 45, 70

Worked Example

Here are the first five terms of a quadratic sequence
 $6, -4, -22, -48, -82$
Find an expression, in terms of n , for the n th term of the sequence.

Your Turn

Here are the first five terms of a quadratic sequence
 $-14, -25, -38, -53, -70$
Find an expression, in terms of n , for the n th term of the sequence.

Worked Example

The n th term of a sequence is given by $an^2 + bn + c$
The second term is 23, the fourth term is 57 and the sixth term is 107. Find the values of a , b and c .

Your Turn

The n th term of a sequence is given by $an^2 + bn + c$
The fourth term is 34, the seventh term is 124 and the eleventh term is 328. Find the values of a , b and c .

Worked Example

A quadratic sequence has an n th term of $-3n^2 + 2n - 2$
A term in this sequence is equal to -343 .
Find the position of this term.

Your Turn

A sequence has an n th term of $-2n^2 - 5n + 1$
A term in this sequence is equal to -816 .
Find the position of this term.

Worked Example

Here are the first five terms of a sequence.

$-11, -14, -13, -81$

An expression for the n th term of this sequence is

$2n^2 - 9n - 4$.

Find an expression for the n th term of a sequence whose first five terms are $-99, -126, -117, -729$

Your Turn

Here are the first five terms of a sequence.

$-8, -5, 2, 13, 28$

An expression for the n th term of this sequence is

$2n^2 - 3n - 7$.

Find an expression for the n th term of a sequence whose first five terms are

$56, 35, -14, -91, -196$

Fill in the Gaps

Sequence	Type	n^{th} term	10 th term	11 th term	12 th term	30 th term	Is 60 in the sequence?
8, 11, 14, 17, ...							
4, 11, 20, 31, ...							
			67	74	81		
-4, -10, -16, -22, ...							
0, 11, 28, 51, ...							
		$n^2 + 12n - 4$					
3, 7, 15, 27, ...							
		$4n - 8$					
		$4n^2 + n$					
-3, 0, 5, 12, ...							
	Linear		56		66		
	Linear				70	178	

Fill in the Gaps

eg	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	-1	2	5	8	...	71	$u_n =$	$k =$
(a)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	12	17	22	27	...	162	$u_n =$	$k =$
(b)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	8	6	4	2	...	-96	$u_n =$	$k =$
(c)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	3.5	6.5	9.5	12.5	...	123.5	$u_n =$	$k =$
(d)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	-0.3	-0.1	0.1	0.3	...	2.7	$u_n =$	$k =$
(e)	u_1	u_2	u_3	u_4	...	u_k	$u_n = 8n - 2$	$k =$
					...	430	$u_n = 8n - 2$	$k =$
(f)	u_1	u_2	u_3	u_4	...	u_k	$u_n = 4 - 0.5n$	$k =$
					...	-7.5	$u_n = 4 - 0.5n$	$k =$
(g)	u_1	u_2	u_3	u_4	...	u_k	$u_n = \frac{1}{4}n + \frac{3}{8}$	$k =$
					...	$8\frac{1}{8}$	$u_n = \frac{1}{4}n + \frac{3}{8}$	$k =$
(h)	u_1	u_2	u_3	u_4	...	u_k	$u_n = -3n - 2$	$k =$
					...	-194	$u_n = -3n - 2$	$k =$
(i)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
	5.8		7.4		...	20.2	$u_n =$	$k =$
(j)	u_1	u_2	u_3	u_4	...	u_k	$u_n =$	$k =$
		-5.2		-15.6	...	-130	$u_n =$	$k =$

Extra Notes

3 Iterations

Worked Example

Show that the equation $3 + 5x - 2x^2 - x^3 = 0$ has a solution between -4 and -3 .

Your Turn

Show that the equation $x^3 + 4x^2 - x - 5 = 0$ has a solution between 1 and 2.

Worked Example

Show that the equation $\sqrt{2x+4} - 3 = 0$ has a solution between 2 and 3.

Your Turn

Show that the equation $\frac{3}{\sqrt{2+x}} - 4 = 0$ has a solution between -1.5 and -1 .

Worked Example

Find the solution to the equation $x^3 + x^2 - 0.5 = 0$ between 0 and 1, to 1 decimal place.

Your Turn

Find the solution to the equation $x^3 - 5x - 1 = 0$ between 2 and 3, to 1 decimal place.

Worked Example

An approximate solution to the equation $x^3 - 2x^2 - x - 1 = 0$ can be found using the iteration formula $x_{n+1} = \sqrt[3]{2x_n^2 + x_n + 1}$. Starting with $x_0 = 5$, find the value of x_6 . Give your answer correct to 3 decimal places.

Your Turn

The equation $3x^3 + 8x^2 + 7x + 2 = 0$ can be rearranged to give the iterative formula $x_{n+1} = \frac{-3(x_n)^3 - 8(x_n)^2 - 2}{7}$

Starting with $x_0 = -2$, find the value of x_5 . Give your answer correct to 3 decimal places.

Worked Example

Use the iterative formula $x_{n+1} = \sqrt{3 - x_n}$ to find a solution to the equation $x^2 + x - 3 = 0$ to 3 decimal places. Starting with $x_0 = 1$.

