



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2026 Mathematics 2027

Unit 21 Tasks – Part 1

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KING EDWARD VI
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BIRMINGHAM

Year 11

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Unit 21 Tasks – Part 2

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Year 11

2026 Mathematics 2027

Unit 21 Tasks – Part 3

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- 3 [Combinations and Permutations](#)
- 4 [Capture-Recapture](#)

1 Advanced Statistics

Fluency Practice

learn by heart

Histograms are used for **continuous data**. The frequency is represented by the **area** of each bar, and the vertical scale is **frequency density**, calculated for each bar using the formula:

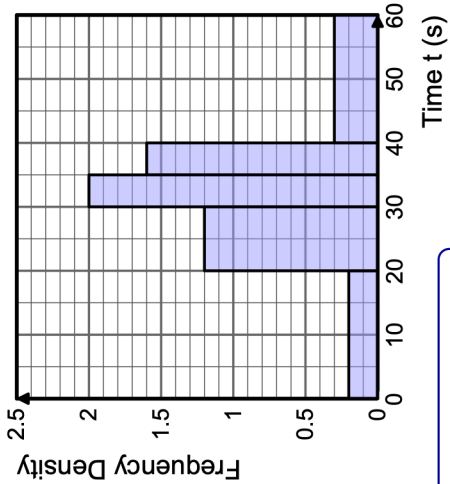
$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

example

Time, t (s)	Frequency	Class Width	Frequency Density
0 < t ≤ 20	4	20	0.2
20 < t ≤ 30	12	10	1.2
30 < t ≤ 35	10	5	2
35 < t ≤ 40	8	5	1.6
40 < t ≤ 60	6	20	0.3

The class width is the range of times included in the row.

$$\text{Frequency Density} = \frac{\text{Frequency}}{\text{Class Width}}$$

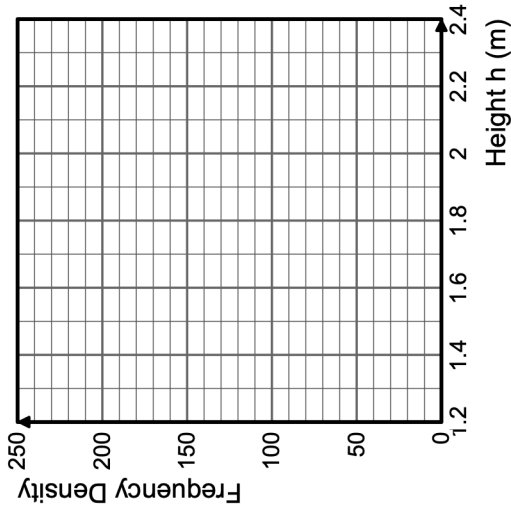


exercise

1. The data in the table shows the heights of a group of people.

Work out the frequency densities and complete the histogram.

Height h (m)	Frequency	Frequency Density
1.4 ≤ h < 1.6	12	
1.6 ≤ h < 1.7	20	
1.7 ≤ h < 1.8	17	
1.8 ≤ h < 2.1	12	

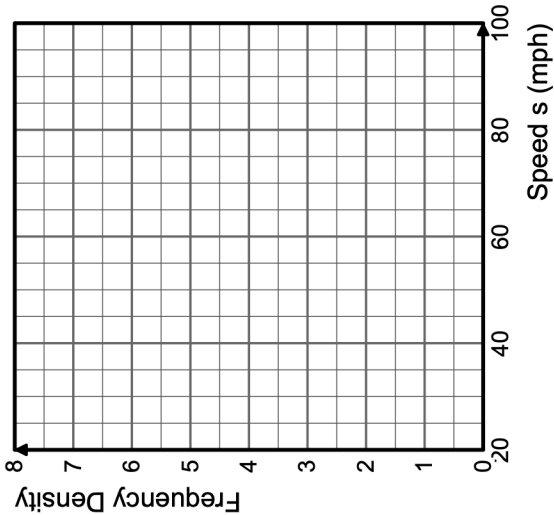


Fluency Practice

2. The data in the table shows the speeds of vehicles passing a point on a motorway.

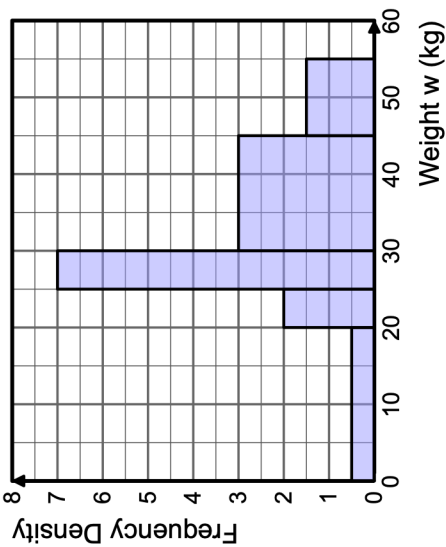
Complete the histogram.

Speed s (mph)	Frequency	Frequency Density
$30 < s \leq 55$	16	
$55 < s \leq 65$	23	
$65 < s \leq 70$	35	
$70 < s \leq 75$	32	
$75 < s \leq 95$	8	



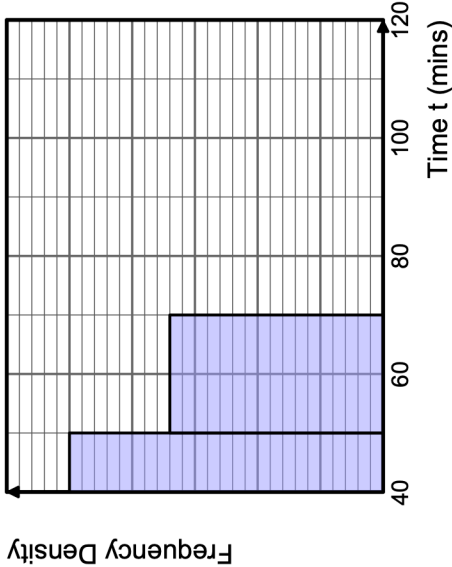
3. The histogram shows the weights of a number of children.
Complete the frequency table.

Weight w (kg)	Frequency
$0 < w \leq 20$	
$20 < w \leq 25$	
$25 < w \leq 30$	
$30 < w \leq 45$	
$45 < w \leq 55$	



4. The table and histogram show the time taken by runners to complete a race. Complete the table and histogram.

Time t (mins)	Frequency
$40 < t \leq 50$	25
$50 < t \leq 70$	
$70 < t \leq 100$	81
$100 < t \leq 120$	12



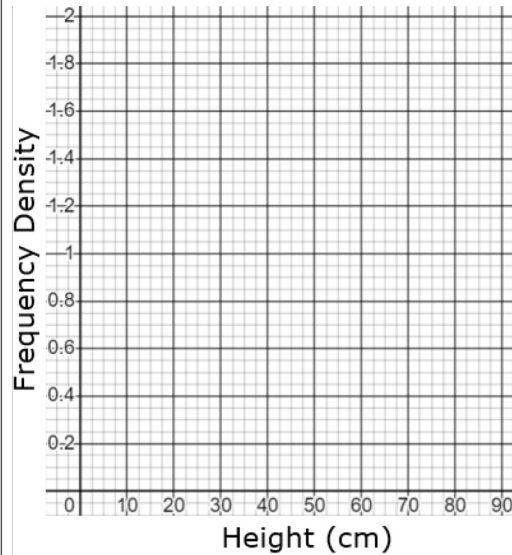
Fluency Practice

Plotting Histograms

(a)

Plot a histogram from the data shown in the table.

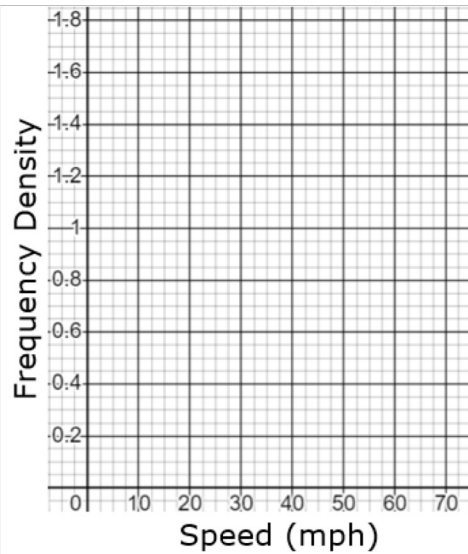
Height (cm)	Frequency	Frequency Density
$0 < h \leq 20$	24	1.2
$20 < h \leq 40$	32	1.6
$40 < h \leq 80$	44	
$80 < h \leq 90$	14	



(b)

Plot a histogram from the data shown in the table.

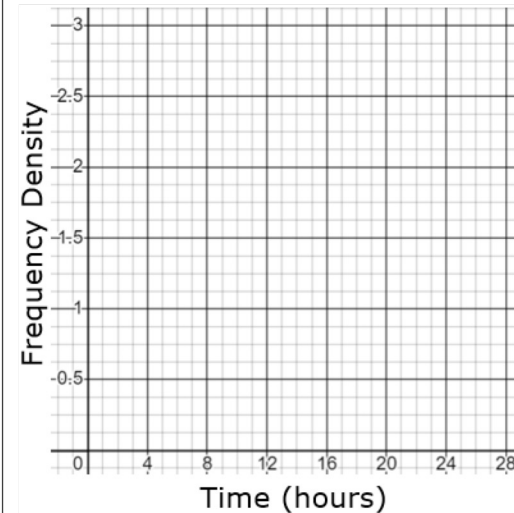
Speed (mph)	Frequency	Frequency Density
$0 < s \leq 10$	5	
$10 < s \leq 35$	30	
$35 < s \leq 50$	24	
$50 < s \leq 70$	16	



(c)

Plot a histogram from the data shown in the table.

Time (hours)	Frequency	Frequency Density
$0 < h \leq 4$	3	
$4 < h \leq 8$	5	
$8 < h \leq 16$	22	
$16 < h \leq 22$	15	
$22 < h \leq 25$	6	



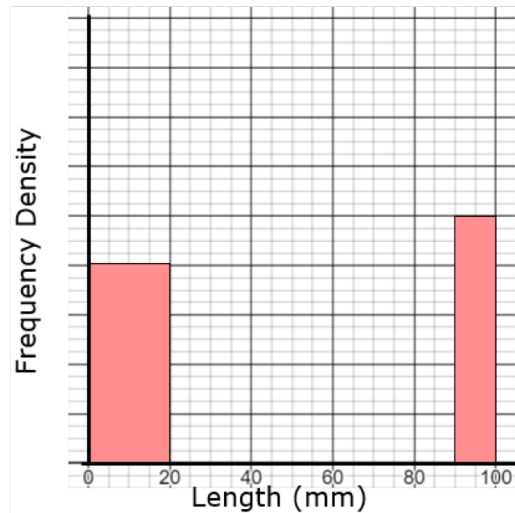
Fluency Practice

Plotting Incomplete Histograms

(a)

Complete the table and histogram.

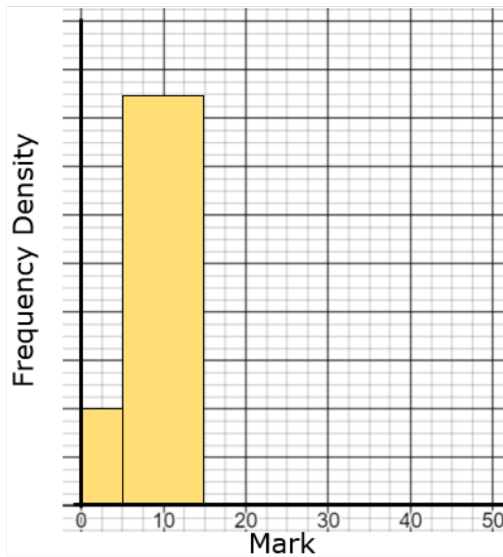
Length (mm)	Frequency	Frequency Density
$0 < h \leq 20$	16	0.8
$20 < h \leq 60$	48	
$60 < h \leq 90$	45	
$90 < h \leq 100$		



(b)

Complete the table and histogram.

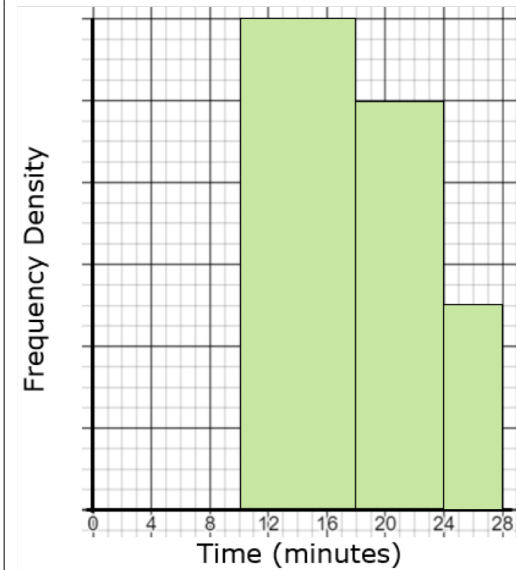
Mark	Frequency	Frequency Density
$0 < s \leq 5$	2	
$5 < s \leq 15$		
$15 < s \leq 30$	27	
$30 < s \leq 50$	22	



(c)

Complete the table and histogram.

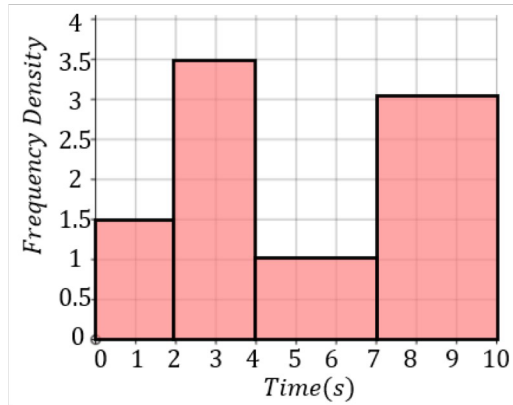
Time (minutes)	Frequency	Frequency Density
$0 < h \leq 4$	2	
$4 < h \leq 10$	9	
$10 < h \leq 18$		
$18 < h \leq 24$		
$24 < h \leq 28$	5	



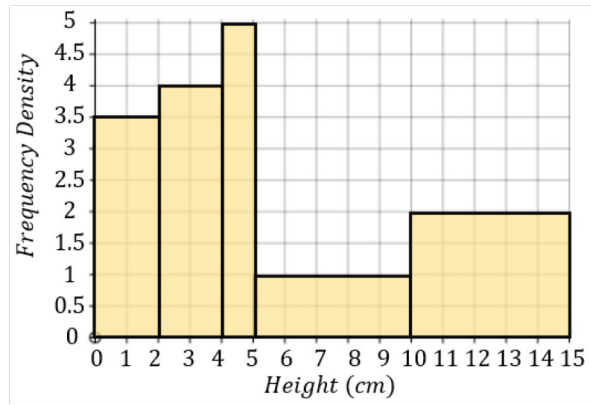
Fluency Practice

Finding the Median from a Histogram

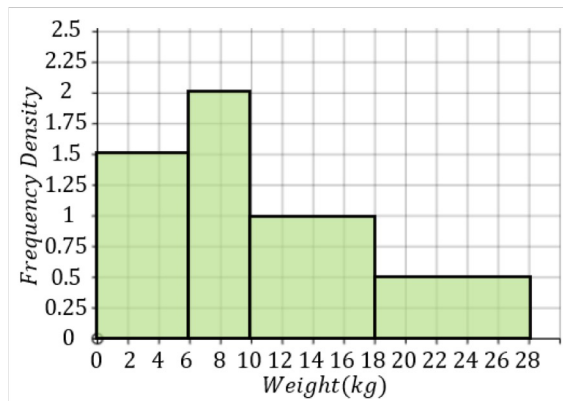
(a)



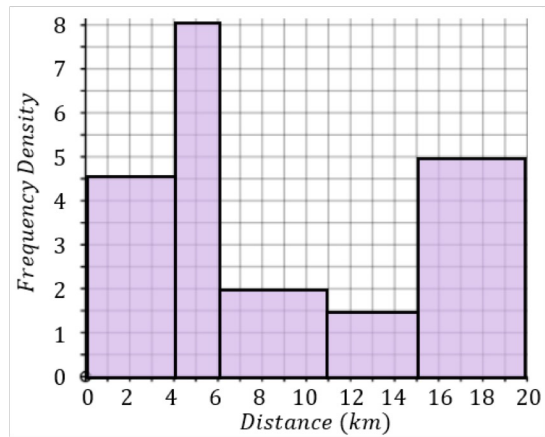
(b)



(c)

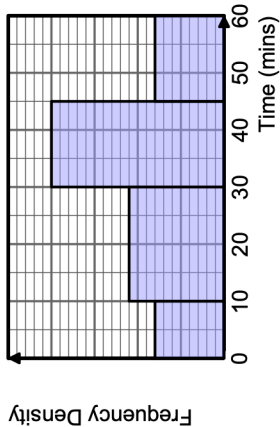


(d)



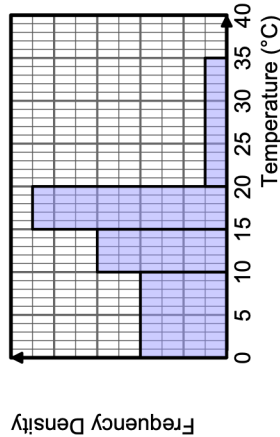
Fluency Practice

1.



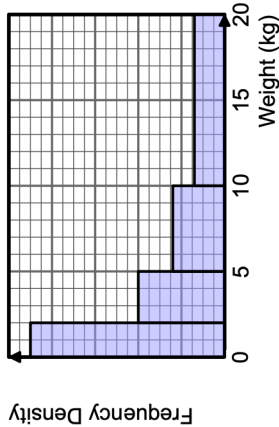
Twelve people waited more than 45 minutes.
How many people waited between 10 and 30 minutes?

2.



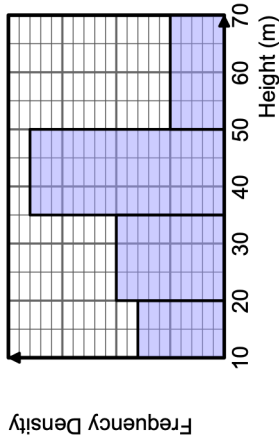
Twelve days had an average temperature between 10°C and 15°C.
How many days were surveyed in total?

3.



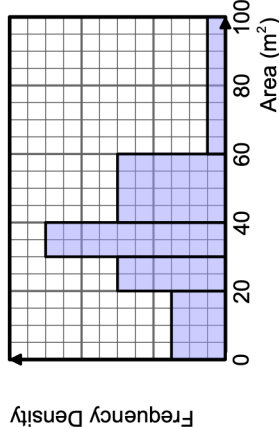
Twelve parcels weighed between 2kg and 5kg.
How many parcels weighed less than 5kg?

4.



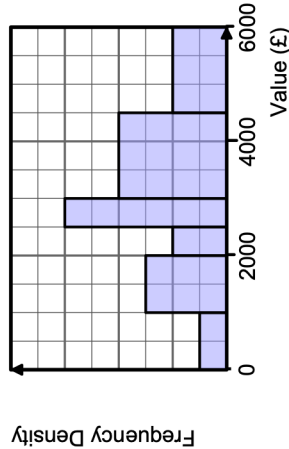
Ten rockets had a height greater than 50m.
How many rockets had a height between 20m and 50m?

5.



Forty rooms have a floor area between 20m² and 40m².
Estimate the number of rooms with a floor area greater than 50m².

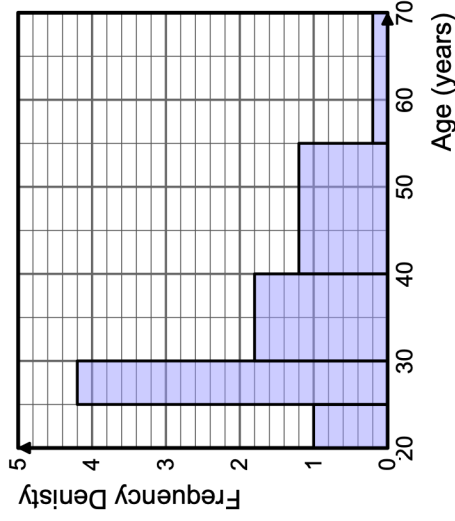
6.



Five used cars were valued at less than £1000.
Estimate the number of used cars valued at less than £4000.

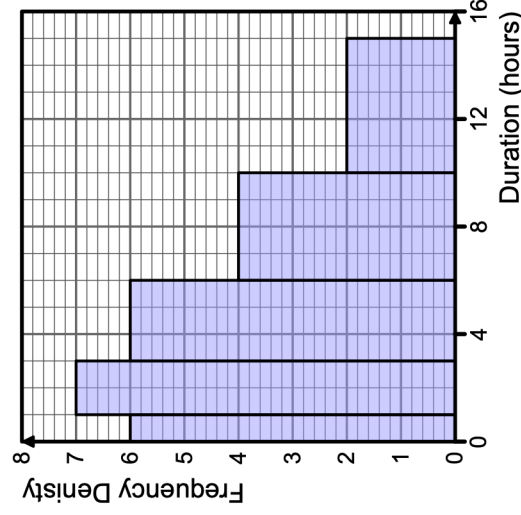
Fluency Practice

7. The histogram shows the ages of the workers at a company.



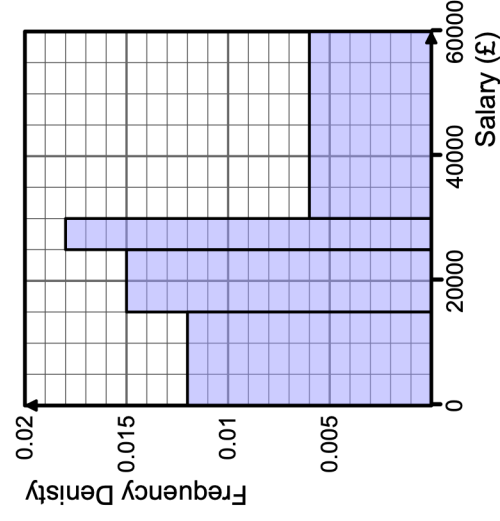
- How many workers are between 20 and 30 years old?
- True or false: there are more workers aged between 30 and 40 than between 40 and 55.
- 60% of the workers at the company are female. Work out the number of female workers.

8. The histogram shows the duration of 64 flights offered by an airline.



- How many of the flights are 3 hours or less in duration?
- Work out an estimate for the number of flights that take between 2 and 6 hours.
- Work out an estimate for the median flight time.

9. In a survey, 600 workers were asked about their salaries. The results are shown in the histogram.



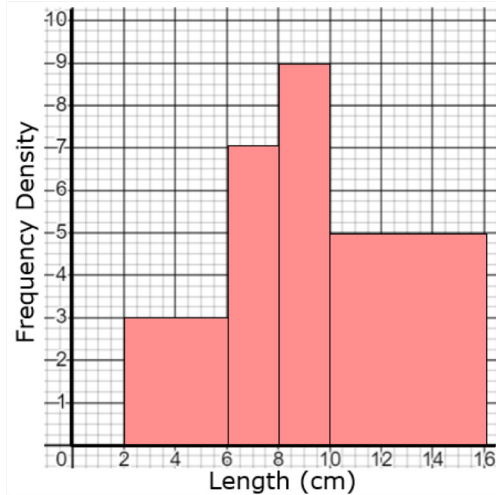
- Work out an estimate for the median salary of the workers.
A worker is to be chosen at random.
- Work out the probability that their salary will be at most £15,000.
- Work out an estimate for the probability that their salary is at least double the median salary.

Fluency Practice

Interpreting Histograms

(a)

The histogram shows some information about the lengths of some sycamore leaves.

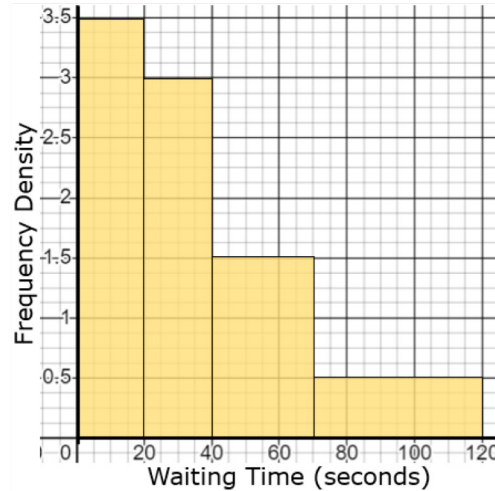


(a) Find the total number of leaves that were measured.

(b) Find an estimate of the number of leaves that were less than 7 cm in length.

(b)

The histogram shows some information about the waiting times in a queue at a supermarket.

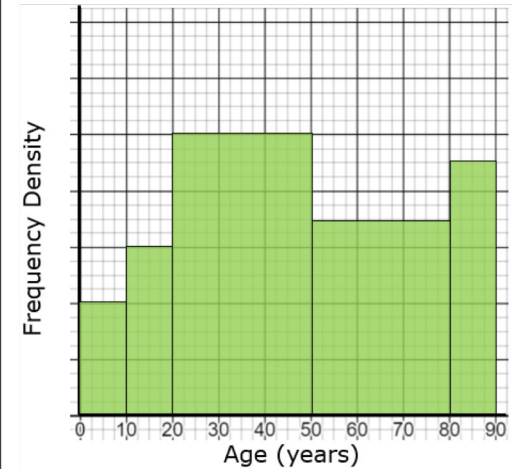


(a) Find an estimate of the proportion of customers who waited more than 50 seconds.

(b) Find an estimate of the median waiting time.

(c)

The histogram shows some information about the ages of people visiting a library. Ten of the people who visited the library were less than 20 years old.



(a) Find the number of people visiting the library who are over 50 years old.

(b) Find an estimate of the median age of the people visiting the library.

Fluency Practice

(a)

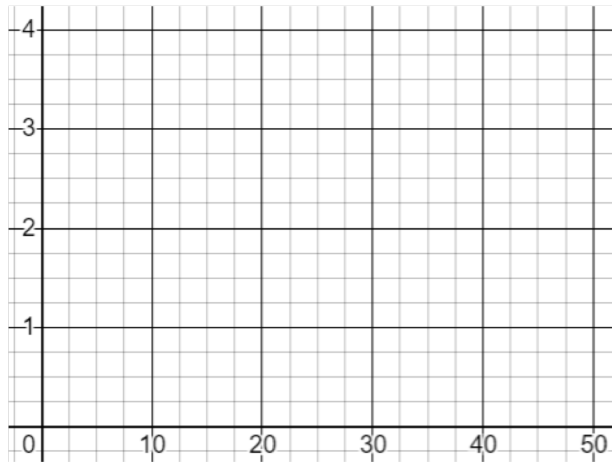
The distance travelled to work by 50 employees is recorded in a grouped frequency table.

(a) Calculate the frequency density for each class.

Distance (km)	Frequency		
$0 < d \leq 5$	12		
$5 < h \leq 10$	16		
$10 < h \leq 20$	10		
$20 < h \leq 30$	7		
$30 < h \leq 50$	5		

(b) Plot a histogram.

(c) Use your histogram to estimate the number of people who travel at least 12 km to work.



(b)

The house prices of 100 houses in a village are recorded in a grouped frequency table.

(a) Use the information in the table to calculate frequency densities and plot a histogram.

(b) Use your histogram to estimate the percentage of houses that cost less than £270 000.

House Price (£ thousands)	Frequency
$0 < p \leq 100$	6
$100 < p \leq 200$	22
$200 < p \leq 250$	18
$250 < p \leq 300$	35
$300 < p \leq 500$	15
$500 < p \leq 1000$	4

(c)

The time taken, in minutes, by 50 students to solve a maths puzzle is recorded in a grouped frequency table.

(a) Plot a histogram to represent this data.

(b) Use your histogram to find the median time taken.

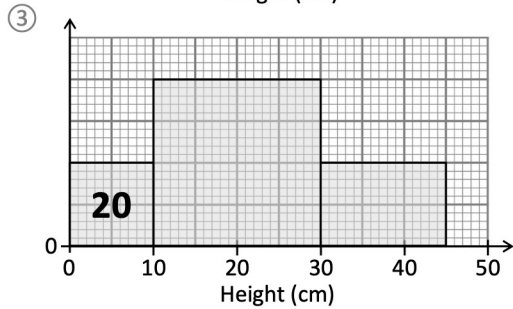
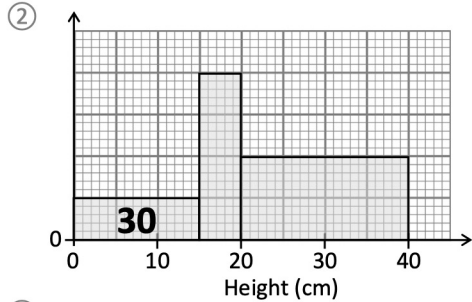
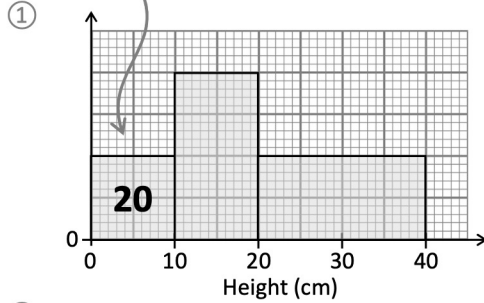
Time Taken (minutes)	Frequency
$0 < t \leq 2$	4
$2 < t \leq 4$	7
$4 < t \leq 5$	10
$5 < t \leq 6$	12
$6 < t \leq 7$	11
$7 < t \leq 12$	6

Fluency Practice

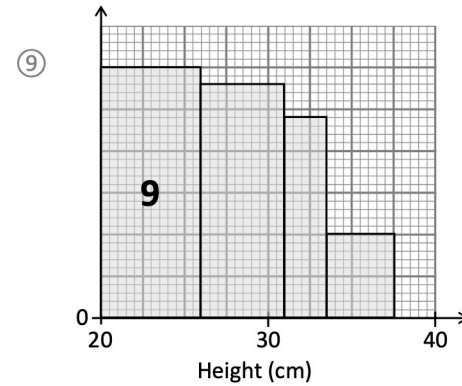
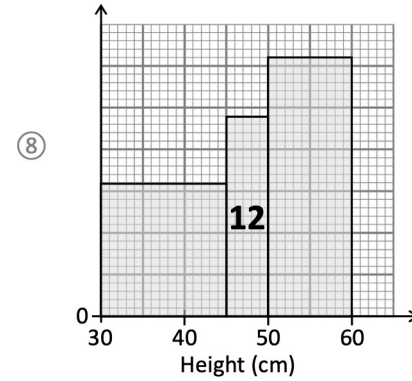
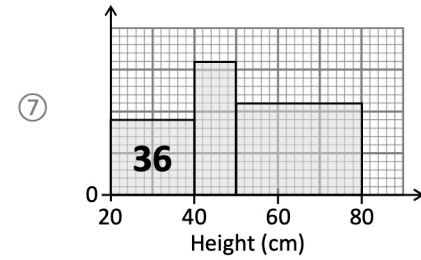
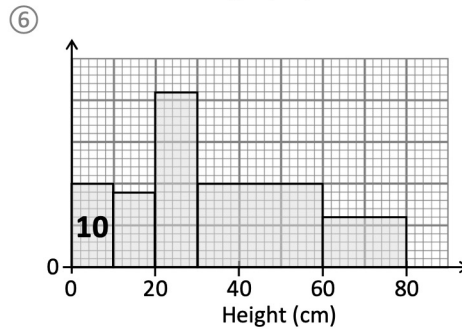
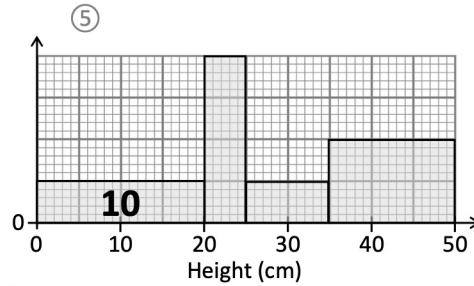
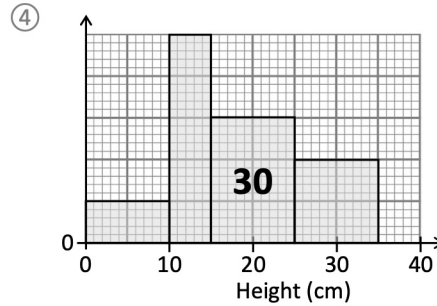


Total Pop.?

20 of the population had a height of 10 cm or less.



The **area** of a histogram represents **frequencies**.
On each histogram one group frequency has been given.
Find the **total population** for each data set.



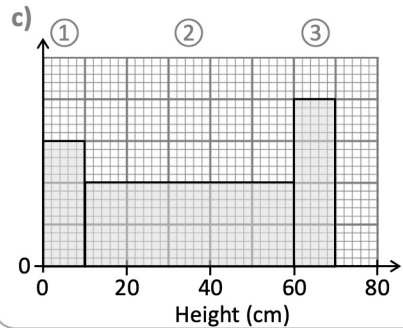
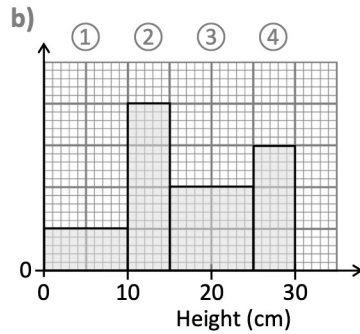
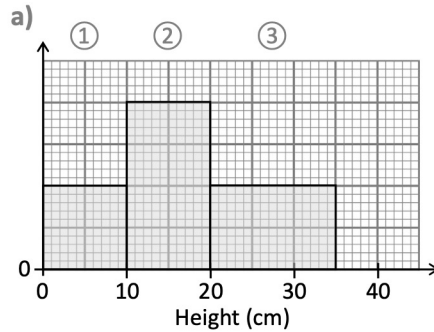
Fluency Practice



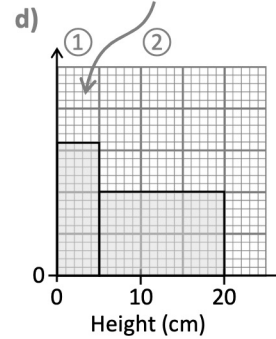
Workin' Backwards

Calculate the frequency for each group in each histogram.

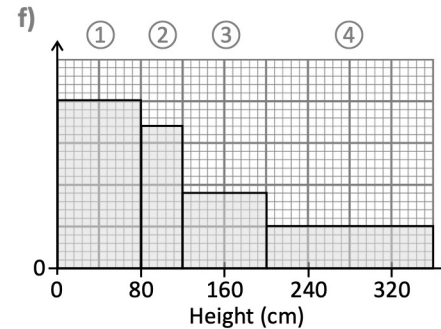
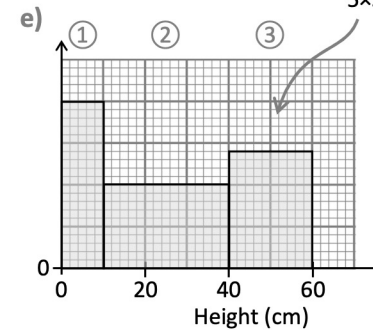
What *method* will you use?



d) How many large (5x5) squares are in this bar?
Express this as a decimal & sum the **total area**.



As a decimal, how many 5x5 squares are in this bar?



Fluency Practice

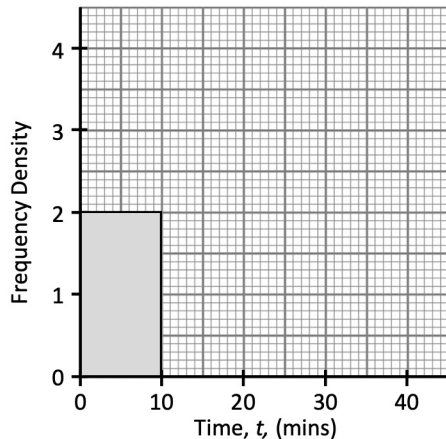
Histograms

1) People were surveyed about the time it took to get to their local shop.

Time, t , (mins)	Frequency	Class Width	Frequency Density
$0 < t \leq 10$	20		
$10 < t \leq 15$	15		
$15 < t \leq 20$	20		
$20 < t \leq 40$	30		

Notice that the classes (groups) are different sizes.
To represent them fairly, we must show how *relatively* dense the class is.
We do this using 'Frequency Density': $\text{Frequency} \div \text{Class Width}$

- Complete Class Width & Frequency Density for each class.
- Use the data to complete the histogram.



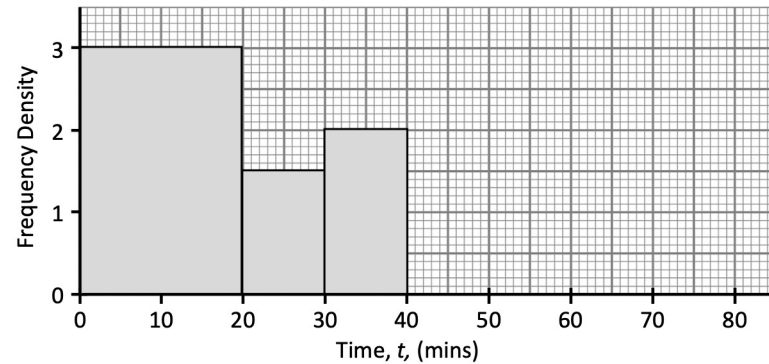
How many people is one square worth?

Estimate:

- How many people took 5 minutes or less to walk to work.
- How many people took between 30 & 40 minutes to walk to work.

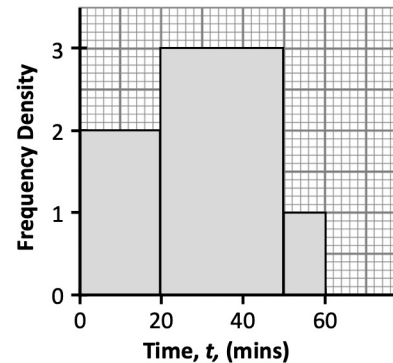
Why are these estimates?

2) Students were surveyed about the time they spent doing homework last night. Complete the graph & table.



Time, t , (mins)	Frequency	Class Width	Frequency Density
$0 < t \leq 20$			
$20 < t \leq 30$			
$30 < t \leq 40$			
$40 < t \leq 80$	40	40	1

3) As a logic test, adults were given a puzzle to complete.
The histogram shows the results. Complete the frequency table.



Time, t , (mins)	Freq.
$0 < t \leq 20$	
$20 < t \leq 50$	
$50 < t \leq 60$	

Which group is the median in?

How far into that group
is the median?

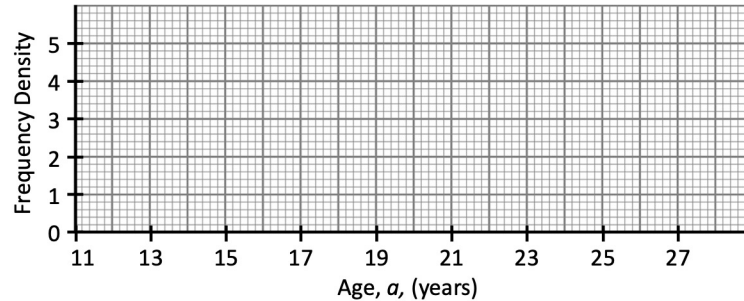
Mark the median on the histogram.
What is its value?

Fluency Practice

Histograms

1) Shoppers were surveyed about their age. Complete the histogram.

Age a , (years)	Frequency	Cumulative Frequency	Class Width	Frequency Density
$11 < a \leq 16$	5			
$16 < a \leq 24$	20			
$24 < a \leq 27$	15			



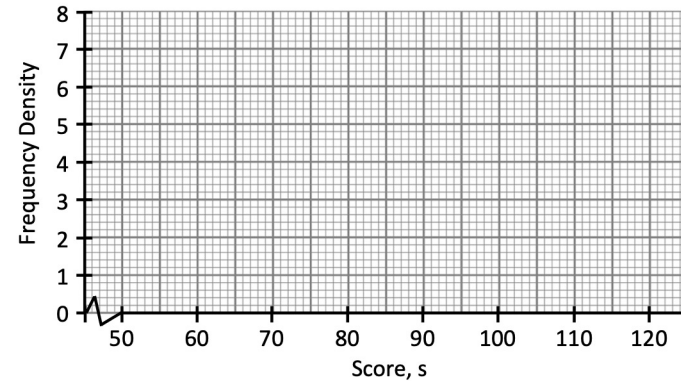
How many people is one square worth?

Complete the Cumulative Frequency column.

- In which class is the median value (20th value)?
How far into the class is the median value?
Mark the median value on the histogram. What is the median age?
Why is this an estimate?
- In which class is the lower quartile (10th)?
How far into the class is the lower quartile?
Use the histogram to estimate the lower quartile.
- In which class is the upper quartile?
How far into the class is the upper quartile?
Use the histogram to estimate the upper quartile.
- Estimate the inter-quartile range.

2) 200 students were given an English test.
Complete the histogram.

Score, s	Frequency	Cumulative Frequency	Class Width	Frequency Density
$50 < s \leq 60$	40			
$60 < s \leq 80$	20			
$80 < s \leq 90$	80			
$90 < s \leq 120$	60			



- What is the modal group?
 - Calculate an estimate for the median.
 - Calculate the lower and upper quartiles.
 - What is the interquartile range?
 - Estimate how many students got less than 55 marks
 - Estimate how many students got more than 115 marks.
- 3) 20% of the students got an 'outstanding' score.
- How many students got an outstanding score?
 - What score was the boundary for 'outstanding'?

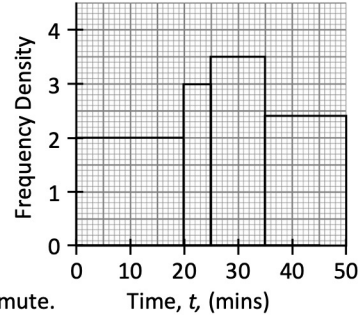
Fluency Practice

Histograms: Reading

1) The histogram shows the time taken by workers to commute.

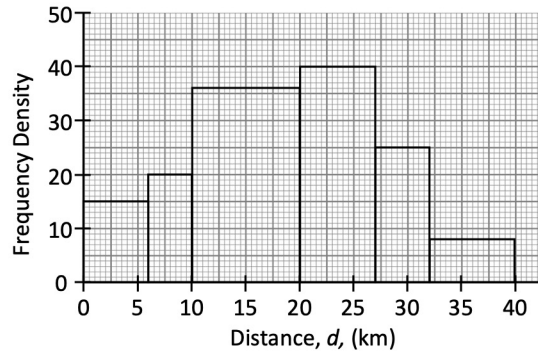
a) Complete the frequency table.

Time, t , (mins)	Freq.
$0 < t \leq 20$	
$20 < t \leq 25$	
$25 < t \leq 35$	
$35 < t \leq 50$	



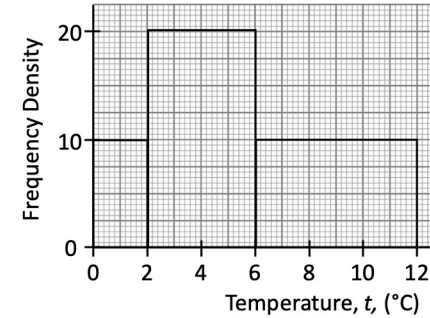
b) Calculate an estimate for the mean time taken to commute.

2) The histogram shows the distance workers commute.



- How many people commute ...less than 10 km?
- ...more than 20 km?
- ...between 10 km and 15 km?
- ...less than 2 km?
- ...more than 22 km?
- ...more than 7 km?

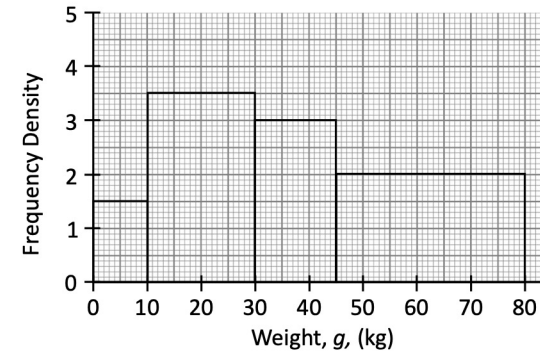
3) The histogram shows temperatures on different winter days.



Use the histogram to estimate...

- ...the median.
- ...the lower quartile.
- ...the upper quartile.
- What is the inter-quartile range?

4) The histogram shows the weight of different packages.



- Estimate the median.
- Estimate the inter-quartile range.
- Estimate the mean.

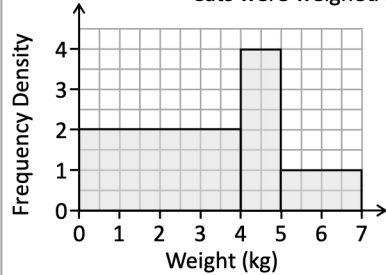
Fluency Practice



Histogram Populations Find the total size of each population.

①

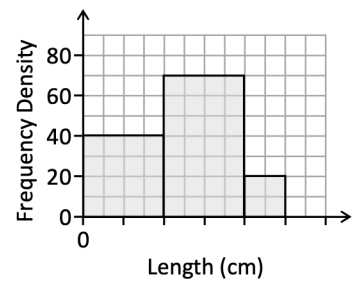
Cats were weighed at a rescue centre.



Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 4$		4	2
$4 < w \leq 5$			
$5 < w \leq 7$			

④

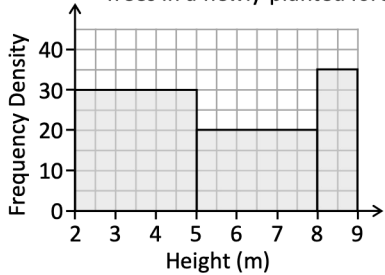
A sample of the snail population on St. Hilda Island was measured.



Length l (cm)	Freq.	CW	FD
$0 < l \leq 5$	80		

②

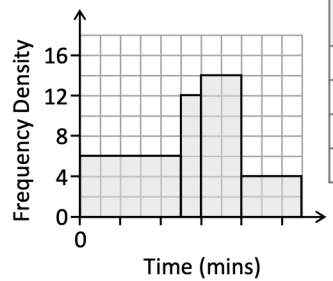
Trees in a newly-planted forest were measured.



Height h (m)	Freq.	Class Width	Frequency Density
$2 < h \leq 5$			
$5 < h \leq 8$			
$8 < h \leq 9$			

⑤

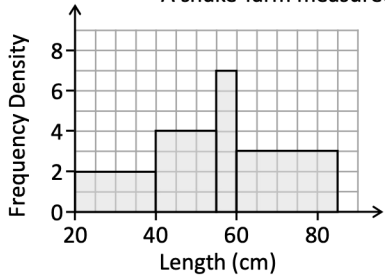
Participants in a global experiment were timed completing a complex puzzle.



Time t (mins)	Freq.	CW	FD
	300		

③

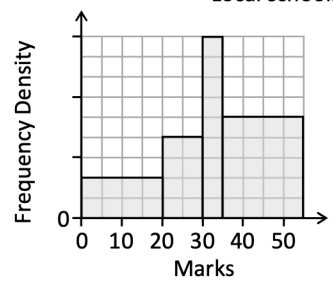
A snake-farm measured its serpents.



Length l (m)	Freq.	CW	FD
$20 < l \leq 40$			
$40 < l \leq 55$			

⑥

Local schools compiled English test scores.



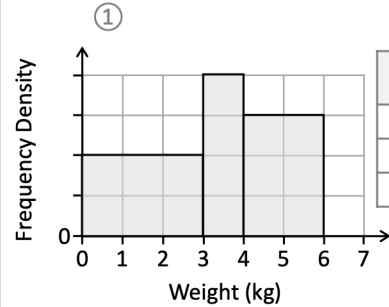
Marks, m	Freq.	CW	FD
	120		

Fluency Practice

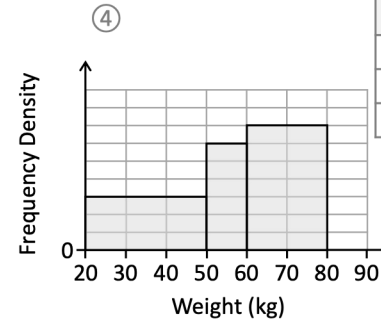


Histogram Populations: Missing Frequency Density

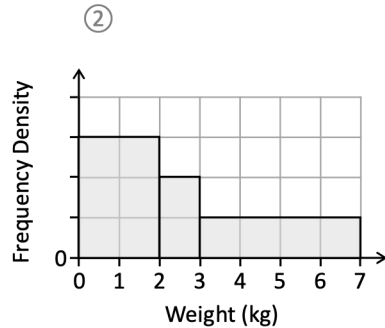
Use frequency tables to find the **total population** from each graph.



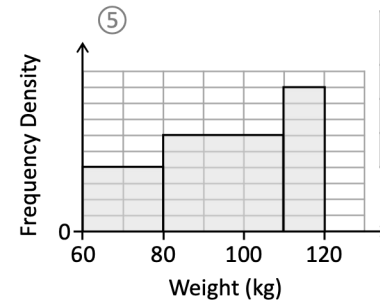
Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 3$	6		
$3 < w \leq 4$			
$4 < w \leq 6$			



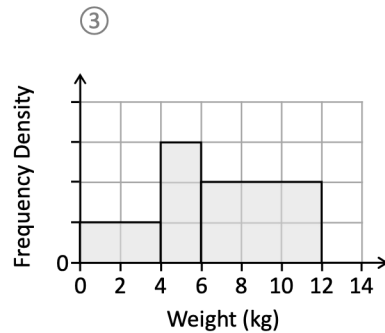
Weight w (kg)	F	CW	FD
$20 < w \leq 50$			
$50 < w \leq 60$	30		
$60 < w \leq 80$			



Weight w (kg)	Freq.	CW	FD
$0 < w \leq 2$	12		
$2 < w \leq 3$			
$3 < w \leq 7$			



Weight w (kg)	F		
$60 < w \leq 80$			
$80 < w \leq 110$	36		
$110 < w \leq 120$			



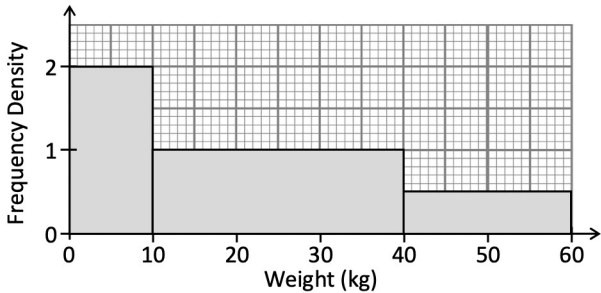
Weight w (kg)	Freq.	CW	FD
$0 < w \leq 4$			
$4 < w \leq 6$	30		
$6 < w \leq 12$			



Fluency Practice

A scientist records catch-weights on a fishing boat.
Estimate the median.