Your Turn

Use the iterative formula $x_{n+1} = \frac{1-4x_n^3}{3}$ to find a solution to the equation $4x^3 + 3x - 1 = 0$ to 2 decimal places. Starting with $x_0 = 0$.

Worked Example

Show that $x^3 - 2x^2 - x - 1 = 0$ can be rearranged to
 $x = \sqrt[3]{2x^2 + x + 1}$

Your Turn

Show that $x^3 - 2x + 5 = 0$ can be rearranged to give
 $x = \sqrt[3]{2x - 5}$

Worked Example

The value of a car at the start of year n is V_n . The value at the start of the following year is V_{n+1} where $V_{n+1} = kV_n$. A car was purchased as new in 2020 for £3,200. The same car was sold in 2022 for £2,048. Work out the value of the depreciation constant k .

Your Turn

At the start of year n , the number of animals in a population is P_n . At the start of the following year, the number of animals in the population is P_{n+1} where $P_{n+1} = kP_n$. At the start of 2017 the number of animals in the population was 4000. At the start of 2019 the number of animals in the population was 3610. Find the value of the constant k .

Extra Notes

4 Algebraic Proof

Fluency Practice

Forming Expressions If n is any integer,
how can we form algebraic expressions to describe
these types, sequences, sums & products of numbers?

A number	
An even number	
An odd number	
Two consecutive numbers	
Two consecutive even numbers	
Two consecutive odd numbers	
The sum of two consecutive numbers	
The sum of two consecutive even numbers	
The sum of two consecutive odd numbers	
A number squared	
The square of an even number	
The square of an odd number	
The product of two consecutive numbers	
The product of two consecutive even numbers	
The product of two consecutive odd numbers	

Worked Example

A number is given as $5n - 8$ where n is an integer.
Write down the expression for the next consecutive integer.

Your Turn

A number is given as $-2n + 13$ where n is an integer.
Write down the expression for the next consecutive integer.

Worked Example

- a) An odd number is given as $-2n + 13$ where n is an integer. Write down the expression for the next consecutive odd number.
- b) An even number is given as $-2n - 4$ where n is an integer. Write down the expression for the next consecutive even number.

Your Turn

- a) An even number is given as $-2n + 14$ where n is an integer. Write down the expression for the next consecutive even number.
- b) An odd number is given as $-2n - 9$ where n is an integer. Write down the expression for the next consecutive odd number.

Worked Example

- a) Given that n is an integer. Prove that $(2n - 5)(4n - 9) - 10$ is always an odd number.
- b) Given that n is an integer. Prove that $(4n - 3)^2 + 9$ is always an even number.

Your Turn

- a) Given that n is an integer. Prove that $(4n + 9)(4n - 1) - 3$ is always an even number.
- b) Given that n is an integer. Prove that $(2n - 7)^2 + 2$ is always an odd number.

Worked Example

Given that n is a positive integer. Prove that $(2m + 3)^2 - (2m + 2)^2 - 1$ is always divisible by 4.

Your Turn

Given that n is a positive integer. Prove that $(2m + 2)^2 - (2m - 4)^2 - 12$ is always divisible by 24.

Worked Example	Your Turn
Prove that $(2x - 7)(4x - 7) - (2x - 2)(2x - 7) + 1$ is a perfect square.	Prove that $(4y - 7)(5y - 1) - (2y + 3)(2y - 3) + 7y$ is a perfect square.

Worked Example

Prove algebraically that the sum of any four consecutive integers is not divisible by 4.

Your Turn

Prove algebraically that the sum of any six consecutive integers is divisible by 3.

Worked Example

Prove algebraically that the sum of the squares of any three consecutive integers is always two more than a multiple of 3.

Your Turn

Prove algebraically that the sum of the squares of any four consecutive integers is always two more than a multiple of 4.

Worked Example

Prove algebraically that the sum of four consecutive even integers is always divisible by 4.

Your Turn

Prove algebraically that the sum of three consecutive odd integers is always divisible by 3.

Worked Example	Your Turn
a) Prove algebraically that the sum of the squares of two consecutive even integers is always divisible by 4. b) Prove algebraically that the sum of the squares of two consecutive odd integers is always 2 more than a multiple of 4.	a) Prove algebraically that the sum of the squares of three consecutive odd integers is always 1 less than a multiple of 12. b) Prove algebraically that the sum of the squares of three consecutive even integers always has a remainder of 8 when divided by 12.

Worked Example	Your Turn
a) Prove algebraically that the sum of any two odd integers is always even. b) Prove algebraically that the difference of any two even integers is always even.	a) Prove algebraically that the sum of any two even integers is always even. b) Prove algebraically that the difference of any two odd integers is always even.

Worked Example

Given that the difference of the squares of 2 consecutive even integers equals 68, find the value of the smallest integer.

Your Turn

Given that the difference of the squares of 2 consecutive odd integers equals 112, find the value of the smallest integer.

Worked Example

A sequence has the n^{th} term $n^2 - 6n + 10$. By completing the square, show that every term is positive.

Your Turn

A sequence has the n^{th} term $n^2 - 10n + 27$. By completing the square, show that every term is positive.

Worked Example

Show that for any integer n , $n^2 + n$ is always even.

Your Turn

Prove that $n(n - 1) + 1$ is odd for all integers n .

Worked Example

I think of a two-digit number. I then reverse the digits. Prove that the difference between the two numbers is a multiple of 9.

Your Turn

Prove that the sum of a four-digit number and its reverse is a multiple of 11.

Worked Example

Given that

$$4bx - 3a + 7 - 10ax \equiv -30x - 8$$

Find the values of a and b .

Your Turn

Given that

$$ax + 5b - 8ax + 4bx \equiv -23x + 15$$

Find the values of a and b .

Worked Example

Given that

$$3(4px - q) + 5(px + 3q) \equiv 68x - 60$$

Find the values of p and q .

Your Turn

Given that

$$5(4ax + 3b) - 2(3ax + 2b) \equiv -84x + 66$$

Find the values of a and b .

Worked Example

Given that

$$2x^2 - 4x + p \equiv (x - 3)(qx + r) + 1$$

where p , q and r are integers, find the values of p , q , and r

Your Turn

Given that

$$(x + 3)(lx + m) + 9 \equiv 3x^2 + 5x + k$$

where k , l and m are integers, find the values of k , l , and m

Worked Example

Given that

$$(2y - 1)^2 + ay + 7 = (2y + b)(2y + 4)$$

where a and b are integers, find the value of a and the value of b .

Your Turn

Given that

$$(2x + 1)^2 - 12x + r = (2x + s)(2x - 2)$$

where r and s are integers, find the value of r and the value of s .

Worked Example

$$3x^2 - 3bx + 16a \equiv 3(x - a)^2 + 5$$

Work out the two possible pairs of values of a and b

Your Turn

$$2x^2 - 2bx + 7a \equiv 2(x - a)^2 + 3$$

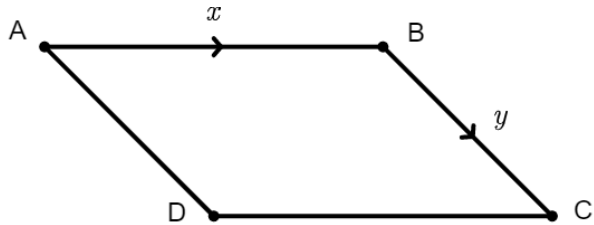
Work out the two possible pairs of values of a and b

Extra Notes

5 Advanced Vectors

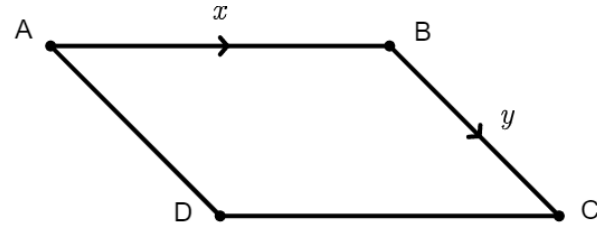
Worked Example

ABCD is a parallelogram.
Express $\overrightarrow{DB}$ in terms of x and y .



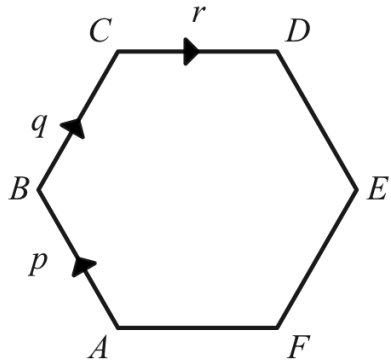
Your Turn

ABCD is a parallelogram.
Express $\overrightarrow{CA}$ in terms of x and y .



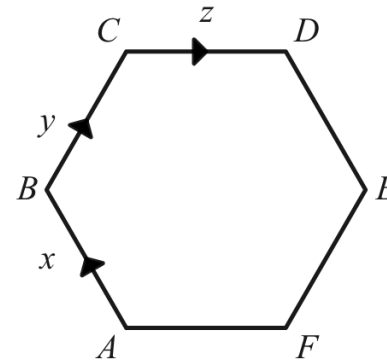
Worked Example

Express $\overrightarrow{DF}$ in terms of p , q and r .



Your Turn

Express $\overrightarrow{BF}$ in terms of x , y and z .

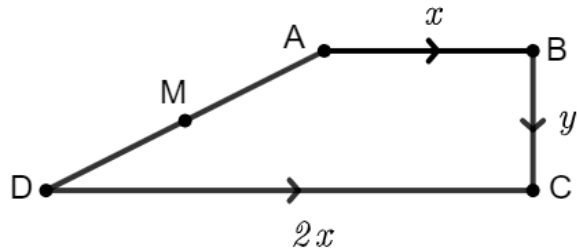


Worked Example

ABCD is a trapezium.

M is the midpoint of AD.

Find $\overrightarrow{MA}$ in terms of x and y .

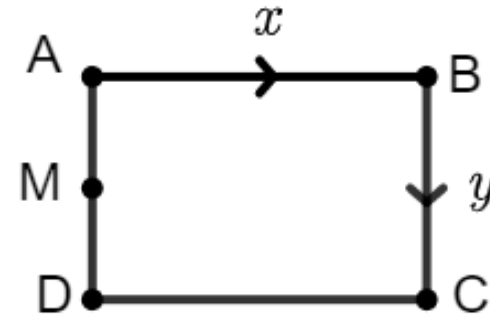


Your Turn

ABCD is a rectangle.