Weight w (kg)	Frequency	Class Width	Frequency Density
$0 < w \leq 10$			
$10 < w \leq 40$			
$40 < w \leq 60$			

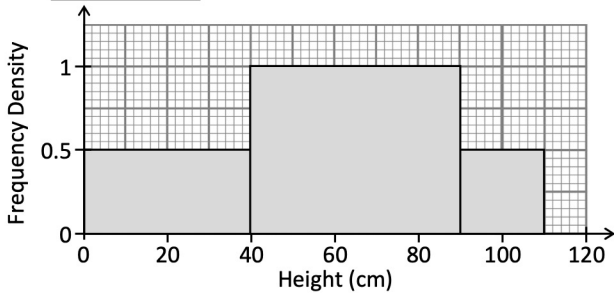


Histograms: Quartiles



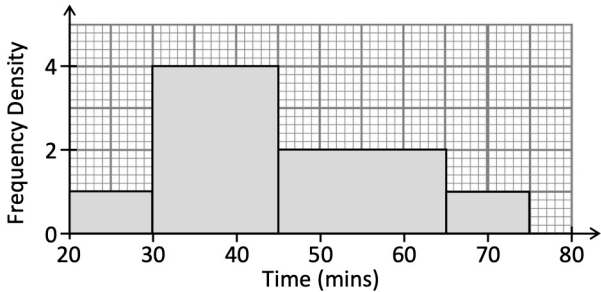
A scientist samples corn heights from a field.
Estimate the median.

Height, h (cm)	Frequency	CW	FD
$0 < h \leq 40$			
$40 < h \leq 90$			
$90 < h \leq 110$			

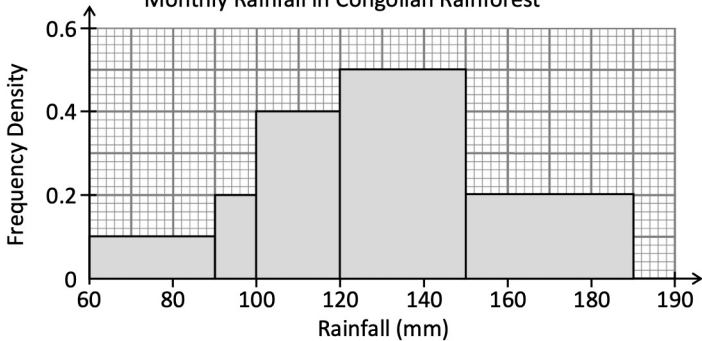


A fast-food delivery company checks delivery times.
Estimate the interquartile range.

Time, t (mins)	Frequency		



Monthly Rainfall in Congolian Rainforest



Estimate the interquartile range.

Fluency Practice

Overspeed



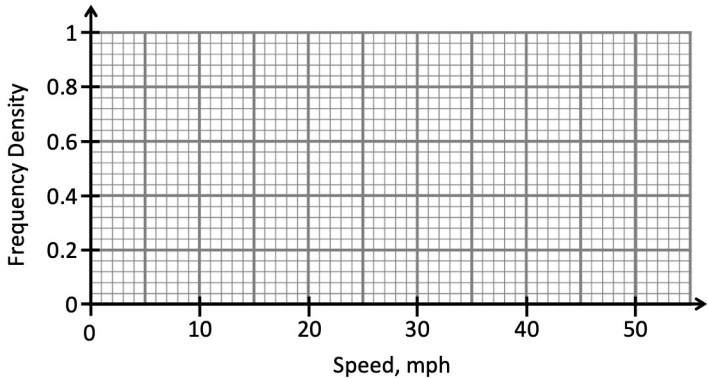
A local council samples car speeds within a 30 mph zone.



27	20	10	26	16	15	3
36	11	28	18	12	8	14
21	7	26	31	42	24	48
22	6	29	30	9	35	

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Speed, s (mph)	Tally	Frequency	Class Width	Frequency Density
Underspeed	$0 < s \leq 10$				
Expected	$10 < s \leq 30$				
Acceptable	$30 < s \leq 35$				
Speeding	$35 < s \leq 50$				



The council may impose a 20 mph speed limit. Use the graph to **estimate** how many cars were travelling above this speed.

How can we calculate an estimate for the mean?

Overspeed



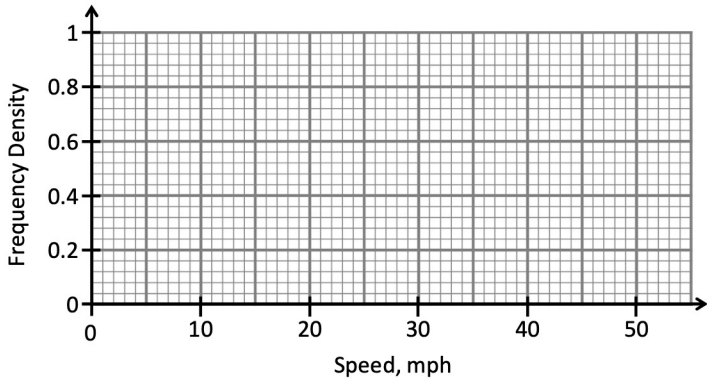
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Expected	$10 < s \leq 30$				
Acceptable	$30 < s \leq 35$				
Speeding	$35 < s \leq 50$				



The council may impose a 20 mph speed limit. Use the graph to **estimate** how many cars were travelling above this speed.

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Fluency Practice

Sugar Rush



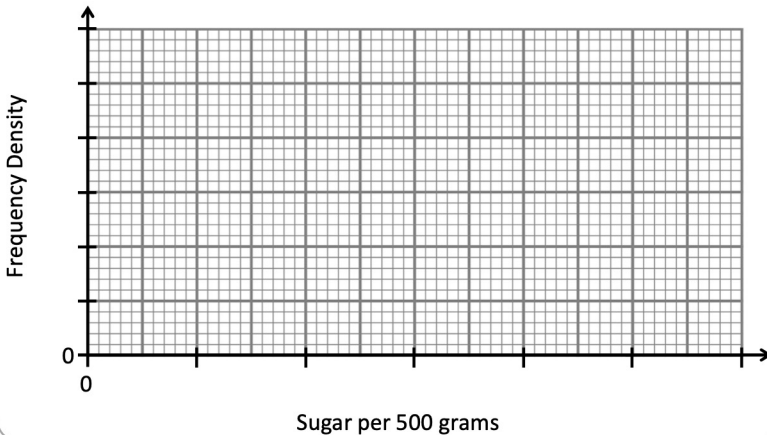
Food scientists measure the grams of sugar in 500 grams of different snacks.



31	81	26	14	42	37	93	11
87	28	96	23	80	12	6	95
33	10	18	100	35	38		

Tally & summarise the data in the frequency table.
Use the information to create a histogram.

	Sugar, s (g)	Tally			
Negligible	$0 < s \leq 10$				
Acceptable	$10 < s \leq 40$				
Unhealthy	$40 < s \leq 80$				
Unacceptable	$80 < s \leq 100$				



According to health experts, snacks should have less than 5% sugar.
Use the graph to estimate the percentage of snacks above this limit.

Mark an estimate for the mean sugar level on the histogram.

Sugar Rush



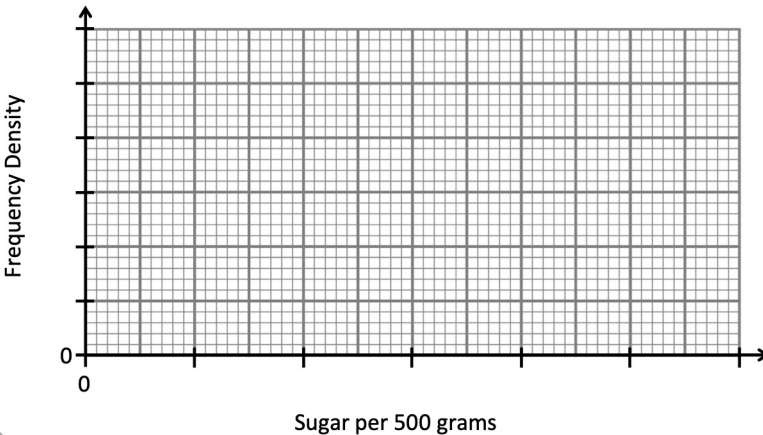
Food scientists measure the grams of sugar in 500 grams of different snacks.



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87	28	96	23	80	12	6	95
33	10	18	100	35	38		

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Use the information to create a histogram.

	Sugar, s (g)	Tally			
Negligible	$0 < s \leq 10$				
Acceptable	$10 < s \leq 40$				
Unhealthy	$40 < s \leq 80$				
Unacceptable	$80 < s \leq 100$				



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Mark an estimate for the mean sugar level on the histogram.

Fluency Practice

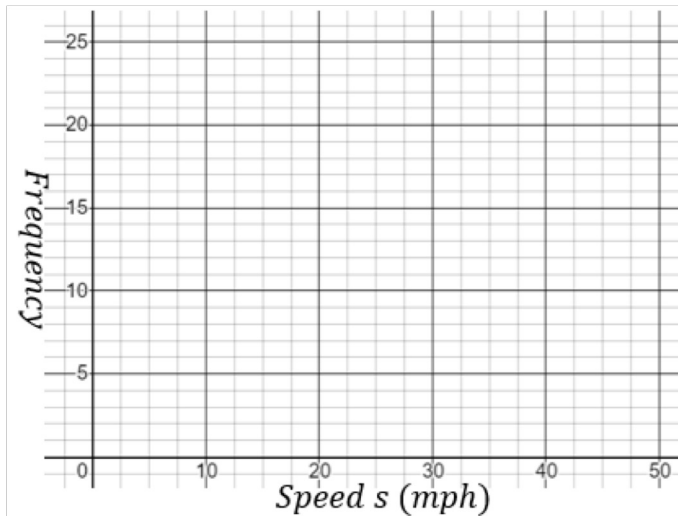
Plotting Frequency Polygons

(a)

The table shows the speeds of 60 motorbikes on a road.

Speed s (mph)	Frequency
$0 < s \leq 10$	3
$10 < s \leq 20$	11
$20 < s \leq 30$	24
$30 < s \leq 40$	15
$40 < s \leq 50$	7

Draw a frequency polygon to represent this information.

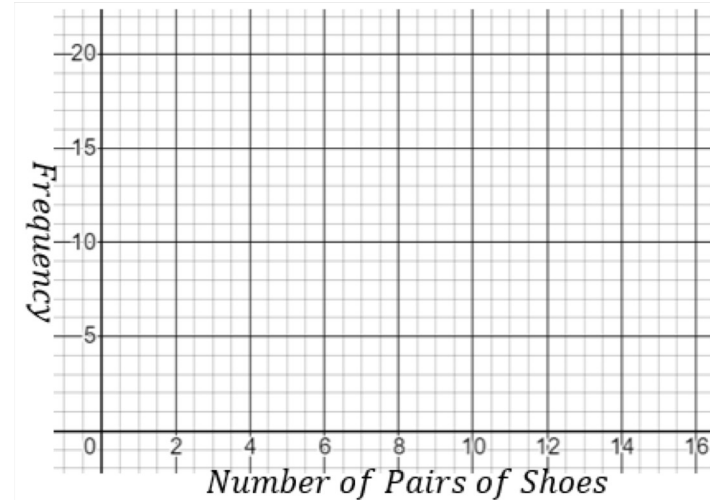


(b)

The table shows the number of pairs of shoes owned by 50 students.

Number of Pairs of Shoes	Frequency
1 – 3	6
4 – 6	16
7 – 9	18
10 – 12	7
13 – 15	3

Draw a frequency polygon to represent this information.

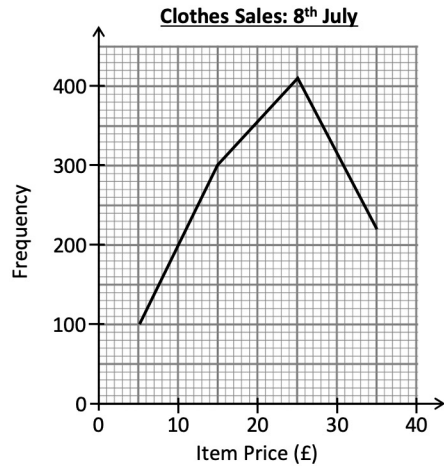


Purposeful Practice

①

Calculating Averages From Graphs

New Trend Clothing Inc. record sales in their stores over one day.



Use the frequency polygon to complete the table & calculate the **mean** item price.

Item Price, p (£)	Frequency	Midpoint	$f \times m$
$0 < p \leq 10$			
$10 < p \leq 20$			
$20 < p \leq 30$			
$30 < p \leq 40$			
Totals			

In which group is the median?

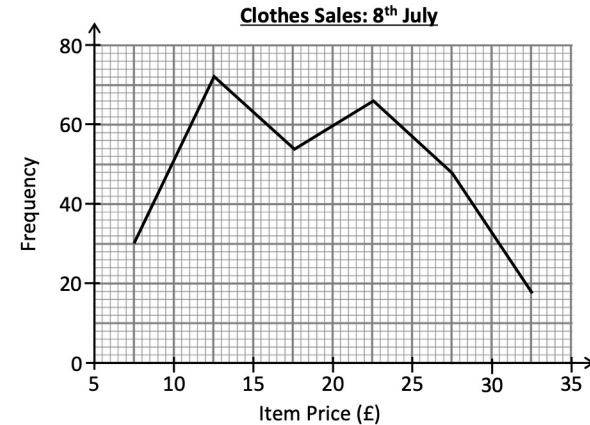
Which is the modal group?



②

Calculating Averages From Graphs

The New Trend store in Forest Valley Mall records its sales on one day.



State the **modal** group, the **mean** item price, and the group that contains the **median** item price.



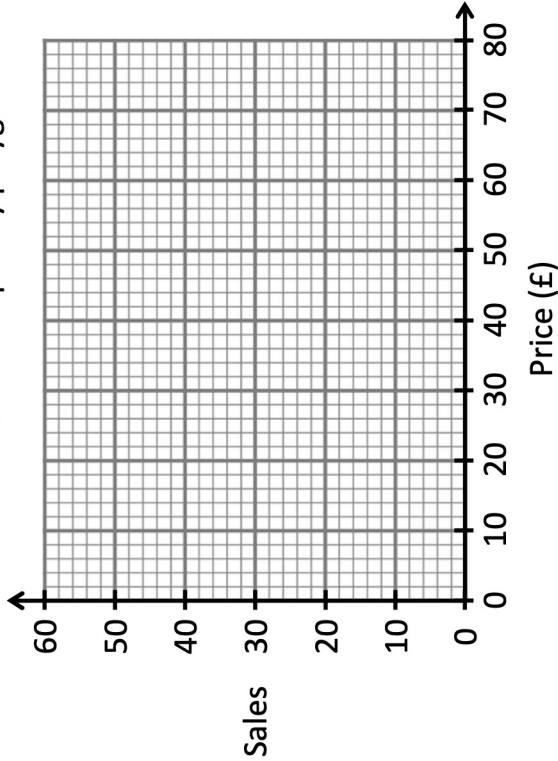
Shop Sales

A clothes shop has 4 price groups: Bargain, Value, Everyday & Luxury.
Which of the groups earnt the shop the most amount of money on Monday?
(multiply sales by the mid-group value to find the total group value)

Monday

	Price p , (£)	Sales	Mid-Group Value (£)	Total Group Value
Bargain	$0 < p \leq 20$	46		
Value	$20 < p \leq 40$	28		
Everyday	$40 < p \leq 60$	32		
Luxury	$60 < p \leq 80$	14		
Totals				

Calculate the total income & the mean price of the items sold.
Plot the data as a frequency polygon.



Plot the data for Friday on the same grid. Calculate a mean price.

Friday

	Price p , (£)	Sales	
Bargain	$0 < p \leq 20$	26	
Value	$20 < p \leq 40$	36	
Everyday	$40 < p \leq 60$	40	
Luxury	$60 < p \leq 80$	56	

What comments can you make comparing the sales on Monday & Friday?

Snack Stores

3 food shops in a shopping centre record their sales over a Saturday.
Complete the tables to find estimates for their total profits.

Pete’s Pizza ‘n’ Slices

Profit p , (£)	Sales
$0 < p \leq 2$	30
$2 < p \leq 4$	34
$4 < p \leq 6$	28
$6 < p \leq 8$	16

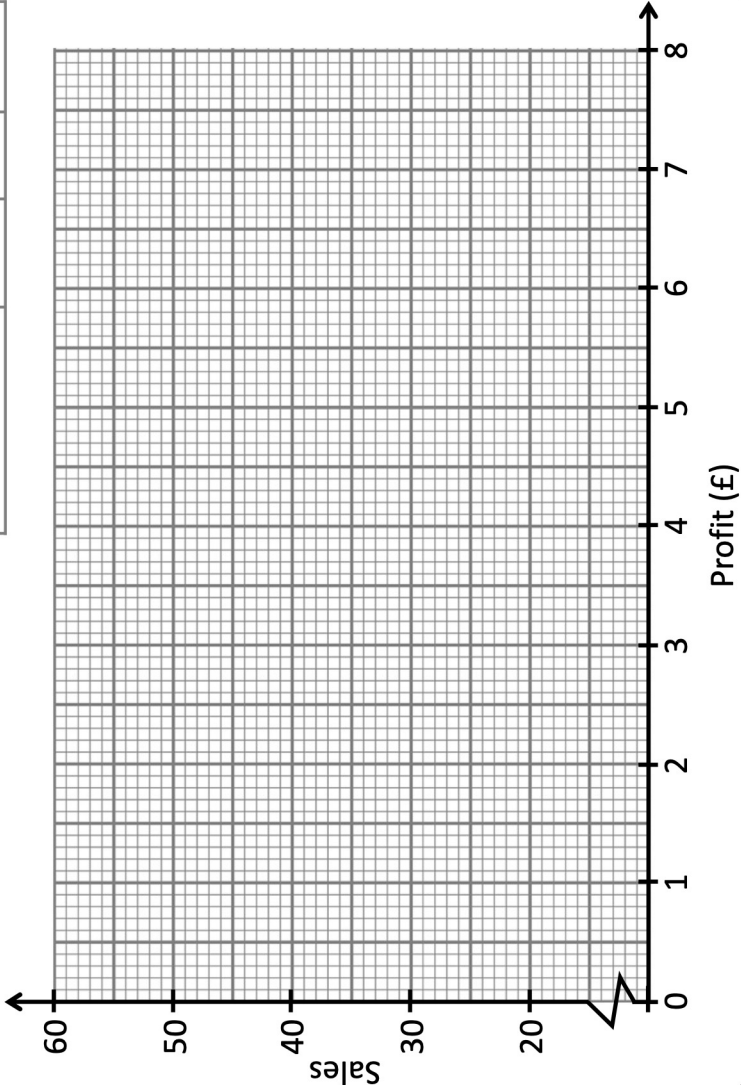
Plot the information as three frequency polygons on the grid.
What comments can you make about the data distributions?
Which shop was most profitable?
Calculate the mean profit for each shop.

Lucy’s Luxury Cakes

Profit p , (£)	Sales
$2 < p \leq 4$	21
$4 < p \leq 6$	24
$6 < p \leq 8$	34

Derek’s Doughnuts

Profit p , (£)	Sales
$0 < p \leq 1$	54
$1 < p \leq 2$	56
$2 < p \leq 3$	51
$3 < p \leq 4$	39



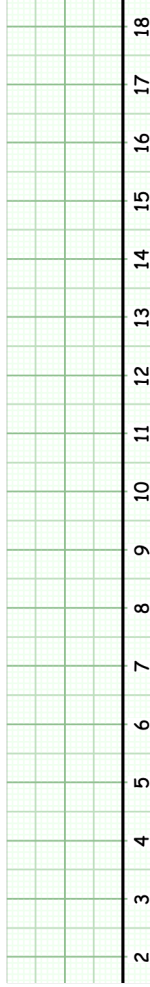
Fluency Practice

Drawing Box Plots

For each set of data draw a box plot on the grid provided.

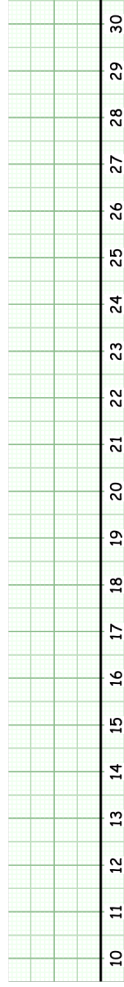
- | | | | | | | | | | | | | |
|--|----|---|---|---|---|---|---|---|---|----|----|----|
| | 1. | 2 | 3 | 4 | 6 | 7 | 8 | 8 | 9 | 13 | 15 | 18 |
|--|----|---|---|---|---|---|---|---|---|----|----|----|

Min =	L.Q. =	Median =	U.Q. =	Max =



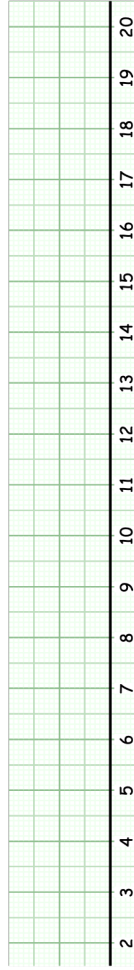
- | | | | | | | | | | | | |
|----|----|----|----|----|----|----|----|----|----|----|----|
| 2. | 11 | 14 | 17 | 18 | 18 | 21 | 22 | 23 | 24 | 27 | 28 |
|----|----|----|----|----|----|----|----|----|----|----|----|

Min =	L.Q. =	Median =	U.Q. =	Max =
-------	--------	----------	--------	-------



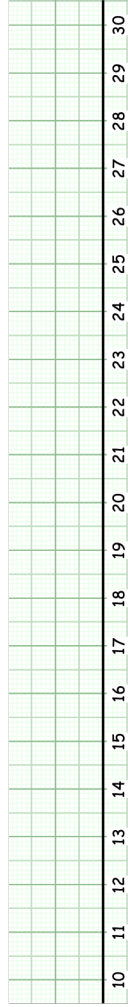
- | | | | | | | | | | |
|----|---|---|---|----|----|----|----|----|----|
| | 3 | 4 | 6 | 10 | 13 | 14 | 16 | 18 | 19 |
| 3. | | | | | | | | | |

Min =		L.Q. =		Median =		U.Q. =		Max =	
-------	--	--------	--	----------	--	--------	--	-------	--



- [illegible]

Min =	L.Q. =	Median =	U.Q. =	Max =



Fluency Practice

5.

5353535454595962626363636464686870

Min =

L.Q. =

Median =

U.Q. =

Max =

50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70

6.

333559111313141416161717191920

Min =

L.Q. =

Median =

U.Q. =

Max =

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

7.

222558111313141415151616191920

Min =

L.Q. =

Median =

U.Q. =

Max =

2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20

8.

404040424245454646474748484949505051515252535354545555565657575858

Min =

L.Q. =

Median =

U.Q. =

Max =

40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60

Fluency Practice

Drawing box plots

1.

Lightest weight	3 g
Lower quartile	5 g
Median	11 g
Upper quartile	18 g
Heaviest weight	23 g

Show this data on a box plot.

2.

Minimum weight	5 g
Lower quartile	12 g
Median	15 g
Upper quartile	26 g
Maximum weight	32 g

Show this data on a box plot.

3.

Minimum length	15.4 cm
Lower quartile	16.8 cm
Median	17.6 cm
Upper quartile	18.2 cm
Maximum length	19.8 cm

Show this data on a box plot.

4.

Minimum weight	42 kg
Lower quartile	48 kg
Median	54 kg
Upper quartile	65 kg
Maximum weight	71 kg

Show this data on a box plot.

5.

Minimum	3
Lower quartile	8
Median	15
Interquartile range	12
Range	19
Upper quartile	
Maximum	

Show this data on a box plot.

6.

Minimum weight	5 g
Lower quartile	22 g
Median	34 g
Interquartile range	18 g
Maximum weight	48 g
Upper Quartile	

Show this data on a box plot.

7.

Minimum	4
Lower quartile	10
Median	22
Interquartile range	16
Range	29
Upper quartile	
Maximum	

Show this data on a box plot.

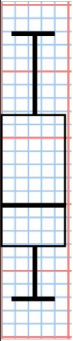
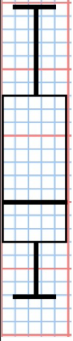
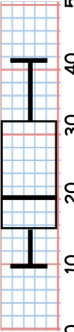
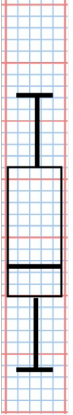
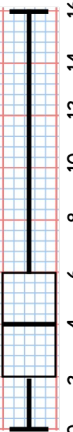
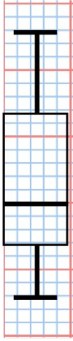
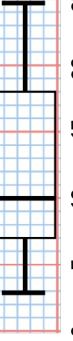
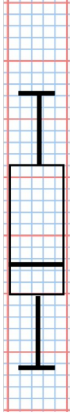
8.

Maximum	58
Upper quartile	50
Median	38
Interquartile range	26
Range	52
Lower quartile	
Minimum	

Show this data on a box plot.

Fluency Practice

Interpreting Box plots

1.		2.	
What is the range?	What is the inter-quartile range?	What is the range?	What is the inter-quartile range?
3.		4.	
What is the range?	What is the inter-quartile range?	What is the range?	What is the inter-quartile range?
5.		6.	
What is the range?	What is the inter-quartile range?	What is the range?	What is the inter-quartile range?
7.	The box plot displays the heights of 60 plants. How many plants were smaller than 12 cm?	8.	The box plot displays the heights of 80 plants. How many plants were shorter than 7cm?
How many plants were taller than 12.7 cm?		How many plants were shorter than 18cm?	
9.		10.	
The box plot displays the test scores of 120 pupils. How many pupils scored less than 16?	The box plot displays the time spent on homework by 32 pupils. How many pupils spent over 40 minutes on their homework?	How many pupils scored more than 42?	How many pupils spent less than 18 minutes on their homework?
11.		12.	
The box plot shows the time spent revising by pupils in a class. If 21 pupils spent more than 2 hours revising, how many pupils were in the class?	The box plot shows the number of behaviour points for each of the pupils in a form. If 18 pupils had less than 6 behaviour points, how many pupils are in the form?		

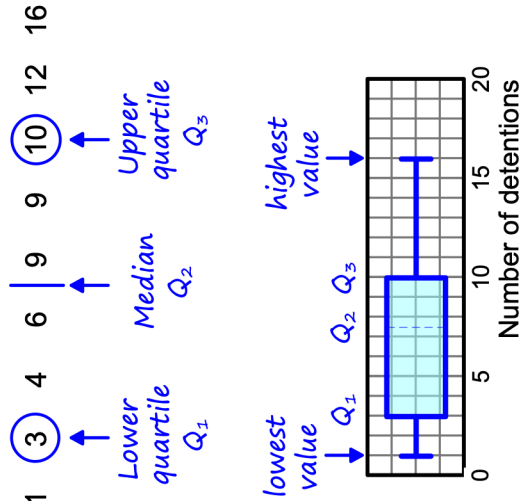
example

The **quartiles** of a data set are the values one quarter and three quarters of the way through the data.

The **interquartile range (IQR)** is the difference between the upper and lower quartiles.

$$\text{IQR} = Q_3 - Q_1$$

The quartiles can be used to draw a **boxplot** for the data. This shows how the data is spread across the range of values.



exercise

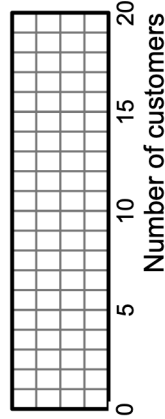
1. For each data set, work out Q_1 , Q_2 , Q_3 and the interquartile range.

- a) 7 9 11 14 16 16 18
- b) 2 2 4 5 7 8 8 9 10 10 11 11 12
- c) 0.4 0.7 1.0 1.5 1.7 1.7 2.2 2.6
- d) 29 31 34 34 35 39 47 51 52 58 65 82

2. Work out the median and quartiles and construct a box plot:

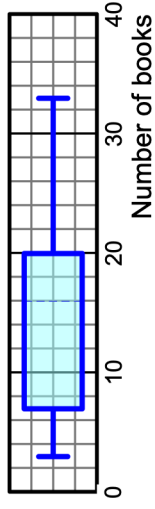
- a) Number of customers each hour
- b) Maximum day temperature ($^{\circ}\text{C}$)

2	3	5	7	7	9	12	12	14	16	19	20	21
9	9	9	13	14	19	23	24	28	28	29	31	31



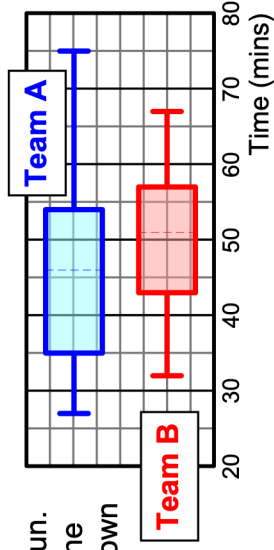
Purposeful Practice

3. A group of people were asked how many books they read in a year. The boxplot shows the results.



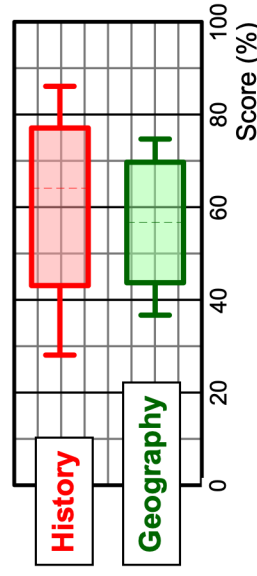
- State the median.
- Work out the range.
- Work out the interquartile range.
- What percentage of the people had read more than 20 books?
- What percentage of the people had read more than 7 books?

4. Two teams took part in a charity run. The times taken for members of the teams to complete the run are shown in the box plots.



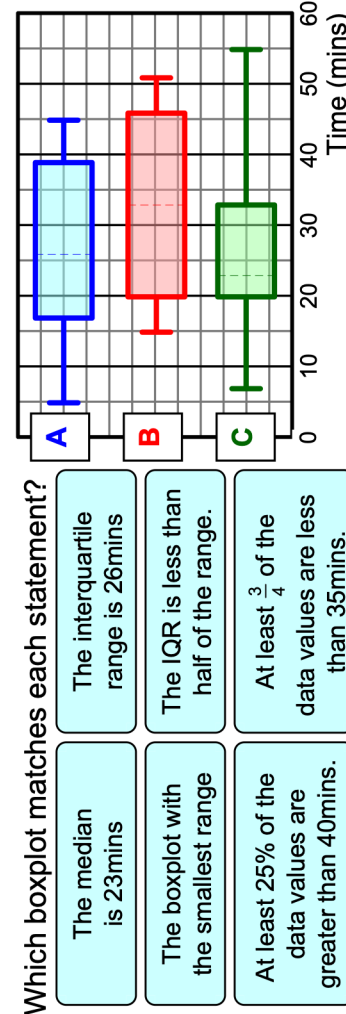
- On average, which team's members were quicker? Justify your answer.
- Which team had a smaller overall spread of times? Justify your answer.
- Which team had a smaller spread of the middle 50% of times.

5. The results of a class of pupils in exams for two subjects are shown in the box plots.



Compare the distributions of exam results in the two subjects.

6. Which boxplot matches each statement?



The median is 23mins

The interquartile range is 26mins

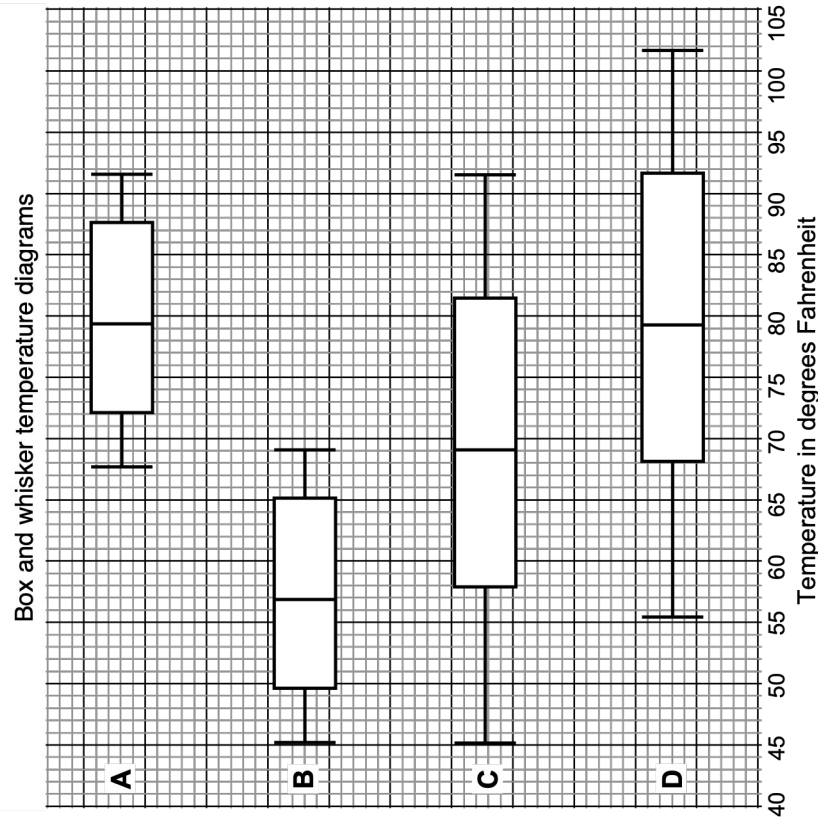
The boxplot with the smallest range

The IQR is less than half of the range.

At least 25% of the data values are greater than 40mins.

At least $\frac{3}{4}$ of the data values are less than 35mins.

Purposeful Practice



2. Which of the four box diagrams shows the Washington temperatures?
Explain how you decided.

3. Which of the four box diagrams shows the California temperatures?
For which months of the year is the maximum monthly temperature for California between the upper and the lower quartiles?

Explain how you figured it out.

Problem Solving

Sketch it!

Class 9Y2 are taking a maths test. What would the **box plot of their results** look like if...

a) ...the test was far too easy.



b) ...there was a high median & a small interquartile range.



c) ...the distribution was symmetrical.



d) ...the class did well, except George who was late!



e) ...the results were very consistent.



f) ...the results were perfectly distributed.



g) ...the results were low scoring & inconsistent.



h) ...50% of the students got 100%.



i) ...the test was too difficult for most, but Mary got 100%!



j) ...the interquartile range was 20% of the range.



k) ...the lowest result was shared by 25% of the students.



l) ...only the top 50% of students had consistent results.



What interesting results & distributions can you sketch?

Problem Solving

Results Day

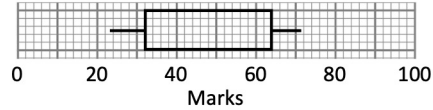
Match each classes Science Test Results with a box plot & complete both. Every class has 32 students.

10X1

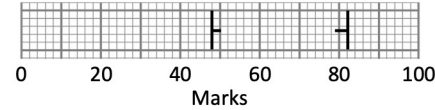
Lowest Score	28
Greatest Score	
Lower Quartile	50
Upper Quartile	
Median	60

a

50% of the class scored more than 52.



75% of the class scored more than 58.



10Y1

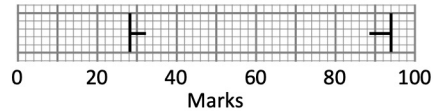
Lowest Score	10
Greatest Score	78
Lower Quartile	32
Upper Quartile	
Median	

10X2

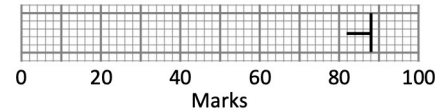
Lowest Score	14
Greatest Score	98
Lower Quartile	
Upper Quartile	76
Median	

b

25% of the class scored more than 80.



The interquartile range was 22.



10Y2

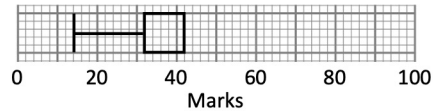
Range	72
Greatest Score	74
Lower Quartile	
Upper Quartile	
Median	54

10X3

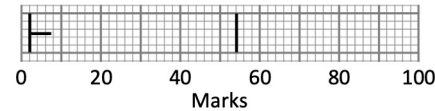
Lowest Score	48
Range	
Lower Quartile	
Upper Quartile	72
Median	68

c

50% of the class scored less than 36.



The middle 50% of students scored between 38 and 68 marks.



10Y3

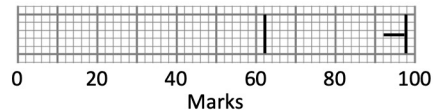
Lowest Score	14
Range	48
Lower Quartile	
Upper Quartile	
Median	

10X4

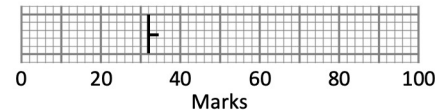
Lowest Score	
Range	64
Lower Quartile	
Upper Quartile	66
Median	58

d

8 students scored less than 28.



24 students scored more than 34.



10Y4

Lowest Score	18
Greatest Score	88
Lower Quartile	
Upper Quartile	48
Median	34

What comments can you make about the results? (Median, Maximums, Minimums, Consistency, Range)

Problem Solving



Comparing Classes 1

2 classes took a science test. We want to compare the scores for the boys in each class (9Y4 & 9X4).
The scores are in the table.



Place the scores in ascending order to identify each quartile (Lower, Median, Upper).
Plot this information as a box plot.

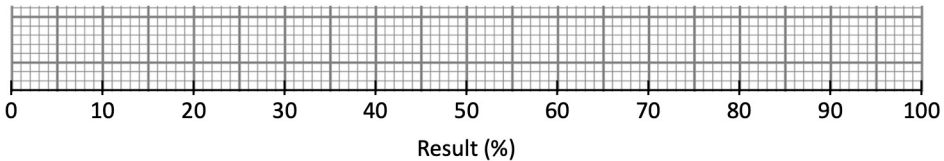
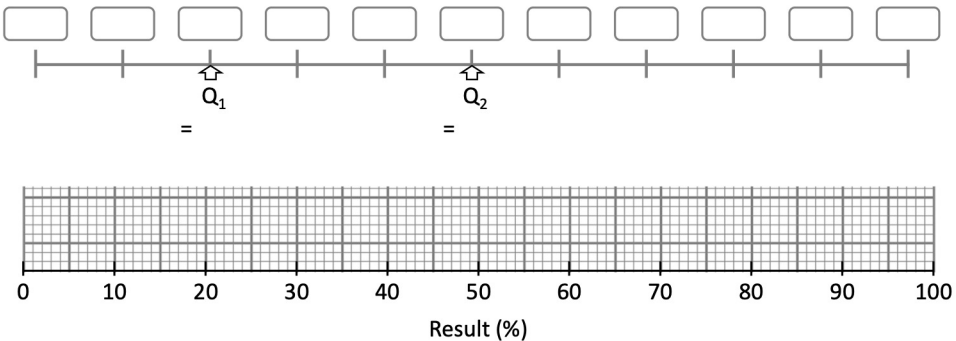
What does the shape of each box plot show?
What comparisons can we make between the classes?

**Class 9Y4
(boys)**

28	44	40
52	56	60
62	26	45
40	54	

**Class 9X4
(boys)**

31	43	58
54	58	31
91	18	54
42	56	40
70	32	90



Problem Solving



Comparing Classes 2

2 classes took a science test. We want to compare the scores for the girls in each class (9Y4 & 9X4).
The scores are in the table.



Place the scores in ascending order to identify each quartile (Lower, Median, Upper).
Plot this information as a box plot.

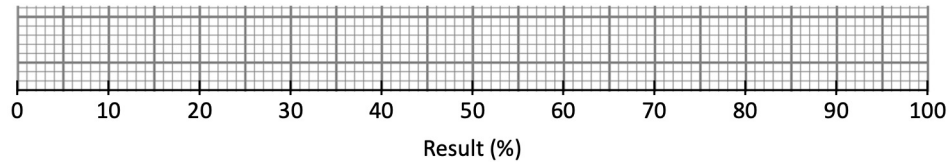
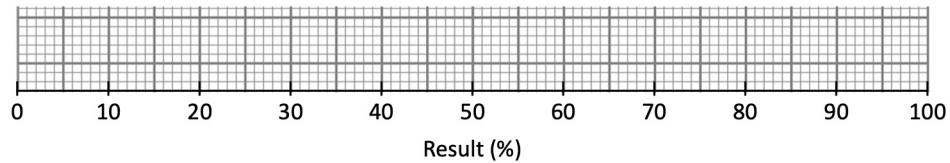
What does the shape of each box plot show?
What comparisons can we make between the classes?

**Class 9Y4
(girls)**

80	72	36
35	15	78
42	79	12
74	71	74
14		

**Class 9X4
(girls)**

42	58	42
52	50	56
46	42	98
62	51	46

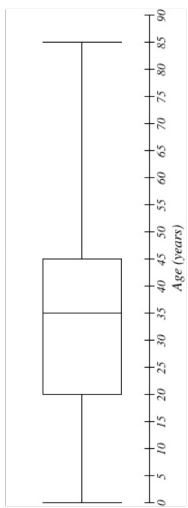
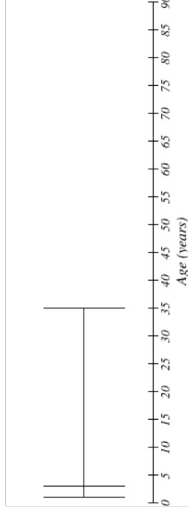
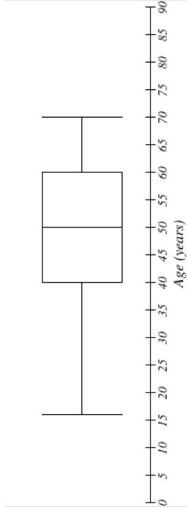
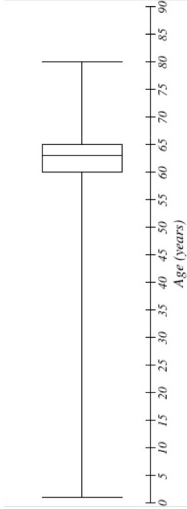
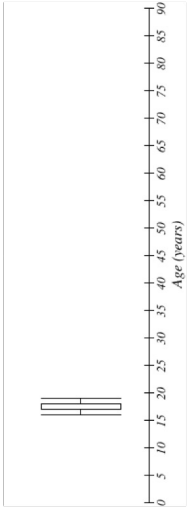
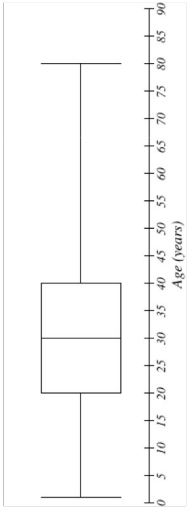


Problem Solving

6 parties were held in the Village Hall last month. They were:

- | | | |
|--|-------------------------|----------------------------------|
| An 18 th birthday party | A retirement party | A wedding reception |
| A 20 th wedding anniversary party | A christening reception | A 3 rd birthday party |

These box plots show the ages of the party guests. Underneath each box plot, write down which party it represents and explain how you know.



Problem Solving

A company has 72 employees, all of whom are at least 20 years of age.

The number of employees aged from 20 to 30 is equal to the number of employees aged over 50.

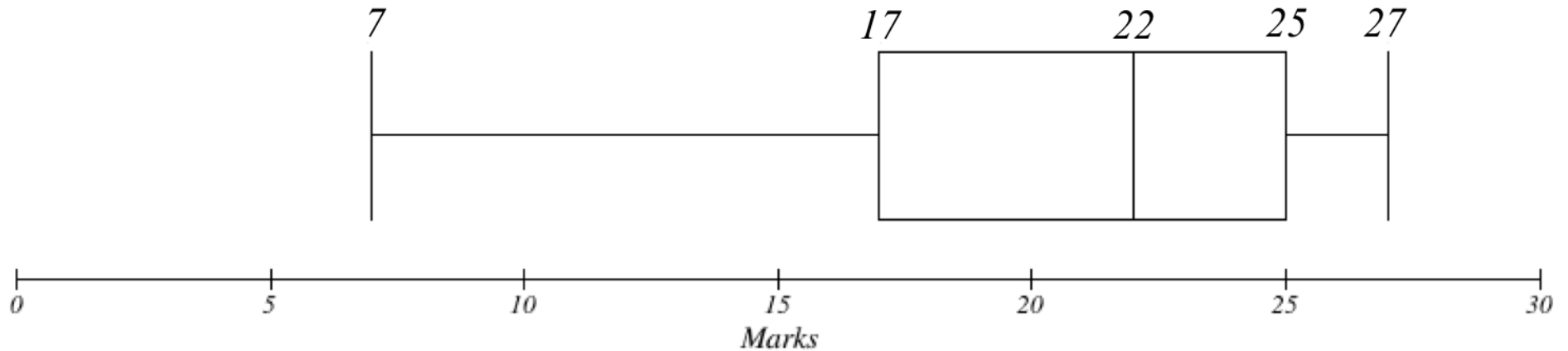
The number of employees aged over 30 but no more than 40 is twice the number of employees aged over 50.

One third of the company's employees are over 40.

The company's oldest employee is exactly 60.

Construct a cumulative frequency graph for the ages of the employees.

Problem Solving



The boxplot above shows the distribution of marks scored in a test by pupils in a Geography class. There are 31 students in the class.

Find upper and lower bounds for the **mean** score achieved by the 31 students.

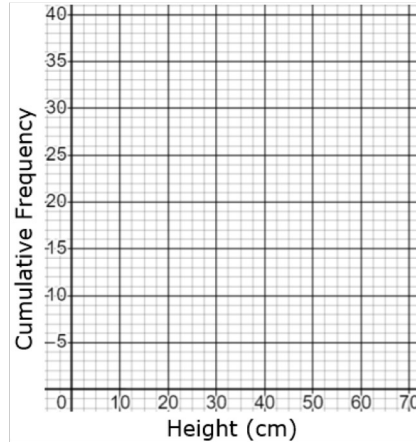
Fluency Practice

Plotting Cumulative Frequency Graphs

(a)

Plot a cumulative frequency graph from the data shown in the table.

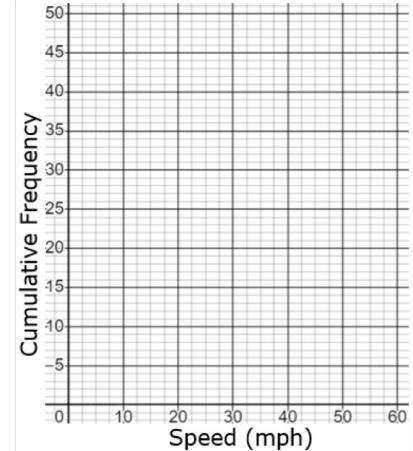
Height (cm)	Cumulative Frequency
$10 < h \leq 20$	3
$10 < h \leq 30$	8
$10 < h \leq 40$	19
$10 < h \leq 50$	29
$10 < h \leq 60$	35
$10 < h \leq 70$	40



(b)

Plot a cumulative frequency graph from the data shown in the table.

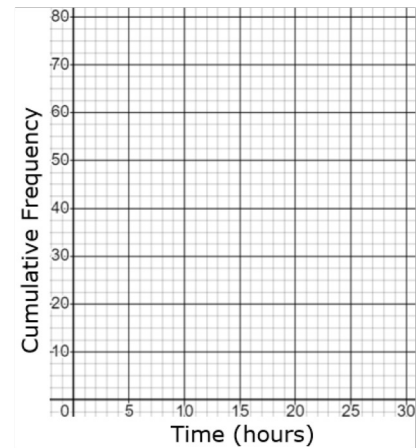
Speed (mph)	Cumulative Frequency
$0 < s \leq 10$	6
$0 < s \leq 20$	10
$0 < s \leq 30$	22
$0 < s \leq 40$	35
$0 < s \leq 50$	44
$0 < s \leq 60$	50



(c)

Plot a cumulative frequency graph from the data shown in the table.

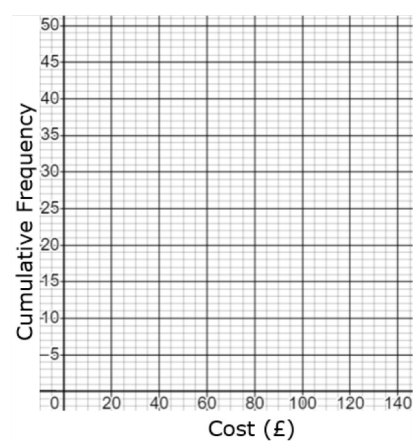
Time (hours)	Cumulative Frequency
$0 < h \leq 5$	9
$0 < h \leq 10$	21
$0 < h \leq 15$	38
$0 < h \leq 20$	60
$0 < h \leq 25$	73
$0 < h \leq 30$	80



(d)

Plot a cumulative frequency graph from the data shown in the table.

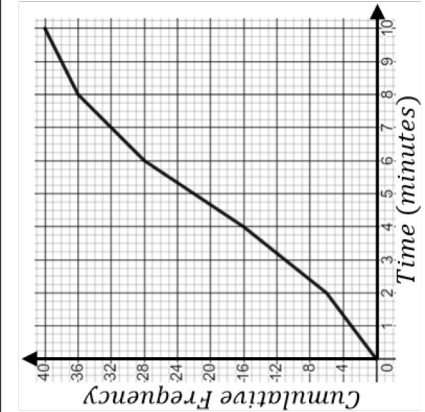
Cost (£)	Cumulative Frequency
$20 < c \leq 40$	4
$20 < c \leq 60$	11
$20 < c \leq 80$	19
$20 < c \leq 100$	36
$20 < c \leq 120$	45
$20 < c \leq 140$	48



Fluency Practice

Finding Frequencies from Cumulative Frequency Graphs

(a)



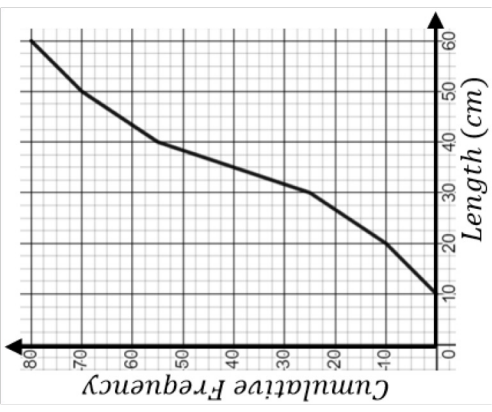
Complete the cumulative frequency table.

Time (minutes)	Cumulative Frequency
0 < t ≤ 2	6
2 < t ≤ 4	16
4 < t ≤ 6	
6 < t ≤ 8	
8 < t ≤ 10	40

Complete the grouped frequency table.

Time (minutes)	Frequency
0 < t ≤ 2	6
2 < t ≤ 4	10
4 < t ≤ 6	
6 < t ≤ 8	
8 < t ≤ 10	

(b)



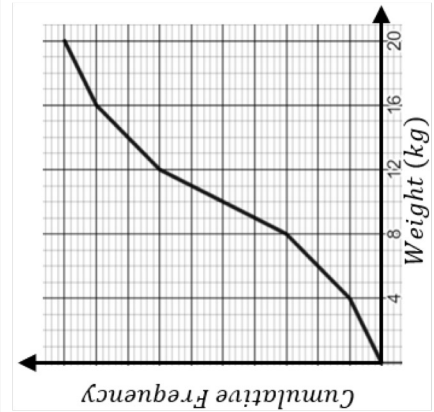
Complete the cumulative frequency table.

Length (cm)	Cumulative Frequency
0 < l ≤ 10	10
10 < l ≤ 20	20
20 < l ≤ 30	30
30 < l ≤ 40	50
40 < l ≤ 50	70
50 < l ≤ 60	80

Complete the grouped frequency table.

Length (cm)	Frequency
0 < l ≤ 10	
10 < l ≤ 20	
20 < l ≤ 30	
30 < l ≤ 40	
40 < l ≤ 50	
50 < l ≤ 60	

(c)



Complete the cumulative frequency table.

Weight (kg)	Cumulative Frequency
0 < w ≤ 4	4
4 < w ≤ 8	8
8 < w ≤ 12	12
12 < w ≤ 16	16
16 < w ≤ 20	20

Complete the grouped frequency table.

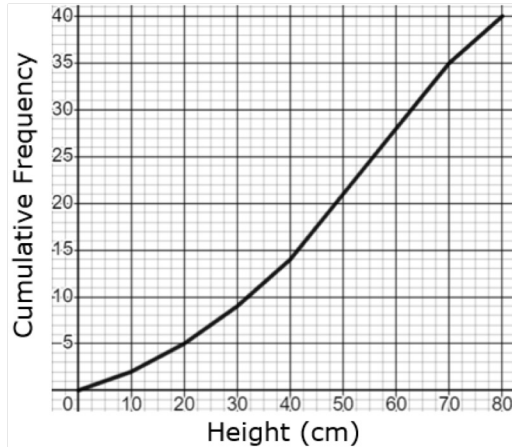
Weight (kg)	Frequency
0 < w ≤ 4	
4 < w ≤ 8	10
8 < w ≤ 12	
12 < w ≤ 16	
16 < w ≤ 20	

Fluency Practice

Reading Cumulative Frequency Graphs

(a)

The cumulative frequency graph shows the height of 40 plants in *cm*.



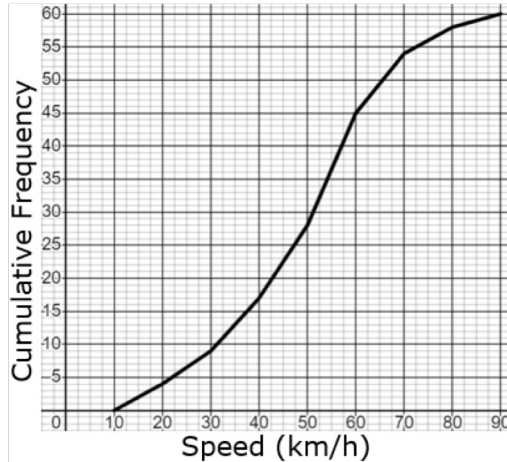
(a) Find the median height of the plants.

(b) Find the interquartile range of the heights of the plants.

(c) Find the number of plants whose height is less than 30 *cm*.

(b)

The cumulative frequency graph shows the speed of 60 cars in *km/h*.



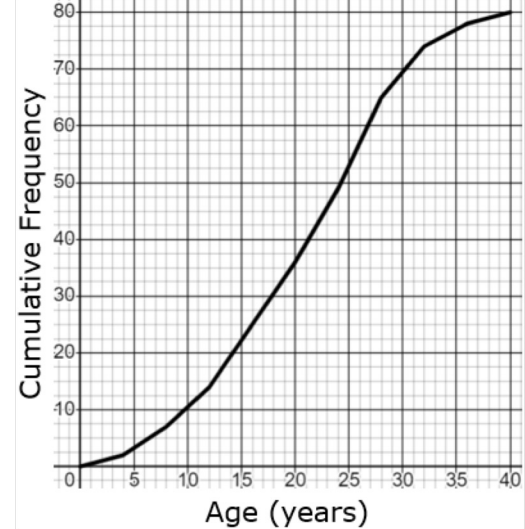
(a) Find the interquartile range of the speeds.

(b) Find the number of cars travelling less than 45 *km/h*.

(c) The speed limit is 70 *km/h*. What fraction of cars are breaking the speed limit?

(c)

The cumulative frequency graph shows the age of 80 apple trees in years.



(a) Find the number of trees that are at least 25 years old.

(b) Oscar says "30% of apple trees are less than 15 years old". Is Oscar correct? You must show working.

Fluency Practice

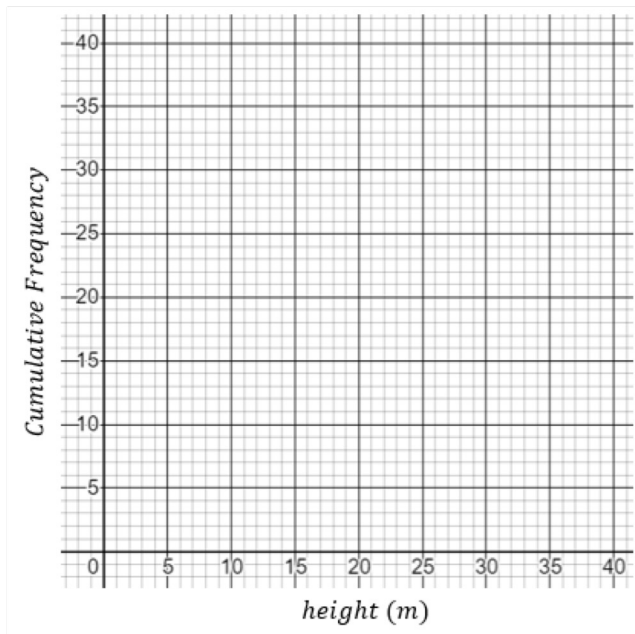
(a)

The heights of 40 oak trees are recorded in a frequency table.

(a) Use the information to calculate cumulative frequencies and plot a cumulative frequency graph.

(b) Use your cumulative frequency graph to find the median and interquartile range of the heights of the oak trees.

Height (m)	Frequency
$10 < h \leq 15$	5
$15 < h \leq 20$	9
$20 < h \leq 25$	13
$25 < h \leq 30$	7
$30 < h \leq 35$	4
$35 < h \leq 40$	2



(b)

The time taken by 100 students to complete a jigsaw is recorded in a grouped frequency table.

(a) Use the information to calculate cumulative frequencies and plot a cumulative frequency graph.

(c) Use your cumulative frequency graph to find the median and interquartile range of the times taken.

Time (minutes)	Frequency
$30 < t \leq 40$	4
$40 < t \leq 50$	11
$50 < t \leq 60$	20
$60 < t \leq 70$	28
$70 < t \leq 80$	19
$80 < t \leq 90$	13
$90 < t \leq 100$	5

(c)

The History test score of 60 students is recorded in a grouped frequency table.

(a) Plot a cumulative frequency graph.

(b) Find the median score in the History test.

(c) Use you graph to estimate the number of students who scored less than 36 marks.

(d) Use your graph to find the minimum score required to come in the top 10% of students in the test.

Test Score	Frequency
$0 < s \leq 10$	1
$10 < s \leq 20$	4
$20 < s \leq 30$	6
$30 < s \leq 40$	12
$40 < s \leq 50$	17
$50 < s \leq 60$	13
$60 < s \leq 70$	5
$70 < s \leq 80$	2

learn by heart

The **cumulative frequency** works like a 'running total'. It tells you the number of pieces of data up to and including a particular value.

example

The length of a number of television adverts is given in the frequency table.
Work out the cumulative frequencies.

Frequency Table

Time t (s)	Frequency
$0 < t \leq 10$	7
$10 < t \leq 20$	12
$20 < t \leq 30$	10
$30 < t \leq 40$	8
$40 < t \leq 50$	3

Cumulative Frequency Table

Time t (s)	Cumulative Frequency
$t \leq 10$	7
$t \leq 20$	19
$t \leq 30$	29
$t \leq 40$	37
$t \leq 50$	40

The second row tells you that 19 adverts lasted less than or equal to 20 seconds.

exercise

1. The widths of some birds are recorded and shown in the frequency table.

Width w (mm)	Frequency
$10 < w \leq 20$	2
$20 < w \leq 30$	6
$30 < w \leq 40$	14
$40 < w \leq 50$	18
$50 < w \leq 60$	7
$60 < w \leq 70$	4

Width w (mm)	Cumulative Frequency
$w \leq 20$	
$w \leq 30$	
$w \leq 40$	
$w \leq 50$	
$w \leq 60$	
$w \leq 70$	

- a) Complete the cumulative frequency table for the data.
- b) How many of the eggs had a width less than or equal to 40mm?
- c) How many eggs had widths up to and including 60mm?
- d) How many eggs were included in the data?

Fluency Practice

2. The hourly wages of the workers at a company are shown in the tables.

Hourly wage W (£)	Frequency
$4 \leq W < 8$	15
$8 \leq W < 12$	11
$12 \leq W < 16$	5
$16 \leq W < 20$	

Hourly wage W (£)	Cumulative Frequency
$W < 8$	
$W < 12$	
$W < 16$	
$W < 20$	33

a) Complete the tables.

b) How many workers are at the company?

c) How many workers earn less than £8 per hour?

d) How many workers earn £8 or more per hour?

3. The air temperature at midnight on a number of days was recorded. The data is shown in the cumulative frequency table.

Temperature T (°C)	Cumulative Frequency
$T \leq 0$	3
$T \leq 5$	9
$T \leq 10$	17
$T \leq 15$	21
$T \leq 20$	25

a) How many times was the temperature less than or equal to 5°C?

b) How many times was the temperature greater than 15°C?

c) How many times was the temperature in the range $10^\circ\text{C} < T \leq 15^\circ\text{C}$?

d) How many times was the temperature in the range $5^\circ\text{C} < T \leq 15^\circ\text{C}$?

4. The test scores of some pupils are shown in the cumulative frequency table.

Score, $x\%$	Cumulative Frequency
$x \leq 20$	2
$x \leq 40$	11
$x \leq 60$	25
$x \leq 80$	34
$x \leq 100$	40

a) How many pupils scored more than 80%?

b) How many pupils scored more than 40%?

c) How many pupils scored between 20% and 60%?

d) What fraction of the pupils scored 60% or less?

e) What percentage of the pupils scored 80% or less?

5. At a company, 30% of the workers earn less than £22,000. 88% of the workers earn less than £48,000. What percentage of workers have:

a) $salary \geq £48,000$

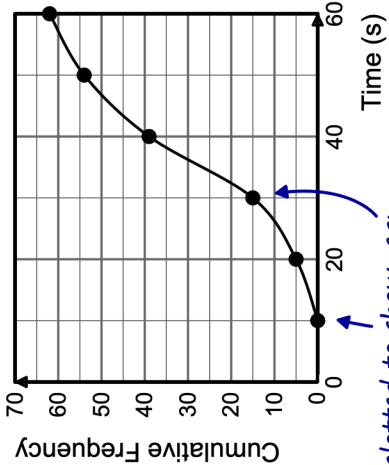
b) $£22,000 \leq salary < £48,000$

Fluency Practice

example

Construct a cumulative frequency graph for this data set:

Time t (s)	Frequency	Cumulative Frequency
$10 \leq t < 20$	5	5
$20 \leq t < 30$	10	15
$30 \leq t < 40$	24	39
$40 \leq t < 50$	15	54
$50 \leq t < 60$	8	62



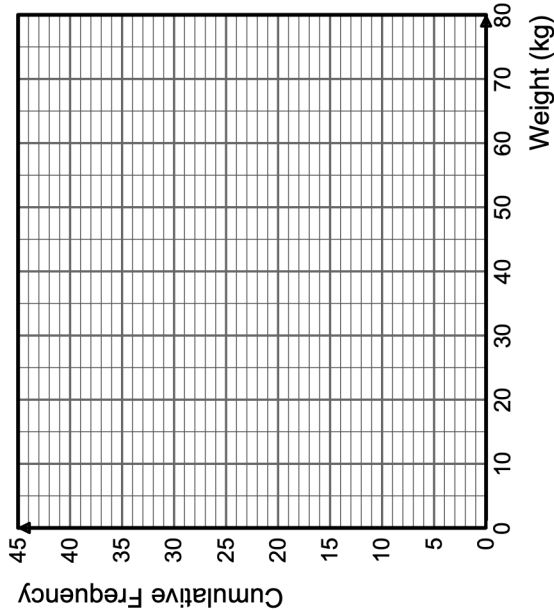
exercise

1. The frequency table shows the weights of 40 dogs.

- a) Complete the cumulative frequency table and graph.
b) Use your graph to estimate the number of dogs that weighed less than 35kg.

Weight w (kg)	Frequency
$0 \leq w < 20$	6
$20 \leq w < 30$	9
$30 \leq w < 40$	14
$40 \leq w < 60$	7
$60 \leq w < 80$	4

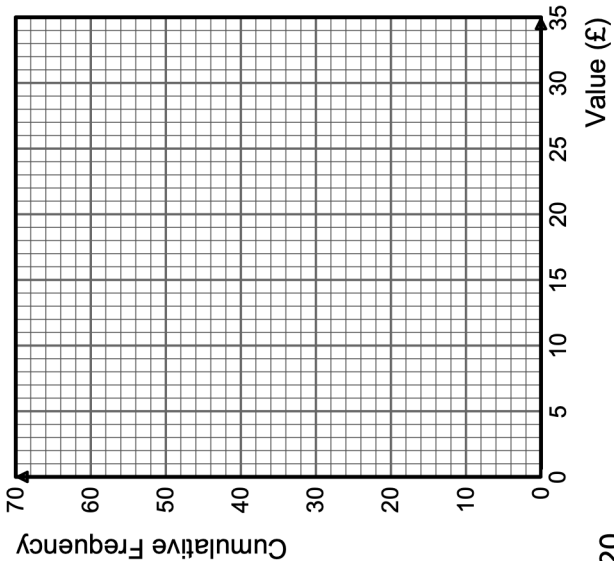
Weight w (kg)	Cumulative Frequency
$w < 20$	6
$w < 30$	15
$w < 40$	
$w < 60$	
$w < 80$	



Fluency Practice

2. The table shows the values of 60 collectable stamps.
a) Complete the table and cumulative frequency graph.

Value v (£)	Freq.	C. Freq.
$0 \leq v < 5$	22	
$5 \leq v < 10$	17	
$10 \leq v < 15$	8	
$15 \leq v < 20$	5	
$20 \leq v < 25$	4	
$25 \leq v < 30$	4	

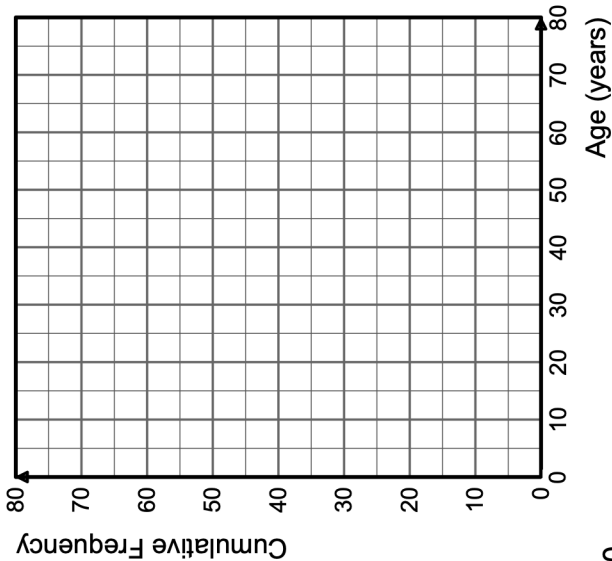


- b) Use your graph to estimate the number of stamps:

- (i) valued at less than £8.
- (ii) valued at £8 or more.
- (iii) valued between £8 and £20.

3. The table shows the ages of people watching a film at the cinema.
a) Complete the table and cumulative frequency graph.

Age a (years)	Freq.	C. Freq.
$0 \leq a \leq 5$	6	
$5 < a \leq 10$	20	
$10 < a \leq 20$	8	
$20 < a \leq 30$	8	
$30 < a \leq 40$	17	
$40 < a \leq 60$	9	
$60 < a \leq 80$	6	



- b) Use your graph to estimate the number of people:

- (i) aged 25 or less.
- (ii) aged 50 or less.
- (iii) aged between 25 and 50.
- (iv) aged more than 35.

Fluency Practice

example

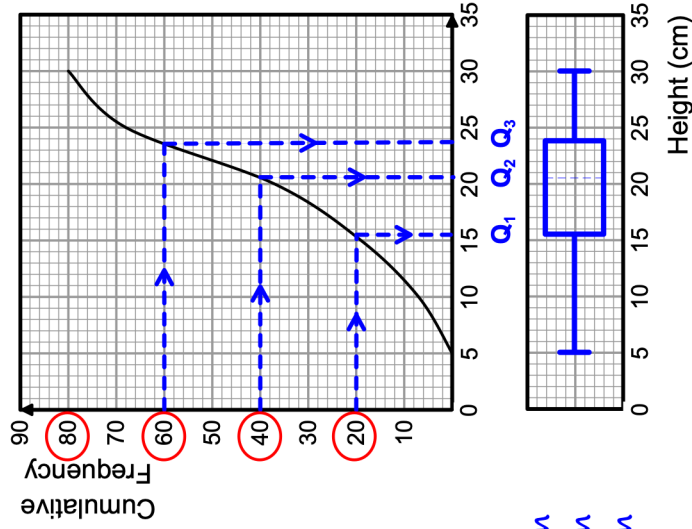
The cumulative frequency graph shows the heights of some plants. Find estimates for Q_1 , Q_2 and Q_3 and draw a box plot for the data.

*There are 80 pieces of data.
The quartiles can be found at:*

$$Q_1 : \frac{1}{4} \times 80 = 20\text{th value}$$

$$Q_2 : \frac{1}{2} \times 80 = 40\text{th value}$$

$$Q_3 : \frac{3}{4} \times 80 = 60\text{th value}$$



$$Q_1 = 16\text{cm}$$

$$Q_2 = 21\text{cm}$$

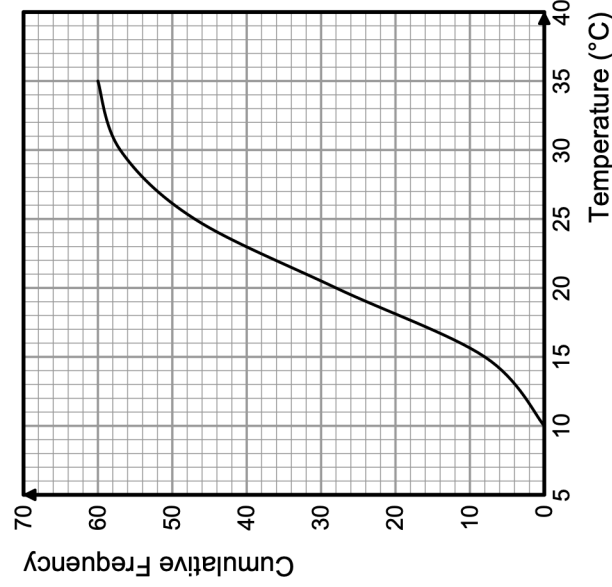
$$Q_3 = 24\text{cm}$$

exercise

1. The cumulative frequency graph shows the maximum temperature recorded on 60 days.

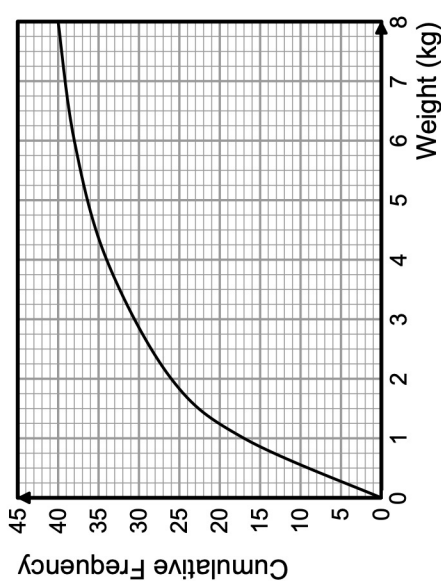
Find estimates for:

- a) the median
- b) Q_1
- c) Q_3
- d) the interquartile range
- e) the range

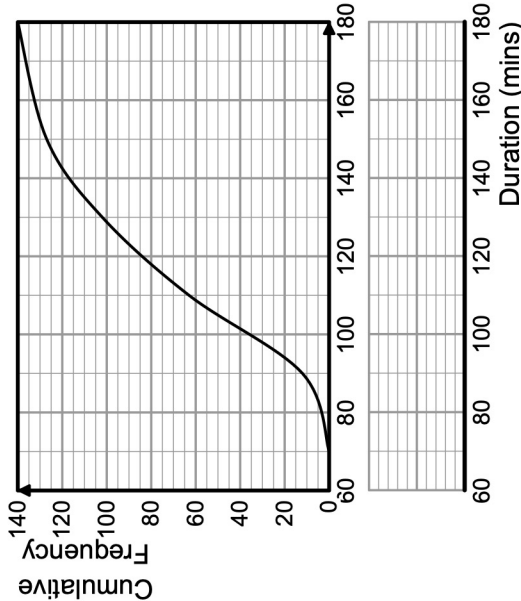


Fluency Practice

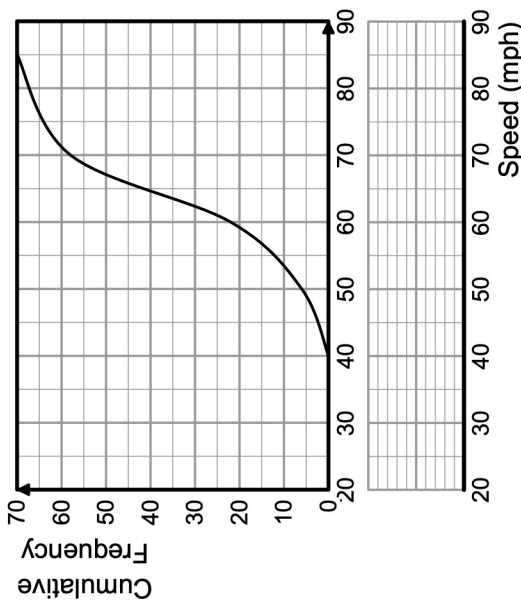
2. The cumulative frequency graph shows the weight of a number of parcels.
- a) Find estimates for:
- i) the lower quartile
 - ii) the median
 - iii) the upper quartile
- b) True or false? 80% of the parcels weighed less than 4kg.



3. The cumulative frequency graph shows the duration of a number of films, in minutes.
- Find estimates for the median and quartiles and construct a boxplot in the space below the graph.

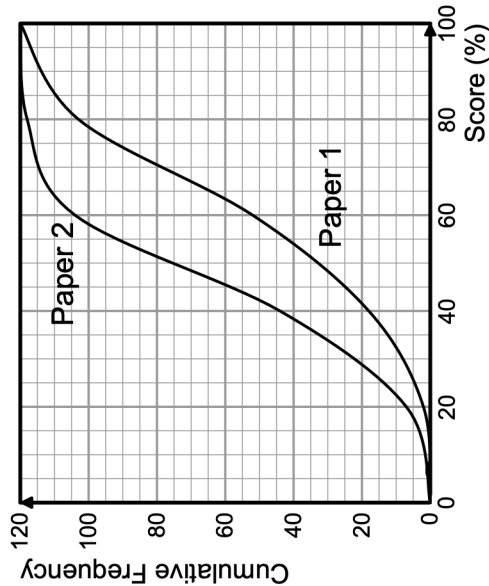


4. The cumulative frequency graph shows the speeds of vehicles on a motorway.
- Find estimates for the median and quartiles and construct a boxplot in the space below the graph.



Fluency Practice

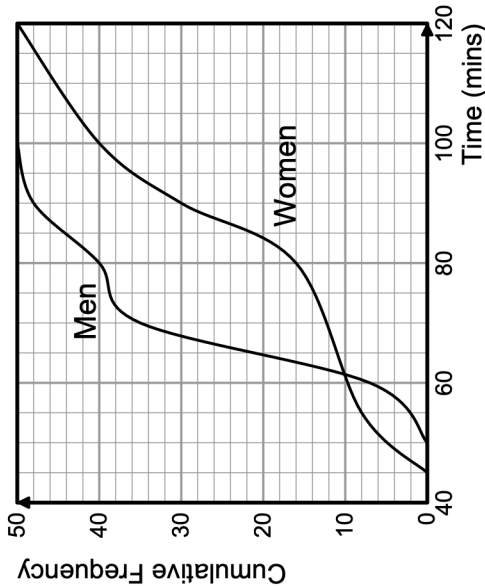
5. The cumulative frequency graph shows the scores of a year group for two exam papers.



a) Which paper did the pupils find easier? Justify your answer.

b) Certificates were given to pupils who achieved over 70% on a paper. Estimate the number of certificates given for each paper.

6. The cumulative frequency graph shows the times taken by 50 men and 50 women to complete a race.

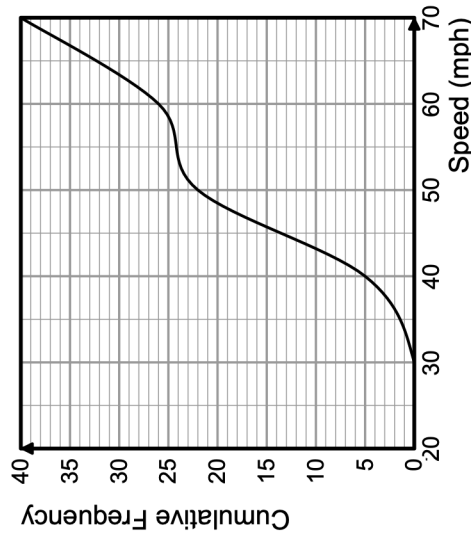


a) Complete the table:

	Men	Women
Median		
Range		
IQR		

b) Compare the two data sets.

7. The cumulative frequency graph shows the speeds of some vehicles on a road.



One of the cars is to be chosen at random. Work out estimates for these probabilities:

a) $P(\text{speed} \leq 45)$

b) $P(\text{speed} \leq 65)$

c) $P(45 < \text{speed} \leq 65)$

Page 93

Figure 1 displays four cumulative frequency plots (A, B, C, D) and their corresponding box plots (E, F, G, H). The x-axis for all plots is 'Score' (0 to 100), and the y-axis is 'Cumulative Frequency' (0 to 60).

A Cumulative frequency plot showing a distribution that is skewed to the right. The curve starts at (0,0) and rises steeply, then levels off towards (100,60).

B Cumulative frequency plot showing a distribution that is skewed to the left. The curve starts at (0,0) and rises gradually, then levels off towards (100,60).

C Cumulative frequency plot showing a symmetric, bell-shaped distribution. The curve starts at (0,0) and rises smoothly to (100,60).

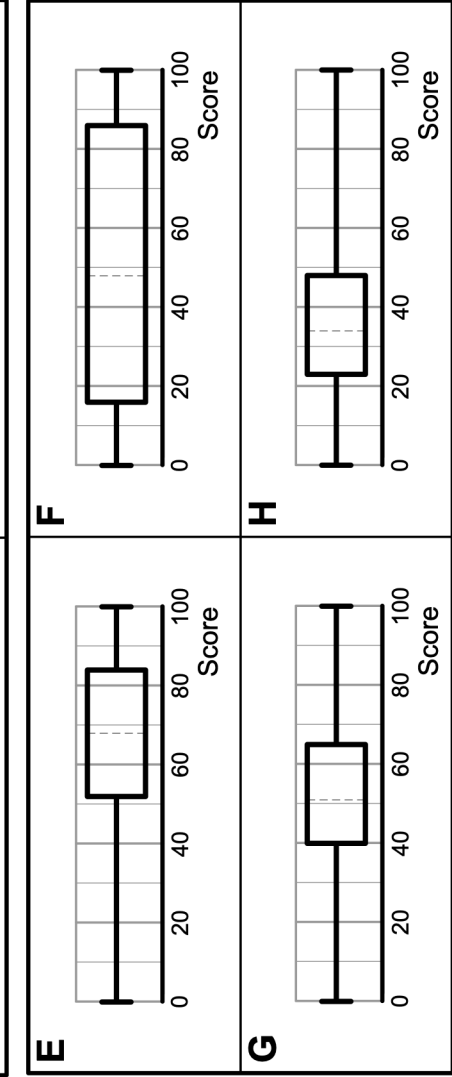
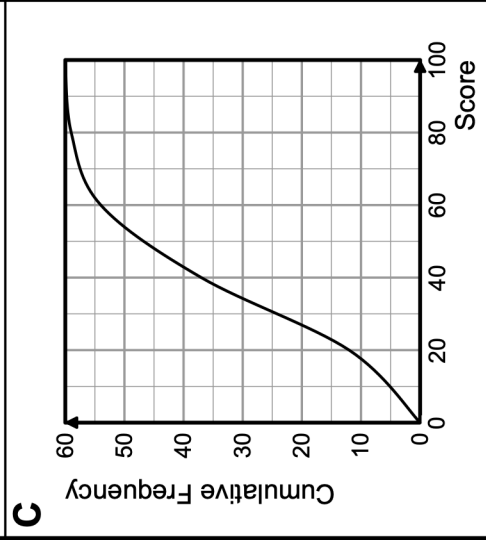
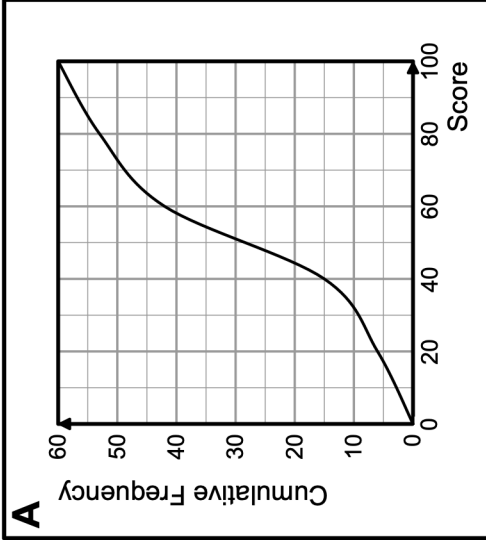
D Cumulative frequency plot showing a distribution that is skewed to the left. The curve starts at (0,0) and rises steeply, then levels off towards (100,60).

E Box plot corresponding to distribution A. The median is approximately 25, the interquartile range (IQR) is from 15 to 35, and the range is from 0 to 50.

F Box plot corresponding to distribution B. The median is approximately 75, the IQR is from 65 to 85, and the range is from 50 to 100.

G Box plot corresponding to distribution C. The median is approximately 50, the IQR is from 40 to 60, and the range is from 20 to 80.

H Box plot corresponding to distribution D. The median is approximately 25, the IQR is from 15 to 35, and the range is from 0 to 50.

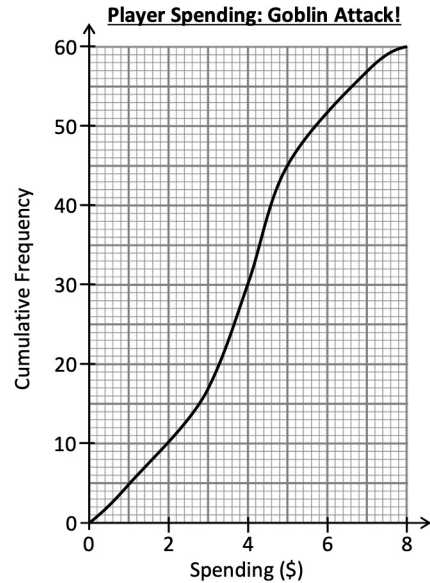


Purposeful Practice

①

Calculating Averages From Graphs

For players that spend money in their game, Apex Games records how much they spend in one day.



Use the graph to estimate the **median & interquartile range** for spending.

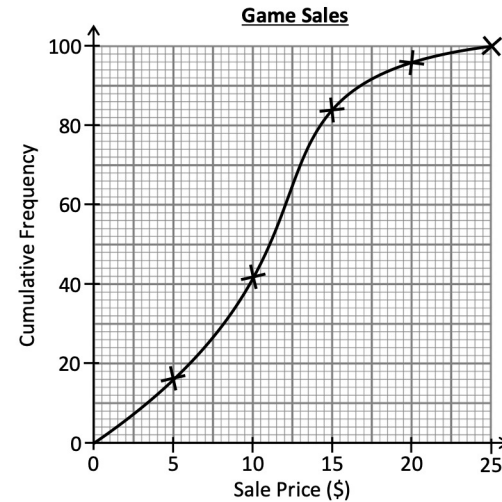
Use the graph to complete the table & calculate the **mean** spending.

Spending, s (\$)	Frequency	Midpoint	$f \times m$
$0 < s \leq 2$			
$2 < s \leq 4$			
$4 < s \leq 6$			
$6 < s \leq 8$			
Totals			

②

Calculating Averages From Graphs

Apex Games has published over 20 games. Over time, they change prices up & down to increase demand or increase profit. They record all the sales made at different price-points.



Use the graph to estimate the **median & interquartile range** for sale prices.

Use the graph to calculate the **mean** sale price.

Purposeful Practice

Interpreting the Shapes of Cumulative Frequency Graphs

Each of these cumulative frequency graphs show the test scores for different science classes.

①

How can we describe the results represented by each graph?

How do the results relate to the median?

Nearly all students scored near the median.

Most students scored very high or very low marks.

Most students scored high marks

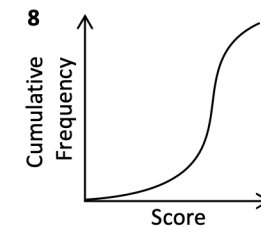
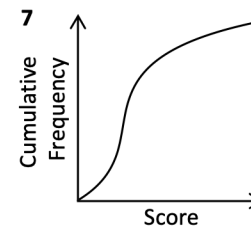
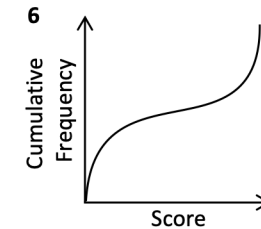
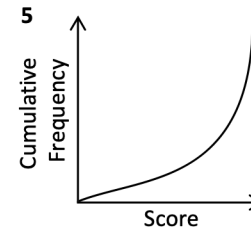
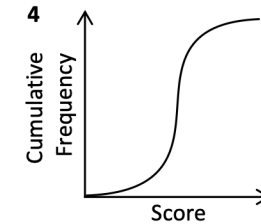
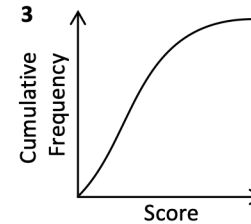
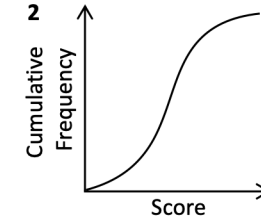
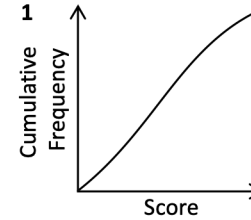
High median score.

Few students scored high marks

Low median score.

Most students scored near the median.
There are equal amounts of high & low scores.

Even distribution of students across the range of scores.



Purposeful Practice

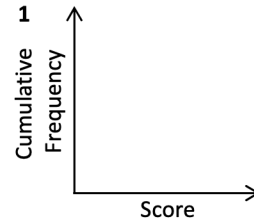
Sketching the Shape of Cumulative Data

8 classes took a science test & their results are described below.

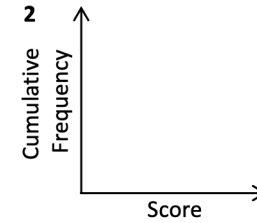
Sketch a cumulative frequency curve for each description.

Think about the rate the cumulative frequency increases above and below the median.

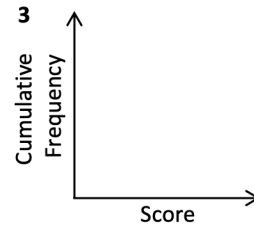
Even distribution of students across the range of scores.



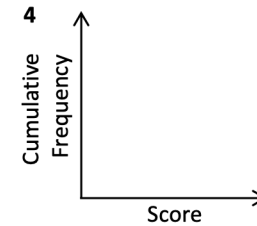
Most students scored near the median. There are equal amounts of high & low scores.



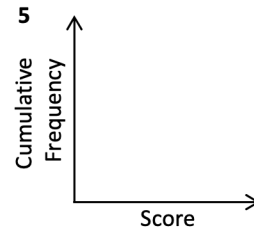
Few students scored high marks



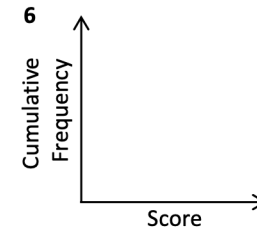
Nearly all students scored near the median.



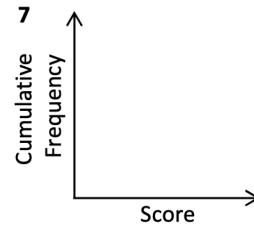
Most students scored high marks



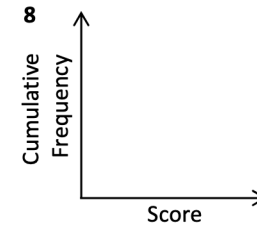
Most students scored very high or very low marks.



Low median score.



High median score.



③

Purposeful Practice

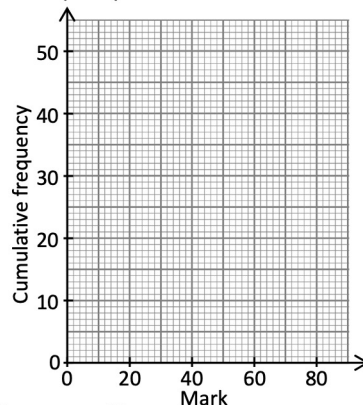
Cumulative Frequency Curves

1) 50 students took a Geography test. Their results were put into 4 groups.

- A) Complete the table to find the cumulative frequency.
B) Plot the table data on the graph.

Remember we plot each group at its **highest** value.

Mark	Frequency	Cumulative Frequency	Coord.
0-20	5		
21-40	22		
41-60	14		
61-80	9		

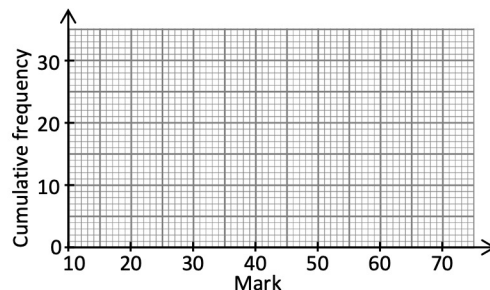


- C) Join the points with a curve.
Could we join points with straight lines instead?
Should we start the curve at (0, 0)?

- D) We want to find the median. With grouped data we can estimate and use $n \div 2$. Plot a line from $n \div 2$ to the curve. What is the estimate for the median?

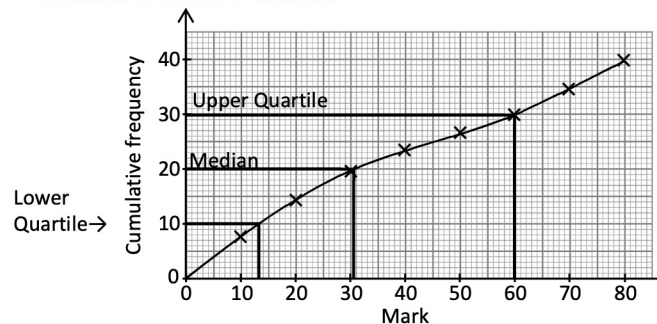
Mark (m)	F.	C.F.
$11 \leq m \leq 20$	2	
$21 \leq m \leq 30$	7	
$31 \leq m \leq 40$	9	
$41 \leq m \leq 50$	8	
$51 \leq m \leq 60$	3	
$61 \leq m \leq 70$	1	

2) 30 students take a Science test.
Their results are put into 6 groups.



- A) Find an estimate for the median

3) Mr Smith has drawn a cumulative frequency graph for the results of his Year 9 students.



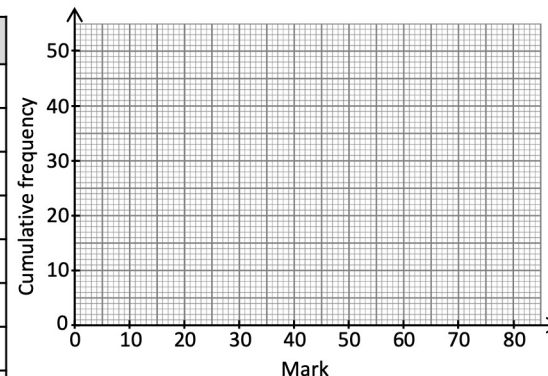
Mr Smith wants to measure the spread of the data, but he wants to eliminate extreme values and focus on the middle 50%. To do this he marks the first quarter (Lower-Quartile) and third quarter (Upper Quartile) of the data.

- A) What is the interquartile range?

4) The data for Mr Smith's Year 10 students is in the table.

- A) Complete the cumulative frequency graph.
B) Find the median and the interquartile range.

Mark (m)	F.	C.F.
$0 \leq m \leq 10$	2	
$11 \leq m \leq 20$	3	
$21 \leq m \leq 30$	9	
$31 \leq m \leq 40$	13	
$41 \leq m \leq 50$	14	
$51 \leq m \leq 60$	6	
$61 \leq m \leq 70$	2	
$71 \leq m \leq 80$	1	



- C) What comparisons can you make between Mr Smith's Year 9 and Year 10 students?

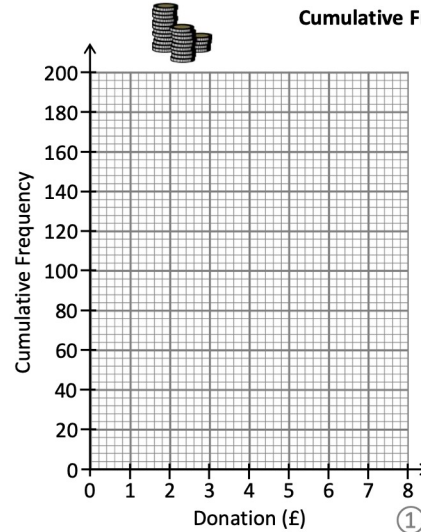
Purposeful Practice

A charity recorded the size of some donations it received.

Complete a cumulative frequency graph to show this information.

Donation, d (£)	Frequency	Cumul. Freq.
$0 < d \leq 2$	20	
$2 < d \leq 4$	60	
$4 < d \leq 6$	80	
$6 < d \leq 8$	40	

Estimate the **median** donation size by plotting a horizontal line from 100 on the y-axis.



Cumulative Frequency Graphs

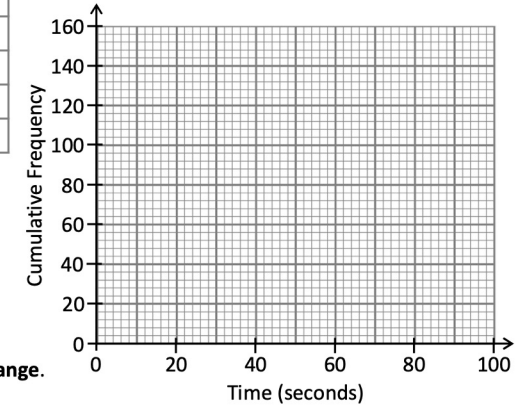
Time, t (seconds)	Freq.	CF
$0 < t \leq 20$	20	
$20 < t \leq 40$	40	
$40 < t \leq 60$	80	
$60 < t \leq 80$	16	
$80 < t \leq 100$	4	

Use the graph to estimate the upper quartile (horizontal line from 120) & lower quartile (horizontal line from 40).

Use these readings to calculate the **interquartile range**.

Customer support recorded how long customers had to wait before their phone call was answered.

Plot a cumulative frequency graph to show this data.

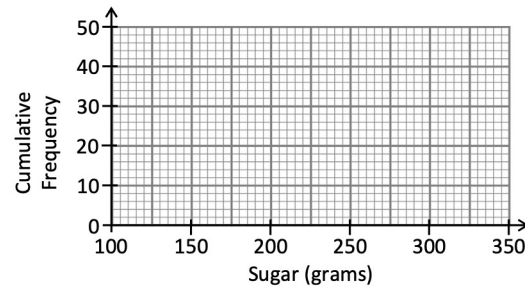


A food scientist records the sugar content in 1 kg of different snacks.

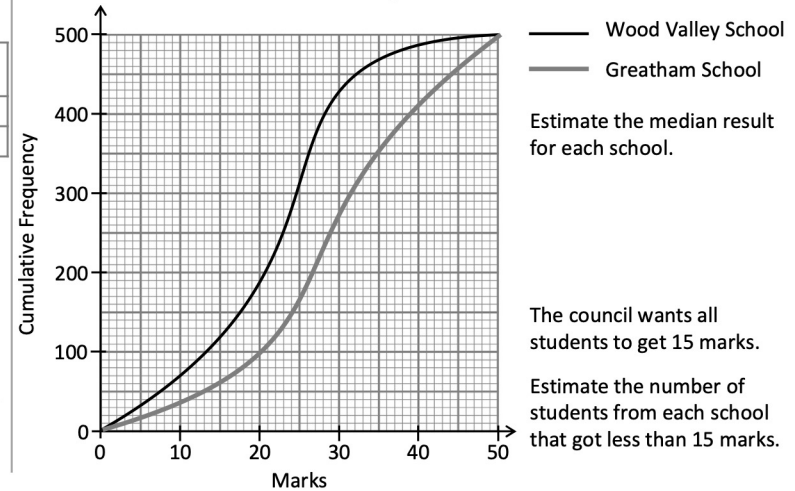
Plot a cumulative frequency graph to show this data.

Sugar, s (grams)	$100 < s \leq 150$	$150 < s \leq 200$	$200 < s \leq 250$	$250 < s \leq 300$	$300 < s \leq 350$
Freq.	4	10	14	6	6

Use the graph to estimate the median & the interquartile range.



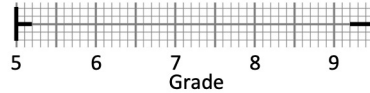
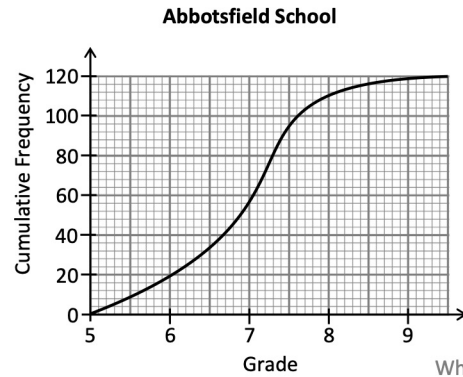
A local council records English test results from two schools.



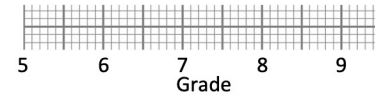
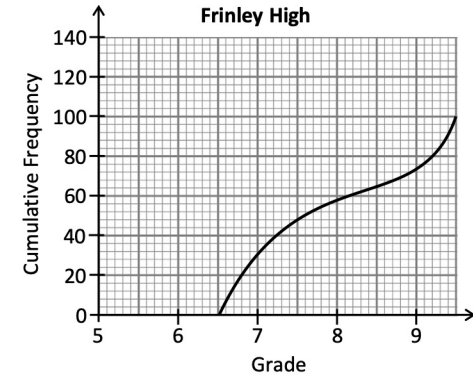
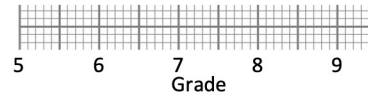
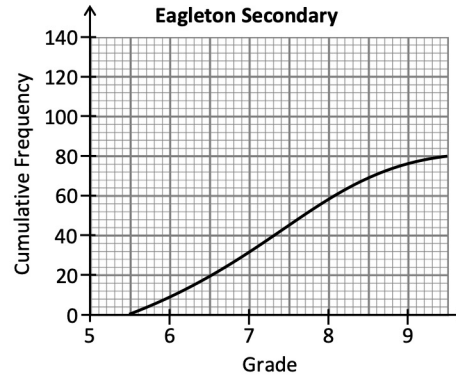
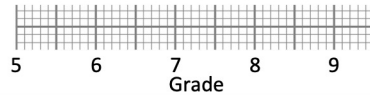
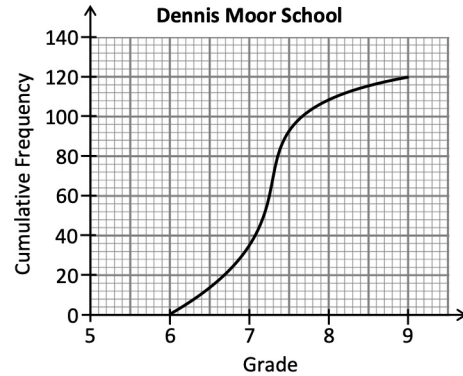
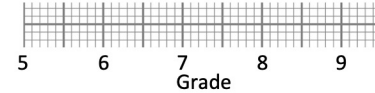
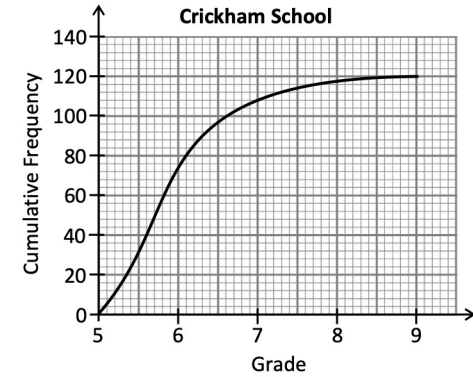
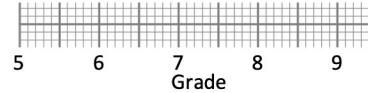
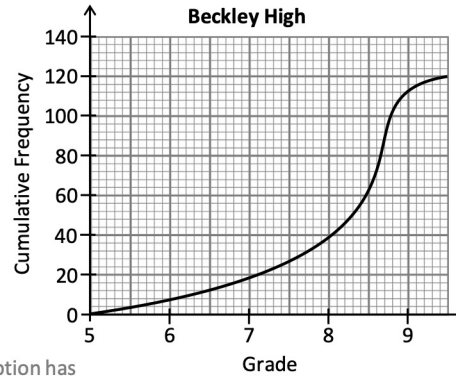
Purposeful Practice

Cumulative Frequency Graphs & Box Plots

Students in 6 schools take an English exam (graded 5.0 to 9.5). Complete the box plot for each school.
Which school did best? Which was most consistent? What comments can you make about the maximums & minimums?



What assumption has been made?



Purposeful Practice

Cumulative Frequency Graphs & Box Plots

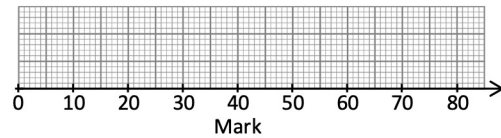
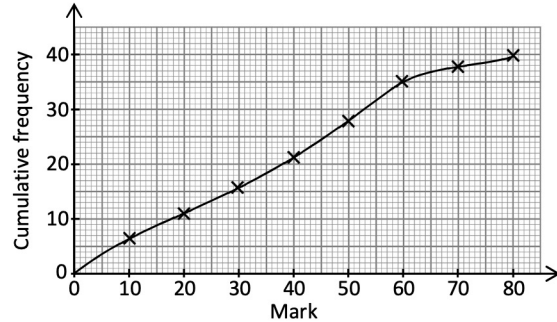
- 1) The cumulative frequency graph below shows the results of a Year 8 science test.