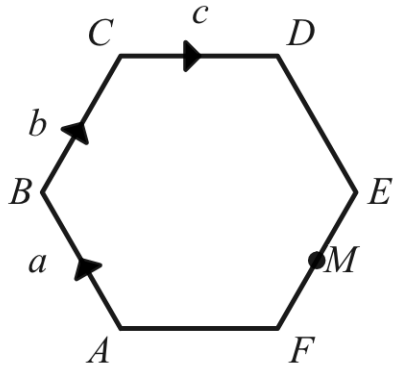
M is the midpoint of AD.

Find $\overrightarrow{MA}$ in terms of x and y .



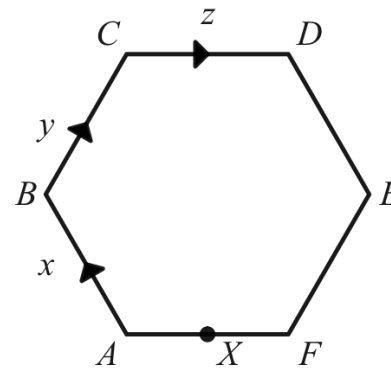
Worked Example

The point M is the midpoint of EF .
Express $\overrightarrow{DM}$ in terms of a , b and c .



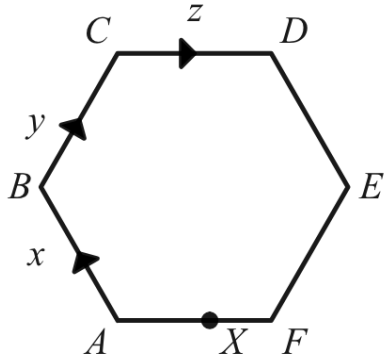
Your Turn

The point x is the midpoint of FA .
Express $\overrightarrow{EX}$ in terms of x , y and z .



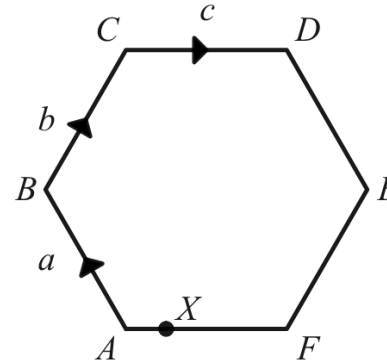
Worked Example

The point X shares the line segment FA in the ratio $2 : 3$.
Express $\overrightarrow{EX}$ in terms of x , y and z .



Your Turn

The point X shares the line segment FA in the ratio $3 : 1$.
Express $\overrightarrow{CX}$ in terms of a , b and c .



Fill in the Gaps

Point X divides the vector $\overrightarrow{AB}$ in the ratio given to create vectors $\overrightarrow{AX}$ and $\overrightarrow{XB}$.

$\overrightarrow{AB}$	Ratio $AX : XB$	$\overrightarrow{AX}$	$\overrightarrow{XB}$
$3a$	$1 : 2$	a	$2a$
$3a + 3b$	$2 : 1$	$2a + 2b$	
$4a - 4b$	$3 : 1$		
$5a + 10b$	$3 : 2$		
$10a - 15b$	$1 : 4$		
a	$2 : 1$	$\frac{2}{3}a$	
$a + b$	$1 : 2$		$\frac{2}{3}a + \frac{2}{3}b$
$a - b$	$3 : 1$		
$2a + b$	$4 : 1$		
$a - 4b$	$3 : 2$		
	$1 : 3$	$\frac{1}{4}a - \frac{1}{4}b$	
$2a - 3b$			$\frac{4}{3}a - 2b$
		$\frac{6}{5}a + \frac{3}{10}b$	$\frac{4}{5}a + \frac{1}{5}b$

Worked Example

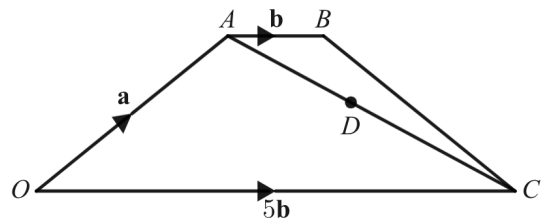
$OABC$ is a trapezium

$$\overrightarrow{OA} = a$$

$$\overrightarrow{AB} = b$$

$$\overrightarrow{OB} = 5b$$

D is the point on AC such that $AD : DC = 3 : 4$



Find, in terms of a and b , the vector $\overrightarrow{OD}$

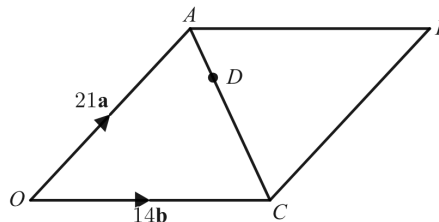
Your Turn

$OABC$ is a parallelogram

$$\overrightarrow{OA} = 21a$$

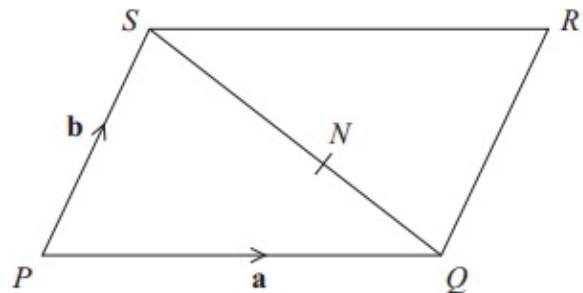
$$\overrightarrow{OB} = 14b$$

D is the point on AC such that $AD : DC = 2 : 5$



Find, in terms of a and b , the vector $\overrightarrow{DB}$

Worked Example



$PQRS$ is a parallelogram.