Josh got the highest mark in the class with 76.

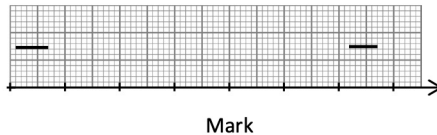
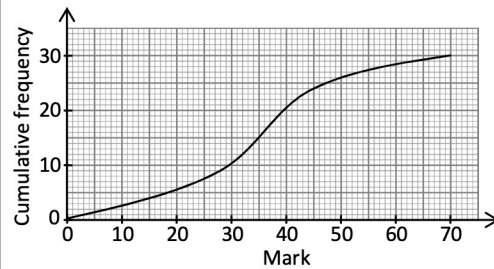
Samantha got the lowest with 7.

A) Find the Median, Upper Quartile and Lower Quartile from the graph.

B) Combine all this data to create a box plot.



- 2) This cumulative frequency graph shows the results of a Year 10 science test.



A) Use the data to complete the box plot.

B) What comparisons can you make between the Year 8s and the Year 10s?

- 3) These two tables show the results for Year 7 and Year 10 end-of-year English tests.

Year 7
Min = 8
Max = 99

Mark (m)	F.	C.F.
$0 \leq m < 20$	15	
$21 \leq m < 40$	22	
$41 \leq m < 60$	18	
$61 \leq m < 80$	11	
$81 \leq m < 100$	4	

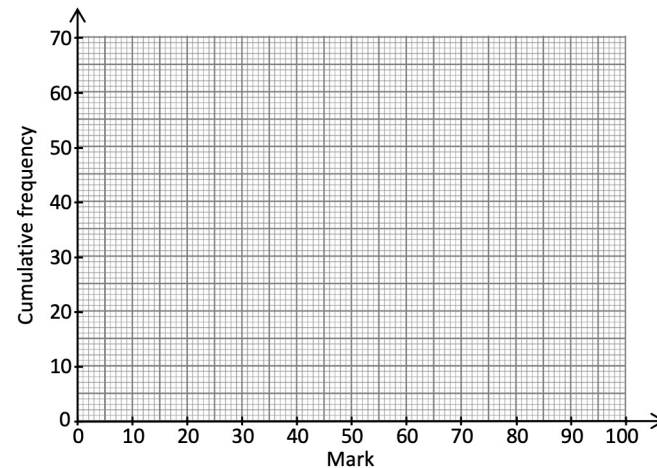
Year 10
Min = 0
Max = 92

Mark (m)	F.	C.F.
$0 \leq m \leq 10$	1	
$11 \leq m \leq 20$	1	
$21 \leq m \leq 30$	7	
$31 \leq m \leq 40$	12	
$41 \leq m \leq 50$	16	
$51 \leq m \leq 60$	13	
$61 \leq m \leq 70$	15	
$71 \leq m \leq 80$	2	
$81 \leq m \leq 90$	2	
$91 \leq m \leq 100$	1	

A) Plot both sets of data on the single grid.

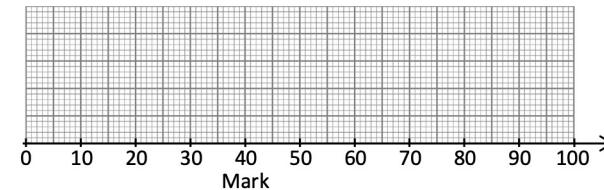
B) Use the graphs to plot two box plots on the same grid.

C) Compare the data. Which class did best?



Year 7

Year 10



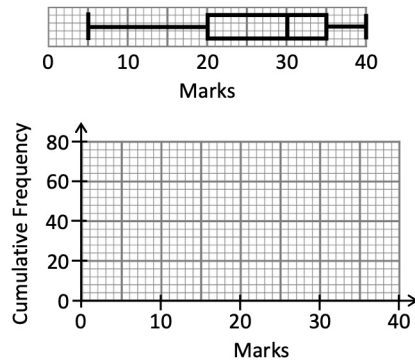
Purposeful Practice

①

80 Year 10 students take a science test.

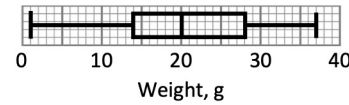
The data was not grouped to create the box plot.

Which points can we plot to create a cumulative frequency curve/polygon?



Converting Box Plots to Cumulative Frequency Graphs

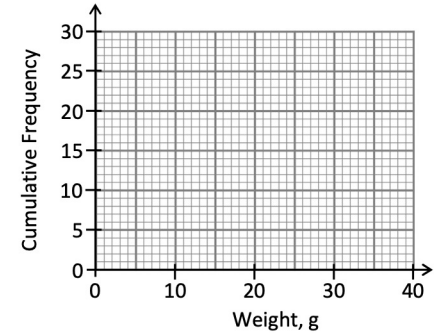
Pirate Penny weighs the 30 diamonds she has plundered!



(She doesn't group the data.)

Plot the data on the grid.

What shape do we expect a cumulative frequency curve to have outside the interquartile range?



②

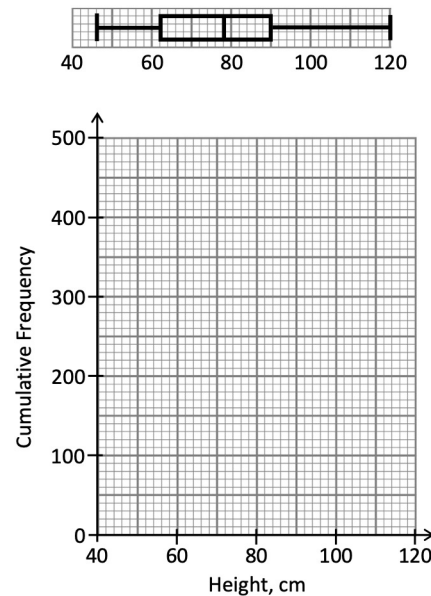
③

A scientist takes a sample of 400 stalks of wheat and measures the heights.

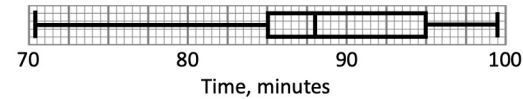
To create the box plot, they group the data and use the minimum & maximum.

Plot a cumulative frequency curve.

Where do you think it should meet the x-axis?



Race times for 400 charity runners in the Green Park 20K.

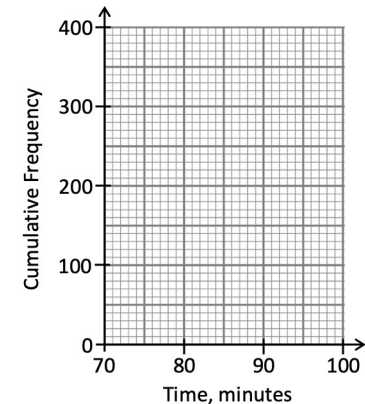


The data was sorted into 3 groups to calculate the median & the quartiles.

Plot a cumulative frequency curve.

How can we use this curve to estimate the mean finishing time for the runners?

Is a curve or a polygon more accurate?



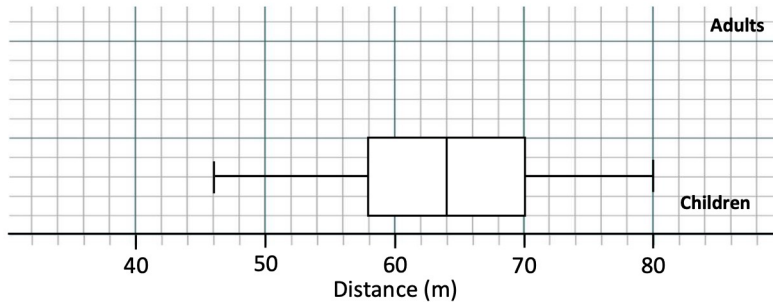
④

Purposeful Practice

1. In an experiment, 50 children and 50 adults were given 10 seconds to run as far as possible. The results for the children are shown below.

The slowest adult went 42 m, the fastest went 88 m. The adults' median was the same as the children. The lower quartile was 54 m and the upper quartile was 74 m.

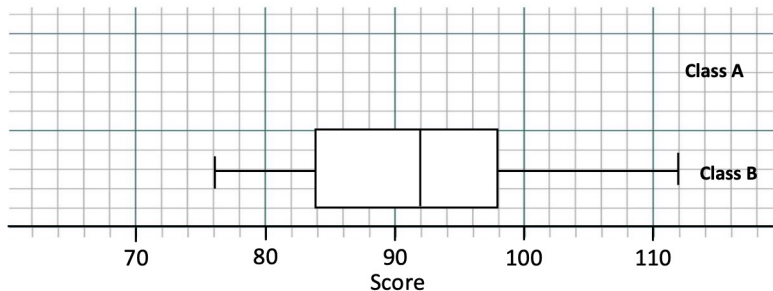
- Complete a box plot to show the adults' results.
- Which group had the furthest and shortest distances?
- Which group had the smallest range?



2. Two classes took the same physics assessment.

Class A scored a median of 90, a lower-quartile of 80 and an interquartile range of 22. The highest score was 108 and the range was 42.

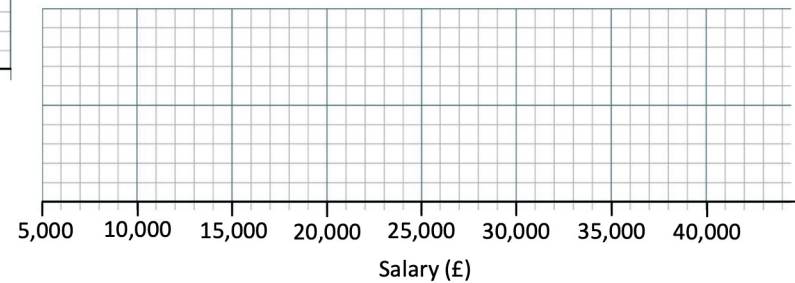
- Complete a box plot for Class A. Which class was more consistent?



Box Plots

- The table below shows data for the salaries of 100 men and 100 women.
 - Complete box plots to show this data.
 - Comment on the difference between men's and women's salaries.

	Lowest Salary	Lower Quartile	Median	Upper Quartile	Highest Salary
Men	£6000	£14,500	£21,000	£23,000	£43,000
Women	£5500	£13,000	£19,000	£22,500	£32,000



- The table below shows the monthly income for 100 individuals.
 - Draw a cumulative frequency graph for this data.
 - Use your graph to estimate the median, as well as Q_1 and Q_2 .
 - The lowest income was £1250, the highest income was 1750.
 - Complete a box plot to show this data.

Income	1201-1300	1301-1400	1401-1500	1501-1600	1601-1700	1701-1800
Frequency	8	22	32	19	14	5

Fluency Practice

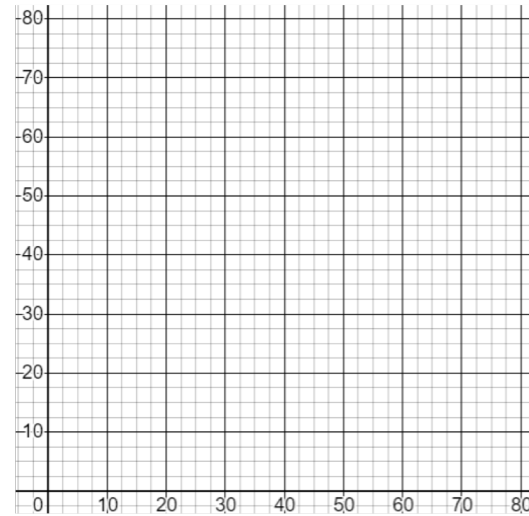
1. The length in mm of 80 leaves is recorded in a grouped frequency table.

Length L (mm)	Frequency
$20 < L \leq 30$	4
$30 < L \leq 40$	7
$40 < L \leq 50$	15
$50 < L \leq 60$	23
$60 < L \leq 70$	22
$70 < L \leq 80$	9

(a) Complete a cumulative frequency table.

Length L (mm)	Cumulative Frequency
$20 < L \leq 30$	
$20 < L \leq 40$	
$20 < L \leq 50$	
$20 < L \leq 60$	
$20 < L \leq 70$	
$20 < L \leq 80$	

(b) Plot a cumulative frequency graph.



(c) Find the median length.

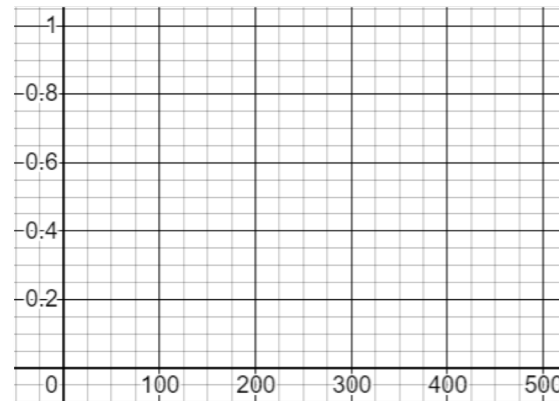
(d) Find the interquartile range of lengths.

(e) Find an estimate for the number of leaves greater than 75 mm in length.

2. The areas in m^2 of 200 gardens are recorded in a grouped frequency table.

Area (m^2)	Frequency		
$0 < A \leq 50$	10		
$50 < A \leq 100$	25		
$100 < A \leq 200$	80		
$200 < A \leq 300$	65		
$300 < A \leq 500$	20		

(a) Plot a histogram.



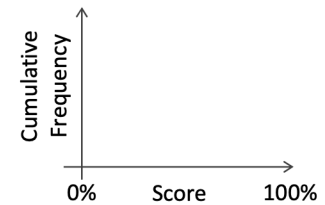
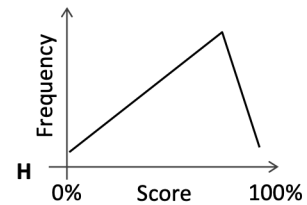
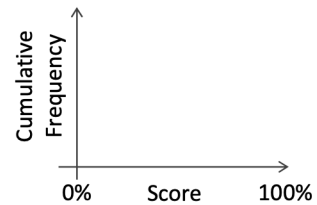
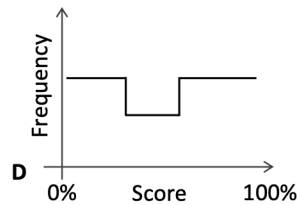
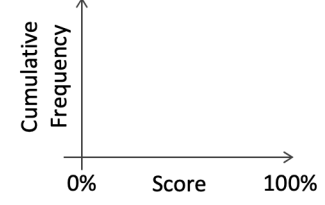
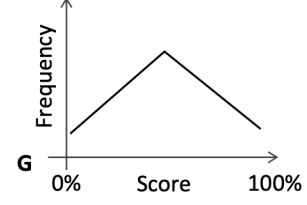
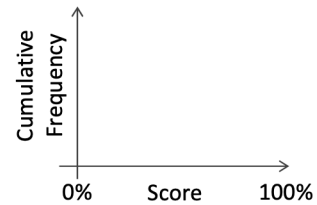
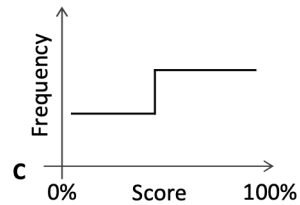
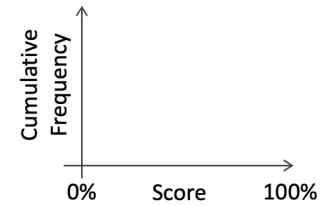
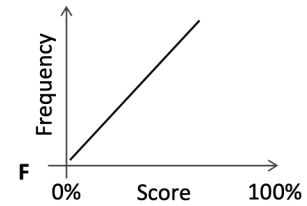
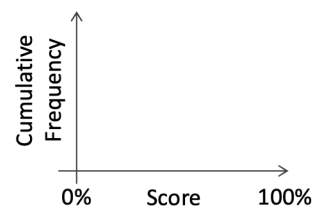
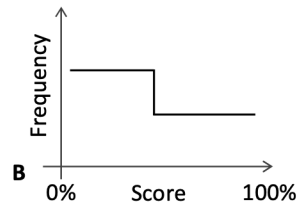
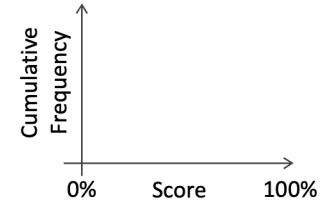
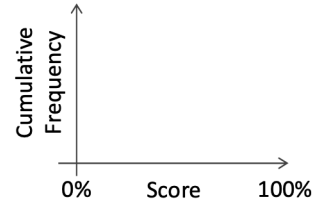
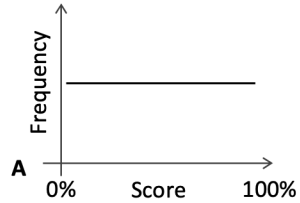
(b) Use your histogram to estimate the number of gardens that are larger than $220 m^2$.

(c) Use your histogram to estimate the median garden size.

Purposeful Practice

Frequency Polygons to Cumulative Frequency Graphs

8 different classes took a science test. Can you complete a cumulative frequency graph for each set of results?



Purposeful Practice

①

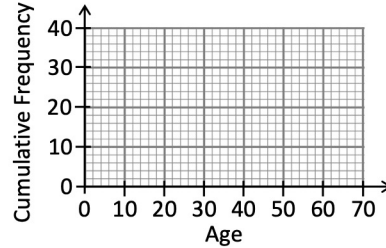
Raw → Grouped

A cinema asked for the ages of people in a screening audience.
Convert the stem & leaf diagram to a cumulative frequency graph.

What groups will you use?

Key 1 | 3 means 13

0	6 6 7 8 9 9
1	2 3 3 3 4
2	1 1 2 5 5 6
3	1 3 3 4 7 7 7 8 9
4	4 4 5 6 6 8 8 9 9
5	1 2 2 2



What is the modal value?

Estimate the median from the graph. Is it accurate?

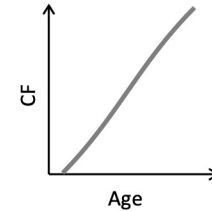
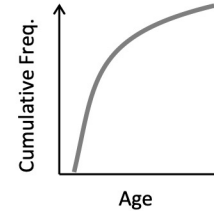
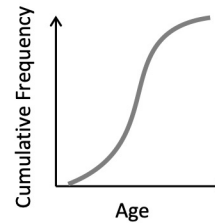
②

Match each stem & leaf diagram with a cumulative frequency graph.

3	2 2 3 4 5 9
4	1 5 5 7 7
5	2 5 5 5
6	3 4 9
7	6

0	2 2 5
1	1 5 5 7
2	2 2 3 4 4 9
3	2 4 4 6 6 6 8 7 9 9
4	1 1 2 3 4 5 5 8
5	2 3 3 7 8
6	7 7 9

2	6 6 7 8 9 9
3	2 3 3 3 4
4	1 1 2 5 5 6
5	1 3 3 4 7 7 7
6	4 4 5 8 8 9 9
7	1 2 2 2 8 9 9
8	1 2 2 2 8



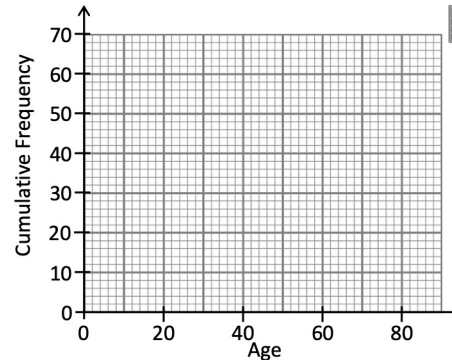
③

To decide who to advertise to, a theatre samples the audiences for both performances on a Saturday.

Plot both sets of data as cumulative frequency curves.

Key 6 | 2 | 3 means 26 for afternoon
23 for evening

afternoon	evening
3 2 2	1 1 7 7 8 9 9
7 6 5 5 5	2 2 2 3 5 5 8 8 8 9
8 8 7 5 4 1 1	3 7 7 8 8 8 8 8 9 9 9 9
8 7 6 5 2 2 1	4 1 2 4 4 5 5 7 8 8
9 8 7 4 3 3 3 2	5 1 1 2 3 6 7 7
8 8 8 5 5 4 4 2 2 1 1 1	6 2 2 2 3 7 8
9 9 9 8 7 7 7 6 6 6	7 1 1 1 2 2 3 4
4 3 3 2 2 2 1	8 2 3 6 6



Use this frequency table to estimate a mean age for the evening audience.

Age			
$10 < a \leq 30$			
$30 < a \leq 50$			
$50 < a \leq 70$			
$70 < a \leq 90$			

Use the graph to estimate the **difference** in median age between the audiences.

Are the groups representative of the raw data they contain?

Would you analyse the data differently if one audience member was actually 20?

Can you think of a quick method to find the mean from the raw data?

Frequency Diagrams with...

30 students have their heights measured to the nearest foot. Draw a **cumulative frequency diagram** for these data.

Height (to nearest foot)	Height (inequality)	Frequency	
4	$\leq h <$	12	
5		14	
6		4	

100 random numbers between 0 and 10 are rounded to the nearest prime. Draw a **histogram** for these data.

Number (to nearest prime)	Number (inequality)	Frequency	
2	$\leq n <$	15	
3		24	
5		24	
7		30	
11		7	

Bounds

The heights of 30 sunflowers are truncated to a whole number of metres. Draw a **histogram** for these data.

Height (truncated)	Height (inequality)	Frequency	
0	$\leq h <$	3	
1		17	
2		10	

The side lengths of 30 cubes are measured to the nearest cm. Draw a **frequency polygon** for the **volumes** of these cubes.

Side Length (to nearest cm)	Volume (inequality)	Frequency	
1	$\leq V <$	11	
2		9	
3		10	

2 Advanced Probability

Fluency Practice

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 10 \text{ inclusive}\}$$

$$A = \{1 \text{ to } 5 \text{ inclusive}\}$$

$$B = \{\text{even numbers}\}$$

$$C = \{3 \text{ to } 7 \text{ inclusive}\}$$

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 12 \text{ inclusive}\}$$

$$A = \{\text{prime numbers}\}$$

$$B = \{\text{multiples of } 6\}$$

$$C = \{\text{multiples of } 3\}$$

Illustrate with a Venn diagram.

$$\xi = \{a \text{ to } j \text{ inclusive}\}$$

$$A = \{h, i, j\}$$

$$B = \{a, c, e, g, i\}$$

$$C = \{e, f, g, h, i\}$$

Illustrate with a Venn diagram.

$$\xi = \{10 \text{ to } 20 \text{ inclusive}\}$$

$$A = \{\text{multiples of } 2\}$$

$$B = \{\text{multiples of } 3\}$$

$$C = \{\text{multiples of } 5\}$$

Illustrate with a Venn diagram.

$$\xi = \{1 \text{ to } 15 \text{ inclusive}\}$$

$$A = \{x: 3 \leq x \leq 9\}$$

$$B = \{\text{odd numbers}\}$$

$$C = \{7, 8, 9, 10, 11\}$$

Fluency Practice

Statement	True or false
$1 \in \{\text{odd numbers}\}$	True
$1 \in \{\text{prime numbers}\}$	
$2 \notin \{\text{odd numbers}\}$	
$\text{Red} \in \{\text{colours}\}$	
$\text{Red} \notin \{\text{colours}\}$	
$9 \in \{\text{prime numbers}\}$	
$\text{Chocolate} \notin \{\text{fruits}\}$	
$\text{scalene} \in \{\text{types of triangle}\}$	
$\text{cylinder} \in \{\text{2D shapes}\}$	

Fluency Practice

$$A = \{1, 9, 15, 23, 100\}$$

B =	$B \subset A?$
$\{1, 9\}$	Yes
$\{0, 1, 9\}$	
$\{23, 9\}$	
$\{100, 1\}$	
$\{9\}$	
$\{23, 9, 1, 100, 5\}$	
$\{0\}$	
$\{\}$ <i>also known as $\emptyset$</i>	
$\{100, 23, 15, 9, 1\}$	

Fluency Practice

For each of the following say if the statement is true or false.

$$A = \{1, 4, 9, 16, 25\} \quad B = \{1, 4\} \quad C = \{1, 8, 27, 64, 125\} \quad D = \{1, 27\} \quad E = \{4, 1\}$$

a) $B \subset A$	b) $A \subset B$	c) $6 \in A$
d) $B = E$	e) $B \subset C$	f) $D \subset C$
g) $27 \in C$	f) $E \subset D$	h) $25 \in A$

Purposeful Practice

1. $\xi = \{4,5,6,7,8\}$
 $A = \{5,6,7\}$
 $B = \{6,7,8\}$

Construct a Venn Diagram to show these sets.

2. $\xi = \{1,2,3,4,5,6,7,8\}$
 $A = \{1,2,3,8\}$
 $B = \{3,4,5,8\}$
 $C = \{1,5,6,8\}$

Construct a Venn Diagram to show these sets.

3. $\xi = \{1,2, \dots, 10\}$
 $A = \text{set of all primes}$
 $B = \text{triangular numbers}$
 $C = 1 \text{ less than multiple of } 4$

Construct a Venn diagram for these sets.

4. You have two sets A and B and $B \subset A$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.

5. You have three sets A , B and C and $A \subset C$. Draw a Venn Diagram (without any numbers) that indicates the relationship between the sets.

- ☠ The **power set** of a set is the set of all possible subsets, including the empty set and itself. E.g.

$$P(\{1,2\}) = \{\emptyset, \{1\}, \{2\}, \{1,2\}\}$$

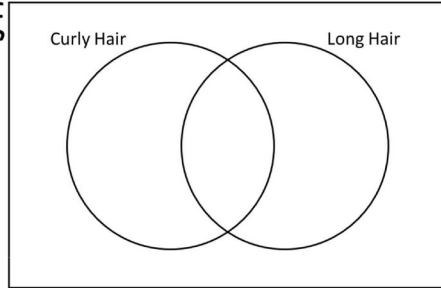
- a. Determine $P(\{1,2,3\})$
b. Determine how many members $P(A)$ has for a set of A of size n .

Fluency Practice

- ① 24 students were surveyed about their hair.
The results are below.

Curly	Long	Short
Chris	Chris	Edward
Fred	Diana	Anna
Anna	Kris	Viv
Tess	Fred	Lisa
Olly	Noel	Will
Mark	Robert	Tess
Iona	Yuna	Ben
Jessica	Zoe	Olly
Ben	Shay	Xena
	Peter	Jessica
	Mark	
	George	
	Iona	
	Hannah	

Complete the Venn diagram using letters for the student's names.



If you pick a student at random, what is the probability...

$$P(\text{Curly Hair}) = \quad P(\text{NOT Curly Hair}) =$$

$$P(\text{Long AND Curly Hair}) =$$

$$P(\text{Long AND NOT Curly Hair}) =$$

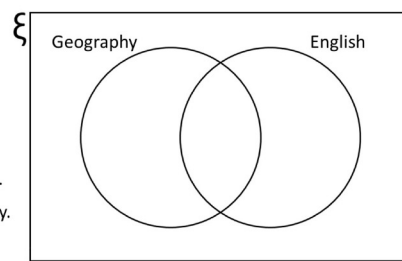
$$P(\text{Curly OR Short Hair}) =$$

②



50 students were asked if they study Geography or English.

- 5 study Geography and not English.
- 25 study English and not Geography.
- 20 study English and Geography.



If I pick a student at random, what is the probability they...

$$P(\text{study English}) =$$

$$P(\text{study Geography}) =$$

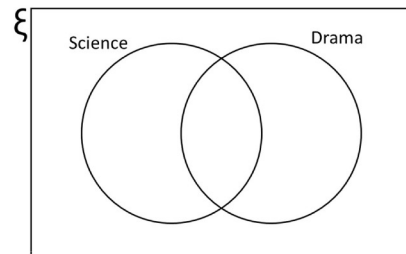
$$P(\text{study Geography AND English}) =$$

③



70 students were asked if they study Science or Drama.

- 13 study Science and Drama.
- 37 study Science and not Drama.
- 8 study Drama and not Science.



If I pick a student at random, what is...

$$P(\text{study Drama}) =$$

$$P(\text{study Science}) =$$

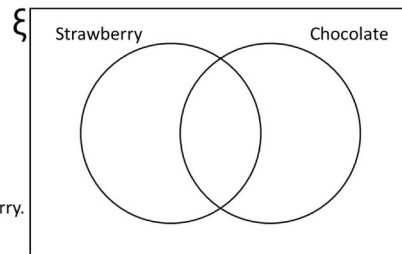
$$P(\text{don't study Science OR Drama}) =$$

④



45 people bought one or two scoops of ice cream.

- 20 bought strawberry.
- 10 didn't buy chocolate or strawberry.
- 30 bought chocolate.



How many people bought both flavours?

If I pick a person at random, what is...

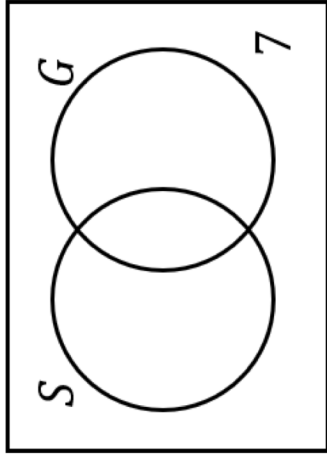
$$P(\text{strawberry}) =$$

$$P(\text{only chocolate}) =$$

$$P(\text{only one flavour}) =$$

Purposeful Practice

There are 30 students in a class. 18 study Spanish. 13 study German. 7 study neither Spanish nor German. Complete the Venn diagram to represent this information and find the number of students who study Spanish and German.



In a group of 25 people, 8 said they had been to the theatre recently. 15 said they had been to the library recently, and 10 said they had been to neither the theatre or the library recently. Draw a Venn diagram to represent this information and find the number of people who had only been to the library recently.

In a class of 30 students, 15 play football and 11 play tennis. Twice as many students play neither sport as play both sports. Draw a Venn diagram to represent this information and find the number of students who play both football and tennis.

There are 50 cars in a car park. 40% of the cars are Fords. Of the Fords, 6 were white. There were three times as many cars that were neither white nor Ford as there were white Fords. Draw a Venn diagram to represent this information.

Find:

- (a) the number of cars that were white, but not Fords.
- (b) the number of cars that were Fords, but not white.

Purposeful Practice

In a group of 100 students, 42 study Statistics, 40 study Mathematics, and 50 study Physics. 21 study Mathematics and Physics, 19 study Statistics and Mathematics and 5 study all three.

Draw a Venn diagram to represent this information.

- (a) How many students study only **one** of these subjects?
- (b) How many students study none of these subjects?

A group of 200 adults were asked which types of magazines they read. Their replies showed that 82 read Sports magazines, 80 read Garden magazines, and 84 read Fashion magazines. 36 read Sports magazines and Garden magazines. 31 read Sports magazines and Fashion magazines. 25 read Garden magazines and Fashion magazines. 14 read all three magazines.

- (a) How many adults read Sports and Garden magazines, but not Fashion magazines?
- (b) How many adults reads exactly two of these types of magazine?

There are 3 clubs - chess, drama and art. All the members of a group of 35 students belong to at least one club. 8 of the students belong to only art club. 6 of the students belong to all 3 clubs. 3 of the students belong to chess and art clubs but not to drama club. 18 of the students belong to art club. 3 of the students belong only to chess club. 4 of the students belong only to drama club.

- (a) How many students belongs to chess club and to drama club but not to art club?
- (b) How many students belong to chess club?

Purposeful Practice

1• Draw a Venn diagram to represent each set of information

a•

There are 80 students.
9 students study French (F) and German (G).
35 students only study French.
2 students do not study French or German.

b•

50 children were eating lunch.
11 students only had chips (C).
20 students only had a burger (B).
13 students had both chips and a burger.

c•

In a group of 24 students,
13 like only oranges (O).
2 like only bananas (B).
4 don't like oranges or bananas.

d•

There are 32 students.
24 students study History (H), Geography (G) or both.
15 students only study History.
5 students only study Geography.

2• Draw a Venn diagram to represent each set of information

a•

40 people were asked if they had a cat (C) or a dog (D).
10 people owned both pets.
17 people owned a cat.
14 people didn't own either pet.

b•

110 workers were asked if they liked tea (T) or coffee (C).
90 workers liked tea.
41 workers liked coffee.
25 workers liked tea and coffee.

c•

In a class of 24 students,
12 students play the piano (P).
13 students play the guitar (G).
4 students play neither instrument.

d•

100 people were asked if they had visited Rome (R) or Paris (P).
38 had been to Rome.
80 had been to Paris.
11 had not been to either city.

3• Draw a Venn diagram to represent each set of information

a•

32 students were asked which club they belong to.
6 students are only part of the art club (A).
2 students are only part of the chess club (C).
3 students are only part of the drama club (D).
5 students belong to all three clubs.
2 students are part of the art and chess clubs, but not the drama club.
15 students are part of the art and drama clubs, but not the chess club.
7 students are not part of any of the clubs.

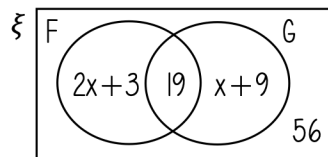
b•

60 people were asked which fruits they liked.
40 people said they liked melons (M).
42 people said they liked pears (P).
25 people said they liked strawberries (S).
6 people said they liked all three fruits.
20 people said they liked melons and pears, but not strawberries.
10 people said they liked pears and strawberries, but not melons.
6 people said they liked melons and strawberries, but not pears.

4• Find the value of x , and then find the frequency of each region.

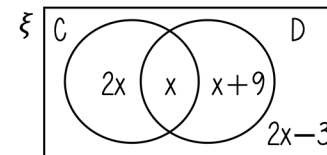
a•

150 students were asked whether they studied French or German.



b•

30 people were asked whether they had a dog (D) or a cat (C).



Purposeful Practice

Patrons are reminded to *define their variables*; i.e. if you introduce an x somewhere then you must define what it is (for example “Let x be the number of students in school who don’t define their variables”).

1. 100 people are surveyed as to whether they enjoy rugby or football.
70 like football.
60 like rugby.
45 like both.
 - (a) Draw a Venn diagram of the situation.
A person from the survey is selected at random.
 - (b) Find the probability they like neither rugby nor football.
 - (c) Find the probability they like only one of the two sports.
2. Fifty people are surveyed as to whether they read the Guardian or the Telegraph. Forty read the Telegraph. Thirty read the Guardian. Five read neither.
 - (a) Draw a Venn diagram of the situation.
 - (b) Find the number of people who read both the Guardian and the Telegraph.
A person from the survey is selected at random.
 - (c) Find the probability they read the Telegraph but not the Guardian.
 - (d) Find the probability they read only one paper.
3. Sixty A level students are surveyed as to whether they study maths or history. Fifty study maths. Twenty study history. Seven read neither.
 - (a) Draw a Venn diagram of the situation.
 - (b) Find the number of students who study both maths and history.
A student from the survey is selected at random.
 - (c) Find the probability they read history but not maths.
 - (d) Find the probability they read neither subject.
4. A college has 80 students in Year 12.
20 students study Biology.
28 students study Chemistry.
30 students study Physics.
7 students study both Biology and Chemistry.
11 students study both Chemistry and Physics.
5 students study both Physics and Biology.
3 students study all 3 of these subjects.
 - (a) Draw a Venn diagram to represent this information.
A Year 12 student at the college is selected at random.
 - (b) Find the probability that the student studies Chemistry but not Biology or Physics.
☐
 - (c) Find the probability that the student studies Chemistry or Physics or both.

Purposeful Practice

5. A group of 100 students are asked if they like folk music, rock music or soul music.
- All students who like folk music also like rock music.
No students like both rock music and soul music.
75 students do not like soul music.
12 students who like rock music do not like folk music.
30 students like folk music.
- (a) Draw a Venn diagram to illustrate this information.
(b) State two of these types of music that are mutually exclusive.
Find the probability that a randomly chosen student
- (c) does not like folk music, rock music or soul music,
(d) likes rock music,
(e) likes folk music or soul music.
6. The following shows the results of a survey on the types of exercise taken by a group of 100 people.
- 65 run
48 swim
60 cycle
40 run and swim
30 swim and cycle
35 run and cycle
25 do all three
- (a) Draw a Venn Diagram to represent these data.
Find the probability that a randomly selected person from the survey
- (b) takes none of these types of exercise,
(c) swims but does not run,
(d) takes at least two of these types of exercise.
7. There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.
- 112 take systems support,
70 take developing software,
81 take networking,
35 take developing software and systems support,
28 take networking and developing software,
40 take systems support and networking,
4 take all three extra options.
- (a) Draw a Venn diagram to represent this information.
A student from the course is chosen at random. Find the probability that the student
- takes
- (b) none of the three extra options,
(c) networking only.

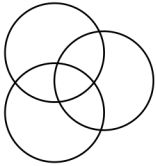
Purposeful Practice

8. A person's blood group is determined by whether or not it contains any of 3 substances A, B and C. A doctor surveyed 300 patients' blood and produced the table below.

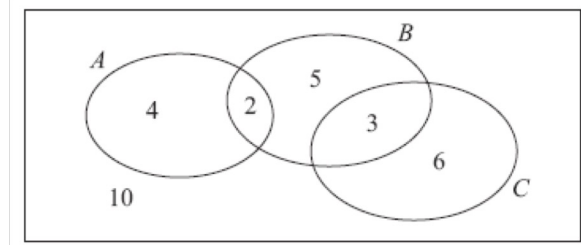
Blood contains	No. of patients
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A, B and C	10
A and B but not C	3

- (a) Draw a Venn diagram to represent this information.
- (b) Find the probability that a randomly chosen patient's blood contains substance C. Patients whose blood contains none of these substances are called universal blood donors.
- (c) Find the probability that a randomly chosen patient is a universal blood donor.

Problem Solving

A group of friends are comparing whether they received homework from Maths, English and PE.	James had more homework than Miya.	Ellie had homework from all three subjects. Rory and Jack only had one piece each.
 You may find this useful.	Seven students in total have Maths homework but Miya isn't one of them.	Tom received homework from Maths and PE but not English.
Four students received two pieces of homework.	Two of the group just have PE homework and they are both male.	Katie is the only girl with no homework.
Rory has more homework than Aidan.	Maddie is one of two people who just have Maths homework.	Sophie and Rory do not have PE homework.
Wayne has less homework than Ashley.	Nobody in the group just has English homework.	None of the girls has exactly the same homework as another girl.
Five students must write an essay for English.	What is the probability that one of the students chosen at random has three pieces of homework?	There are 7 boys and 5 girls.

Purposeful Practice



One of these students is selected at random.

(a) Show that the probability that the student reads more than one magazine is $\frac{1}{6}$. (2)

(b) Find the probability that the student reads *A* or *B* (or both). (2)

(c) Write down the probability that the student reads both *A* and *C*. (1)

Given that the student reads at least one of the magazines,

(d) find the probability that the student reads *C*. (2)

1. [JMC 2002 Q11] The Pythagoras School of Music has 100 students. Of these, 60 are in the band and 20 are in the orchestra. Given that 12 students are in both the band and the orchestra, how many are in neither the band nor the orchestra?

2. [Edexcel S1 May 2010 Q4] The Venn diagram in Figure 1 shows the number of students in a class who read any of 3 popular magazines *A*, *B* and *C*.

3. [Edexcel S1 Jan 2010 Q4] There are 180 students at a college following a general course in computing. Students on this course can choose to take up to three extra options.

112 take systems support,

70 take developing software,

81 take networking,

35 take developing software and systems support,

28 take networking and developing software,

40 take systems support and networking,

4 take all three extra options.

Purposeful Practice

- (a) Draw a Venn diagram to represent this information. (5)

A student from the course is chosen at random.

- Find the probability that the student takes
(b) none of the three extra options, (1)
(c) networking only. (1)

Students who want to become technicians take systems support and networking. Given that a randomly chosen student wants to become a technician,

- (d) find the probability that this student takes all three extra options. (2)

4. [Edexcel S1 May 2008 Q5] A person's blood group is determined by whether or not it contains any of 3 substances A , B and C .
A doctor surveyed 300 patients' blood and produced the table below.

Blood contains	No. of Patients
only C	100
A and C but not B	100
only A	30
B and C but not A	25
only B	12
A , B and C	10
A and B but not C	3

- (a) Draw a Venn diagram to represent this information. (4)
(b) Find the probability that a randomly chosen patient's blood contains substance C . (2)
Harry is one of the patients. Given that his blood contains substance A ,
(c) find the probability that his blood contains all 3 substances. (2)
Patients whose blood contains none of these substances are called universal blood donors.
(d) Find the probability that a randomly chosen patient is a universal blood donor. (2)

5. [Edexcel S1 Jan 2008 Q5] The following shows the results of a wine tasting survey of 100 people.
96 like wine A , 93 like wine B ,
96 like wine C , 92 like A and B ,
91 like B and C , 93 like A and C ,
90 like all three wines.

- (a) Draw a Venn Diagram to represent these data. (6)
Find the probability that a randomly selected person from the survey likes
(b) none of the three wines, (1)
(c) wine A but not wine B , (2)
(d) any wine in the survey except wine C , (2)
(e) exactly two of the three kinds of wine. (2)
Given that a person from the survey likes wine A ,
(f) find the probability that the person likes wine C . (3)

6. [Edexcel S1 May 2006 Q6] A group of 100 people produced the following information relating to three attributes. The attributes were wearing glasses, being left-handed and having dark hair.

Glasses were worn by 36 people, 28 were left-handed and 36 had dark hair. There were 17 who wore glasses and were left-handed, 19 who wore glasses and had dark hair and 15 who were left-handed and had dark hair. Only 10 people wore glasses, were left-handed and had dark hair.

- (a) Represent this on a Venn diagram. (6)

A person was selected at random from this group.

- Find the probability that this person
(b) wore glasses but was not left-handed and did not have dark hair, (1)
(c) did not wear glasses, was not left-handed and did not have dark hair, (1)
(d) had only two of the attributes, (2)
(e) wore glasses, given they were left-handed and had dark hair. (3)

7. [Edexcel S1 Jan 2005 Q5] Articles made on a lathe are subject to three kinds of defect, A , B or C . A sample of 1000 articles was inspected and the following results were obtained.

31 had a type A defect
37 had a type B defect
42 had a type C defect
11 had both type A and type B defects
13 had both type B and type C defects
10 had both type A and type C defects
6 had all three types of defect.

- (a) Draw a Venn diagram to represent this. (6)
Find the probability that a randomly selected article from this sample had
(b) no defects, (1)
(c) no more than one of these defects. (2)

An article selected at random from this sample had only one defect.

- (d) Find the probability that it was a type B defect. (2)

Two different articles were selected at random from this sample.

- (e) Find the probability that both had type B defects. (2)



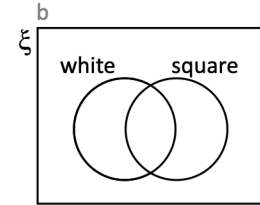
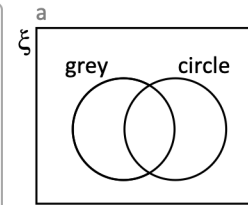
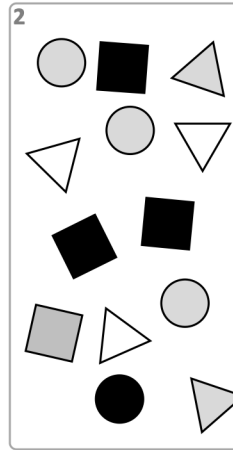
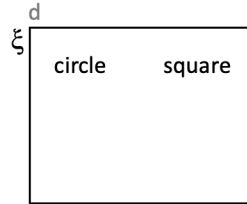
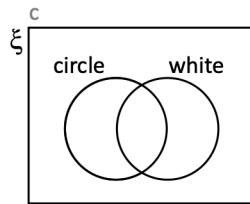
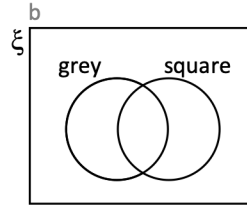
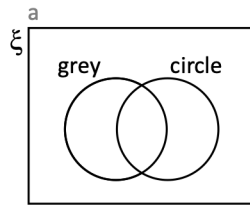
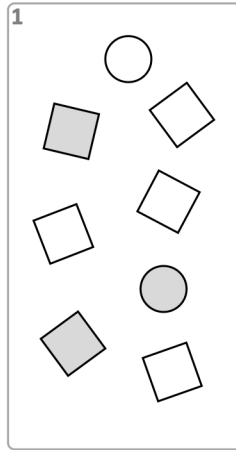
[SMC 2011 Q17] Jamie conducted a survey on the food preferences of pupils at a school and discovered that 70% of the pupils like pears, 75% like oranges, 80% like bananas and 85% like apples. What is the smallest possible percentage of pupils who like all four of these fruits?

- A. At least 10% B. At least 15%
C. At least 20% D. At least 25%
E. At least 70%

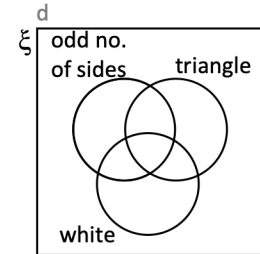
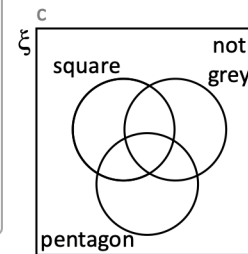
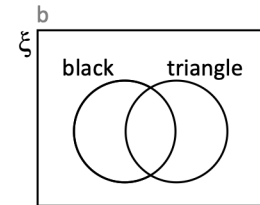
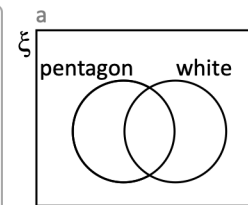
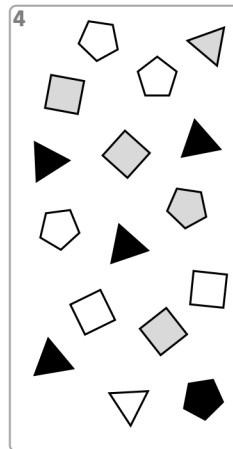
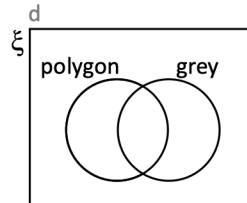
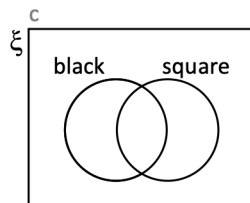
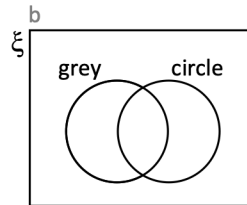
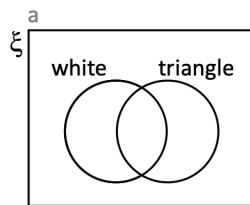
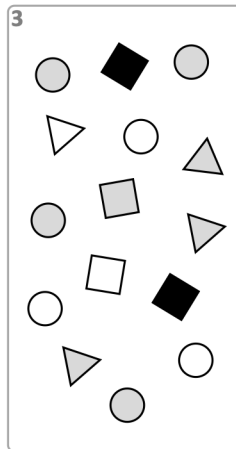
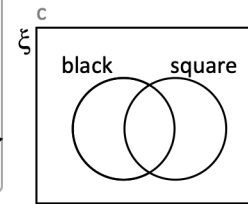
Fluency Practice

Creating Venns

For each *population* of shapes, complete the 4 Venn diagrams.



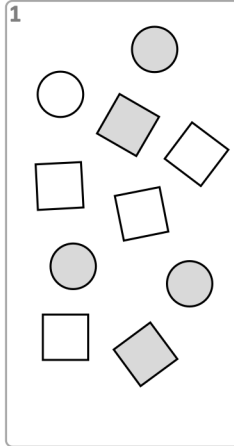
Is this a correct diagram?



☆ How else could we draw these last two Venn diagrams? ☆

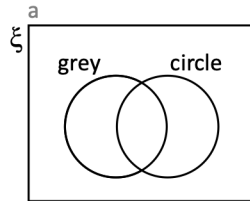
Fluency Practice

If a shape is picked from this bag...



Creating Venns

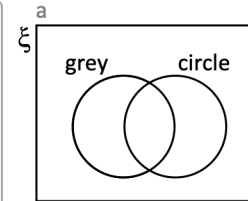
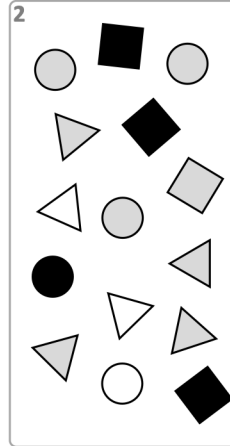
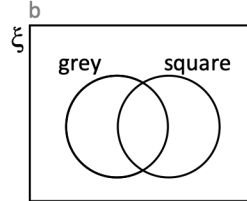
Each diagram shows a different bag of coloured shapes.
Complete the Venn diagrams & use them to calculate each probability.



What is... $P(\text{grey shape}) =$

$P(\text{grey circle}) =$

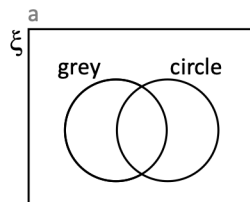
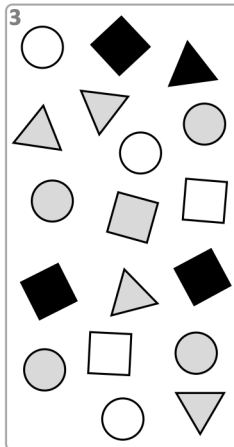
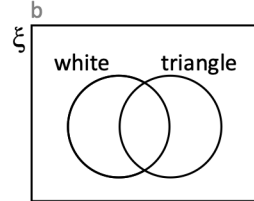
$P(\text{grey square}) =$



What is... $P(\text{grey circle}) =$

$P(\text{not a white triangle}) =$

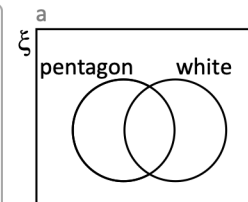
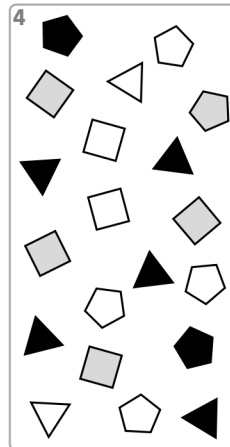
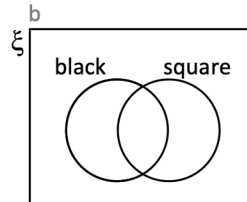
$P(\text{not grey and not a circle}) =$



What is... $P(\text{a grey circle}) =$

$P(\text{not a black square}) =$

A square has been picked from the bag.
What is the probability it is black?

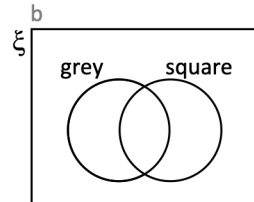


As a percentage, what is...

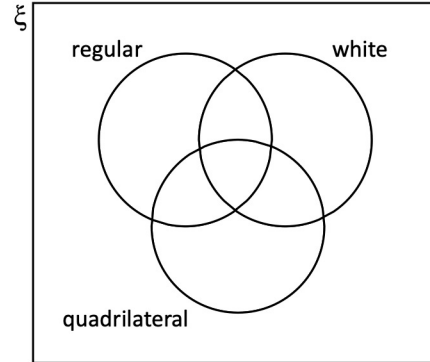
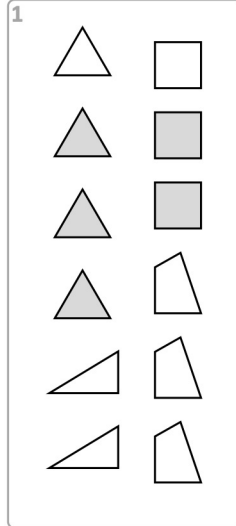
$P(\text{not a white pentagon}) =$

$P(\text{a grey shape is a square}) =$

$P(\text{a square is not grey}) =$



Fluency Practice

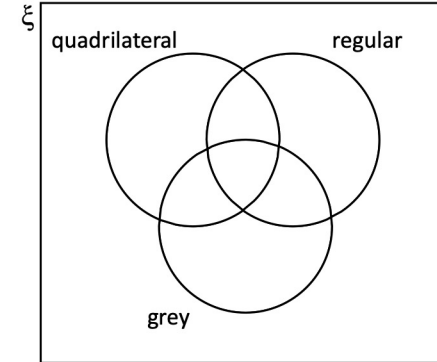
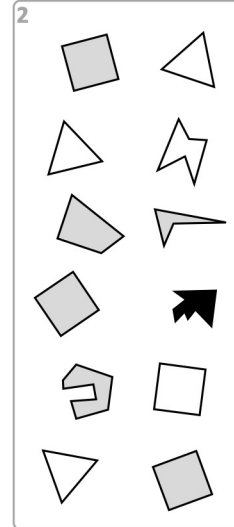


Shape
Venns

If a shape is picked randomly, what is:

$P(\text{a regular white quadrilateral})$

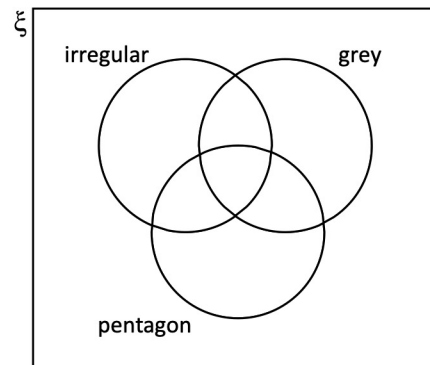
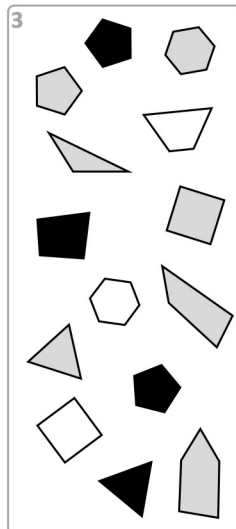
$P(\text{a white quadrilateral})$



If a shape is picked randomly, what is:

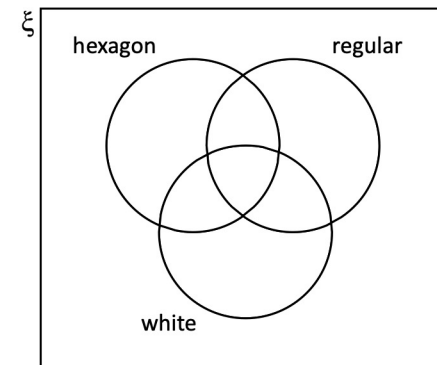
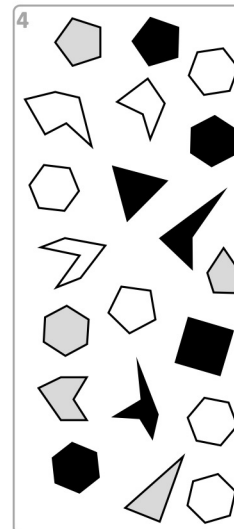
$P(\text{a irregular grey quadrilateral})$

$P(\text{a regular non-grey shape})$



A pentagon has been picked,
what is $P(\text{it is grey})$?

A grey shape has been picked,
what is $P(\text{it is an irregular pentagon})$?



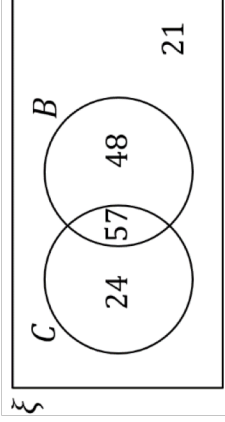
What is...

$P(\text{the shape is white, given that it is regular})$?

$P(\text{an irregular shape is not white})$

Purposeful Practice

The Venn diagram shows information of 150 patients in a local surgery. They were asked if they took any medication for cholesterol (C) or blood pressure (B).



A patient is chosen at random.

- (a) Work out the probability that a patient took neither medication.
- (b) Work out the probability that a patient took cholesterol not but blood pressure medication.
- (c) Given that the patient took blood pressure medication, what is the probability that they also took cholesterol medication?

90 people in a sports club were surveyed. 19 play tennis and squash. 50 play tennis. 32 play squash.

- (a) Represent this with a Venn diagram. One person is chosen at random.
- (b) Work out the probability that the person chosen does not play tennis
- (c) Work out the probability that the person chosen plays tennis or squash or both.
- (d) Given that the person plays tennis, work out the probability that they also play squash.

In a group of 40 children there are 19 who can swim and 16 who can ride a bike. There are 5 children who can swim and ride a bike.

- (a) Draw a Venn diagram. A child is selected at random.
- (b) Find the probability that this child cannot swim or ride a bike. Another child is selected at random.
- (c) Given that this child can ride a bike, work out the probability that this child can swim.

Purposeful Practice

90 children were asked they had in their packed lunch. Their replies are as follows: 38 had sandwich, crisps and cake. 60 had sandwich and a cake. 52 had sandwich and crisps. One student had crisps and cake only, and 5 students had a sandwich only. 10 students had none of these items in their packed lunch.

- (a) Show this on a Venn diagram.
- (b) Find the probability that a child chosen at random has both crisps and cake in their packed lunch.
- (c) Given that a child had a sandwich, find the probability that this child also had crisps.

100 people were asked which sports they watched on television. Here are the results.

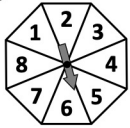
36 people watched cricket, 28 people watched rugby, 36 people watched football, 17 people watched both cricket and rugby, 19 people watched both cricket and football, 15 people watched both rugby and football, and 10 people watched all three sports.

- (a) Draw a Venn diagram.
- (b) One of the 100 people is selected at random. Find the probability that they watch none of these sports.
- (c) Given that a person watches cricket, find the probability that this person also watches football.
- (d) Given that a person watches at least one of the sports, find the probability that this person watches all three.

Fluency Practice

The General Addition Law

①

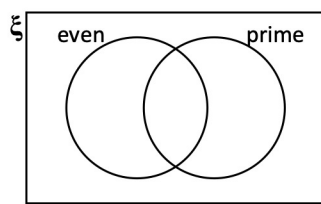


A fair spinner.

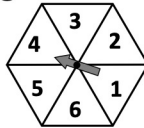
$P(\text{even}) =$

$P(\text{prime}) =$

$P(\text{even or prime}) =$



②

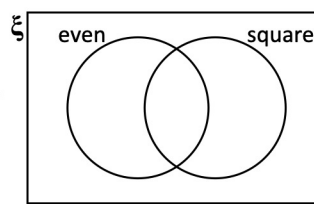


A fair spinner.

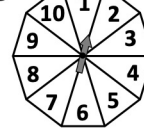
$P(\text{even}) =$

$P(\text{square}) =$

$P(\text{even or square}) =$



③

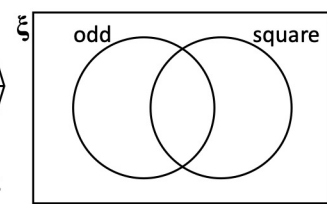


A fair spinner.

$P(\text{odd}) =$

$P(\text{square}) =$

$P(\text{even or square}) =$



④

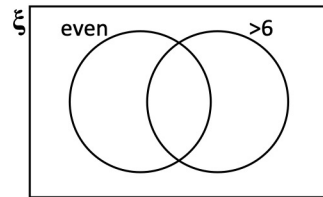


10 cards: 1 to 10

$P(\text{even}) =$

$P(\text{greater than 6}) =$

$P(\text{even or greater than 6}) =$



⑤

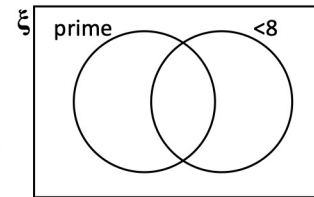


15 cards: 1 to 15

$P(\text{prime}) =$

$P(\text{less than 8}) =$

$P(\text{prime or less than 8}) =$



⑥

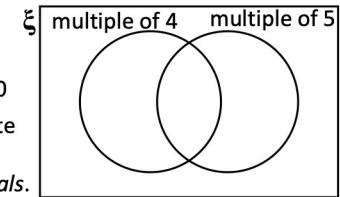


20 cards: 1 to 20
You can complete
the Venn
diagram with *totals*.

$P(\text{multiple of 4}) =$

$P(\text{multiple of 5}) =$

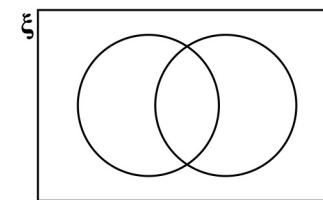
$P(\text{multiple of 4 or multiple of 5}) =$



⑦

A bag contains
50 spheres with
the numbers
1 to 50.

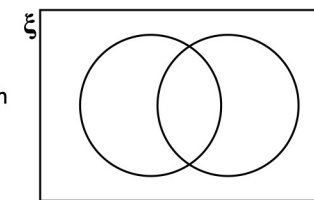
$P(\text{multiple of 7 or even}) =$



⑧

A bag contains
100 spheres with
the numbers
1 to 100.

$P(\text{multiple of 4 or a multiple of 10}) =$

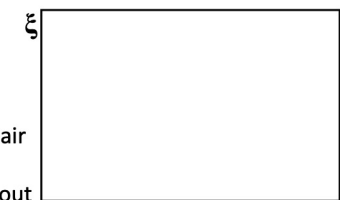


⑨



30 students:
12 with blonde hair
13 with glasses
10 students without
blonde hair or glasses

If we pick a student that is blonde,
what is the probability they wear glasses?



Fluency Practice

conditional probability
and venn diagrams

(1)

30 students in a class
20 have a brother
16 have a sister
12 have a brother but not a sister

what is the probability of picking a
student at random that has a brother,
given that they have a sister?

(2)

a survey of 125 students
asked whether they have a cat or a
dog in their home
61 had a cat
68 had a dog
13 had neither

what is the probability of picking a
student at random that has a cat,
given that they have a dog?

(3)

50 students
went to an activity camp
38 did climbing
13 did climbing and canoeing
4 did neither climbing or canoeing

what is the probability of picking a
student at random that did canoeing,
given that they did not do climbing?

(4)

a discrete survey of 90 students
asked if they have a laptop or a
tablet, or both, or neither
40 had a laptop
57 had a tablet
23 had neither

what is the probability of picking a
student at random that has a laptop,
given that they own just one of these
devices?

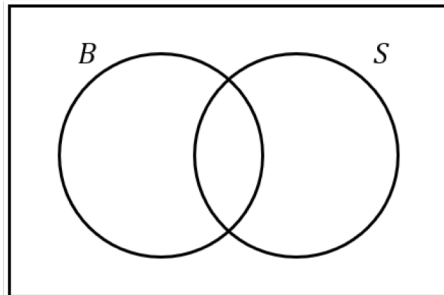
Purposeful Practice

Venn Diagrams with Algebra

(a)

Some children are asked about whether they have any sisters or brothers. Six children said they had both sisters and brothers. Eight children said they had no sisters or brothers. Seven children said they had brothers but no sisters. x children said they had sisters.

(i) Complete the Venn diagram, giving numbers in terms of x where needed.

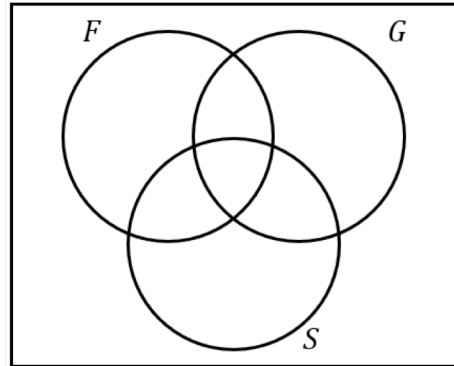


(ii) Given that there are 30 children in total, find the value of x using algebra.

(b)

Some students are asked whether they study French, German or Spanish. Four students study all three languages. Six study both French and German, and x students study both French and Spanish. No students study both German and Spanish but not French. 21 students study French. 15 students study Spanish. 18 students study German. All students must study at least one language.

(i) Complete the Venn diagram, giving numbers in terms of x where needed.

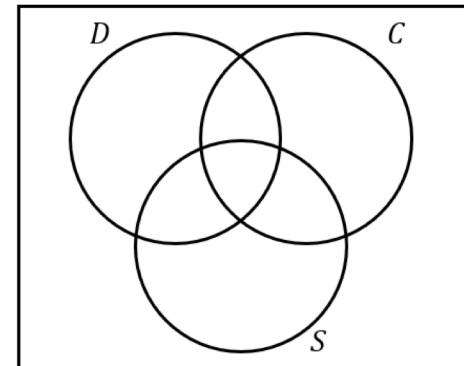


(ii) Given that there are 40 students in total, find the value of x using algebra.

(c)

Some TV viewers are asked whether they enjoy drama, comedy or sport programmes. Six viewers enjoy none of them. x viewers enjoy all three programmes. 15 viewers enjoy both drama and sport. 30 viewers enjoy both comedy and drama. 21 viewers enjoy both sport and comedy. 43 viewers enjoy comedy. 56 viewers enjoy sport. 38 viewers enjoy drama.

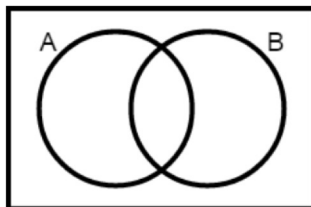
(i) Complete the Venn diagram, giving numbers in terms of x where needed.



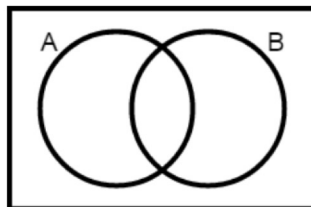
(ii) Given that there are 90 viewers in total, find the value of x using algebra.

Fluency Practice

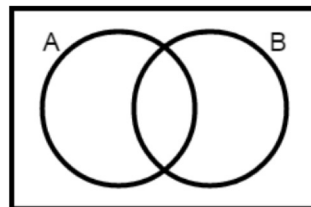
shade the regions corresponding to the sets



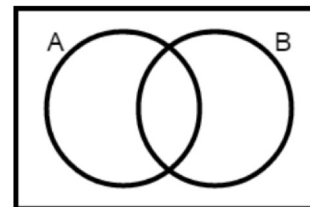
A



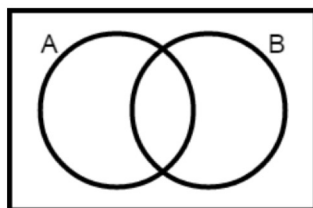
$A \cup B$



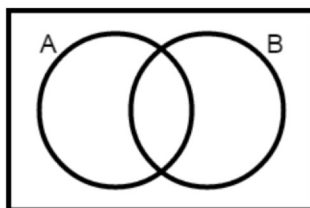
$A \cap B$



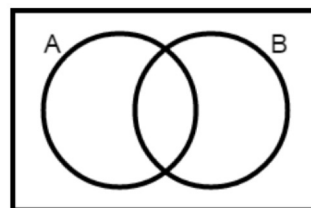
B'



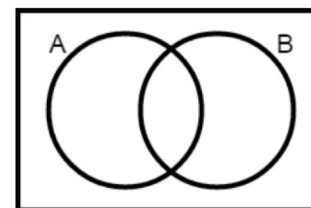
$A \cup B'$



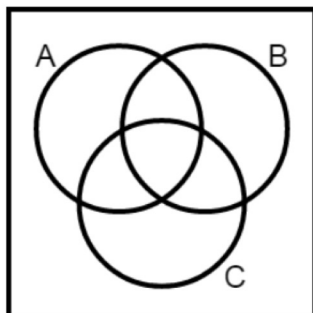
$A' \cap B$



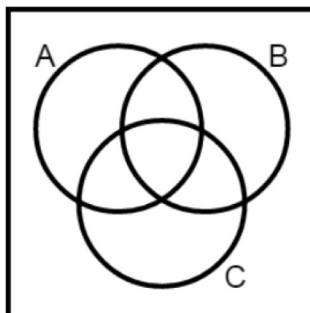
$A' \cap B'$



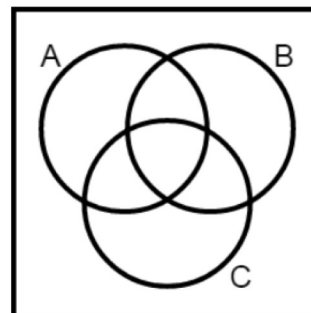
$(A \cup B)'$



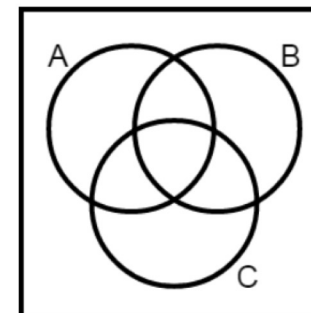
$A \cap C$



C'

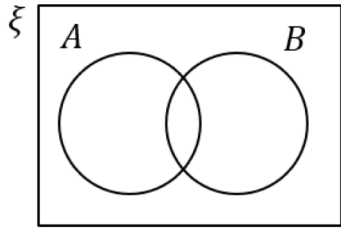
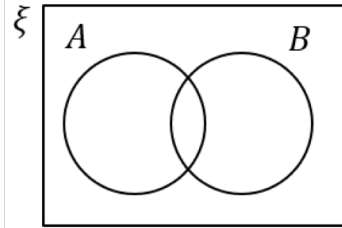
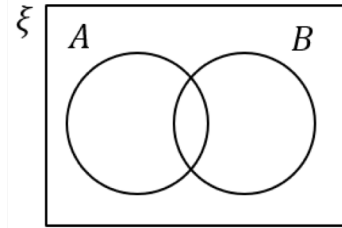
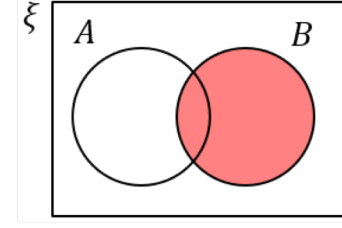
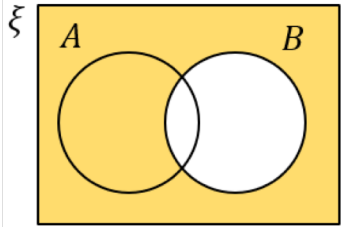
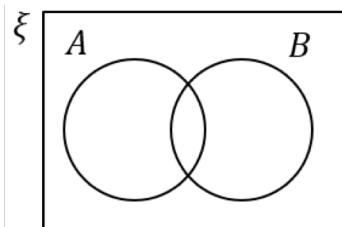
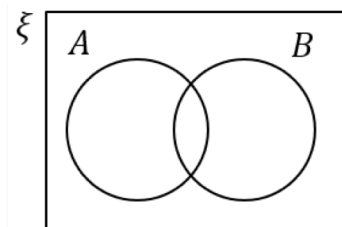
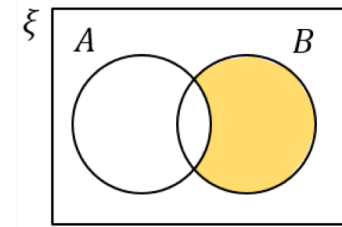
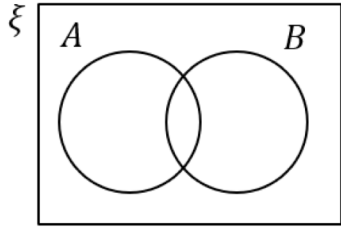
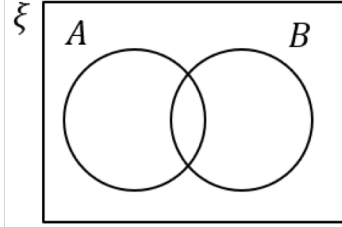
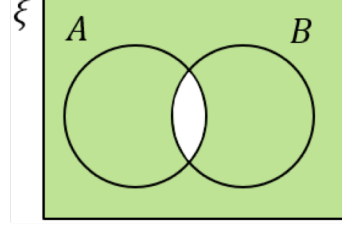
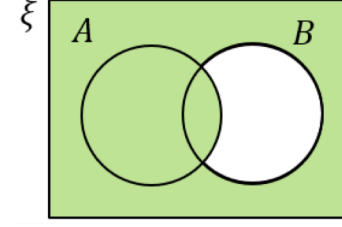


$B \cup C$



$A \cap C'$

Fluency Practice

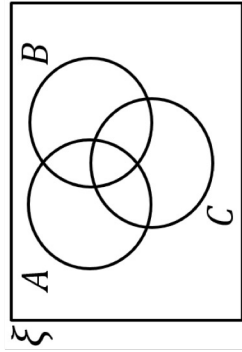
Shading Two Set Venn Diagrams			
(a)	(b)	(c)	(d)
Shade A' 	Shade $A \cap B$ 	Shade $A \cup B$ 	Describe the region shaded 
(e)	(f)	(g)	(h)
Describe the region shaded 	Shade $A \cap B'$ 	Shade $A' \cap B'$ 	Describe the region shaded 
(i)	(j)	(k)	(l)
Shade $A' \cup B$ 	Shade $(A \cup B)'$ 	Describe the region shaded 	Describe the region shaded 

Fluency Practice

Shading Three Set Venn Diagrams

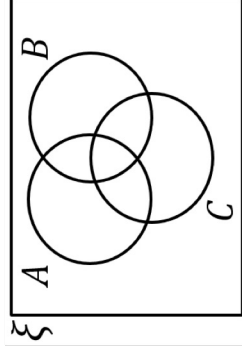
(a)

Shade the region corresponding to $A \cap B$



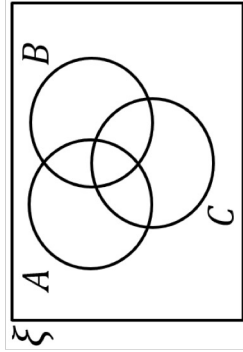
(b)

Shade the region corresponding to $(B \cup C)'$



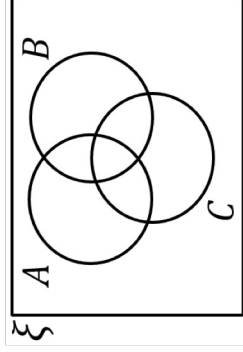
(c)

Shade the region corresponding to $A' \cap B$



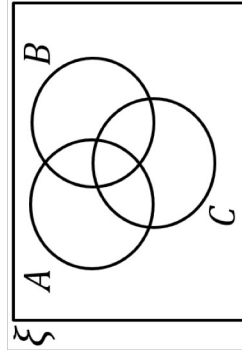
(d)

Shade the region corresponding to $(A \cap B \cap C)'$



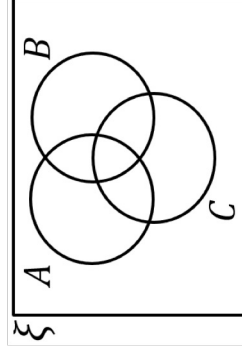
(e)

Shade the region corresponding to $(A \cup B \cup C)'$



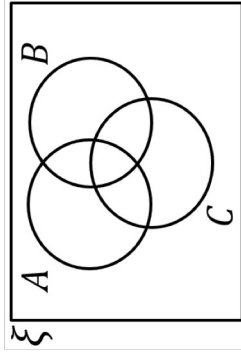
(f)

Shade the region corresponding to $(A \cap C) \cap B'$



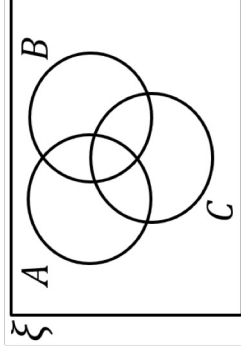
(g)

Shade the region corresponding to $A' \cap (B \cup C)$



(h)

Shade the region corresponding to $(B \cup C) \cup A'$



Fluency Practice

Describe these sets in words.

- (a) $\{4, 8, 12, 16, 20, 24, 28\}$
- (b) $\{1, 4, 9, 16, 25\}$
- (c) $\{\text{Europe, Asia, Africa, ...}\}$
- (d) $\{1, 2, 3, 4, 6, 12\}$

List the elements of the sets:

- (a) Multiples of 7 less than 30
- (b) Months of the year
- (c) Factors of 25

$A = \{\text{factors of } 6\}$

$B = \{\text{prime numbers less than } 20\}$

$C = \{\text{integers from } 1 \text{ to } 10\}$

- (a) List the elements of A
- (b) List the elements of B
- (c) List the elements of C
- (d) Write down $A \cup B$
- (e) Write down $B \cup C$
- (f) Write down $A \cap B$
- (g) Write down $C \cap A$

$A = \{\text{factors of } 6\}$

$B = \{\text{prime numbers less than } 20\}$

$C = \{\text{integers from } 1 \text{ to } 10\}$

State whether the following are true or false?

- (h) $9 \in C$
- (i) $9 \in B$
- (j) $9 \in A$
- (k) $6 \notin C$
- (l) $19 \in B$
- (m) $1 \notin C$

Fluency Practice

Set Notation

$A \cup B$

Union: the combination of A and B.

$A \cap B$

Intersection: The overlap of A and B.

A'

Complement: Not in A.

$\xi = \{ 1 \text{ to } 25 \text{ inclusive} \}$

$A = \{ \text{Factors of } 12 \} = \{ 1, 2, \quad \}$

$B = \{ \text{Factors of } 18 \} =$

$C = \{ \text{Factors of } 32 \} =$

$D = \{ \text{Prime Numbers} \} =$

$E = \{ \text{Multiples of } 4 \} =$

$F = \{ \text{Square numbers} \} =$

$A \cup B = \{ 1, 2, \quad \}$

$A \cup F =$

$A \cap B =$

$A \cap D =$

$D' =$

$A \cup D' =$

$B \cap D' =$

$B \cap C \cap F =$

$B \cup C \cup E =$

$A \cap B \cap C =$

$A \cup B \cap D' =$

Fluency Practice

$$A = \{1, 2, 7, 8, 10\}$$

$$B = \{1, 2, 3, 4\}$$

$$C = \{2, 4, 6, 8\}$$

- (a) List the elements of $A \cap B$
- (b) List the elements of $B \cup C$
- (c) Find $n(B)$
- (d) Find $n(A \cup C)$
- (e) $3 \in B$. True or false?
- (f) $2 \in B \cap C$. True or false?

$$A = \{n, u, m, b, e, r\}$$

$$B = \{e, q, u, a, l\}$$

$$C = \{s, i, x\}$$

- (a) List the elements of $A \cup C$
- (b) List the elements of $B \cap A$
- (c) Explain why $B \cap C = \emptyset$
- (d) Find $n(B \cup C)$
- (e) List the elements of $A \cap B'$
- (f) $d \notin A \cup B \cup C$. True or false?

$$\xi = \{5, 6, 7, 8, 9, 10, 11, 12\}$$

$$A = \{5, 7, 9, 11\}$$

$$B = \{9, 10, 11, 12\}$$

- (a) List the elements of $A \cup B$
- (b) List the elements of B'
- (c) Find $n(B \cap A)$
- (d) $10 \notin B$. True or false?
- (e) List the elements of $(A \cup B)'$
- (f) $11 \in A \cap B$. True or false?
- (g) Find $n(A' \cup B)$
- (h) List the elements of $A \cup B'$

Fluency Practice

For each statement, circle the correct response. Where a statement is false, correct it to make it true.

1	$A = \{3, 4, 5, 6, 7, 8\}$	$5 \notin A$	True	False
2	$\xi = \{1 \text{ to } 20 \text{ inclusive}\}$ $A = \{\text{Multiples of } 3\}$	$15 \in A$	True	False
3	$B = \{2, 4, 6, 8, 10, 12\}$	$n(B) = 6$	True	False
4	$\xi = \{1 \text{ to } 20 \text{ inclusive}\}$ $B = \{\text{Multiples of } 3\}$	$n(B) = 3$	True	False
5	$A = \{3, 4, 5, 6, 7, 8\}$ $B = \{1, 2, 3, 4, 5\}$	$A \cap B = \{3, 4\}$	True	False
6	$A = \{6, 7, 8, 9\}$ $B = \{1, 2, 3, 4, 5\}$	$A \cap B = \emptyset$	True	False
7	$A = \{6, 7, 8, 9\}$ $B = \{1, 2, 3, 4, 5\}$	$n(A \cup B) = 0$	True	False
8	$A = \{2, 4, 6, 8, 10\}$ $B = \{1, 2, 3, 4, 5\}$	$5 \in A \cap B$	True	False
9	$A = \{5, 7, 9, 11\}$ $B = \{3, 4, 5, 6, 7\}$	$A \cap B' = \{9, 11\}$	True	False
10	$A = \{4, 8, 12\}$ $B = \{6, 8, 10, 12, 14\}$	$n(B \cap A') = 2$	True	False
11	$A = \{4, 5, 6\}$ $B = \{3, 4, 5, 6, 7, 8, 9\}$	$B \subset A$	True	False
12	$\xi = \{1 \text{ to } 12 \text{ inclusive}\}$ $A = \{\text{Multiples of } 3\}$ $B = \{3, 4, 5, 6, 7, 8\}$	$n(A' \cap B') = 5$	True	False

Fluency Practice

$\xi = \{\text{Integers from 1 to 12}\}$

$A = \{\text{multiples of 2}\}$

$B = \{\text{multiples of 3}\}$

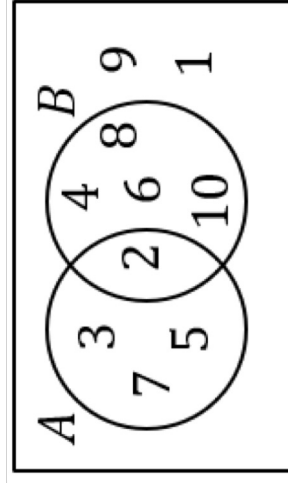
- (a) Show these sets on a Venn diagram
- (b) List the elements of $A \cap B$
- (c) List the elements of A'
- (d) List the elements of $A' \cup B$

$\xi = \{\text{Integers from 1 to 10}\}$

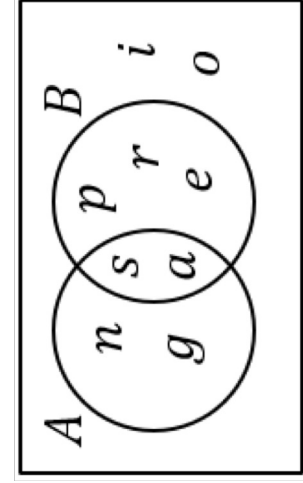
$A = \{\text{Integers five or less}\}$

$B = \{\text{factors of 12}\}$

- (a) Show these sets on a Venn diagram
- (b) List the elements of $A \cup B$
- (c) List the elements of B'
- (d) List the elements of $A' \cap B'$

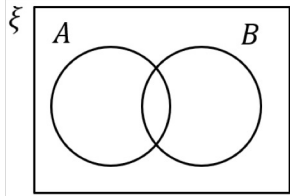
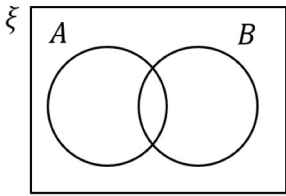
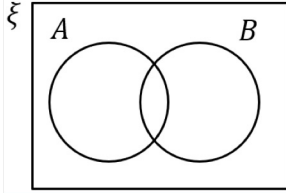
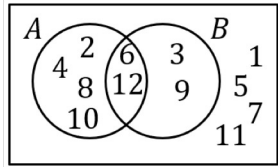
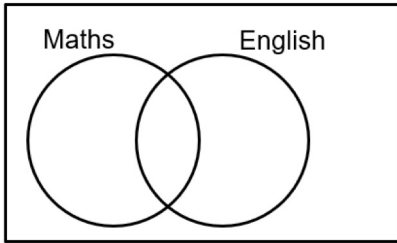
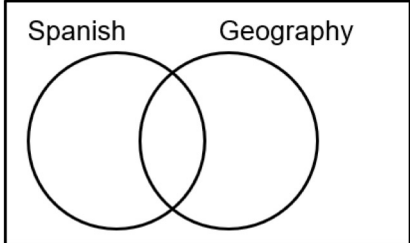


- (a) List the elements of $A \cap B$
- (b) List the elements of $(A \cap B)'$
- (c) List the elements of B'



- (a) List the elements of $A \cup B$
- (b) List the elements of $(A \cup B)'$
- (c) List the elements of $A' \cap B'$

Fluency Practice

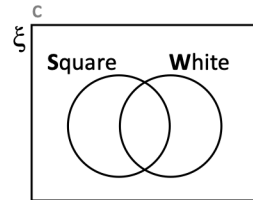
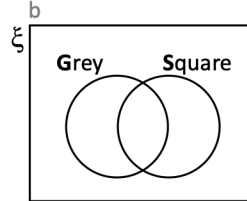
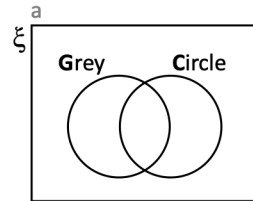
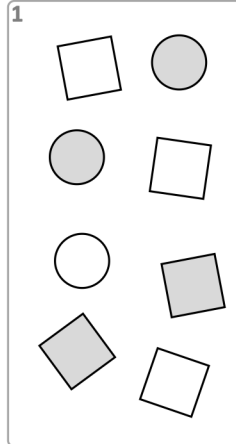
Sets and Venns Revision			
(a) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 2, 3, 4, 5, 6\}$ $B = \{\text{even numbers}\}$ List the members of $A \cap B$	(b) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{\text{factors of } 9\}$ $B = \{\text{multiples of } 4\}$ List the members of $A \cup B$	(c) $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{\text{factors of } 9\}$ $B = \{\text{multiples of } 4\}$ Anna says that $A \cap B = \emptyset$. Is she correct?	(d) $A = \{1, 3, 5, 7, 9\}$ $A \cap B = \{1, 3\}$ $A \cup B = \{0, 1, 2, 3, 4, 5, 7, 9\}$ List the members of B
(e) Shade the region which represents $A \cap B'$ 	(f) Shade the region which represents $A' \cup B$ 	(g) Show in a Venn diagram. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{1, 4, 9, 10\}$ $B = \{2, 4, 6, 8, 10\}$ 	(h) List the members of B' and $A' \cap B'$ 
(i) In a group of 20 students, 11 like Maths and 10 like English. 2 like neither subject. (a) Complete the Venn diagram. (b) How many students like Maths but not English?		(j) There are 32 students in a class. 21 students like Spanish and 15 like Geography. There are twice as many students who like both subjects as like neither. (a) Complete the Venn diagram. (b) How many students like only Spanish?	

Fluency Practice

A shape is
picked from
this bag...

Creating Venns

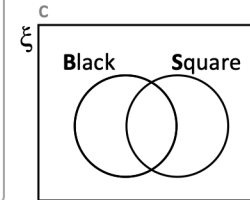
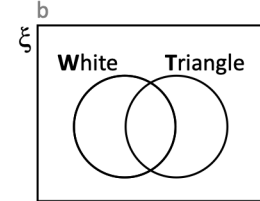
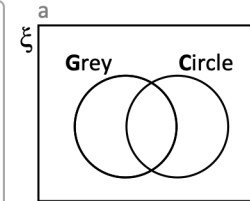
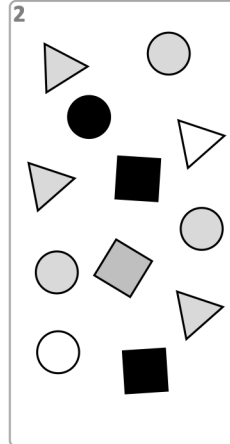
Each diagram shows a different bag of coloured shapes.
Complete the Venn diagrams & use them to calculate each probability.
Shade the diagrams to show the areas defined by the notation.



$$P(G \cup C) =$$

$$P(G \cup S) =$$

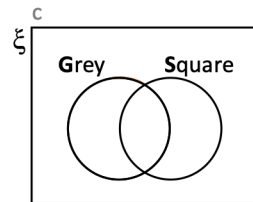
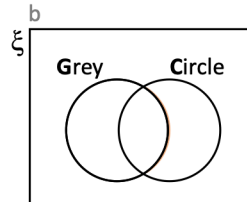
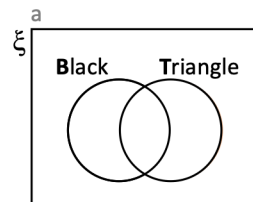
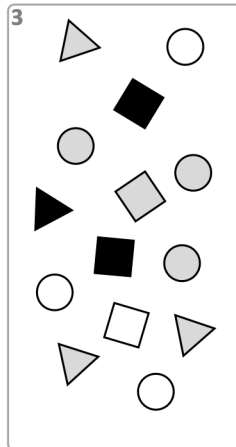
$$P(S \cup W) =$$



$$P(G \cap C) =$$

$$P(W \cap T) =$$

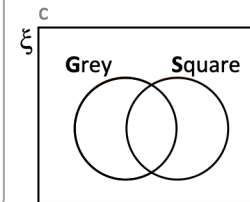
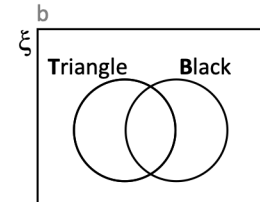
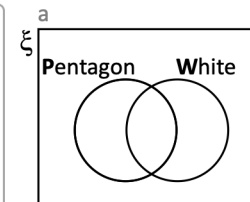
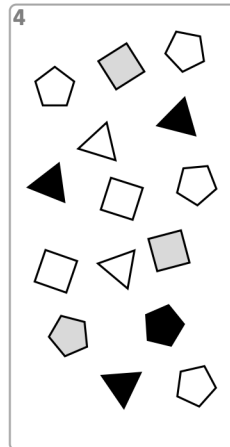
$$P(B \cap S) =$$



$$P(T') =$$

$$P(G') =$$

$$P(S \cup G') =$$



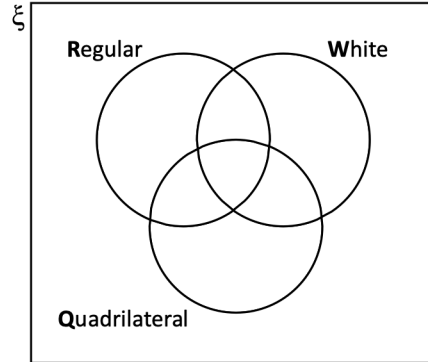
$$P(P \cup W) =$$

$$P(T \cap B) =$$

$$P(G' \cap S') =$$

Fluency Practice

1



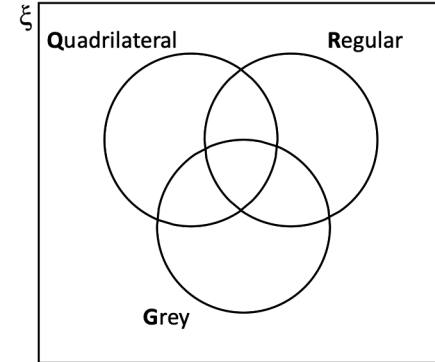
Shape
Venns

Shade $R \cap W \cap Q$

If a shape is picked randomly, what is:

$P(W') =$

2

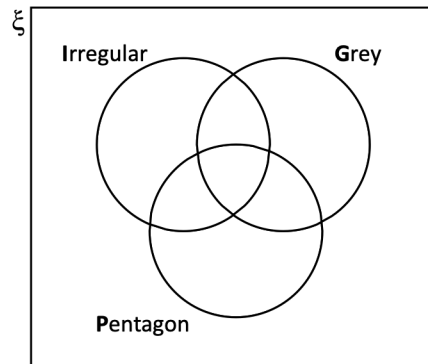


Shade $Q \cap R$

If a shape is picked randomly, what is:

$P(Q \cap R \cap G) =$

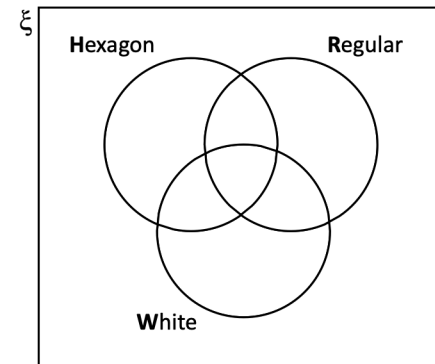
3



Shade $G \cap P$

What is the probability of randomly picking a shape in the set $(G \cap P)'$?

4



Shade $R \cap W$

What is the probability of randomly picking a shape in the set $(H \cap W)'$?

Fluency Practice

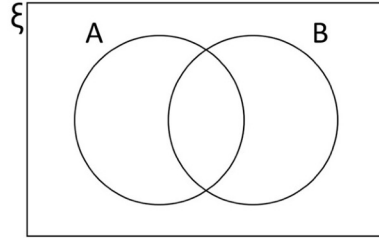
①

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5, 9, 10\}$

$B = \{3, 5, 6, 7, 10\}$

Complete the Venn diagram.



If I pick a number at random what is...

$P(A) =$

$P(A \text{ AND } B) = P(A \cap B) =$

$P(\text{NOT } A) = P(A') =$

$P(A \text{ OR } B) = P(A \cup B) =$

$P(B') =$

$P(A \text{ OR NOT } B) = P(A \cup B') =$

②

Remember!

$A' = \text{NOT } A$

$A \cap B = A \text{ AND } B$

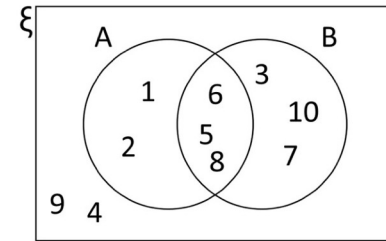
$A \cup B = A \text{ OR } B$

True or False?

$P(A) = \frac{1}{2}$ T / F

$P(B) = \frac{4}{5}$ T / F

$P(A') = \frac{4}{10}$ T / F



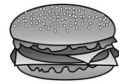
$P(A \cap B) = \frac{3}{10}$ T / F

$P(A \cup B) = \frac{7}{10}$ T / F

$P(A \cup B') = \frac{2}{5}$ T / F

③

50 students were asked about their lunch.



25 students had a burger (B)

35 students had fries (F)

15 students had both.

Complete the Venn diagram.

How many students had neither F or B?

If a student was chosen at random, what is... $P(B \cap F) =$

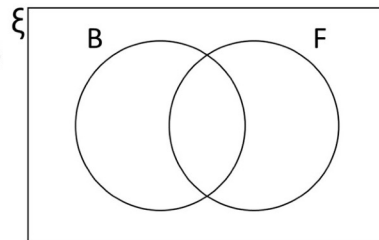
$P(B) =$

$P(F \cup B) =$

$P(F) =$

$P(B') =$

$P(F' \cap B') =$



④

100 students were asked about their school subjects:

15 studied History (H), French (F) and Drama (D).

40 studied Drama and History.

25 studied History and French.

55 studied History.

22 studied French and Drama.

3 studied only Drama.

40 studied French.

Complete the Venn diagram with this information.

$P(H) =$

$P(D') =$

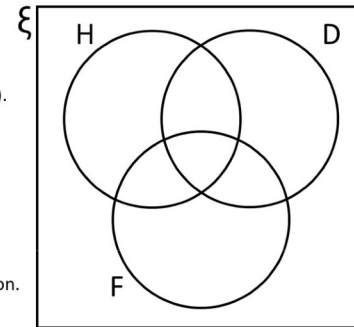
$P(F') =$

$P(D \cap F) =$

$P(H \cap F \cap D) =$

$P(H \cup D) =$

$P(D \cup F) =$



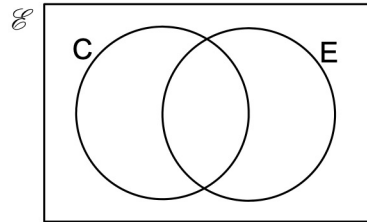
Purposeful Practice

venn diagrams 5

1. 60 people were asked if they read either of two local newspapers, the Chronicle and the Echo.

12 people read both newspapers.
36 people read the Chronicle.
25 people read the Echo.

(a) Write the number of people who belong in each section of the Venn diagram.

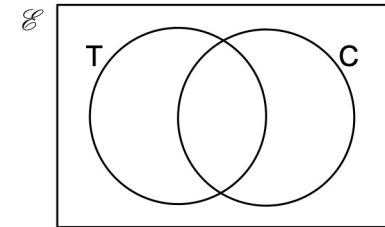


(b) What proportion of people read neither newspaper?

2. 85 people were asked about whether they like tea or coffee.

67 like at least one of the drinks.
29 like tea, but not coffee.
22 like coffee, but not tea.

(a) Write the number of people who belong in each section of the Venn diagram.

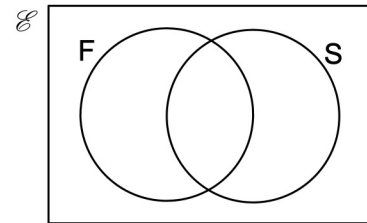


(b) One of the people is chosen at random. Work out $P(T \cap C)$ =

3. At a school, all pupils study French, Spanish or both.

In a class of 30 pupils 21 pupils study French and 13 pupils study Spanish.

(a) Write the number of people who belong in each section of the Venn diagram.



(b) A pupil is picked at random from the class. Work out:

$$P(F \cap S) =$$

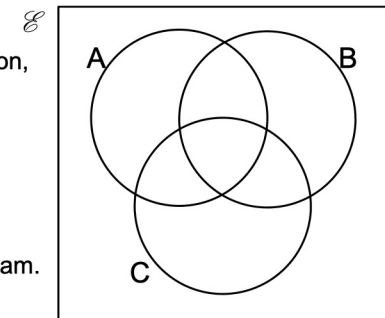
$$P(F') =$$

4. Students at a college can take part in different extra-curricular activities, including Aerobics classes, Badminton and Chess club.

In a group of 140 students,
7 do all three activities,
34 do aerobics and badminton,
22 do aerobics and chess,
19 do badminton and chess,
76 do aerobics,
62 do badminton,
46 do chess.

(a) Complete the Venn diagram.

(b) How many students do none of the three activities?



Purposeful Practice

venn diagrams 6

1. The proportion of elements in three of the sections of a Venn diagram are shown.

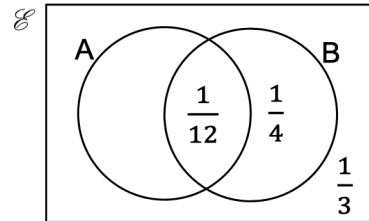
An element is chosen at random. Work out:

(a) $P(A \cup B) =$

(b) $P(B) =$

(c) $P(A') =$

(d) $P(B') =$

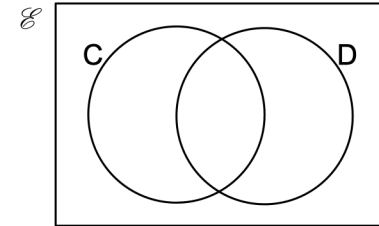


2. In a survey, people were asked about their pets.

34% had a pet cat (C)
42% had a pet dog (D)
11% had both a cat and a dog.

(a) Write the proportion of people who belong in each section of the Venn diagram.

(b) One of the people is chosen at random. Work out the probability that they have a cat, given that they have a dog.



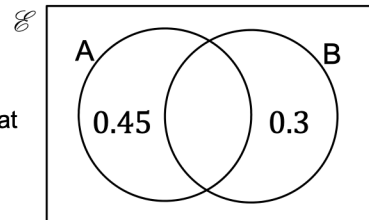
3. The proportion of elements in two of the sections of a Venn diagram are shown.

A fifth of the elements are in neither A nor B.

(a) Complete the Venn diagram.

(b) One of the elements is chosen at random.

Work out the probability that the element is in A, given that it is not in B.

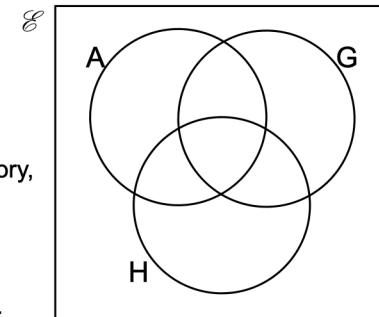


4. Students at a school can choose to study GCSEs in Art, Geography and History.

In a group of students,
12% study all of the subjects,
19% study Art and Geography,
23% study Art and History,
27% study Geography and History,
40% study Art,
54% study Geography,
2% study none of the subjects.

(a) Complete the Venn diagram.

(b) What proportion of the Art students also study History?



Purposeful Practice

venn diagrams 7

The numbers in the Venn diagrams show the proportions of elements that belong in each section. Use the clues to complete them.