N is the point on SQ such that $SN : NQ = 3 : 2$

$$\vec{PQ} = \mathbf{a} \quad \vec{PS} = \mathbf{b}$$

- (a) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, an expression for $\vec{SQ}$.
- (b) Express $\vec{NR}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Your Turn

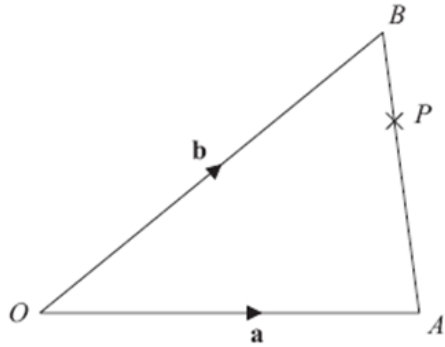


Diagram **NOT**
accurately drawn

OAB is a triangle.

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

.....
(1)

P is the point on AB such that $AP : PB = 3 : 1$

(b) Find $\overrightarrow{OP}$ in terms of **a** and **b**.
Give your answer in its simplest form.

Worked Example

$OABC$ is a parallelogram

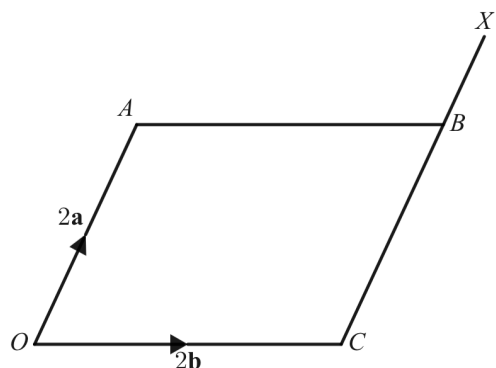
$$\overrightarrow{OA} = 2a$$

$$\overrightarrow{OB} = 2b$$

CBX is a straight line

$$CB : BX = 5 : 2$$

The point Y is such that $\overrightarrow{CY} = 5\overrightarrow{AX}$



Find, in terms of a and b , the vector $\overrightarrow{OY}$

Your Turn

$OABC$ is a parallelogram

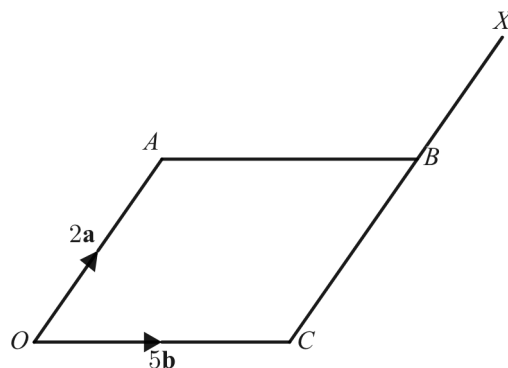
$$\overrightarrow{OA} = 2a$$

$$\overrightarrow{OB} = 5b$$

CBX is a straight line

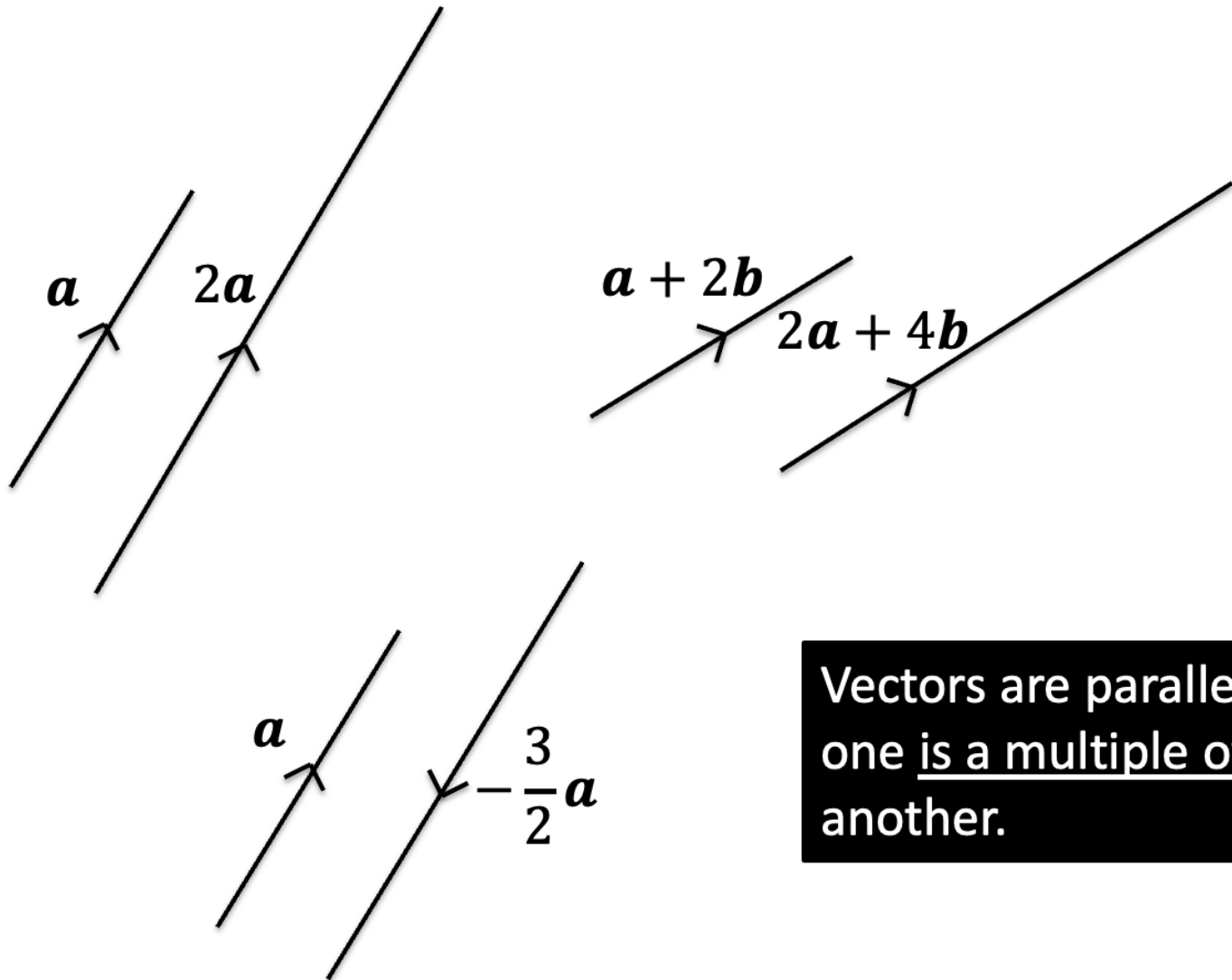
$$CB : BX = 3 : 2$$

The point Y is such that $\overrightarrow{CY} = 4\overrightarrow{AX}$



Find, in terms of a and b , the vector $\overrightarrow{OY}$

Parallel Vectors



Vectors are parallel if one is a multiple of another.

Parallel Vectors

Two vectors are parallel if they are ***multiples*** of each other.

Vector 1	Vector 2	Parallel?	