$P(A \cap B) = \frac{1}{8}$

$P(C) = 0.5$

$P(C \cup D) = 0.6$

$P(E') = P(F)$

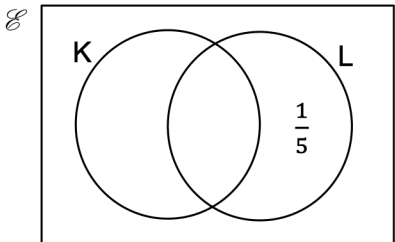
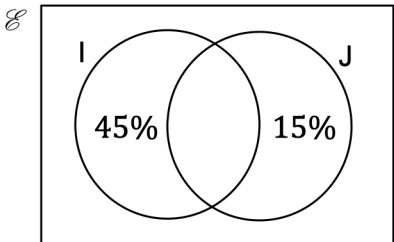
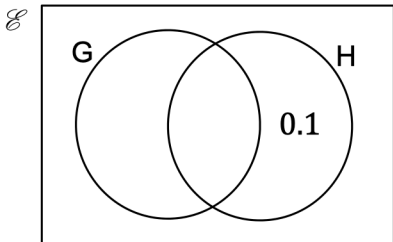
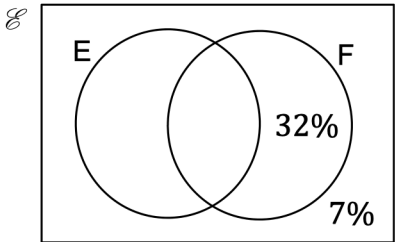
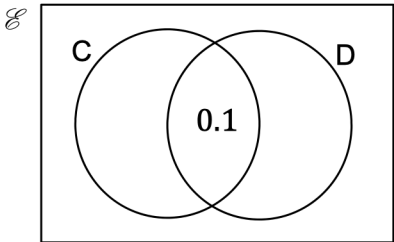
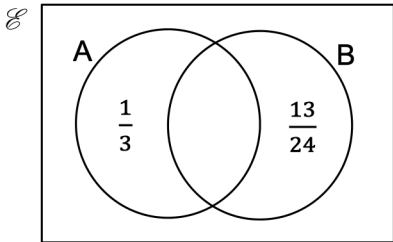
$P(G \cap H) = 0$

$P(G) = 0.3$

$P(I \cup J) = P(I) + P(J)$

$P(K) = P(K')$

$P(L) : P(L') = 2 : 3$

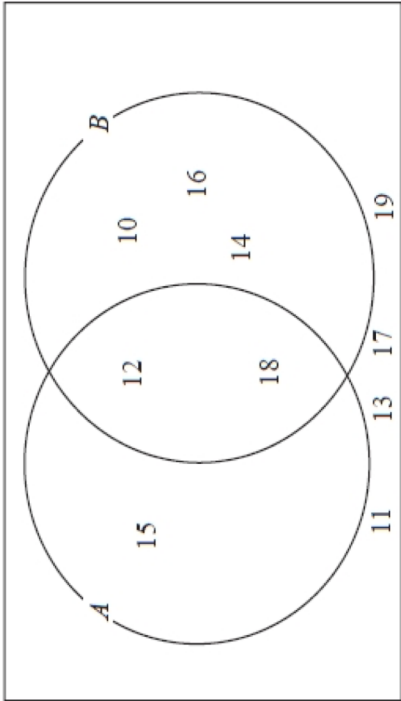


Put a tick in the 5 correct boxes.

	A & B	C & D	E & F	G & H	I & J	K & L
Exhaustive						
Mutually Exclusive						
Independent						

Exam Questions

Q1.
Here is a Venn diagram.



(a) Write down the numbers that are in set

(i) $A \cup B$

(ii) $A \cap B$

(2)

One of the numbers in the diagram is chosen at random.

(b) Find the probability that the number is in set A'

(2)

(Total for question = 4 marks)

Exam Questions

Q2.

Sami asked 50 people which drinks they liked from tea, coffee and milk.

- All 50 people like at least one of the drinks
- 19 people like all three drinks.
- 16 people like tea and coffee but do **not** like milk.
- 21 people like coffee and milk.
- 24 people like tea and milk.
- 40 people like coffee.
- 1 person likes only milk.

Sami selects at random one of the 50 people.

(a) Work out the probability that this person likes tea.

.....
(4)

(b) Given that the person selected at random from the 50 people likes tea, find the probability that this person also likes exactly one other drink.

.....
(2)

(Total for question = 6 marks)

Exam Questions

Q3.

There are 80 students at a language school.

All 80 students speak at least one language from French, German and Spanish.

9 of the students speak French, German and Spanish.

19 of the students speak French and German.

28 of the students speak French and Spanish.

17 of the students speak Spanish and German.

45 students speak French.

50 students speak Spanish.

(a) Draw a Venn diagram to show this information.

(3)

One of the 80 students is selected at random.

(b) Find the probability that this student speaks German but not Spanish.

(1)

Given that the student speaks German,

(c) find the probability that this student also speaks French.

(2)

(Total for question = 6 marks)

Exam Questions

The following results were obtained from a survey concerning the reading habits of students.

60 % read magazine P
 50 % read magazine Q
 50 % read magazine R
 30 % read magazines P and Q
 20 % read magazines Q and R
 30 % read magazines P and R
 10 % read all three magazines

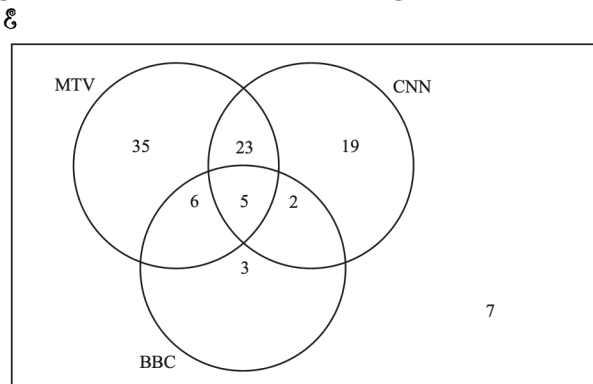
- (a) Represent all of this information on a **Venn diagram**.
- (b) What percentage of students read exactly two magazines?
- (c) What percentage of students read at least two magazines?
- (d) What percentage of students do not read any of the magazines?

A group of 30 children are surveyed to find out which of the three sports cricket (C), basketball (B) or volleyball (V) they play. The results are as follows:

3 children do not play any of these sports
 2 children play all three sports
 6 play volleyball and basketball
 3 play cricket and basketball
 6 play cricket and volleyball
 16 play basketball
 12 play volleyball.

- (a) Draw a **Venn diagram** to illustrate the relationship between the three sports played.
- (b) On your Venn diagram indicate the number of children that belong to each region.
- (c) How many children play only cricket?

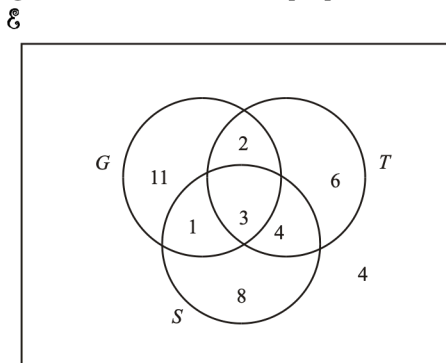
100 students were asked which television channel (MTV, CNN or BBC) they had watched the previous evening. The results are shown in the Venn diagram below.



From the information in the **Venn diagram**, write down the number of students who watched

- (a) both MTV and BBC;
- (b) MTV or BBC;
- (c) CNN and BBC but not MTV;
- (d) MTV or CNN but not BBC.

The sports offered at a retirement village are Golf (G), Tennis (T) and Swimming (S). The Venn diagram shows the numbers of people involved in each activity.



- (a) How many people
 - (i) only play golf?
 - (ii) play both tennis and golf?
 - (iii) do not play golf?
- (b) Shade the part of the **Venn diagram** that represents the set $G' \cap S$.

Exam Questions

In a club with 60 members, everyone attends either on Tuesday for Drama (D) or on Thursday for Sports (S) or on both days for Drama and Sports.

One week it is found that 48 members attend for Drama and 44 members attend for Sports and x members attend for both Drama and Sports.

- (a)
- (i) Draw and label fully a **Venn diagram** to illustrate this information.
 - (ii) Find the number of members who attend for both Drama and Sports.
 - (iii) Describe in words the set represented by $(D \cap S)'$.
 - (iv) What is the **probability** that a member selected at random attends for Drama only or Sports only?

The club has 28 female members, 8 of whom attend for both Drama and Sports.

- (b) What is the **probability** that a member of the club selected at random
- (i) is female and attends for Drama only or Sports only?
 - (ii) is male and attends for both Drama and Sports?

The sets A , B and C are subsets of U . They are defined as follows:

$\mathcal{E} = \{\text{positive integers less than } 16\}$

$A = \{\text{prime numbers}\}$

$B = \{\text{factors of } 36\}$

$C = \{\text{multiples of } 4\}$

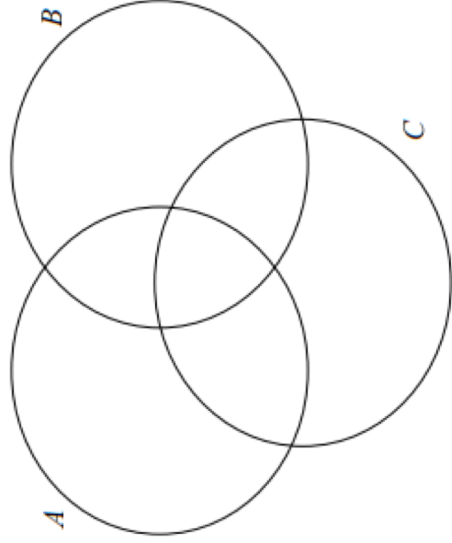
- (a) List the elements (if any) of the following:
- (i) A ;
 - (ii) B ;
 - (iii) C ;
 - (iv) $A \cap B \cap C$.
- (b)
- (i) Draw a **Venn diagram** showing the relationship between the sets U , A , B and C .
 - (ii) Write the elements of sets U , A , B and C in the appropriate places on the **Venn diagram**.
- (c) From the Venn diagram, list the elements of each of the following
- (i) $A \cap (B \cup C)$;
 - (ii) $(A \cap B)'$;
 - (iii) $(A \cap B)' \cap C$.
- (d) Find the **probability** that a number chosen at random from the universal set $\mathcal{E}$ will be
- (i) a prime number;
 - (ii) a prime number, but **not** a factor of 36;
 - (iii) a factor of 36 or a multiple of 4, but **not** a prime number;
 - (iv) a prime number, given that it is a factor of 36.

Exam Questions

The sets A , B , and C are as follows:

$$A = \{1, 2, 3, 5, 6, 7\} \quad B = \{2, 3, 4, 5, 8, 9\} \quad C = \{1, 4, 5, 10\}.$$

(a) Complete the Venn diagram below.



(b) List the elements of each of the following sets.

$$A \cup B = \underline{\hspace{2cm}}$$

$$A \setminus C = \underline{\hspace{2cm}}$$

$$A \cup (B \cap C) = \underline{\hspace{2cm}}$$

(c) Complete the following identity.

$$A \cup (B \cap C) = (A \cup B) \cap (\underline{\hspace{1cm}})$$

Exam Questions

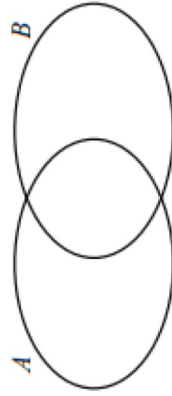
Question 2

(Suggested maximum time: 5 minutes)

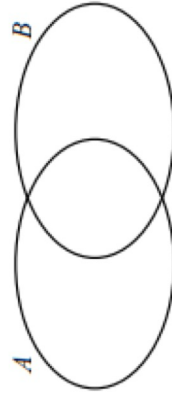
- (a) On the Venn diagram below, **shade in** the region that represents $A \cap B$.



- (b) On the Venn diagram below, **shade in** the region that represents $A \cup B$.



- (c) On the Venn diagram below, **shade in** the region that represents $(A \cup B) \setminus (A \cap B)$.



- (d) Put a tick (✓) in the correct box to show which of the following represents the elements that are **in A but not in B**.

$$B \setminus A$$

☐

$$A + B$$

☐

$$A \setminus B$$

☐

Question 5

(a) John was asked to give an example of a set. He said: "The set of good websites."

[illegible]

- (b) The sets U , A , and B are defined as follows:

A is the set of multiples of 2, from 1 to 10 inclusive

B is the set of multiples of 3, from 1 to 10 inclusive.

-

- [illegible]

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(Suggested maximum time: 10 minutes)

$$A = \{2, 3, 4, 5, 6\}, B = \{2, 4, 6, 8, 10\}, \text{ and } C = \{1, 4, 8, 12, 14\}.$$

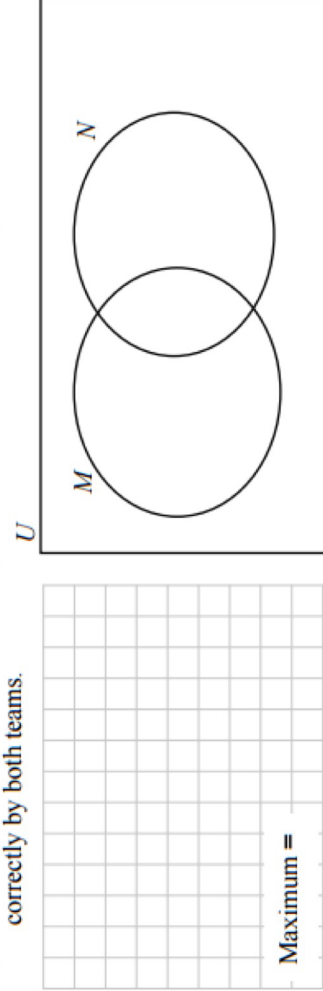
- $A \cap B =$
- _____

$$B \setminus (A \cap C) =$$

$$(B \setminus A) \cup (B \setminus C) =$$



- Minimum =



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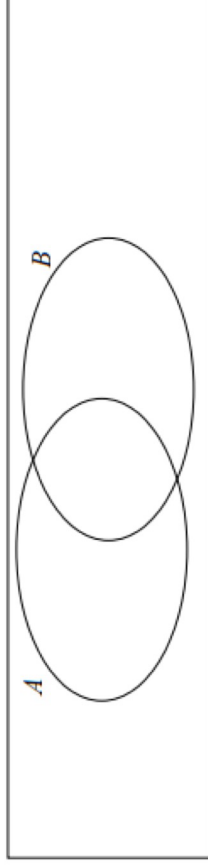
(Suggested maximum time: 5 minutes)

A Venn diagram with two overlapping circles. The left circle is labeled 'G' (Gaelic) and contains the text 'Camogie' and 'Hockey'. The right circle is labeled 'B' (British) and contains the text 'Hurling' and 'Rugby'. The intersection of the two circles contains the text 'Tennis', 'Basketball', and 'Football'.

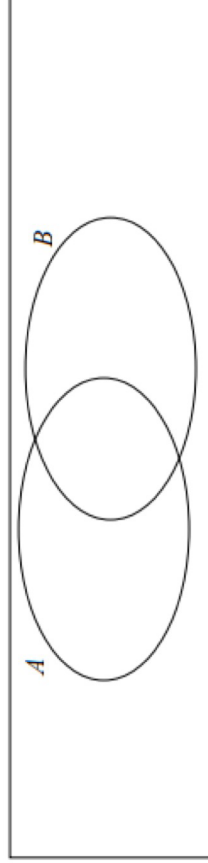
- (c) (i) In the Venn diagram, shade the set $A \cup B$.
- 
- (ii) In the Venn diagram, shade the set $(A \cup B)'$, where $(A \cup B)'$ is the complement of $A \cup B$.
- 

Exam Questions

1. (a) (i) Using the Venn diagram below, shade in the region that represents $A \cap B$.



- (ii) Using the Venn diagram below, shade in the region that represents $B \setminus A$.

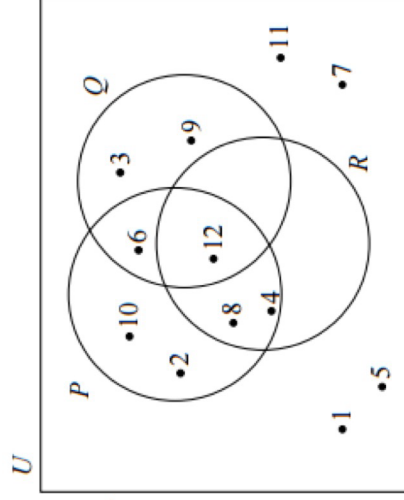


- (b) (i) U is the universal set.

$P = \{2, 4, 6, 8, 10, 12\}$ is the set of multiples of 2 less than 13.

$Q = \{3, 6, 9, 12\}$ is the set of multiples of 3 less than 13.

$R = \{4, 8, 12\}$ is the set of multiples of 4 less than 13.



- (i) List the elements of $\overline{Q \cup R}$.

- (ii) List the elements of P' , the complement of the set P .

- (iii) Write down $\#R$.

- (iv) What is the lowest common multiple of 2, 3 and 4?

Exam Questions

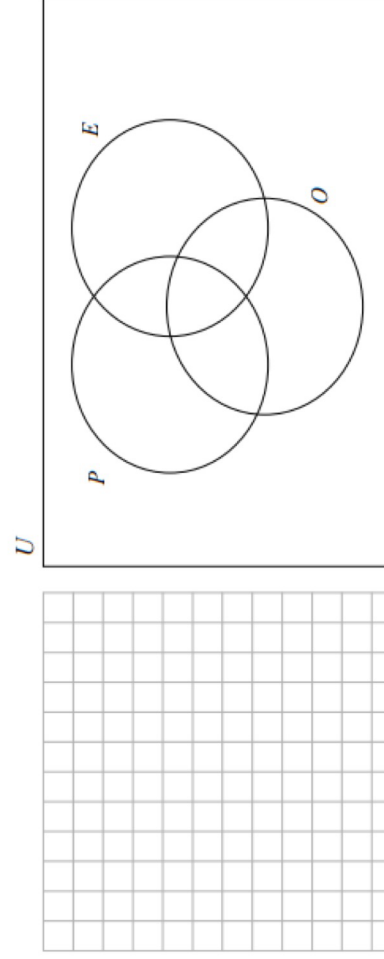
- (c) A survey was carried out in a class to find which of the films A, B or C the students had seen.
- The following data was collected:
- | | | |
|------------------------------|----------------------|----------------------|
| 42% saw film A | 41% saw film B | 45% saw film C |
| 12% saw both A and B | 18% saw both B and C | 15% saw both A and C |
| 15% saw none of these films. | | |
- (i) ~~/~~ Represent this information on a Venn diagram.
- (ii) ~~/~~ What percentage of the students in the class saw all three films?
- (iii) ~~/~~ What percentage of the students in the class saw two or more of the films?

Question 2

(Suggested maximum time: 5 minutes)

$U = \{1, 2, 3, \dots, 12\}$. P is the set of prime numbers less than 12. E is the set of even numbers less than 12. O is the set of odd numbers less than 12.

- (a) Represent these sets on the Venn diagram.



- (b) Name any set on this diagram (after part (a) has been completed) that is a null set.

[illegible]

- (c) If a number is drawn at random from **set P**, what is the probability that it is even?

[illegible]

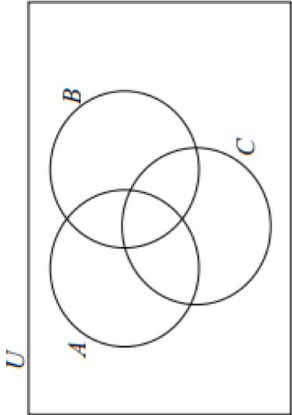
Exam Questions

Question 3

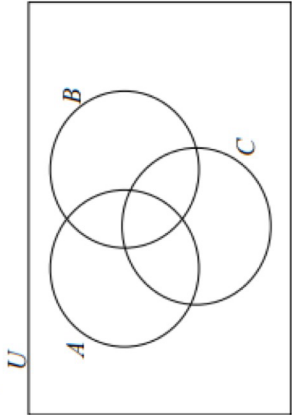
(Suggested maximum time: 15 minutes)

- (a) For diagrams (i) and (ii) below, shade in the named region.

(i)



(ii)



- (b) The box on the right contains six statements, (note: P' is the complement of a set P).

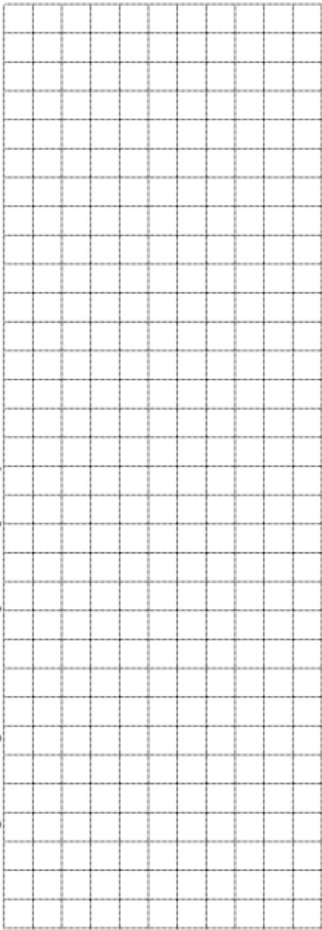
A number of the statements are incorrect.

Write down one incorrect statement.

Statements

- (i) $A \cup B = B \cup A$
- (ii) $(A \cup B) \cup C = A \cup (B \cup C)$
- (iii) $(A \setminus B) \setminus C = A \setminus (B \setminus C)$
- (iv) $(A \cap B)' = U \setminus (A \cap B)$
- (v) $A \setminus B = B \setminus A$
- (vi) $B \setminus (A \cup C) = (B \cup C) \setminus A \setminus C$

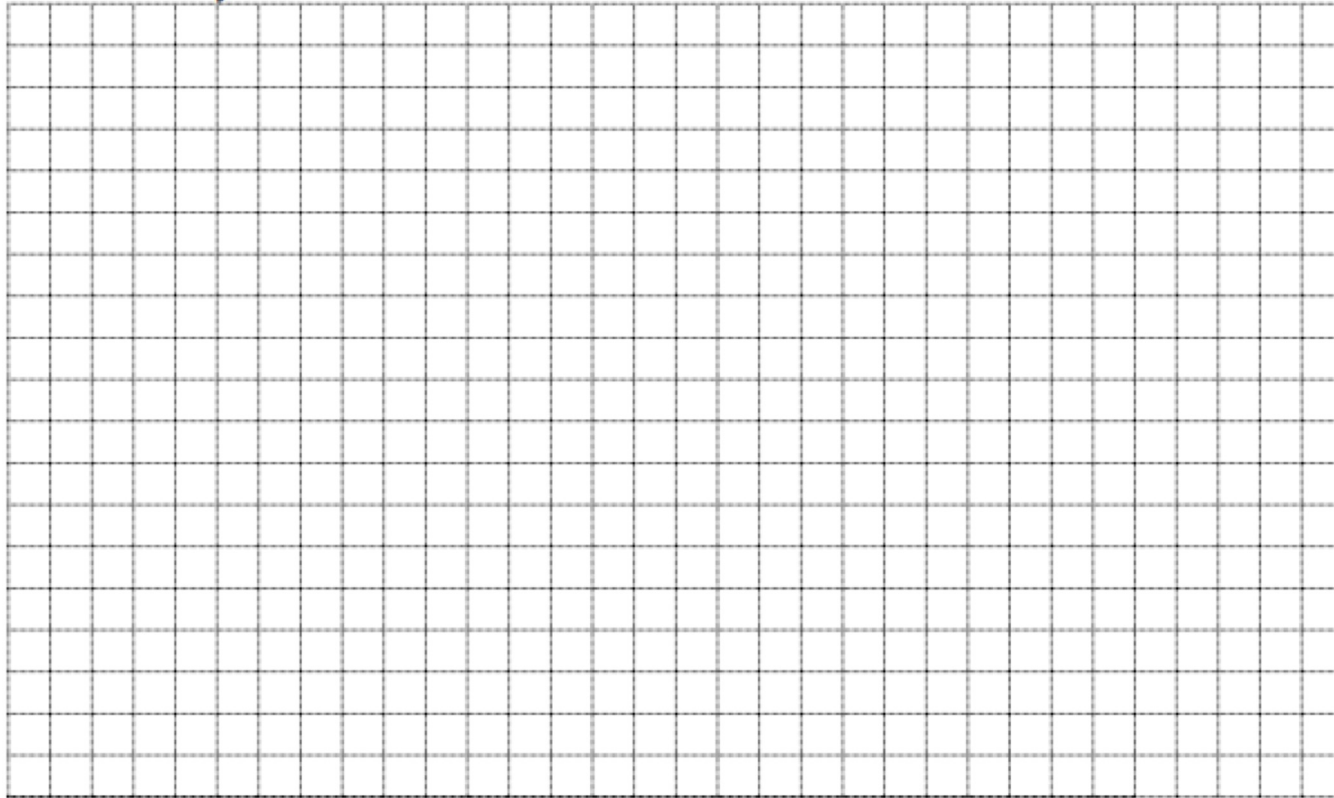
Draw a diagram **or** give an example to explain your choice.



Exam Questions

- (c) A group of 38 students were asked if they had ever been to France or Spain.
The number who had been to Spain only was 3 more than the number who had been to both countries.
Twice as many had been to France as Spain.
4 students had not been to either country.

Find how many had been to both countries.



Page 219

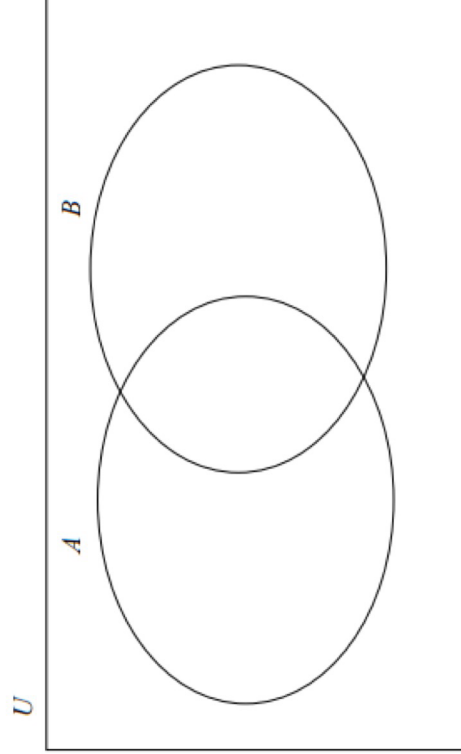
(Suggested maximum time: 5 minutes)

The universal set, $U = \{1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20\}$.

- $$A = \{ \underbrace{\dots}_{\text{...}} \}$$

- $$B = \{ \quad , \quad , \quad , \quad , \quad \}$$

- (c) Fill in the Venn diagram below placing all elements of U in the correct regions.



- $$A \cap B = \{ \}$$

- If an element is in the region $A \cap B$, it has two properties: it is a prime number and it is

- The number 16 is added to the universal set. Place 16 in the correct region in the Venn diagram in part (c) and explain why you placed it there.

Reason:

Exam Questions

(c) In a survey, 60 households were asked if they had a cat (C) or a dog (D).

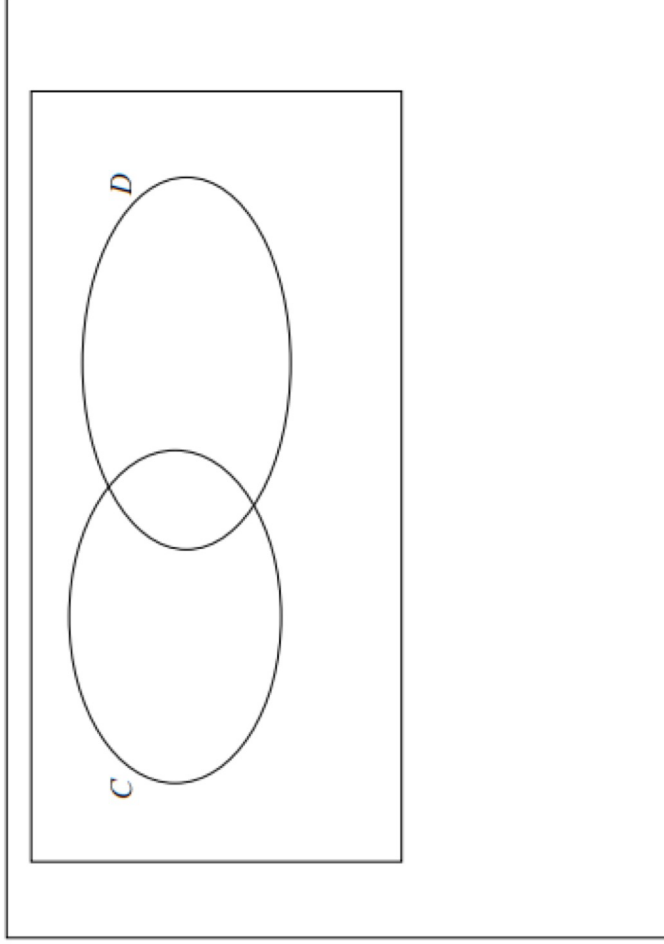
20 said they had a cat.

25 said they had a dog.

12 said they had both a cat and a dog.



(i) Represent this information in the Venn diagram below.



(ii) How many households had only a cat or a dog?

(iii) What percentage of households had neither a cat nor a dog?

Exam Questions

- (b) $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ is the universal set.
 $P = \{3, 5, 6, 8, 10\}$, $Q = \{2, 4, 6, 8, 10, 12\}$ and $R = \{2, 5, 6, 7, 9, 12\}$
are three subsets of U .

(i) Represent the above information on a Venn diagram.

Hence list the elements of:

(ii) $(P \cup Q \cup R)'$

(iii) $(P \cap Q) \setminus R$.

- (c) U is the universal set and P and Q are two subsets of U .
 $\# U = 30$, $\# P = 16$ and $\# Q = 6$.

(i) ✎ Find with the aid of a Venn diagram the minimum value of $\#(P \cup Q)'$.

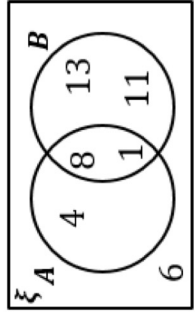
(ii) ✎ Find with the aid of a Venn diagram the maximum value of $\#(P \cup Q)'$.

$\# U = u$, $\# P = p$, $\# Q = q$ and $\#(P \cup Q)' = x$.

(iii) ✎ Show with the aid of a Venn diagram, that if $p > q$ and x is a maximum,
then $u = p + x$.

Fluency Practice

1.

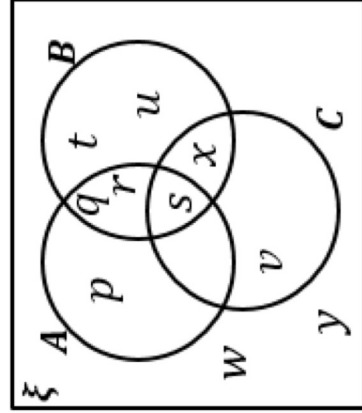


List the numbers in:

- $A \cap B$
- $A \cup B$
- A'
- $A' \cap B$

e. Given that a number is chosen at random, find the probability it is in set B .

2.



Determine the sets:

- $A \cap B$
- $A \cup C$
- $A' \cap C'$
- $A \cap B \cap C$
- $A \cap B' \cap C$
- $(A \cup C) \cap B'$

g. Given that a number is chosen at random from the set $A \cup C$, find the probability it is in set B .

3. A and B are two sets such that:

$$\begin{aligned} n(\xi) &= 20 \\ n(A) &= 14 \\ n(A \cap B) &= 3 \\ n(B') &= 12 \end{aligned}$$

Form a Venn diagram, where the number in each region is the **number** of elements in it.

4. $A = \{f, r, o, s, t\}$
 $B = \{b, a, r, t, o, n\}$

Determine the sets:

- $A \cap B$
- $A \cup B$
- $A' \cap B$

5. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $A = \{\text{all prime numbers}\}$
 $B = \{\text{all multiples of } 6\}$
 Determine:
 a. $A \cup B$
 b. $A \cap B$

6. $\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
 $A = \{\text{all even numbers}\}$
 $B = \{\text{all factors of } 8\}$
 Determine:
 a. $A \cup B$
 b. $A \cap B$
 c. $A \cap B'$

7. Construct a Venn Diagram for sets A, B, C (without numbers) such that:

- $A \subset B, B \cap C = \emptyset$
- $A \subset B, A \cap C = \emptyset$

✎ A, B, C are sets such that:

$$\begin{aligned} n(A) &= 20, \\ n(B) &= 20, \\ n(C) &= 20, \\ n(A \cap B) &= 10 \\ n(A \cap C) &= 10 \\ n(B \cap C) &= 10 \\ n(A \cap B \cap C) &= 5 \end{aligned}$$

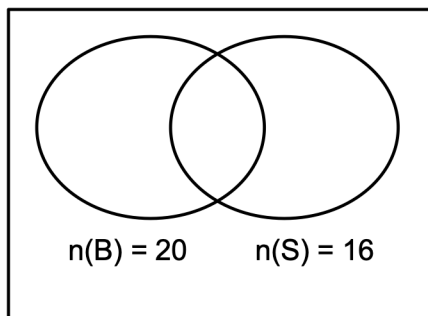
Determine $n(A \cup B \cup C)$.

Fluency Practice

conditional probability
and venn diagrams

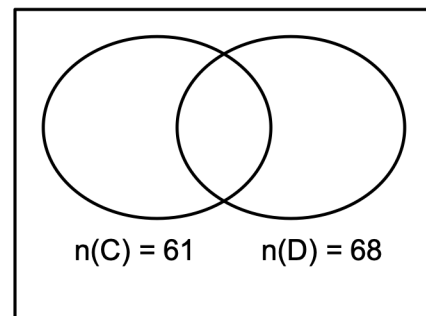
(1)

$$n(E) = 30$$



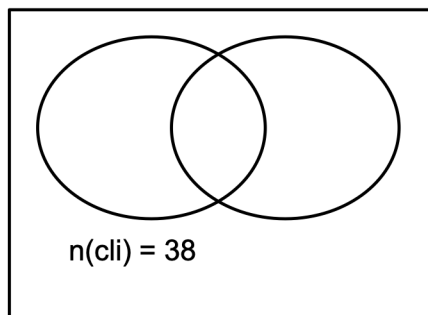
(2)

$$n(E) = 125$$



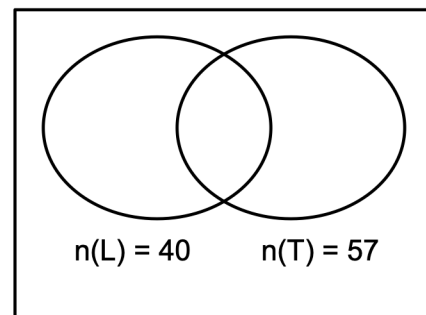
(3)

$$n(E) = 50$$



(4)

$$n(E) = 90$$



Purposeful Practice

A garage has 50 cars for sale. 15 of the cars have both air conditioning and cruise control. 31 of the cars have air conditioning. 17 of the cars have cruise control. Draw a Venn diagram to represent this information.

- (a) How many cars do not have air conditioning or cruise control?
- (b) How many cars have air conditioning but not cruise control?

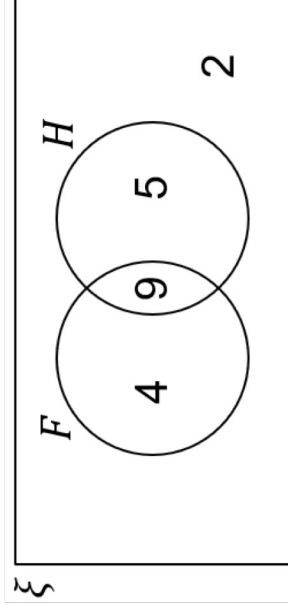
90 people in a sports club were surveyed. 19 play tennis and squash. 50 play tennis. 32 play squash. Draw a Venn diagram to represent this information.

- (a) How many people do not play squash, but do play tennis?
- (b) How many people play neither squash nor tennis?

There are 16 people in a queue. Two are wearing both a hat and a scarf. Eight are wearing a hat. Seven are wearing neither a hat nor a scarf. Draw a Venn diagram to represent this information.

- (a) How many people are wearing a scarf?
- (b) How many people are wearing either a hat or a scarf or both?

The Venn shows how many students in a class play football (F) or hockey (H).

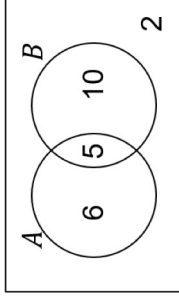
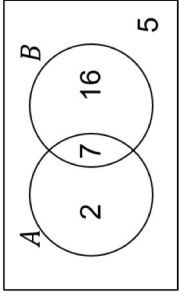
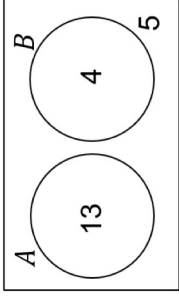


- (a) How many students play both sports?
- (b) Write down $n(H')$
- (c) Write down $n(F' \cap H)$

Fluency Practice

Set Notation and Venn Diagrams

In each question, the values represent the number of elements in each set.

(a)	(b)	(c)
 Write down: (a) $n(A \cap B)$ (b) $n(A)$ (c) $n(B')$ (d) $n(\xi)$	 Write down: (a) $n(B)$ (b) $n(A')$ (c) $n(A \cup B)$ (d) $n(A \cap B')$	 Write down: (a) $n(A \cap B)$ (b) $n(A')$ (c) $n(\xi)$ (d) $n(A' \cap B')$
(d)	(e)	(f)
$n(A \cap B) = 5$ $n(A) = 13$ $n(A \cup B) = 16$ $n(\xi) = 20$ Draw a Venn diagram to represent this information.	$n(A \cap B) = 7$ $n(A) = 13$ $n(B) = 10$ $n(B') = 10$ Draw a Venn diagram to represent this information.	$n(A \cap B) = 10$ $n(A \cup B) = 18$ $n(A) = 13$ $n(\xi) = 20$ Draw a Venn diagram to represent this information.
(g)	(h)	(i)
$n(A \cup B) = 20$ $n(A') = 15$ $n(A \cap B) = 6$ $n(B) = 16$ Draw a Venn diagram to represent this information.	$n(B) = 7$ $n(A) = 10$ $n(A \cup B) = 12$ $n(\xi) = 20$ Draw a Venn diagram to represent this information.	$n(A' \cap B') = 10$ $n(\xi) = 25$ $n(B) = 11$ $n(A) = 8$ Draw a Venn diagram to represent this information.

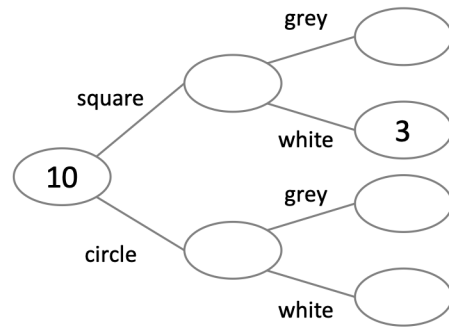
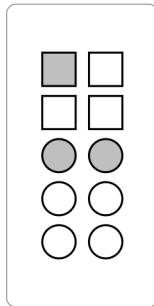
Fluency Practice

A1 $A = \{2, 3, 5, 7, 11\}$ $B = \{3, 5, 7, 9\}$ List $A \cup B$	A2 $A = \{\text{factors of } 100\}$ $B = \{\text{multiples of } 5\}$ List $A \cap B$	A3 $A = \{c, h, i, n, a\}$ $B = \{i, t, a, l, y\}$ List $A \cup B$	A4 $A = \{s, u, p, e, r\}$ $B = \{c, o, m, p, u, t, e, r\}$ List $A \cap B$
B1 $A = \{1, 3, 6, 10, 15\}$ $B = \{3, 6, 9, 12\}$ Find $n(A \cup B)$	B2 $S = \{s, q, u, a, r, e\}$ $V = \{a, e, i, o, u\}$ Find $n(S \cap V)$	B3 $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $A = \{2, 3, 5\}$ Find $n(A')$	B4 $M = \{2, 4, 6, 8, 10\}$ $N = \{1, 3, 5, 7, 9\}$ Find $n(M \cap N)$
C1 $A = \{2, 4, 6, 8, 10\}$ $B = \{1, 3, 5, 7, 9\}$ Explain why $A \cap B = \emptyset$	C2 $A = \{\text{multiples of } 5\}$ $D = \{\text{prime numbers}\}$ Is it true that $A \cap D = \emptyset$?	C3 $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $A = \{\text{even numbers}\}$ $B = \{\text{multiples of } 3\}$ $x \in B \text{ and } x \notin A$ What are the possible values of x ?	C4 $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{1, 3, 5, 7\}$ $B = \{2, 4, 6, 8\}$ $x \in \mathcal{E} \text{ and } x \notin A \cup B$ What is the value of x ?
D1 $\mathcal{E} = \{\text{even numbers}\}$ $A = \{2, 4, 6, 8, 10\}$ B is such that $A \cap B = \{4, 8\}$ and $n(B) = 3$. List a possible set B .	D2 $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7\}$ $A = \{2, 3, 4, 5\}$ C is such that $A \cap C = \{4, 5\}$ and $n(C) = 4$. List a possible set C .	D3 $\mathcal{E} = \{2, 4, 6, 8, 10, 12, 14\}$ $A = \{2, 6, 8, 12\}$ C is such that $A \cap C = \emptyset$ and $n(C) = 3$. List set C .	D4 $A = \{3, 4, 5\}$ $A \cup B = \{1, 2, 3, 4, 5\}$ B has as few members as possible. List set B .

Fluency Practice

FPR Frequency Trees

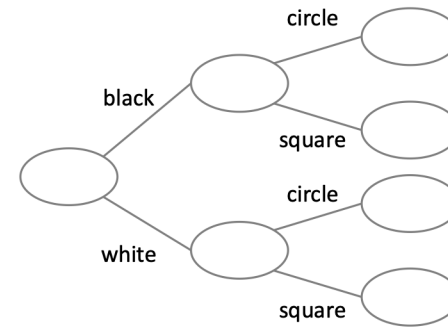
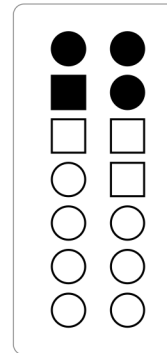
1) Complete the frequency tree for this group of shapes.



What fraction of the shapes are white squares?

Express the ratio grey circles : white circles

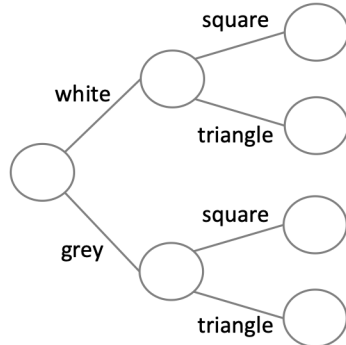
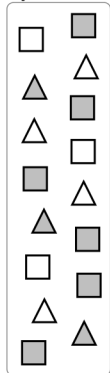
2) Complete the frequency tree.



What fraction of the shapes are black?

What percentage of the white shapes are squares?

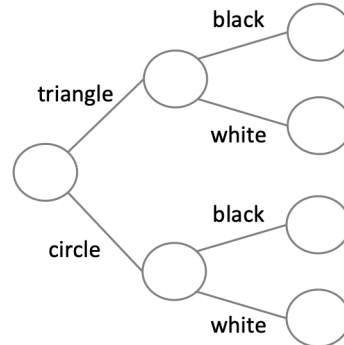
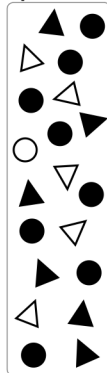
3)



What percentage of the grey shapes are triangles?

Express the ratio white shapes : grey squares

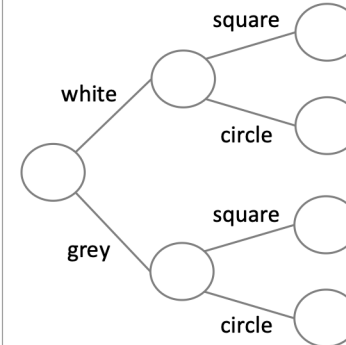
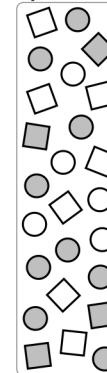
4)



What percentage of the shapes are black?

Express the ratio circles : black triangles

5)



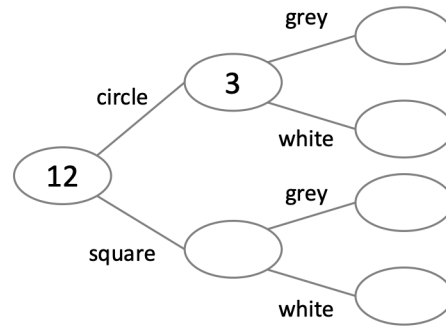
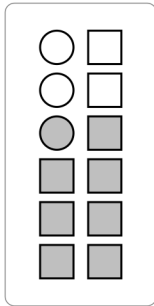
What fraction of the circles are white?

What % of the shapes are grey or circles?

Fluency Practice

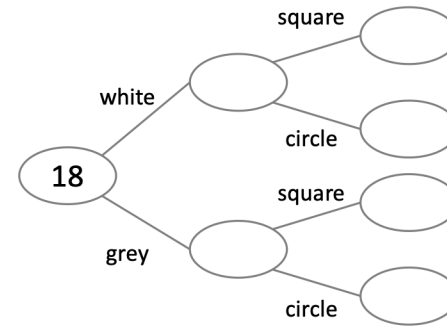
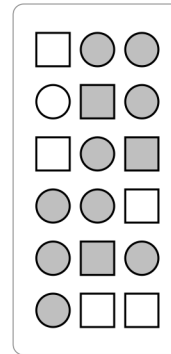
Frequency Trees

1) Complete the frequency tree for this group of shapes.



If a shape is picked randomly from the group,
what is the probability it is a grey square?

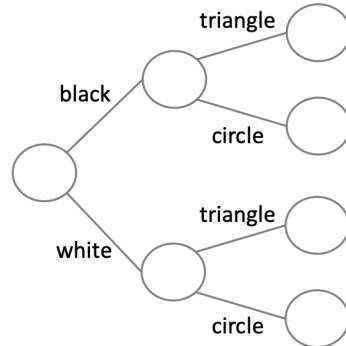
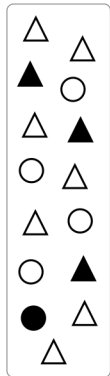
2) Complete the frequency tree.



What is the probability of picking...
... a white square?

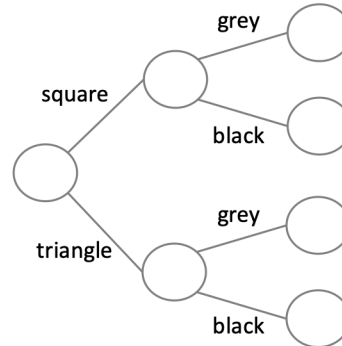
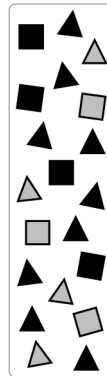
... a grey circle?

3) Complete the frequency tree.



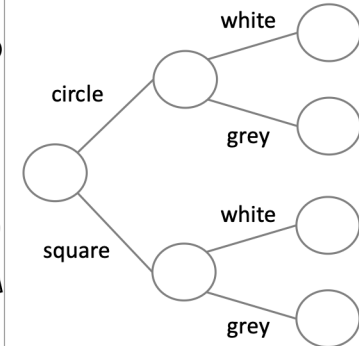
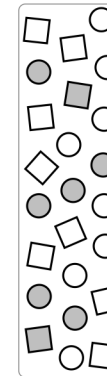
What is... Probability (picking a black shape)?
Probability (picking a circle)?

4) Complete the frequency tree.



A square is picked from the group.
What is the probability it is grey?

5) Complete the frequency tree.

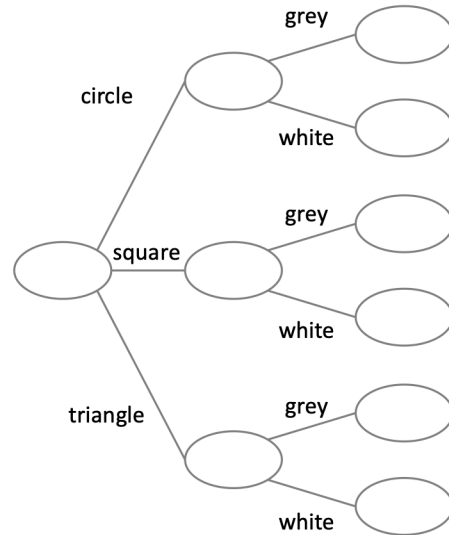
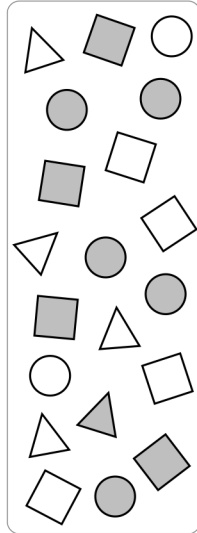


A grey shape is picked from the group.
What is the probability it is a square?

Fluency Practice

Frequency Trees

1) Complete the frequency tree for this group of shapes.



If we pick a shape at random, what is...

P(square)?

P(grey circle)?

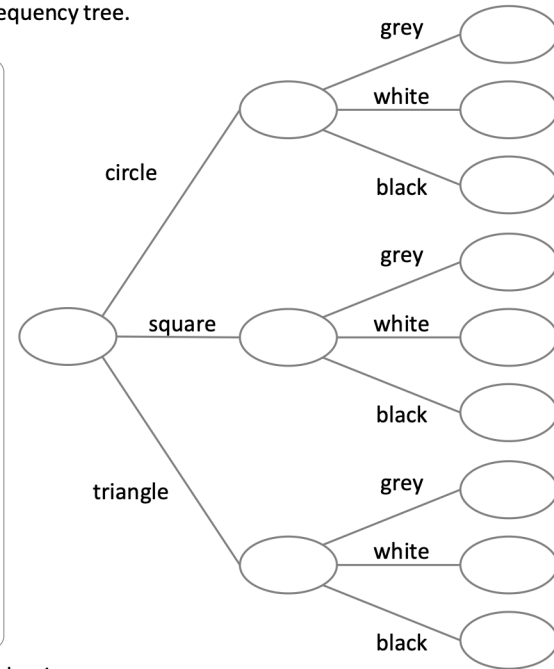
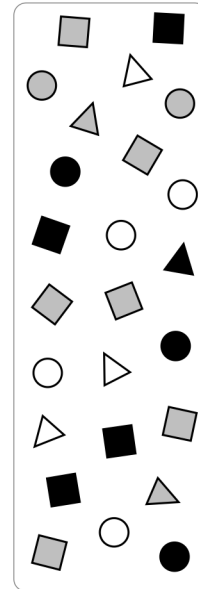
P(white shape)?

What is the probability a circle or a square is picked?

A triangle is picked, what is the probability it is grey?

A grey shape is picked, what is the probability it is a triangle?

2) Complete the frequency tree.



As a percentage, what is...

P(triangle)?

P(white circle)?

P(a grey circle or a black triangle)?

P(a black shape **given that** it is a square)?

P(a square **given that** it is black)?

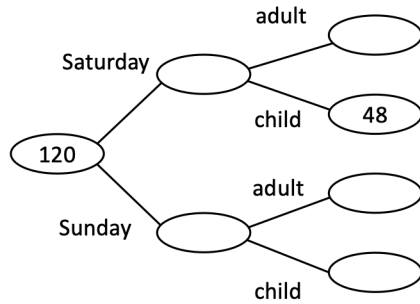
What is the probability a white shape or a triangle is picked?

Fluency Practice

a) A shop recorded the customers it had over the weekend.

$\frac{2}{3}$ of the customers came on Saturday.

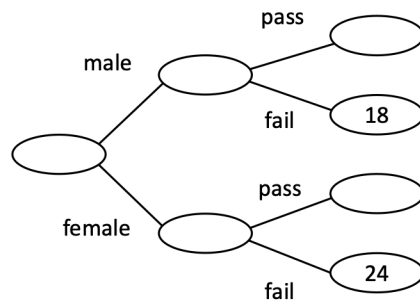
30% of Sunday customers were children.



What fraction of the customers were adults?

d) For a science test, the ratio of passing students to failing students was 5 : 3

$\frac{4}{9}$ of the female students failed.



What fraction of the students were male?

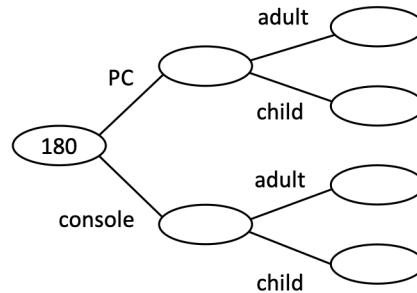
Frequency Trees: FPR

b) A game company compiled data on players.

The ratio of PC to console users was 4 : 5

$\frac{3}{5}$ of console users were children.

25% of PC users were adults.

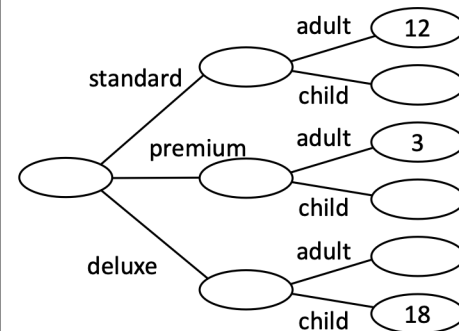


What fraction of the players were children?

e) $\frac{5}{7}$ of deluxe cinema tickets were sold to adults.

75% of standard tickets were sold to children.

The ratio of adult tickets to child tickets was 4 : 7

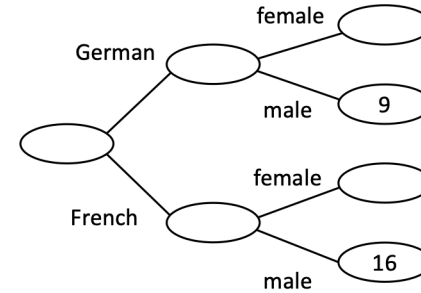


What fraction of the tickets were deluxe?

c) Students were surveyed.

$\frac{3}{8}$ of the students that studied German were male.

The ratio of male to female French students was 2 : 5



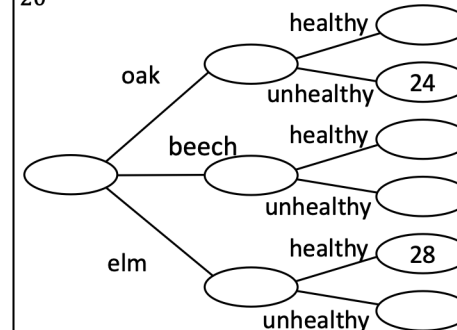
What percentage of the students studied French?

f) 15% of trees surveyed were unhealthy oaks.

The ratio of healthy elms to healthy oaks was 7 : 10

The ratio beech : elm was 5 : 7

$\frac{7}{20}$ of the beech trees were healthy.

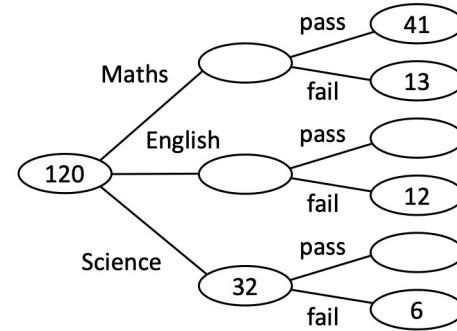
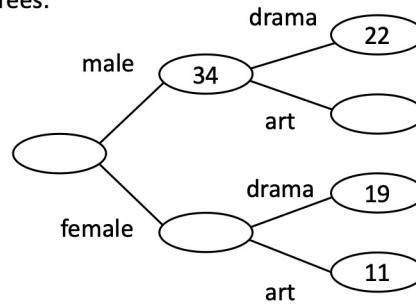
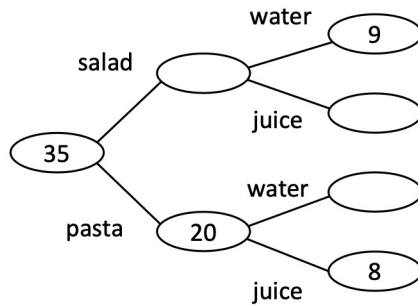


Which species was healthiest?

Fluency Practice

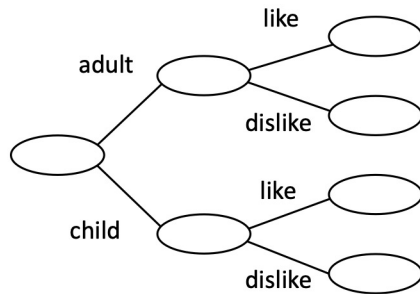
① Frequency Trees

1) Complete the frequency trees.



2) A computer game company surveyed 40 players about a game update.

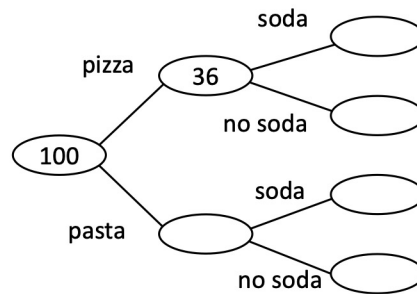
7 adults liked the update.
14 of the 22 children asked liked the update.



What fraction of the adults disliked the update?

3) A restaurant recorded if customers ordered a soda with their meal.

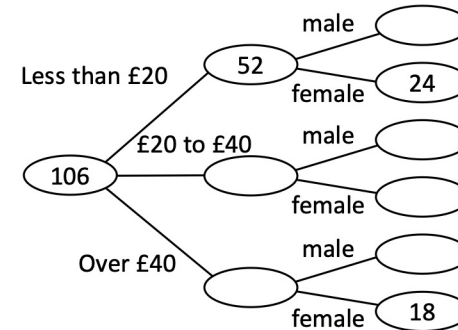
A third of those that had pizza had soda.
34 people ordered pasta with a soda.



What fraction of those surveyed had pasta but no soda?

4) A clothes shop recorded the gender & the money spent by customers spent in a day.

Twice as many customers spent less than £20, than spent over £40.
Of the customers that spent £20-£40, three-quarters were female.



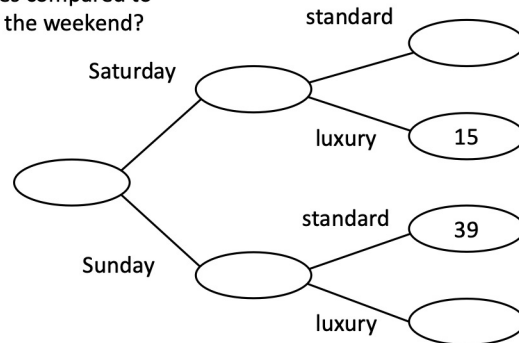
What fraction of customers...
...were male?
...spent over £20?

Fluency Practice

1) A cake shop sells 'standard' (£2) & 'luxury' cakes (£3).

On Sunday, a quarter of the customers ordered a luxury cake.
Over the weekend, a third of the customers ordered a luxury cake.

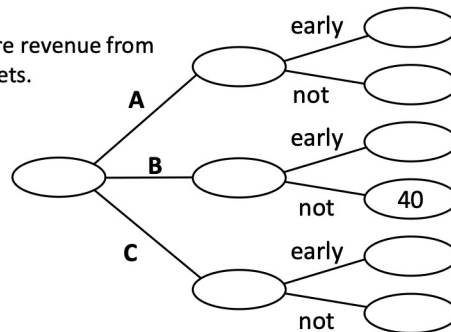
How much more revenue did the shop make
from standard cakes compared to
luxury cakes over the weekend?



3) There are 3 types of seating at a concert **A**: £30 **B**: £20 **C**: £15
Tickets bought early get a 20% discount.

Three-fifths of the 80 **A** tickets were bought early.
12 more people bought **B** tickets early than **A** tickets early.
133 people bought tickets early.
For **C** tickets, the ratio of those bought early to on the door was 1 : 5

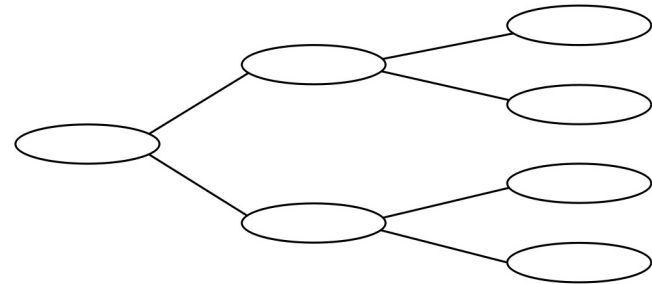
Show that the concert made more revenue from
C tickets than **A** tickets.



2) Mike recorded the meals 120 students had at lunch.

14 students had a sandwich and no snack.
Two-thirds of the students had a sandwich.
Three-quarters of the students had a snack.
24 students had a snack but no sandwich.

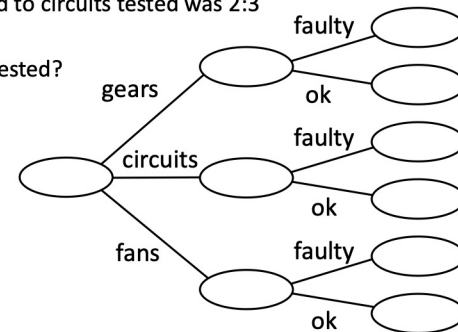
What fraction of students had no sandwich and no snack?
Complete a frequency tree to help.



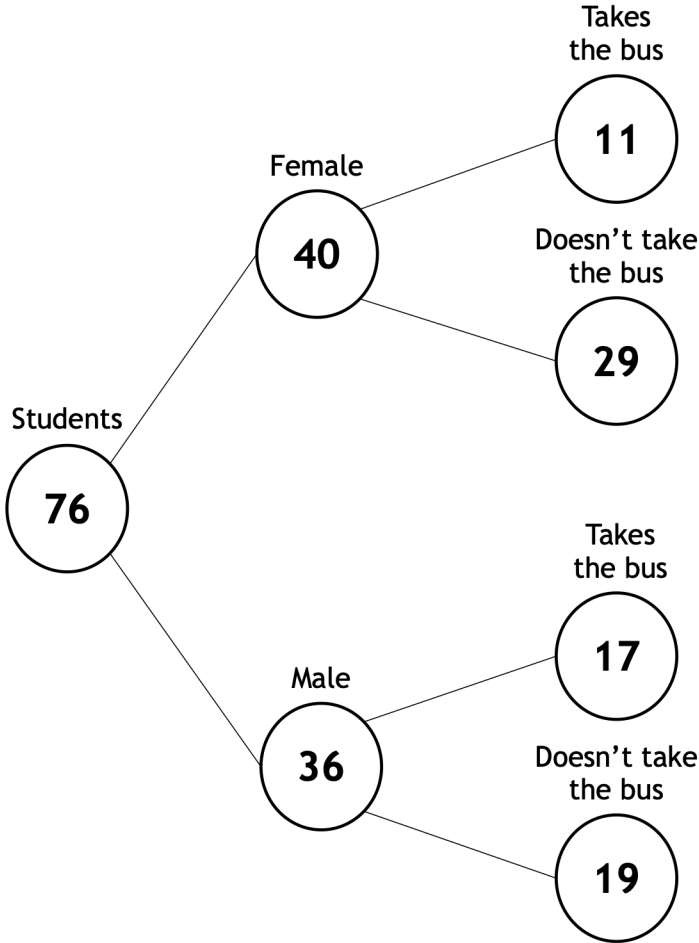
4) A robot factory makes gears, circuits & fans.

After testing, two-thirds of the gears were faulty
& one-fifth of the circuits were faulty.
60% of the items tested worked correctly.
56 fans that were tested worked correctly.
30% of the items tested were gears.
The ratio of gears tested to circuits tested was 2:3

How many items were tested?



Fluency Practice

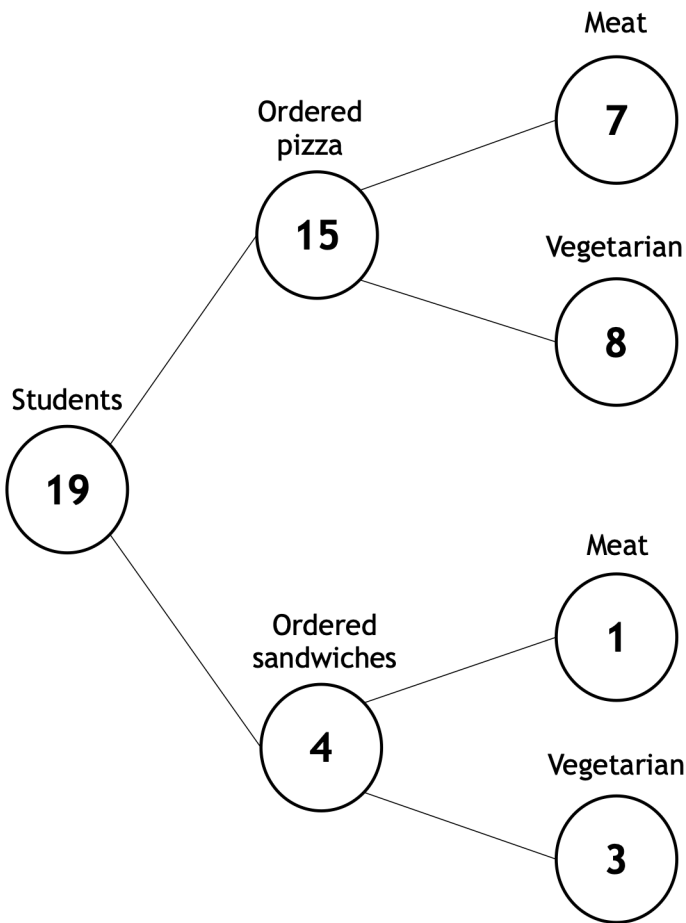


A frequency tree diagram shows the number of students that take the bus to school and don't.

Complete the table.

What fraction of students are male?	
What fraction of females take the bus?	
What fraction of students are females that take the bus?	
What fraction of males take the bus?	
What fraction of students are males who don't take the bus?	
What fraction of students don't take the bus?	
What fraction of males don't take the bus?	

Fluency Practice

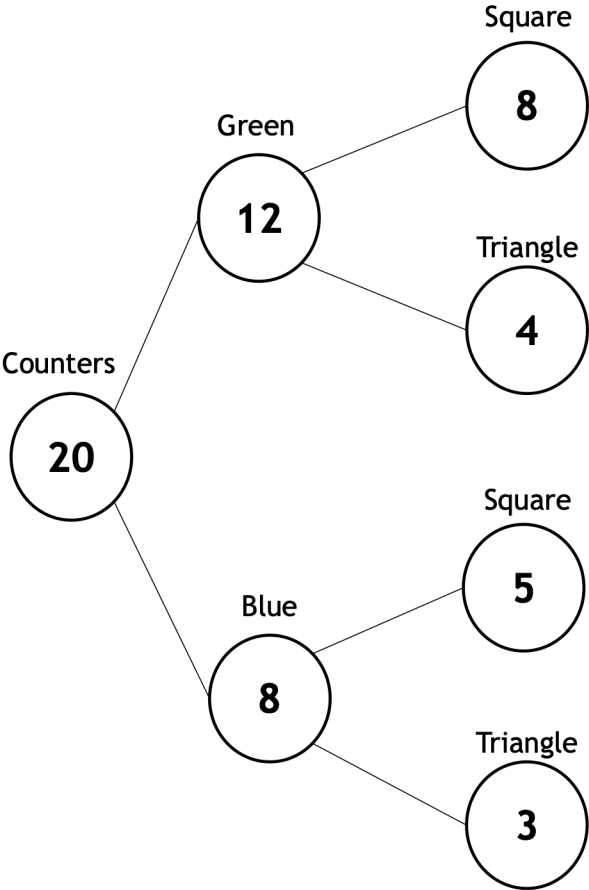


A frequency tree diagram shows the number of students that ordered meals and whether they are vegetarian or not.

Complete the table.

	$\frac{8}{15}$
What fraction of students ordered vegetarian pizza?	
	$\frac{1}{19}$
What fraction of those that ordered sandwiches are not vegetarian?	
	$\frac{15}{19}$
What fraction of students that ordered pizza are not vegetarian?	
	$\frac{3}{4}$

Fluency Practice

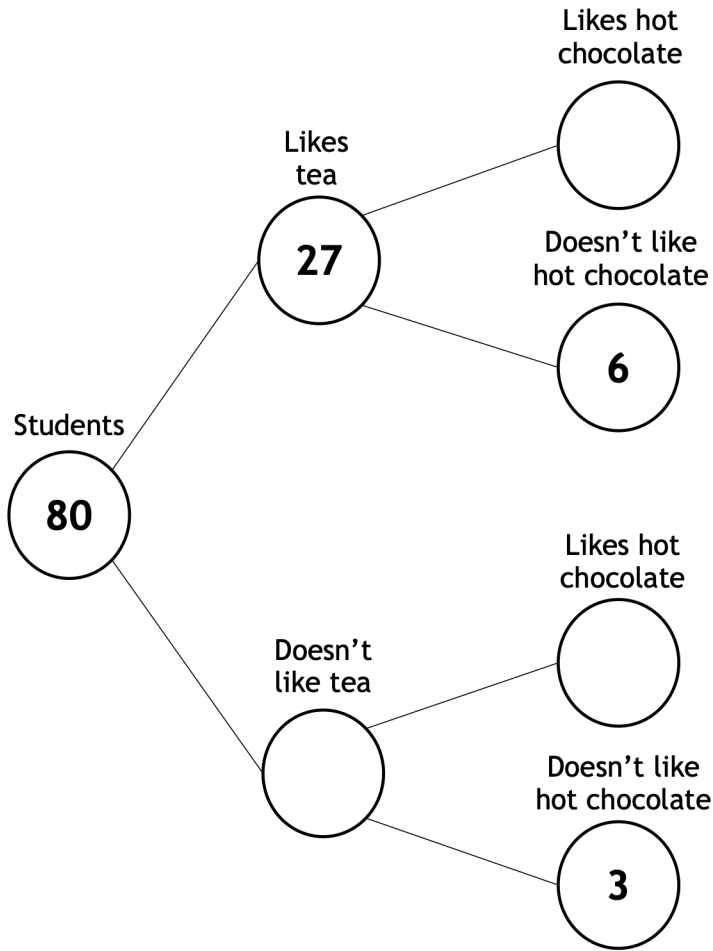


A frequency tree diagram shows the number of counters and their colour and shape.

Complete the table.

What fraction of blue counters are squares?	
	$\frac{3}{20}$
What fraction of blue counters are triangles?	
	$\frac{13}{20}$
What fraction of squares are green?	
What fraction of greens are squares?	
	$\frac{1}{3}$

Fluency Practice

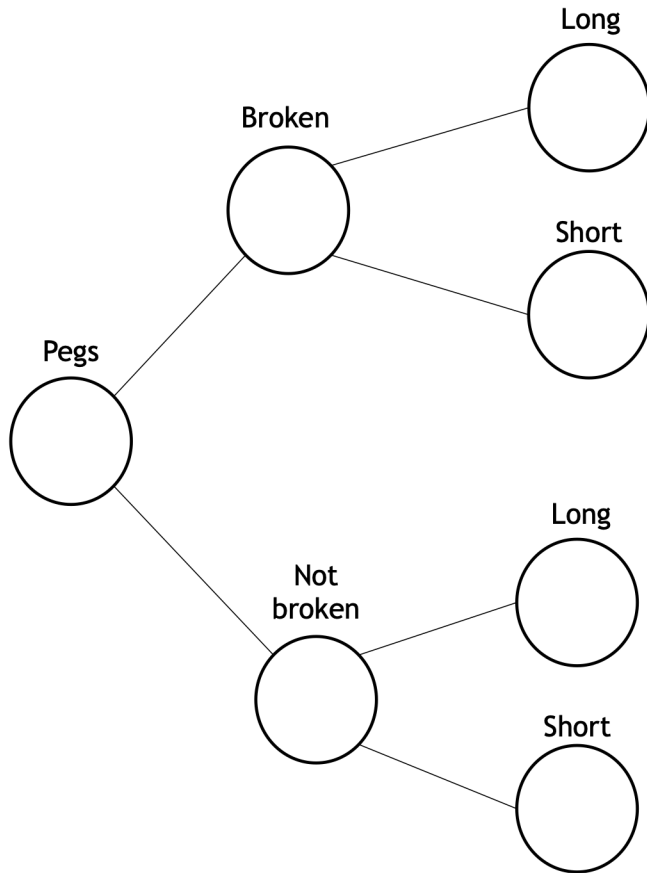


A frequency tree diagram shows the number of students that like hot chocolate and like tea.

Complete the frequency tree and the table

	$\frac{3}{80}$
	$\frac{53}{80}$
	$\frac{21}{80}$
	$\frac{3}{40}$
	$\frac{3}{53}$
	$\frac{5}{8}$
	$\frac{2}{3}$

Fluency Practice



The information below describes the number of short and long tent pegs that are broken or not broken in shop stockroom.

Complete the frequency tree

$\frac{53}{150}$ of the pegs are broken

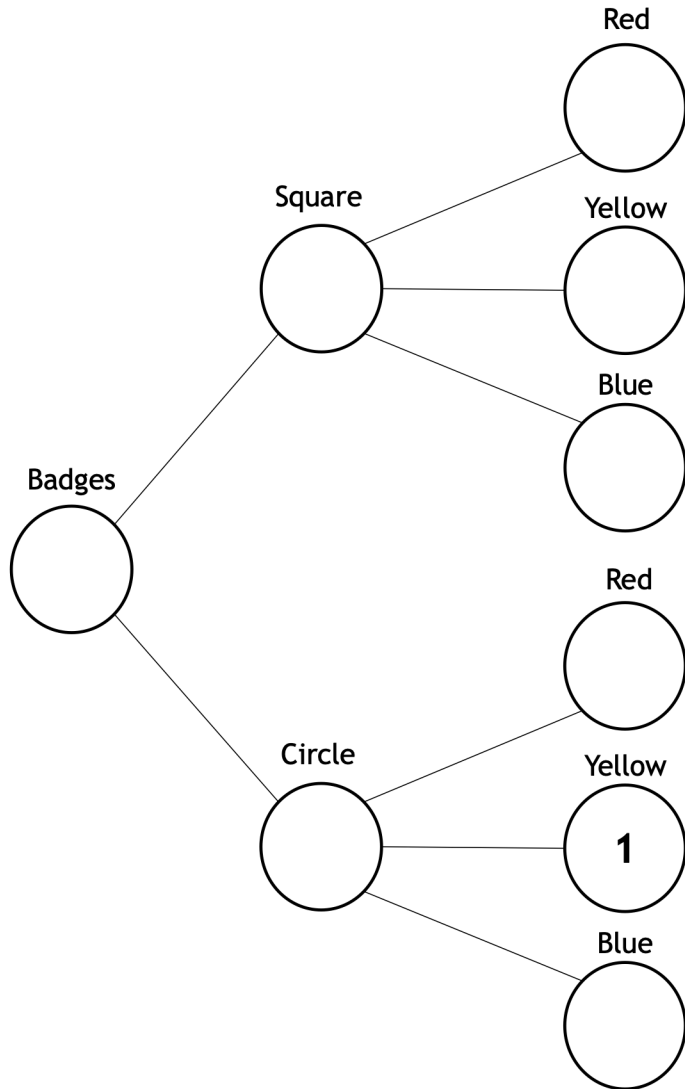
$\frac{51}{150}$ of the of the pegs are long and not broken

$\frac{3}{10}$ of the pegs are long and broken

There are 96 long pegs

$\frac{4}{26}$ of the short pegs are broken

Fluency Practice



A company has ordered badges with their logo on.
There is a choice of two shapes and three colours.

Use the information below to
complete the frequency tree

$\frac{1}{4}$ of the badges are circles

$\frac{1}{4}$ of the badges are red

$\frac{1}{4}$ of the of the red logos are circles

$\frac{1}{4}$ of the circles are red

$\frac{1}{4}$ of the squares are yellow

$\frac{1}{4}$ of the badges are yellow

$\frac{1}{4}$ of the circles are yellow

$\frac{1}{4}$ of the blue logos are circles

Fluency Practice

in a small school, all of the 440 students study just one modern foreign language

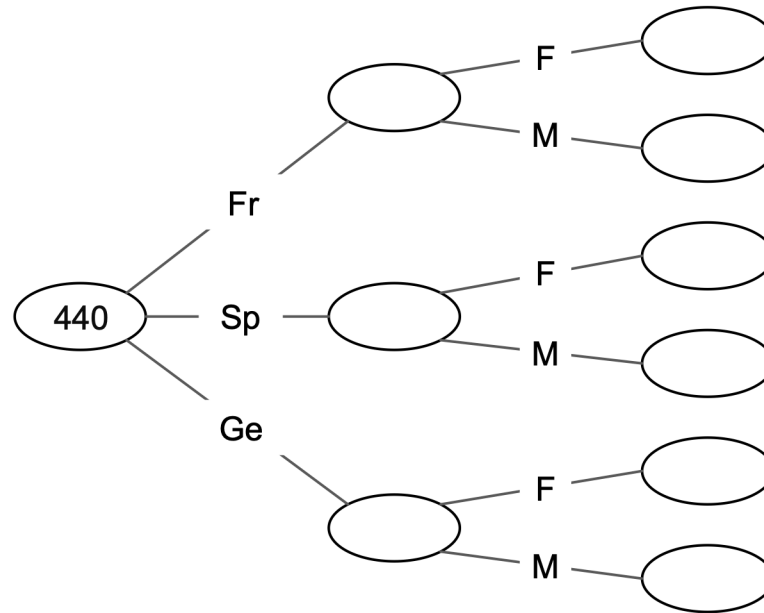
the ratio of numbers of students taking French, Spanish and German is 9 : 5 : 8 (respectively)

33 $\frac{1}{3}$ % of the students that study French are female

40% of the students that study Spanish are male

55% of the students that study German are female

complete the frequency tree:



what percentage of students in
the whole school are male?

[round your answer to the nearest whole number]

Fluency Practice

80 of the students in Y9 take part in an activity during their PE lesson

the same number of students play cricket as play hockey

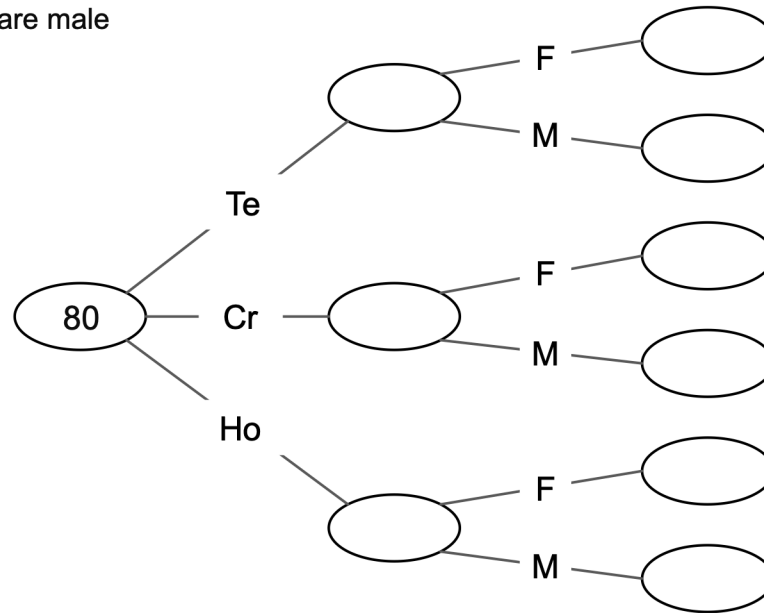
twice the number of students play tennis than play cricket

45% of the students that chose tennis are male

35% of the students that chose cricket are female

45% of the students that chose hockey are male

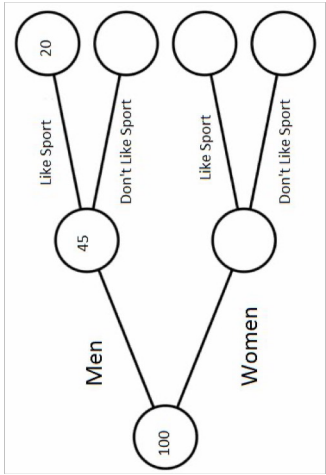
complete the frequency tree:



show that the numbers of girls and boys
in the school are equal

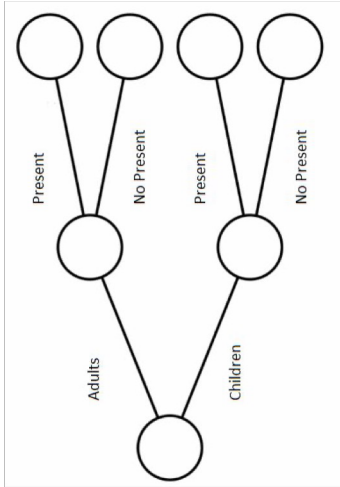
Purposeful Practice

(1) There are 100 people in an office. 45 are men and the rest are women. 20 of the men like sport and the rest of the men don't. 28 of the women like sport and the rest don't. The frequency tree shows some of this information below.



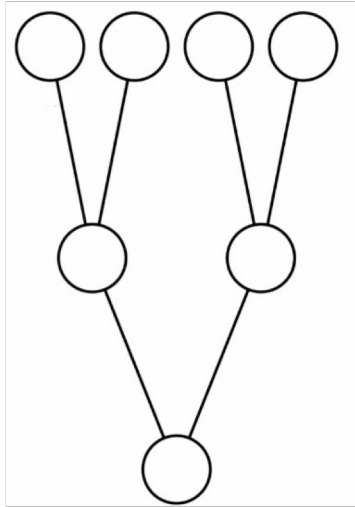
- (a) Complete the frequency tree above.
- (b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
- (c) A man is chosen at random. What is the probability that he liked sport?

(2) There are 80 people at a party. 20 of the people are adults and the rest are children. Of the children 45 take a present and the rest don't. Of the adults, all but one take a present.



- (a) Complete the frequency tree below
- (b) What proportion of the people at the party are adults who have taken a present?
- (c) One person is chosen at random. Find the probability that they were an adult who didn't take a present.

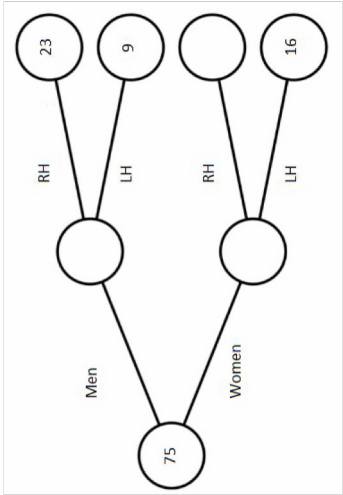
(3) In a box there are pink toys and black toys. There are 90 toys in total. Half of the toys are pink and half of the toys are black. Of the pink toys 20 are plastic and the rest are metal. Of the black toys 10 are plastic and the rest are metal.



- (a) Label and complete the frequency tree below.
- (b) How many more metal toys were there than plastic toys?
- (c) One toy is chosen at random. Find the probability that it's a black plastic toy.
- (d) Write down the ratio of pink toys to black toys.

Purposeful Practice

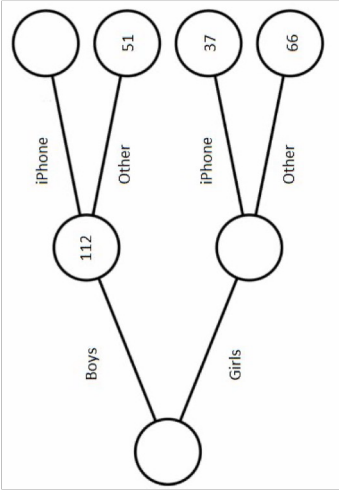
(4) Complete the frequency tree below



(5) There are 90 people at a party. 36 are adults and the rest are children. Of the children 10 take a drink and the rest don't. Of the adults 16 take a drink and the rest don't.
Draw a frequency tree to show this information.

(6) In Y10 there are 250 students. 130 are boys and the rest are girls. 50 boys take pack lunches and 80 girls take pack lunches. Those that don't take pack lunches eat in the canteen. Draw a frequency tree to represent the information.

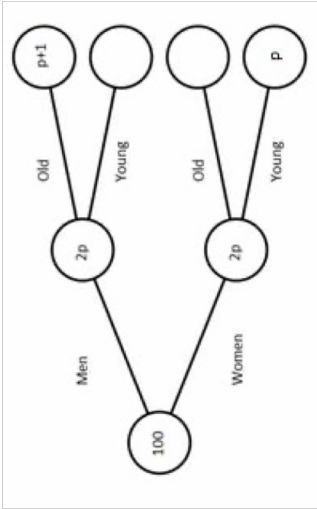
(7) The frequency tree below shows what phone some students at a school owned.
(a) Complete the frequency tree.



(b) One person is chosen at random. Find the probability of the person being a boy with an iPhone.
(c) A girl is chosen at random. Find the probability that she doesn't own an iPhone.
(d) How many more people own 'Other' phones than iPhones?

(8) There are 180 people at a party. 1/3 are men and the rest are women. Of the men half take food and the rest don't. Of the women 20% take food and the rest don't.
(a) Show this information in a frequency tree.
(b) One person is chosen at random. Find the probability that the person is a man who doesn't take food.
(c) A woman is chosen at random. Find the probability that she takes food.

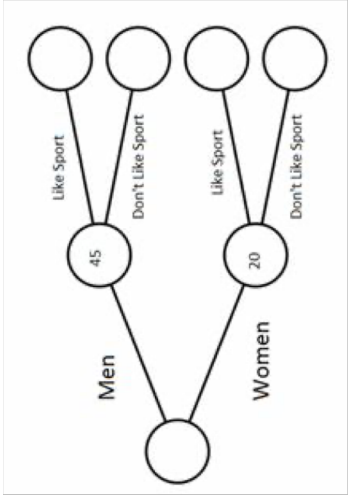
(9) (a) Find the value of p in the frequency tree below.



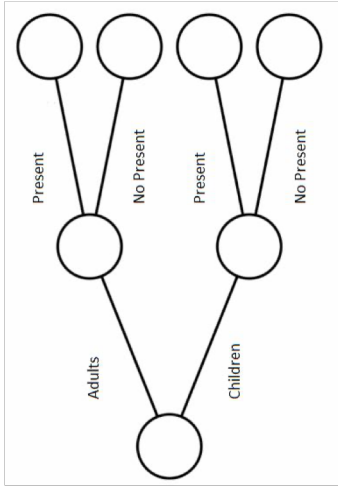
(b) Complete the frequency tree.

Purposeful Practice

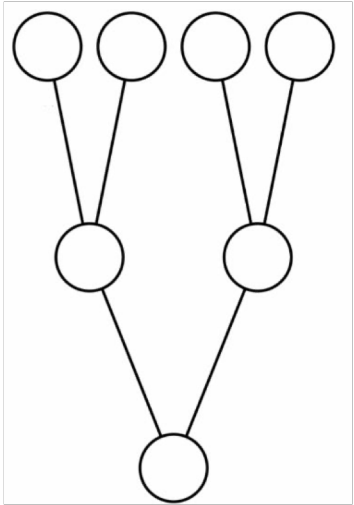
- (1) There are a number of people in an office. 10 of the men like sport and the rest don't. 8 of the women like sport and the rest don't. The frequency tree shows some of this information below.



- (a) Complete the frequency tree above.
(b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
(c) A man is chosen at random. What is the probability that he liked sport?
- (2) There are 92 people at a party. 33 of the people are adults and the rest are children. Of the children 12 take a present and the rest don't. Of the adults, 9 don't take a present.
- (a) Complete the frequency tree below



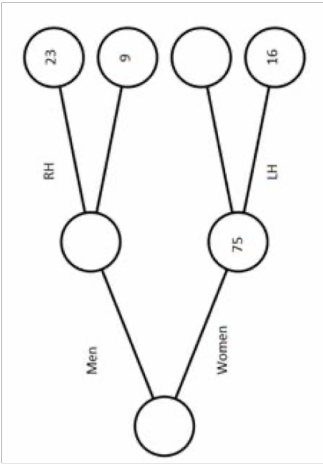
- (b) What fraction of the people at the party are adults who have taken a present?
(c) One person is chosen at random. Find the probability that they were a child who didn't take a present.
- (3) In a box there are blue toys and red toys. There are 112 toys in total. All but 30 of the toys are red. Of the red toys 20 are plastic and the rest are metal. Of the blue toys half are plastic and the rest are metal.
- (a) Label and complete the frequency tree below.



- (b) How many more metal toys were there than plastic toys?
(c) One toy is chosen at random. Find the probability that it's a red plastic toy.
(d) Write down the ratio of metal toys to plastic toys.

Purposeful Practice

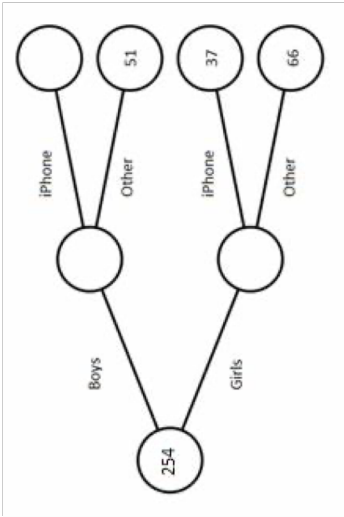
(4) Complete the frequency tree below



(5) There are 80 people at a party. 54 are adults and the rest are children. Of the children 8 take a drink and the rest don't. Of the adults 16 take a drink and the rest don't.
Draw a frequency tree to show this information.

(6) In Y10 there are 180 students. 130 are boys and the rest are girls. 40 boys take pack lunches and 20 girls take pack lunches. Those that don't take pack lunches eat in the canteen. Draw a frequency tree to represent the information.

(7) The frequency tree below shows what phone some students at a school owned.
(a) Complete the frequency tree.

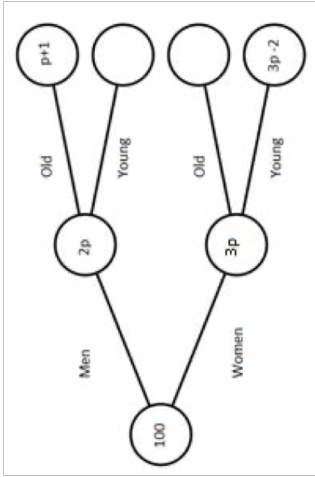


(b) One person is chosen at random. Find the probability of the person being a boy with an iPhone.
(c) A girl is chosen at random. Find the probability that she doesn't own an iPhone.
(d) How many more people own 'Other' phones than iPhones?

(8) There are 120 people at a party. 1/4 are men and the rest are women. Of the men half take food and the rest don't. Of the women 20% take food and the rest don't.

(a) Show this information in a frequency tree.
(b) One person is chosen at random. Find the probability that the person is a man who doesn't take food.
(c) A woman is chosen at random. Find the probability that she takes food.

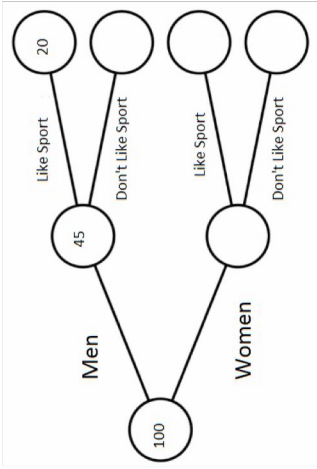
(9) (a) Find the value of p in the frequency tree below.



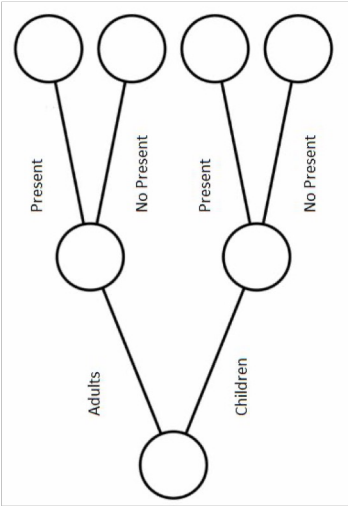
(b) Complete the frequency tree.

Purposeful Practice

- (1) There are 100 people in an office. 45 are men and the rest are women. 20 of the men like sport and the rest of the men don't. 28 of the women like sport and the rest don't. The frequency tree shows some of this information below.

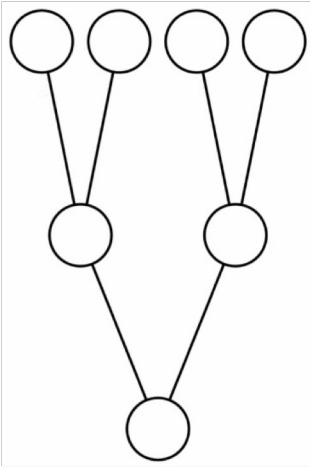


- (a) Complete the frequency tree above.
- (b) One person is chosen at random. Find the probability that the person is a woman who didn't like sport.
- (c) A man is chosen at random. What is the probability that he liked sport?
- (2) There are 80 people at a party. $\frac{1}{4}$ of the people are adults and the rest are children. Of the children 30 take a present and the rest don't. Of the adults, all but one take a present.
- (a) Complete the frequency tree below



- (b) What proportion of the people at the party are adults who have taken a present?
- (c) What percentage of the children took a present?
- (d) One person is chosen at random. Find the probability that they were an adult who didn't take a present.
- (e) Fill in the blank: $P(\text{adult who didn't take a present}) = \frac{19}{80}$
- (3) In a sports club there are 50 people. The ratio of males to females is 2:3. Of the males the ratio of those swimming to not swimming is 7:3. Of the females 60% swim and the rest don't

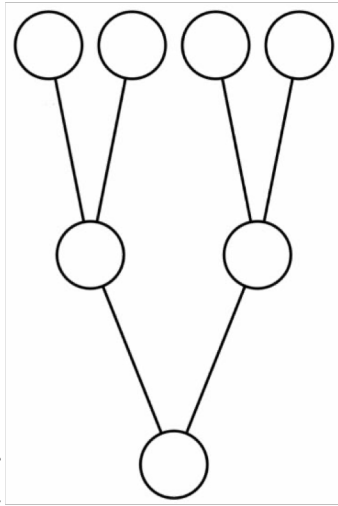
- (a) Use the diagram below to show the information using a frequency tree



- (b) What percentage of the people in the club were males who swam?
- (c) What is the ratio of the women that swim to the men that swim?
- (d) One person is chosen at random. Find the probability that the person is a female who doesn't swim.
- (e) What percentage of the men didn't swim?

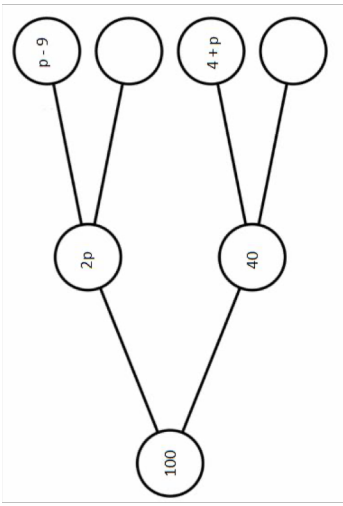
Purposeful Practice

- (4) There are men and women in an office. The ratio of women to men is 4:1. Of the women 25% bring their own lunch. The rest eat in the canteen. Of the men, 6 bring their own lunch and 2 eat in the canteen.
- (a) Show this information using the frequency tree below.



- (b) One person is chosen at random. Find the probability that the person is a man who eats in the canteen.
- (c) A woman is chosen at random. Find the probability that she brings her own lunch.
- (d) What proportion of the people in the office brings their own lunch?
- (5) There are 130 people at a party. 20% are adults and the rest are children. Half of the adults take food and $\frac{2}{13}$ of the children take food.
- (a) Explain why 15% of the people can't be adults.
- (b) Draw a frequency tree in the space below to show the information given.

- (c) Write the ratio of the number of adults bringing food to the number of children bringing food.
- (d) What percentage of party goers were adults who took food?
- (6) In a toy box there are pink toys and black toys. The toys are either electronic or they are manual. The frequency tree shows some information about the toys below.

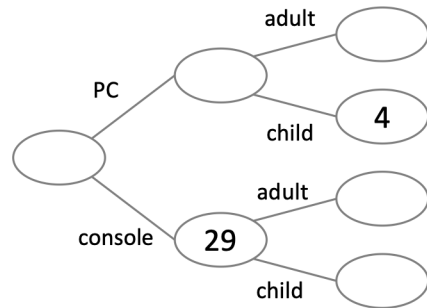
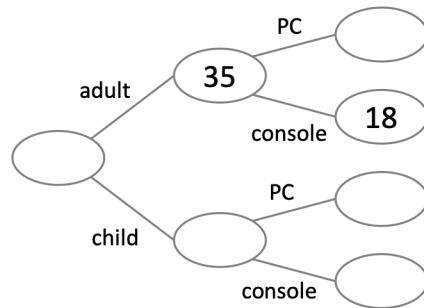


- There are 40 black toys in the box. Of the black toys 6 are manual.
- (a) Complete the frequency tree above.
- (b) One black electronic toy is taken from the box. What proportion of the toys left in the box are pink manual toys?
- (c) Write the ratio of pink toys to black toys in its simplest form.
- (d) n pink toys are removed from the box. What proportion of the toys in the box are now pink?
- (e) $K\%$ of the items are electronic pink toys. Find the value of K .

Purposeful Practice

A computer game company collects data about players: their age & what platform they prefer to use.

	Adult	Child	Total
PC	17		21
Console		11	
Total			50



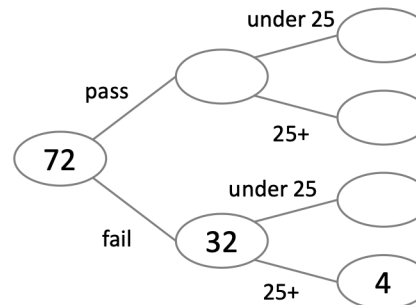
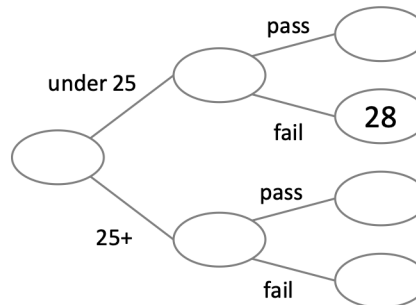
What fraction of the players played on console?

What fraction of the children played on console?

Two-Way Tables & Frequency Trees: Gap Fill

A driving school records the age of students & their success when taking a driving test.

	Pass	Fail	Total
Under 25			
25+	27		
Total	40		

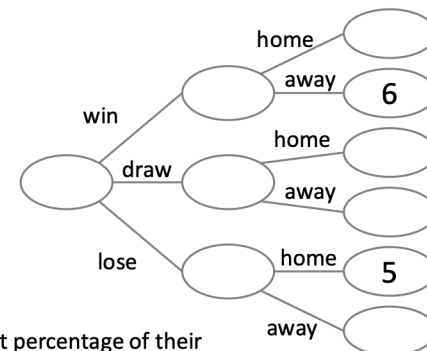
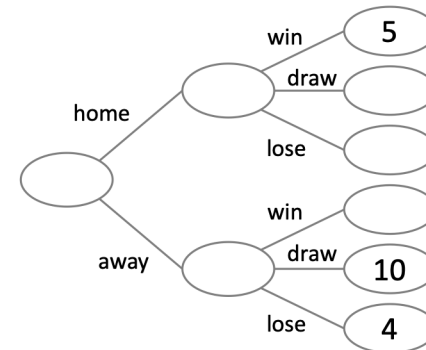


What fraction of the students were 25 or over and failed?

What fraction of those that failed were under 25?

Leesham United look at their results since new owners took over.

	Home	Away	Total
Win			
Draw	6		
Lose			
Total			

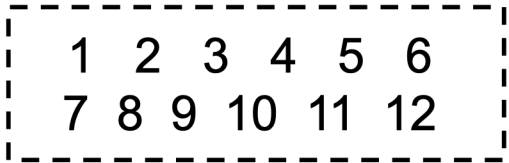


What percentage of their away games did they win?

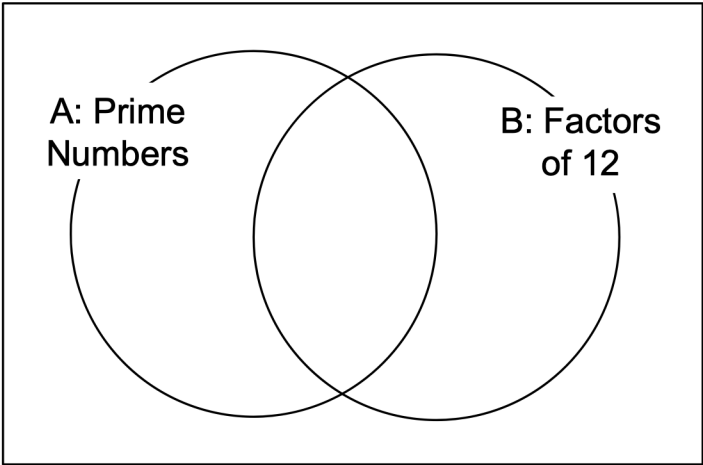
What fraction of the draws were away games?

Fluency Practice

① Place each element in the correct section of the Venn diagram.



Elements



② Shade the TRUE statements.

A has 5 elements	B has the same number of elements as B'
B has 4 elements	B has twice as many elements as $A \cap B$
$A \cap B$ has 9 elements	There are 4 elements in A'
$A \cup B$ has 9 elements	$\frac{1}{4}$ of the elements are outside both circles

③ One of the numbers is chosen at random. Choose the correct probabilities.

$P(A)$	
$P(B)$	
$P(A')$	
$P(B')$	
$P(A \cap B)$	
$P(A \cup B)$	

1
6

75%

7
12

0.5

5
12

1 - P(B)

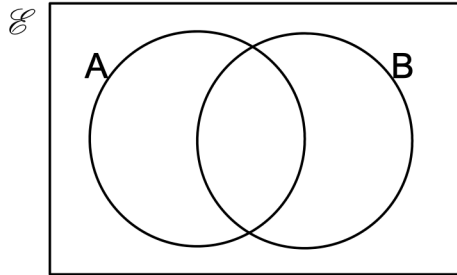
Fluency Practice

1. $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{2, 3, 5, 6\}$

$B = \{1, 3, 5, 7, 9\}$

(a) Complete the Venn diagram.



(b) An element is picked at random. Shade the TRUE statements.

$P(B) = \frac{1}{2}$

$P(A \cap B) = 2$

$P(A \cup B) = 0.7$

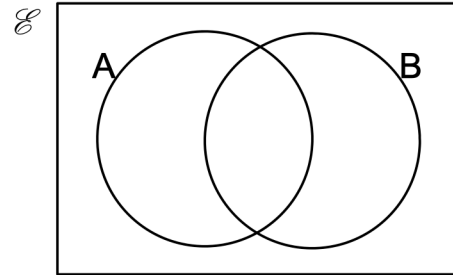
$P(A') = \frac{7}{10}$

2. $\mathcal{E} = \{8, 9, 10, 11, 12, 13, 14, 15, 16\}$

$A = \{\text{even numbers}\}$

$B = \{\text{square numbers}\}$

(a) Complete the Venn diagram.