a	-a	Yes	No
a + b	2a + 2b	Yes	No
a + b	a + 2b	Yes	No
$\frac{1}{2}\mathbf{a} + \mathbf{b}$	a + 2b	Yes	No
2a + 5b	4a + 10b	Yes	No
a + b	a - b	Yes	No
a + b	-a - b	Yes	No
a - b	-a + b	Yes	No
2a + 3b	$\frac{2}{3}\mathbf{a} + \mathbf{b}$	Yes	No

Worked Example

a) The vectors $-3a - b$ and $6a + nb$ are parallel.
Given that a and b are not parallel, find the value of n

b) Vector $p = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and vector $q = \begin{pmatrix} n \\ 7 \end{pmatrix}$

Vector $3p + q$ is parallel to $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$

Find the value of n

Your Turn

a) The vectors $12p - 12q$ and $-6p + dq$ are parallel.
Given that p and q are not parallel, find the value of d

b) Vector $a = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ and vector $b = \begin{pmatrix} l \\ 6 \end{pmatrix}$

Vector $2a + b$ is parallel to $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

Find the value of l

Worked Example

D , E and F are three points on a straight line such that

$$\overrightarrow{DE} = 6x - 3y$$

$$\overrightarrow{EF} = 8x - 4y$$

Find the ratio length of EF : length of DE

Give your answer in its simplest form.

Your Turn

U , V and W are three points on a straight line such that

$$\overrightarrow{UV} = 7a - 3b$$

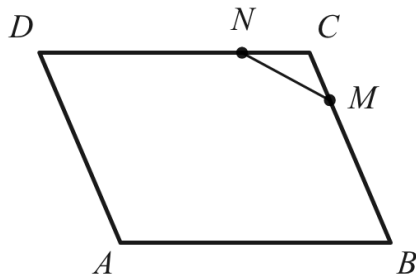
$$\overrightarrow{VW} = \frac{35}{2}a - \frac{15}{2}b$$

Find the ratio length of UV : length of UW

Give your answer in its simplest form.

Worked Example

The diagram shows parallelogram $ABCD$



$$\overrightarrow{AB} = a$$

$$\overrightarrow{AC} = b$$

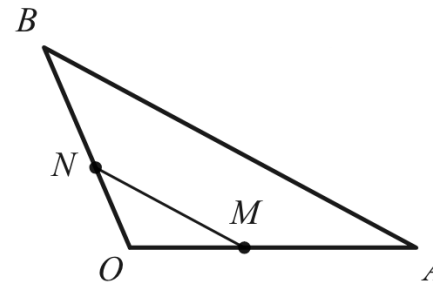
$$BM : MC = 3 : 1$$

$$DN : NC = 3 : 1$$

Use a vector method to prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{BD}$

Your Turn

The diagram shows triangle OAB



$$\overrightarrow{OA} = a$$

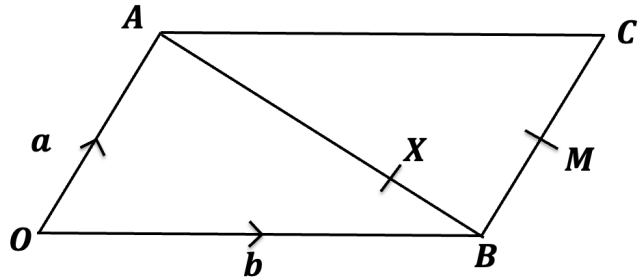
$$\overrightarrow{OB} = b$$

$$OM : MA = 2 : 3$$

$$ON : NB = 2 : 3$$

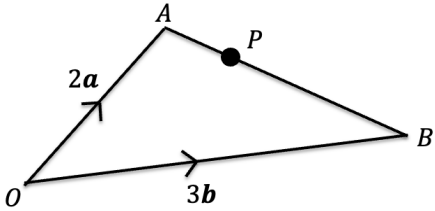
Use a vector method to prove that $\overrightarrow{MN}$ is parallel to $\overrightarrow{AB}$

Worked Example



X is a point on AB such that $AX:XB = 3:1$. M is the midpoint of BC .
Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

Your Turn



- a) Find $\vec{AP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
- b) P is the point on AB such that $AP:PB = 2:3$.
Show that $\vec{OP}$ is parallel to the vector $\mathbf{a} + \mathbf{b}$.

Worked Example

The quadrilateral $OABC$ has vertices O, A, B, C where $\overrightarrow{OA} = 2a + b$, $\overrightarrow{OB} = 3b$ and $\overrightarrow{OC} = -4a + b$

By showing that $\overrightarrow{OA}$ and $\overrightarrow{BC}$ are parallel, prove that the quadrilateral $OABC$ is a trapezium.

Your Turn

The quadrilateral $OABC$ has vertices O, A, B, C where $\overrightarrow{OA} = a + b$, $\overrightarrow{OB} = 2b$ and $\overrightarrow{OC} = -3a - b$

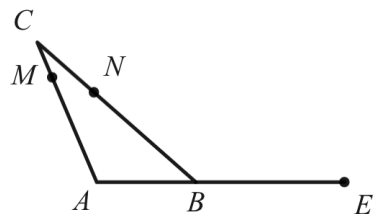
By showing that $\overrightarrow{OA}$ and $\overrightarrow{BC}$ are parallel, prove that the quadrilateral $OABC$ is a trapezium.

Straight Lines

Worked Example

The diagram shows triangle ABC

The point E lies on the straight line through A and B



$$\overrightarrow{AB} = a$$

$$\overrightarrow{AC} = b$$

$$\overrightarrow{AE} = \frac{5}{2}a$$

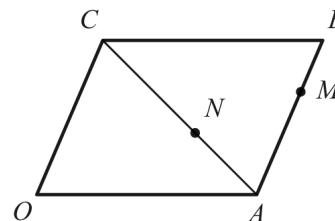
$$AM : MC = 3 : 1$$

$$BN : NC = 9 : 5$$

Use a vector method to prove that M , N and E are collinear.

Your Turn

The diagram shows parallelogram $OACB$



$$\overrightarrow{OA} = 5a$$

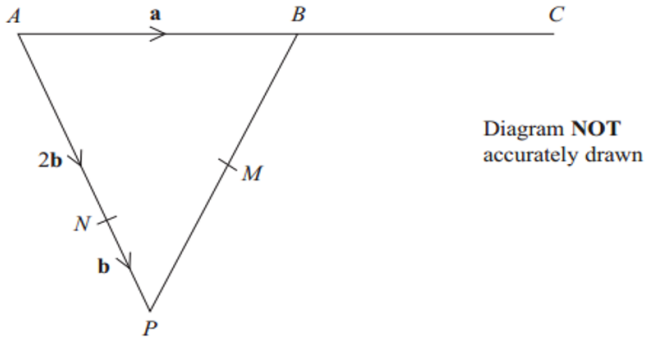
$$\overrightarrow{OC} = 5b$$

$$AM : MB = 2 : 1$$

$$AN : NC = 2 : 3$$

Use a vector method to prove that O , N and M are collinear.

Worked Example

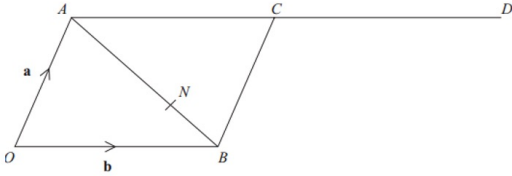


$$\overrightarrow{AN} = 2\mathbf{b}, \quad \overrightarrow{NP} = \mathbf{b}$$

B is the midpoint of AC . M is the midpoint of PB .

- Find $\overrightarrow{PB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.
- Show that NMC is a straight line.

Your Turn



$$\vec{OA} = \mathbf{a} \text{ and } \vec{OB} = \mathbf{b}$$

D is the point such that $\vec{AC} = \vec{CD}$

The point N divides AB in the ratio 2: 1.

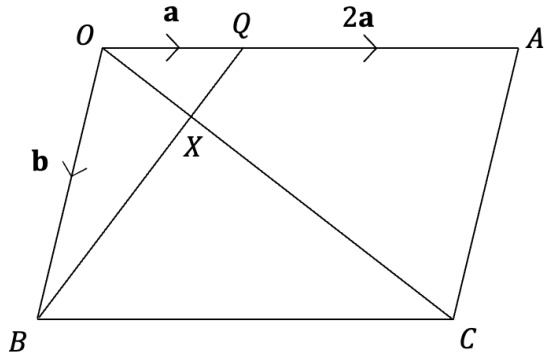
(a) Write an expression for $\vec{ON}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(b) Prove that OND is a straight line.

Vector Proofs

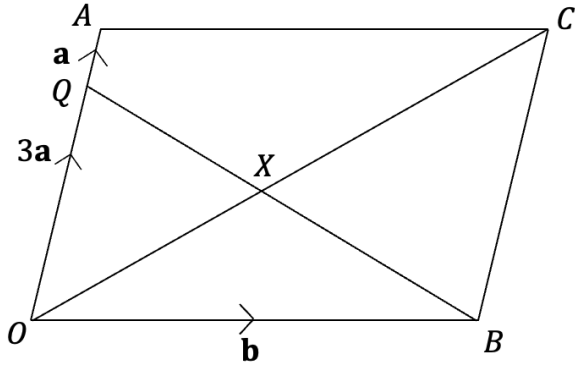
Worked Example

$OACB$ is a parallelogram. Given that OXC and BXQ are straight lines, determine the ratio $OX : XC$.



Your Turn

$OACB$ is a parallelogram. Given that OXC and BXQ are straight lines, determine the ratio $OX : XC$.



Worked Example

OAB is a triangle.

OPM and APN are straight lines.

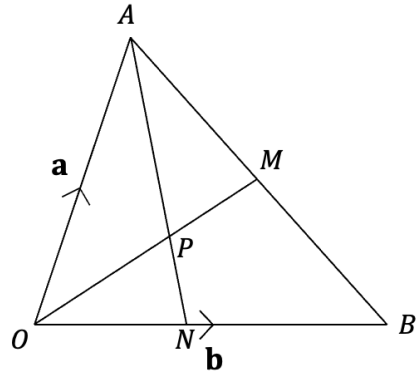
M is the midpoint of AB .

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

$$OP : PM = 3 : 2$$

Work out the ratio $ON : NB$



Your Turn

OAB is a triangle.

OPN and APN are straight lines.

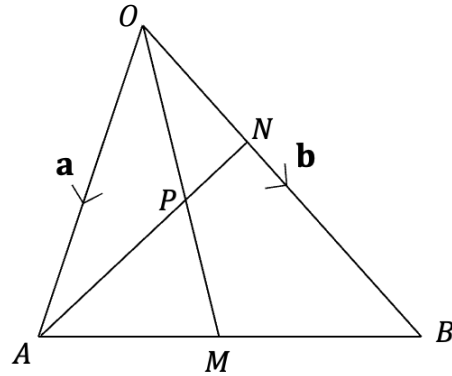
M is the midpoint of OB .

$$\overrightarrow{OA} = \mathbf{a}$$

$$\overrightarrow{OB} = \mathbf{b}$$

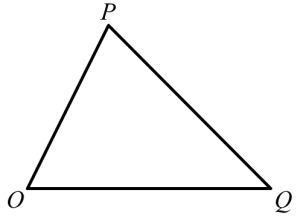
$$OP : PM = 5 : 3$$

Work out the ratio $ON : NB$



Worked Example

The diagram below shows a sketch of triangle OPQ



The point R is such that $OP : PR = 1 : 2$

The point M is such that $PM : MQ = 2 : 3$

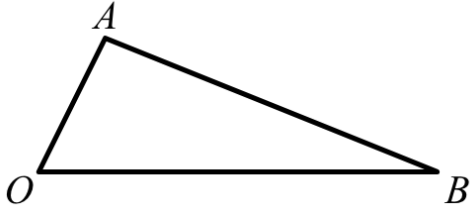
The straight line through R to M cuts OQ at the point N

Let $\overrightarrow{OP} = a$ and $\overrightarrow{OQ} = b$

By first finding $\overrightarrow{RM}$ in terms of a and b , and letting $\overrightarrow{RN} = \lambda \overrightarrow{RM}$, find $ON : NQ$

Your Turn

The diagram below shows a sketch of triangle OAB



The point C is such that $\overrightarrow{OC} = 2\overrightarrow{OA}$

The point M is such that $AM : MB = 3 : 2$

The straight line through C to M cuts OB at the point N

Let $\overrightarrow{OA} = a$ and $\overrightarrow{OB} = b$

By first finding $\overrightarrow{CM}$ in terms of a and b , and letting $\overrightarrow{CN} = \lambda\overrightarrow{CM}$, find $ON : NB$

Extra Notes