(b) An element is picked at random. Work out:

$P(A') =$

$P(B') =$

$P(A \cap B) =$

$P(A \cup B) =$

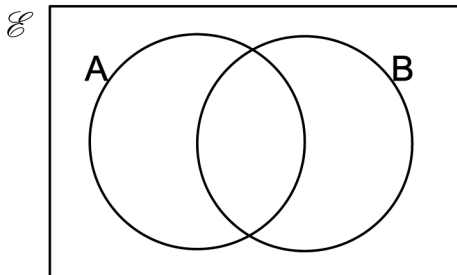
3. $\mathcal{E} = \{5, 6, 7, 8, 9, 10, 11\}$

$A' = \{5, 6, 10\}$

$B = \{6, 8, 10, 11\}$

$A \cap B = \{8, 11\}$

(a) Complete the Venn diagram.



(b) An element is picked at random. Shade the TRUE statements.

$P(A) < P(B)$

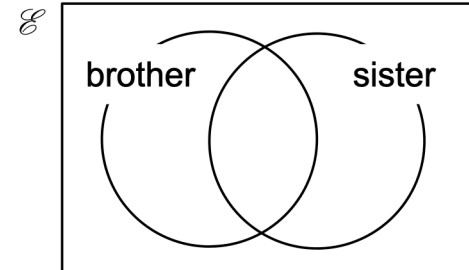
$P(A \cap B) = P(B)$

$P(B) \geq P(B')$

$P(A \cup B) = P(A \cap B)$

4. In a class of 32 pupils,
23 pupils have a brother,
14 pupils have a brother and a sister,
6 pupils have no brothers or sisters.

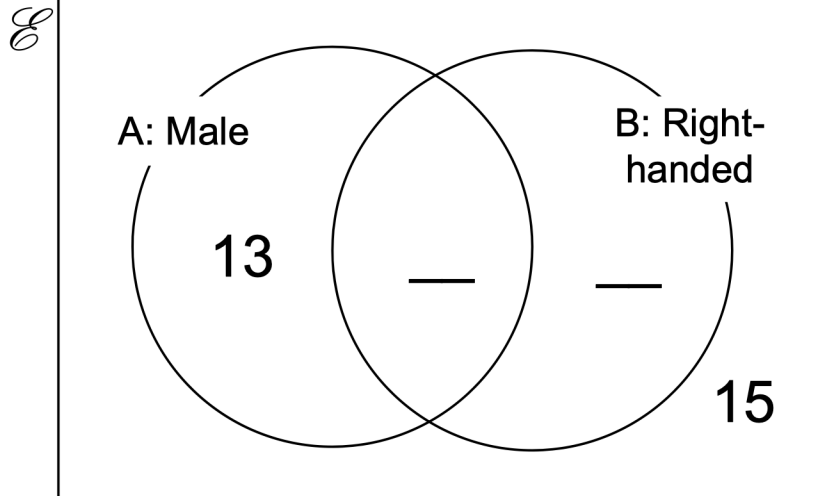
Write the number of pupils who belong in each section of the Venn diagram.



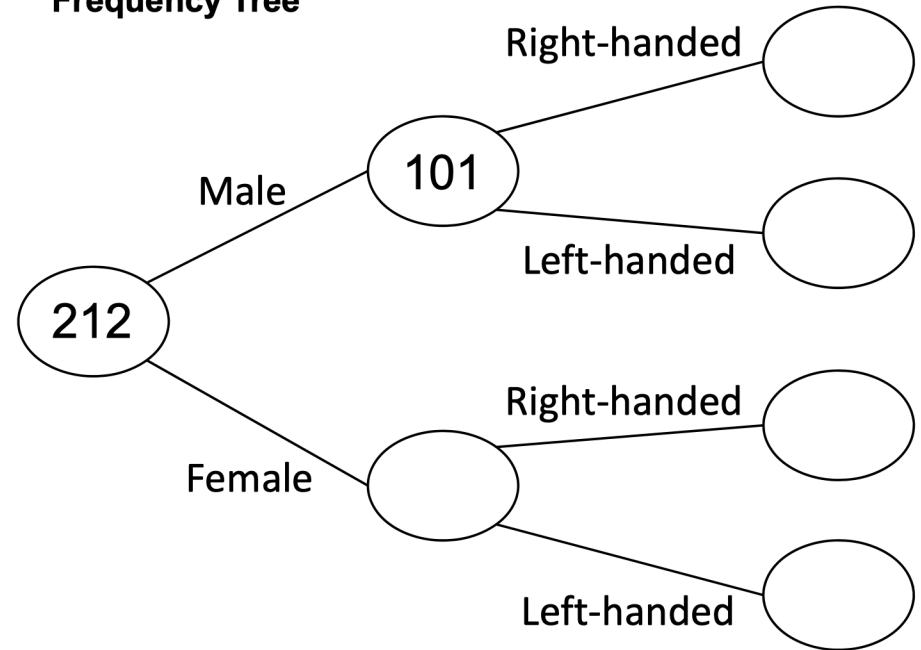
Fluency Practice

The Venn diagram, Frequency Tree and Two-Way Table all show the same information. Complete them all.

Venn diagram – the number of elements belonging to each section is labelled.



Frequency Tree

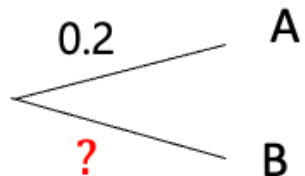


Two-Way Table

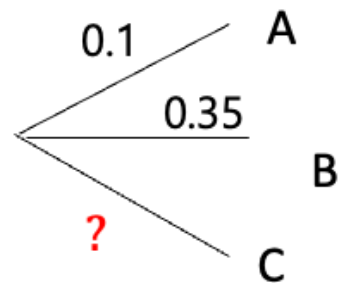
	Male	Female	Total
Right-handed			
Left-handed			
Total			

Fluency Practice

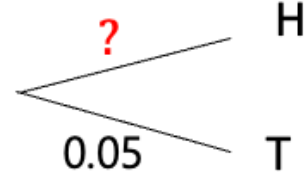
a.



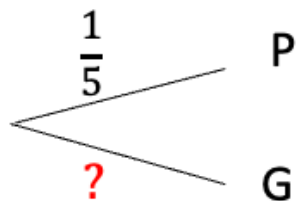
e.



b.

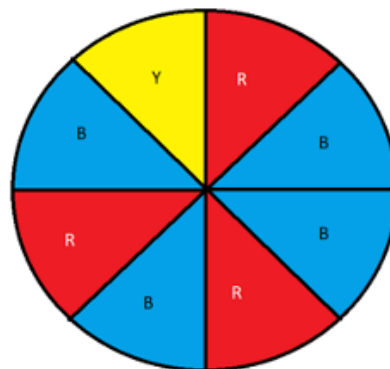


c.

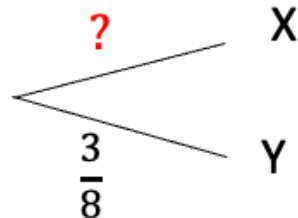


f.

Draw a probability tree for this spinner.



d.



Fluency Practice

0.7 B B'	B $\frac{7}{10}$ B'	$\frac{2}{9}$ B $\frac{4}{9}$ G Y	$\frac{2}{6}$ B G $\frac{1}{3}$ Y
0.07 B B'	B $\frac{7}{9}$ B'	$\frac{2}{7}$ B $\frac{4}{7}$ G Y	$\frac{1}{6}$ B G $\frac{1}{3}$ Y
B 0.07 B'	B $\frac{5}{9}$ G $\frac{2}{9}$ Y	$\frac{2}{7}$ B $\frac{4}{7}$ G Y	B $\frac{1}{4}$ G $\frac{1}{6}$ Y
B 0.27 B'	$\frac{2}{9}$ B $\frac{5}{9}$ G Y	$\frac{2}{6}$ B $\frac{4}{6}$ G Y	B $\frac{3}{4}$ G $\frac{1}{6}$ Y

Fluency Practice

Add or multiply probabilities?

Getting a 6 on a die and a T on a coin.

+

×

Hitting a bullseye or a triple 20.

+

×

Getting a HHT or a THT after three throws of an unfair coin (presuming we've already worked out $P(\text{HHT})$ and $P(\text{THT})$).

+

×

Getting 3 on the first throw of a die and a 4 on the second.

+

×

Bart's favourite colour being red and Pablo's being blue.

+

×

Shaan's favourite colour being red or blue.

+

×

Fluency Practice

Independent?

Event 1

Event 2

Throwing a heads
on the first flip.

Throwing a heads
on the second flip.

No

Yes

It rains tomorrow.

It rains the day
after.

No

Yes

That I will choose
maths at A Level.

That I will choose
Physics at A Level.

No

Yes

Have a garden
gnome.

Being called Bart.

No

Yes

Purposeful Practice



Independent Events?

For Tim, what will happen tomorrow?

Which of these events are **independent** of each other?

How might one event affect the probability of a different event?

It will rain.

Tim's son will pass his maths test.

Tim will have pasta for lunch.

Tim will bump into Sarah.

Tim will be late to work.

Tim's car will break down.

Tim will feel unwell.

Tim will get a pay raise.

Tim will start looking for a new job.

Tim will go to bed early.

Tim will be early for work.

It will be sunny.

Tim will have dinner at a nice restaurant.

Tim will miss his evening running club.

It will be cloudy.

Stockley United will win their football match.

Tim's alarm clock will not work.

Tim's will choose coffee at the café.

Are any of the events **mutually exclusive**? (they cannot both happen)

Purposeful Practice

In a group of 15 boys, 10 like apples, 8 like bananas and 6 like both apples and bananas.

Let A = a boy likes apples and B = a boy likes bananas.

- (a) Show that A and B are not independent.
- (b) Find the probability that a boy likes apples or bananas.

Of the 26 cars on a garage forecourt, 12 had ABS brakes, 16 had a CD-player. 6 had neither ABS brakes nor a CD-player.

Let A = has ABS and B = has CD-player.

- (a) Show that A and B are not independent.
- (b) Find the probability that a car has ABS or CD-player.

In a group of 20 boys, 8 have blue eyes, 10 have fair hair and 4 have both blue eyes and fair hair.

Let A = has blue eyes and B = has fair hair.

- (a) Show that A and B are independent.
- (b) Find the probability that a boy has either blue eyes or fair hair.

6/

There were 38 books on a shelf. 17 had large print, 12 were novels with large print. 10 were not novels and did not have large print.

Let A = is large print and B = is a novel.

- (a) Show that A and B are not independent.
- (b) Find the probability that a book is either large print or is a novel.

In a group of 24 girls, 18 like oranges, 8 like bananas and 4 like neither oranges nor bananas.

Let A = likes oranges and B = likes bananas.

- (a) Show that A and B are independent.
- (b) Find the probability that a girl either likes oranges or bananas.

In a group of 40 people, 24 played cricket and 30 played tennis.

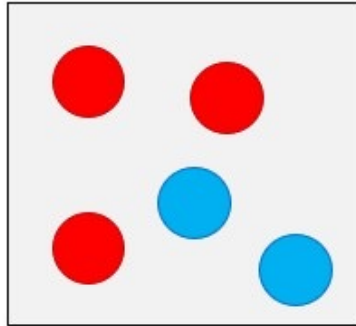
The events 'played cricket' and 'played tennis' are independent. Find the probability that a person plays either cricket or tennis.

In a group of 30 people, 10 read books and 24 read newspapers.

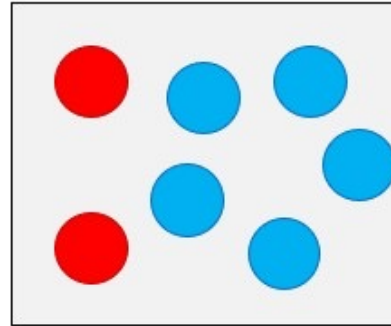
The events 'read books' and 'read newspapers' are independent. Find the probability that a person either reads books or newspapers.

Problem Solving

Leyland picks a marble from each box.



Box A

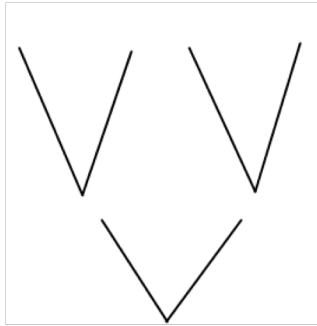


Box B

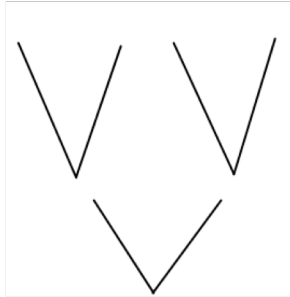
- Does it matter which box Leyland chooses from first?
- Will the order affect the probabilities of the combined outcomes?

Fluency Practice

1. A bag contains 6 blue counters and 4 green counters. One is picked and then replaced. A second is then picked.
Complete the tree diagram below.



2. A box of chocolates contains three dark chocolates and two white chocolates. A chocolate is picked and then replaced. A second chocolate is then picked.
Complete the tree diagram below



3. A wallet contains 2 gold coins and 8 silver coins. A coin is picked at random and then replaced. A second coin is then picked. **Draw a probability tree. What is the probability of getting two silver coins?**

4. Two suitcases each contain 3 hoodies and 9 t-shirts. An item of clothing is picked at random from both suitcases. **Draw a probability tree and calculate the probability of picking two t-shirts.**

5. A fair coin is thrown twice. **What is the probability of getting one head and one tail?**

6. A bag contains 5 red balls and 3 blue. A ball is picked at random and then replaced. A second ball is then picked. **Draw a probability tree. What is the probability that**

- a) Both balls are red?
- b) Both balls are blue?
- c) One of each?

Fluency Practice

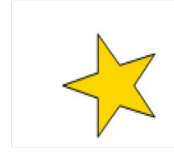
7. There are 4 banana smoothies and three apple smoothies in a box. Jennie takes a random smoothie from a box then replaces it. She then takes another. **What is the probability that the smoothies will be the same flavour?**

8. There are 10 books on a shelf. 7 are fiction and 3 are non-fiction. A member of the public takes a book at random, looks at it, then replaces it. They then take another book. **What is the probability that**

- a) Both books are non-fiction?
- b) Both books are fiction?
- c) one of each?

9. The probability that a biased coin will show heads is 0.4. It is flipped twice. **What is the probability that the coin will show heads on both flips?**

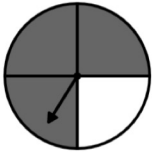
10. Suitcase A contains 3 hats and 7 t-shirts. Suitcase B contains 8 hats and 2 t-shirts. You randomly take an item of clothing from both suitcases. **What is the probability that you will have one hat and one t-shirt?**



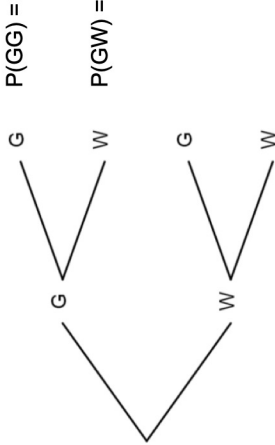
Extension: A bag contains 5 red balls and 3 blue. A ball is picked at random and then **not** replaced. A second ball is then picked. **Draw a tree diagram. What is the probability of picking at least one red ball?**

Fluency Practice

1. A spinner has four equally sized sectors: three grey and one white. The spinner is to be spun twice.



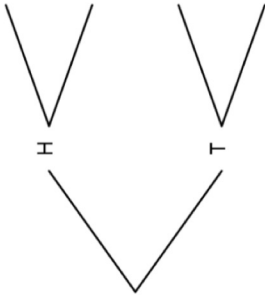
(b) Work out the probability that the spinner will land on the same colour on both spins.



(c) Work out the probability that the spinner will land on grey on at least one of the spins.

2. Simon has a biased coin, which has a probability of 0.68 of landing on heads. He is going to flip the coin twice.

(a) Complete the tree diagram.

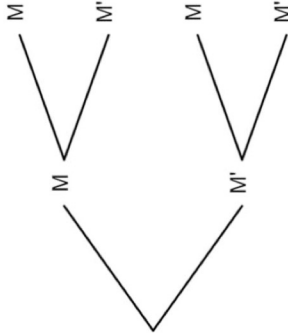


(b) Shade the true statements.

The probability of the coin landing on heads twice is greater than 50%.
The probability of two tails is a quarter of the probability of two heads.
$P(HT) = P(TH)$.
The probability that the coin lands on heads at least once is 0.90 (correct to 2 d.p.)

3. Angela is playing a game with two fair dice. She rolls both dice and wins a point for each die that lands on a multiple of 3.

(a) Complete the tree diagram, in which M stands for a multiple of 3.



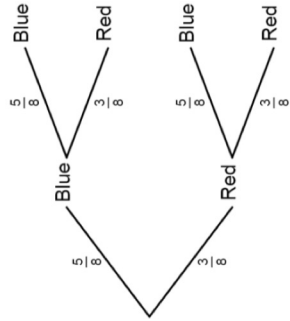
(b) Work out the probability that Angela scores 2 points.

(c) Work out the probability that Angela scores at least 1 point.

(d) Work out the probability that Angela scores no points.

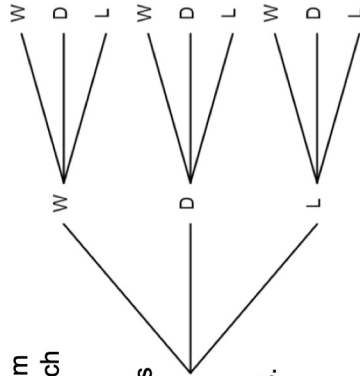
Fluency Practice

4. A pack of cards contains red and blue cards only. Sally is going to pick a card at random, replace it and then pick another. A tree diagram for this situation is shown.



There are 18 red cards in the pack. How many blue cards are there?

5. Liam coaches a football team. Based on their past performance, he works out that the team have a 44% probability of winning each match and a 21% probability of drawing.

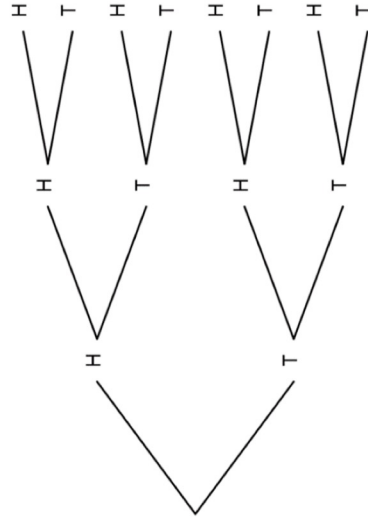


(a) Complete the tree diagram for the team's next two matches.

(b) Work out the probability that the team will not lose either of their next two matches.

6. A fair coin is to be flipped three times.

(a) Complete the tree diagram.



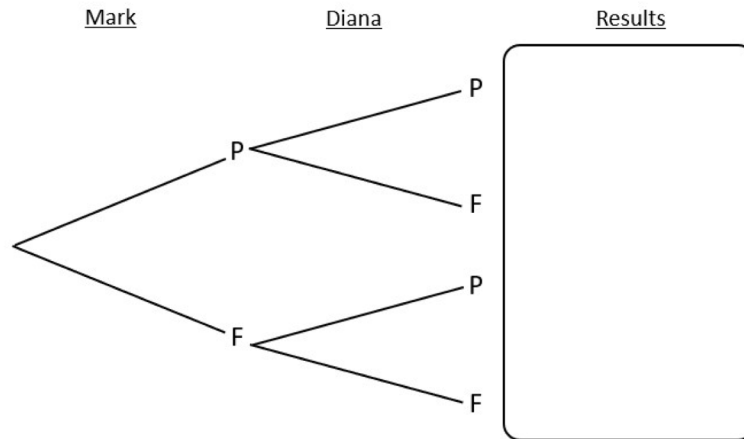
(b) Work out the probability that the coin lands on heads exactly twice.

(c) Work out the probability that the coin lands on tails at least once.

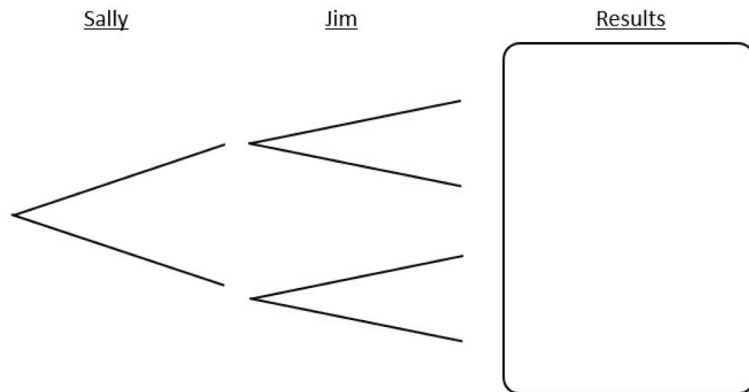
7. John cycles to and from work. He passes a level crossing on his route, which has the gates closed 8% of the time. Work out the probability that on a given day the gates are closed on at least one of his two journeys.

Fluency Practice

1. The probability Mark passes his driving test is $\frac{1}{3}$. The probability Diana passes is $\frac{5}{6}$. Complete the Tree Diagram and find the probability they both pass their tests.



2. The probability Sally passes her history test is $\frac{4}{5}$. The probability Jim passes is $\frac{2}{3}$.
- Complete the Tree Diagram and find the probability they both fail their tests.
 - What is the probability at least one of them passes?



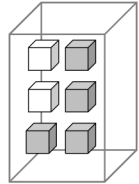
3. The probability Jim passes his maths test is $\frac{7}{10}$. The probability he passes English is $\frac{4}{5}$.
- Complete a Tree Diagram and find the probability he passes both tests.
 - What is the probability he fails both tests?
 - What is the probability he passes only one test?

4. The probability Helen passes her English test is $\frac{4}{9}$. The probability she passes History is $\frac{3}{7}$.
- Complete a Tree Diagram and find the probability she passes at least one test.

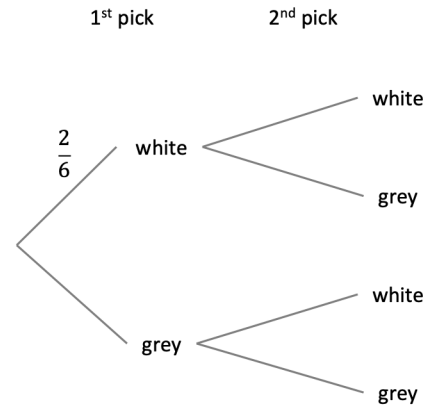
Fluency Practice

Probability Trees

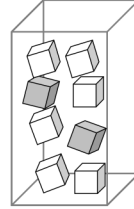
1)



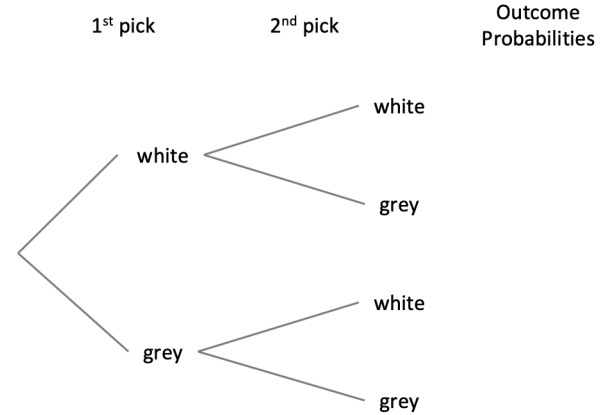
A cube is picked at random from the box.
It is **replaced**, then another cube is picked.
Complete the probability tree & the probability of each outcome.



2)

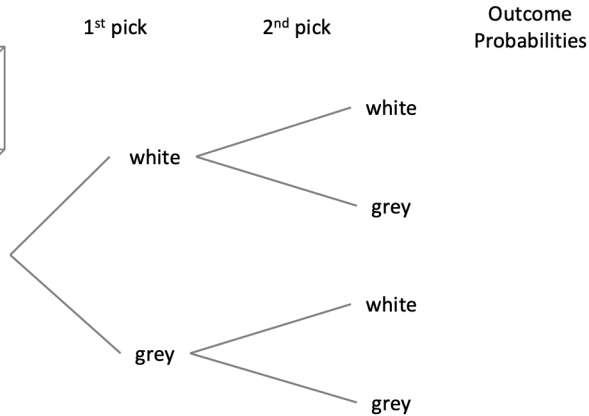
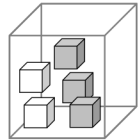


A cube is randomly picked from the box & replaced.
What is the probability 2 grey cubes are picked?
What is the probability 2 white cubes are picked?

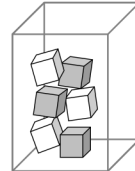


A cube is picked at random from the box.
It is **not replaced** before another cube is picked.
Complete the probability tree & the probability of each outcome.

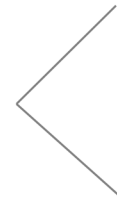
3)



4)



A cube is picked from the box, and **not replaced** before another is picked.
What is the probability 2 white cubes are picked?
What is the probability only 1 grey cube is picked?

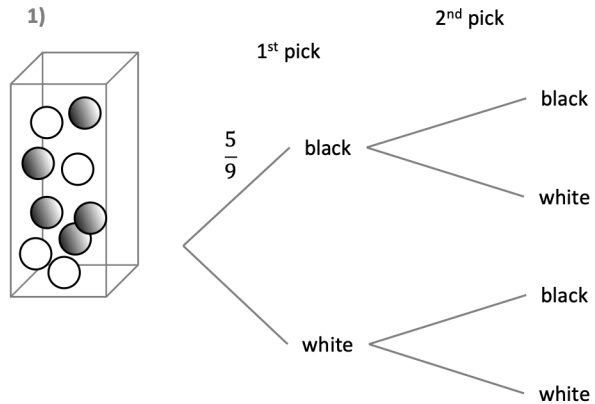


Fluency Practice

Probability Trees

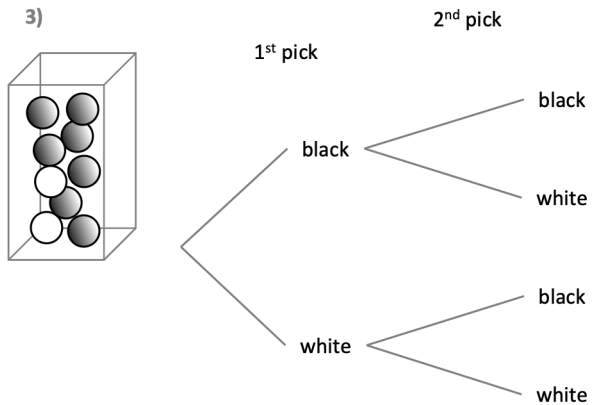
A sphere is randomly picked from the box & replaced.
What is P(two black spheres)?
What is the probability 2 spheres of different colours are picked?

1)



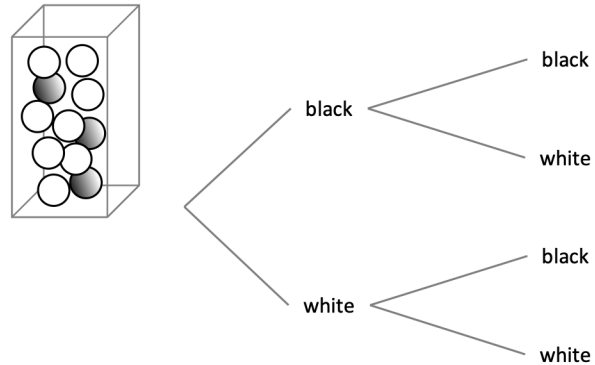
Two spheres are picked from the box (**without** the first being replaced).
What is P(2 white spheres)?
What is the probability at least one of the spheres is white?

3)



A sphere is randomly picked from the box & replaced.
What is P(two spheres of the same colour)?
What is P(at least one black sphere)?

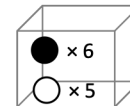
2)



If two disks are picked from this box, what is P(different coloured disks)?

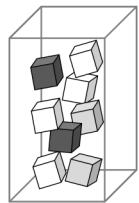
If **three** disks are picked from the box, what is P(3 white disks)?

4)



Fluency Practice

1) A cube is picked at random from the box, then replaced.

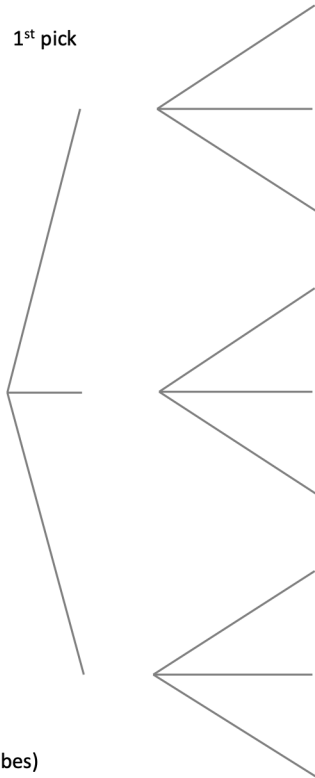


1st pick

2nd pick

Probability Trees

Outcome Probabilities



Calculate:

P(two white cubes)

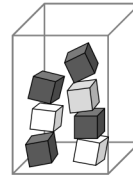
P(**not** two grey cubes)

P(two cubes of the same colour)

P(a white & a grey cube)

P(at least one white cube)

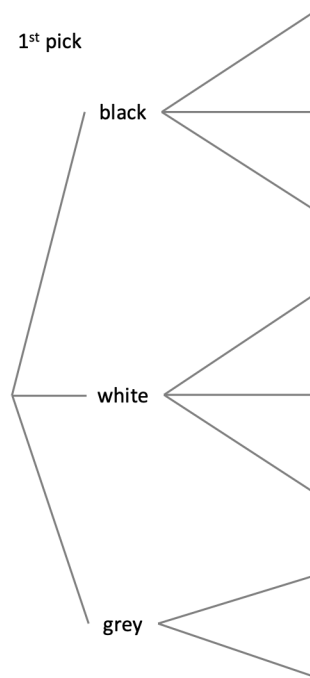
2) Two cubes are picked randomly and simultaneously from the box.



1st pick

2nd pick

Outcome Probabilities



Calculate:

P(two black cubes)

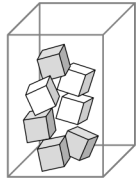
P(**not** two white cubes)

P(two cubes of different colours)

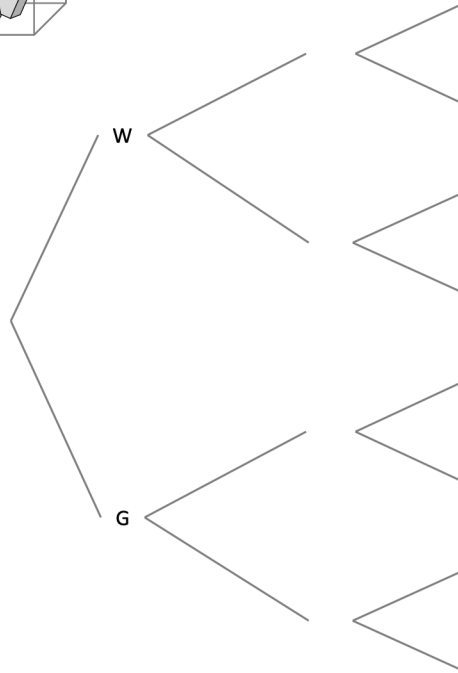
P(a grey cube & a black cube)

P(at least one black cube)

Fluency Practice



- 1) A cube is picked at random from the box, replaced, then another is picked & replaced, then another is picked.
($7 \times 7 \times 7 = 343$)



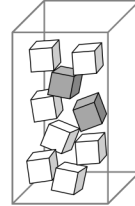
Calculate:

$P(GGG)$

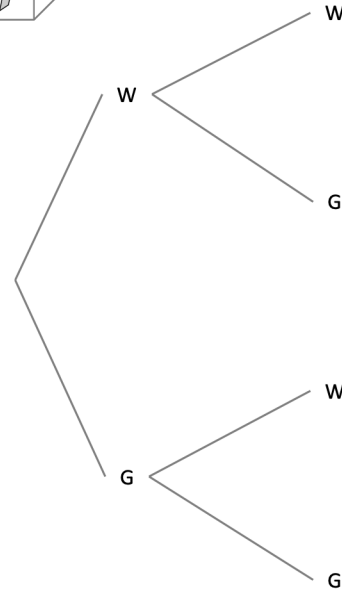
$P(WGW \text{ in order})$

$P(3 \text{ cubes of the same colour})$

$P(\text{exactly 2 white cubes})$



- 2) **Three cubes** are picked at random from the box.
($9 \times 8 \times 7 = 504$)



Calculate:

$P(WWW)$

$P(2 \text{ or more white cubes})$

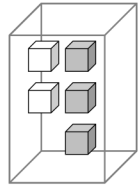
$P(2 \text{ of the cubes are grey})$

If 4 cubes are picked, what is $P(WWWW)$?

Fluency Practice

Probability Trees

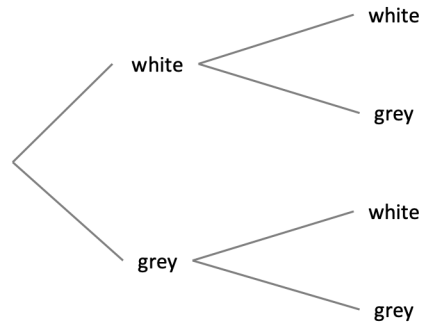
1)



1st pick

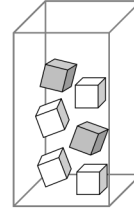
2nd pick

Outcome Probabilities



A cube is picked at random from the box.
It is **replaced**, then another cube is picked.
Complete the probability tree & the probability of each outcome.

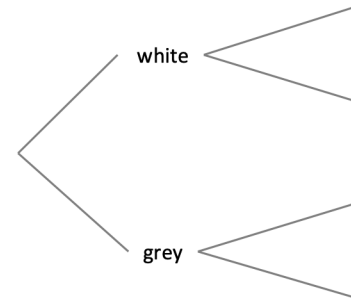
2)



1st pick

2nd pick

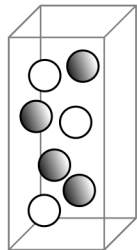
Outcome Probabilities



A cube is randomly picked from the box & replaced.
What is the probability 2 grey cubes are picked?
What is the probability 2 white cubes are picked?

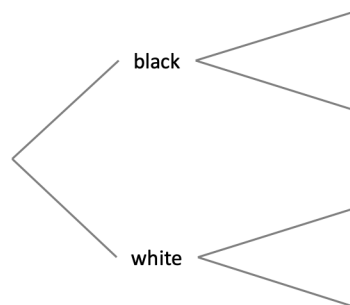
A sphere is randomly picked from the box & replaced.
What is P(two black spheres)?
What is the probability 2 spheres of different colours are picked?

3)

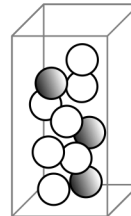


1st pick

2nd pick



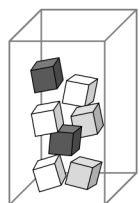
4)



A sphere is randomly picked from the box & replaced.
What is P(two spheres of the same colour)?
What is P(at least one black sphere)?

Fluency Practice

1) A cube is picked at random from the box, then replaced.



1st pick

black

white

grey

2nd pick

Probability Trees
Outcome
Probabilities

Calculate:

P(two white cubes)

P(**not** two grey cubes)

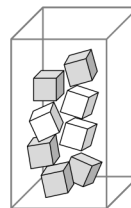
P(two cubes of the same colour)

P(a white & a grey cube)

P(at least one white cube)

2) A cube is picked at random from the box, replaced, then another is picked, replaced, then another is picked.

($8 \times 8 \times 8 = 512$)



W

G

Calculate:

P(GGG)

P(WGW in order)

P(3 cubes of the same colour)

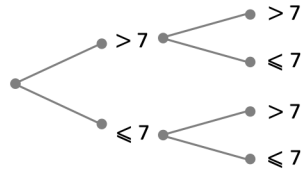
P(exactly 2 grey cubes)

Fluency Practice



10 cards are numbered 1 to 10.
A card is picked randomly & replaced,
before another card is picked.

Complete the probabilities on
each branch of this tree diagram.



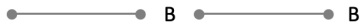
What is the probability
both cards have a value above 7?

Did we need to draw the entire tree diagram?



A bag contains 3 black & 7 red cubes.
Cubes are picked and then replaced.

Use these two branches to calculate:
P(2 cubes of the same colour are picked)



Use two branches to calculate:
P(2 cubes of different colours are picked)

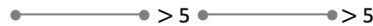


Tree Diagrams: One Branch

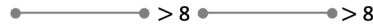
To find specific probabilities,
we can draw only **one branch** of a tree diagram.

10 cards are numbered 1 to 10.
A card is picked randomly & replaced,
before another card is picked.

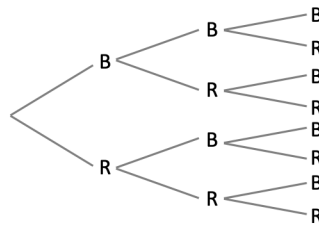
What is the probability both cards have a value above 5?



What is the probability both cards have a value over 8?



This tree diagram shows 3 picks from the same bag.



Calculate:
P(3 cubes of the same colour)

12 cards are numbered 1 to 12.

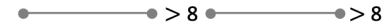
A card is picked randomly & replaced,
before another card is picked.

Complete single branches to calculate these probabilities:

P(both cards are odd)



P(both cards have a value over 8)



P(both cards are not greater than 5)



On the diagram highlight every branch where
only 1 of the 3 cubes picked is red.

Find: P(only 1 of the 3 cubes is red)

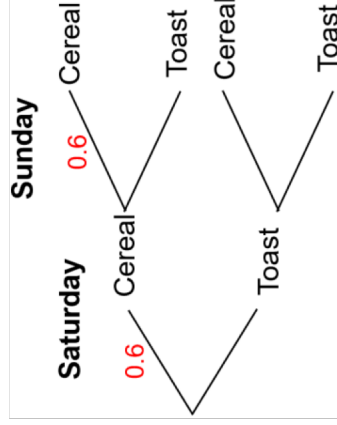
If **4 picks** are made, calculate...

P(only 1 of the cubes is red)

P(exactly 2 of the cubes are red)

Fluency Practice

Ray has cereal or toast for his breakfast. The probability that he has cereal is 0.6. Ray has breakfast on Saturday and Sunday. Complete the tree diagram.



Find the probability that Ray has toast for breakfast on both days.

Zoya throws a biased coin twice. The probability that it lands on heads is 0.55. Represent this with a tree diagram. Find the probability that:

- (a) Zoya gets two heads
- (b) Zoya gets one of each - head and tail

Joy and Peter both sit a French test. The probability of passing the test is $\frac{2}{5}$.

Represent this with a tree diagram. Find the probability that

- (a) only one of them passes the test
- (b) they both fail the test

The probability that Anna gets up late on a Monday is $\frac{1}{3}$. The probability that she gets up late on a Tuesday is $\frac{1}{4}$. Represent this with a tree diagram. Find the probability that Anna gets up late on at least one of the days.

Jamal plays three games of Monopoly against Iris. The probability that Jamal wins Monopoly is 0.4. Find the probability that Jamal wins at least two of the three games.

Purposeful Practice

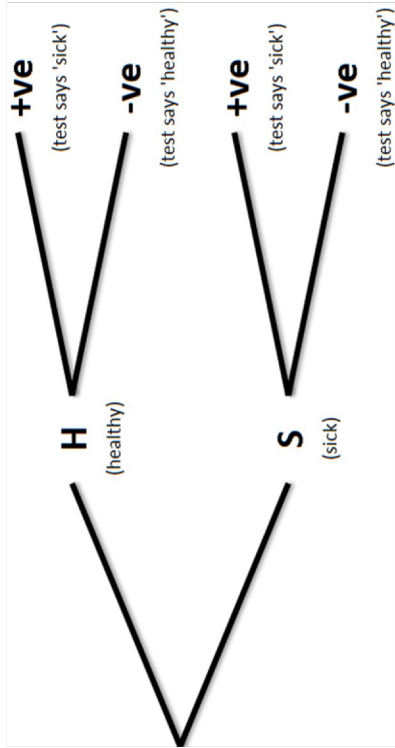


Almost all medical tests have a small chance of a 'false positive' or a 'false negative'.

- **False positive:** The test says you **do** have the condition when you really **don't**.
(*you're fine, but the test thinks you're sick*)
- **False negative:** The test says you **don't** have the condition when you really **do**.
(*you're sick, but the test thinks you're fine*)

- The test for a particular disease has a false positive rate of 5%.
*This means that 5% of people who **don't** have the disease will be told that they **do**.*
- The test has a false negative rate of 1%.
*This means that 1% of people who **do** have the disease will be told that they **don't**.*

Assuming this particular disease affects 10% of people, complete the tree diagram below:



Use the tree diagram to calculate the probabilities for each of the four outcomes:

	Healthy	Sick
Tested Positive	<i>You're fine, and the test was correct</i>	<i>You're sick, and the test was correct</i>
Tested Negative	<i>You're fine, but the test thinks you're sick FALSE POSITIVE: You may get treatment you don't need, or be worried for no reason.</i>	<i>You're sick, but the test thinks you're fine FALSE NEGATIVE: You may not get the treatment you need.</i>

If the test says I'm sick, what is the chance that I really am?

Hint: Imagine 1000 people take the test. Work out how many would test positive, and out of those people, how many are really sick?

Purposeful Practice

WORDED: PROBABILITY

You may want to use a probability tree diagram to help answer these questions.

- 1) Kelly goes ocean fishing. In that area there is a 30% chance that a caught fish is a barracuda. What is the probability Kelly catches two barracuda in a row?
- 2) James plays an online computer game. The probability of him winning any match is 0.6. What is the probability he wins his first match, then loses his second match?
- 3) In a factory, 5% of the finished TVs have a defect that needs to be fixed before they are shipped. As a fraction, what is the probability 3 TVs in a row have a defect?
- 4) Woodgreen United will play a home and an away match against Hughton FC. The probability of Woodgreen winning the home match is 0.9. The probability of them winning the away match is 0.6. What is the probability Woodgreen United win only one match?
- 5) A bag contains 12 red discs and 8 green discs. A disc is picked and replaced before another is picked. What is the probability at least one green disc is picked?
- 6) The probability of Shanice winning any darts match is $\frac{5}{7}$. If she plays twice, what is the probability the same person wins both matches?
- 7) In a box there are 3 triangles and 2 squares. Create a probability tree diagram to show the possible results when we randomly pick and replace a shape 3 times. What is the probability 2 of the 3 picks are squares?

Purposeful Practice



Jim thinks about all the probabilities for the next day.
We can assume these events are **independent**.

P Jim will be disappointed with his new haircut. = $\frac{2}{7}$

P Jim's bus to work will be late. = $\frac{1}{5}$

P Jim will bump into an old friend. = $\frac{3}{20}$

P Jim's favourite coffee shop will run out of Jim's favourite coffee. = $\frac{1}{20}$

P Jim will win the lottery. = $\frac{1}{1,000,000}$

P Jim's alarm clock will not work. = $\frac{1}{10}$

P Jim will get a rude email at work. = $\frac{4}{5}$

P It will be sunny after work. = $\frac{3}{4}$

P Jim will get a pay raise at work. = $\frac{5}{12}$

P Jim's football team will win their mid-week match. = $\frac{1}{2}$

P Jim will get rained on. = $\frac{5}{8}$

P Jim's partner will cook his favourite meal for dinner. = $\frac{2}{3}$

P Jim will find a £5 note on the street. = $\frac{3}{100}$

Find the probabilities for these combined events (a tree-diagram can help):

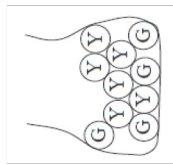
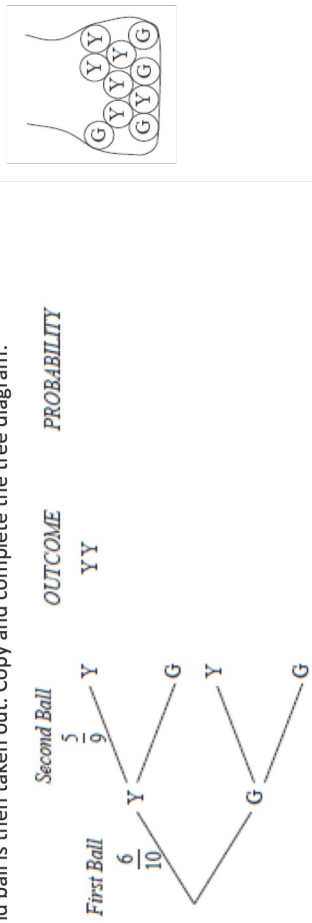
- a) Jim's bus is late but he enjoys some sun after work.
- b) Jim's bumps into an old friend and his football team win their match.
- c) Jim's gets a rude email at work, but he gets his favourite meal in the evening.
- d) Jim's alarm clock will not work, he will get a rude email and he will be disappointed with his new haircut.
- e) Jim's alarm clock works, but the bus to work is late.
- f) Jim doesn't get a pay raise, but he does win the lottery.
- g) Jim's will be happy with his haircut and he doesn't get rained on.
- h) While enjoying some sun after work, Jim will bump into an old friend and then find £5 on the street.

Create possible good & bad days for Jim. How close to a 50% probability can you get?

Describe how tomorrow's events may not be independent.

Purposeful Practice

1. A bag contains 6 yellow balls (Y) and 4 green balls (G). One ball is taken out at random and not put back. A second ball is then taken out. Copy and complete the tree diagram:

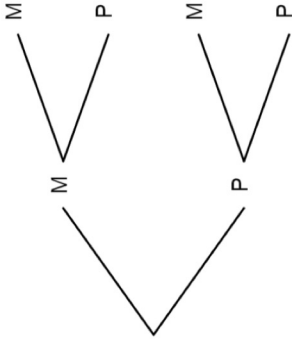


- Calculate P(2 yellow balls)
- Calculate P(2 green balls)
- Calculate P(a green ball and a yellow ball in any order)

2. A drawer contains 8 green socks and 10 blue socks. One sock is taken out of the drawer at random and not replaced. A second sock is then taken out. Draw a probability tree and determine the probability that two socks of the same colour have been taken out of the drawer.
3. A bag contains 9 blue balls and 3 red balls. A ball is selected at random from the bag and its colour is recorded. The ball is not replaced. A second ball is selected at random and its colour is recorded. Draw a tree diagram to represent this information.
- Find the probability that
- both balls selected are red
 - the second ball selected is red
4. A packet contains stamps from three different countries. The packet contains 4 Spanish stamps, 10 French stamps and 6 German stamps. Two stamps are to be removed, without replacement. By drawing a probability tree, calculate the probability that both stamps will be from the same country.
5. In a village, $\frac{3}{5}$ of the pensioners have had a flu jab. If a pensioner has had the flu jab, the probability of catching flu is $\frac{1}{30}$. If a pensioner has **not** had the flu jab, the probability of catching flu is $\frac{7}{10}$.
- Calculate the probability that a pensioner, picked at random from this village, catches flu.
 - A statistician calculated that 120 pensioners from this village are expected to catch flu. Calculate how many pensioners live in the village.
6. After a flood, Debbie finds that all the labels have come off the tins in her cupboard. Debbie knows that she had 5 tins of tomatoes and 7 tins of baked beans. She opens 3 tins at random. What is the probability that she has opened 2 tins of baked beans and one tin of tomatoes?
7. A captain and five remaining passengers remain aboard a sinking ship. There are three lifejackets remaining. The captain knows that three of the passengers cannot swim. In his panic, the captain forgets who the non-swimmers are and hands out the lifejackets randomly to three of the five passengers. What is the probability that the gives the lifejackets to just two of the three non-swimmers?

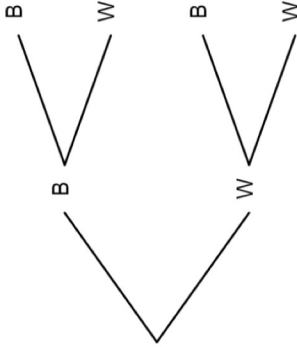
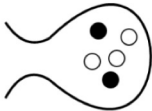
Fluency Practice

1. A box contains 8 milk chocolates and 4 plain chocolates. Linda is going to choose a chocolate at random, eat it, and then choose a second.
(a) Complete the tree diagram.



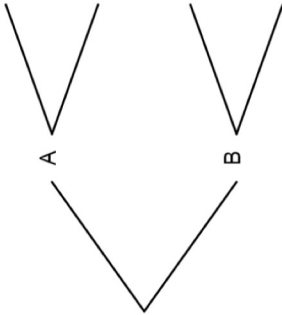
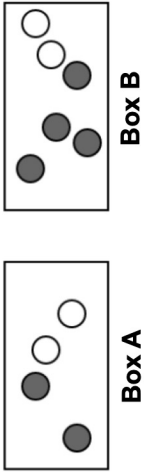
(c) Work out the probability that Linda will choose one of each type of chocolate.

2. A bag contains five counters: three white and two black. Two counters are selected from the bag at random at the same time.
(a) Complete the tree diagram.



(c) Work out the probability that the counters will be different colours.

3. Two boxes contain grey and white marbles, as shown. A box is chosen at random and then a marble from the box is chosen.
(a) Complete the tree diagram.



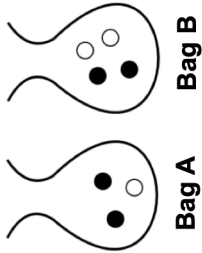
(b) Work out the probability that a grey marble is selected.

(c) Work out the probability that a white marble is selected.

Purposeful Practice

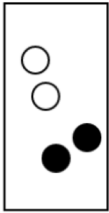
4. Two bags contain black and white counters as shown.

A counter is to be picked at random from Bag A and placed into Bag B. A counter is then to be picked from Bag B.



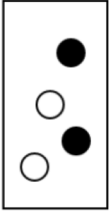
Work out the probability that the counter picked from Bag B will be white.

5. Two boxes contain black and white counters as shown.



A counter will be picked from Box A and placed into Box B.

A counter will then be picked from Box B and placed into Box A.



(a) Work out the probability that Box A will end up containing 3 white counters.

(b) Work out the probability that Box A will end up containing exactly 2 white counters.

6. A jar contains eight 10p coins and six 5p coins. Three coins are picked at random from the jar, without replacement.

(a) Work out the probability that the total value of the three coins is 30p.

(b) Work out the probability that the total value of the three coins is 15p.

(c) Work out the probability that at least one of each type of coin is picked.

7. Each day, there is a 40% chance that Luke will cycle to work, otherwise he will take the bus.

If he cycles, there is a 5% chance he will arrive late. If he takes the bus, there is a 15% chance he will arrive late.

(a) Work out the probability that Luke will arrive late to work on any given day.

(b) Given that Luke was late one day, work out the probability that he had cycled on this day.

8. A particular disease affects 5% of the trees in a forest.

A test on a sample from the tree has a 90% chance of showing a positive result if the tree is diseased. The test has a 15% chance of showing a positive result if the tree is not diseased.

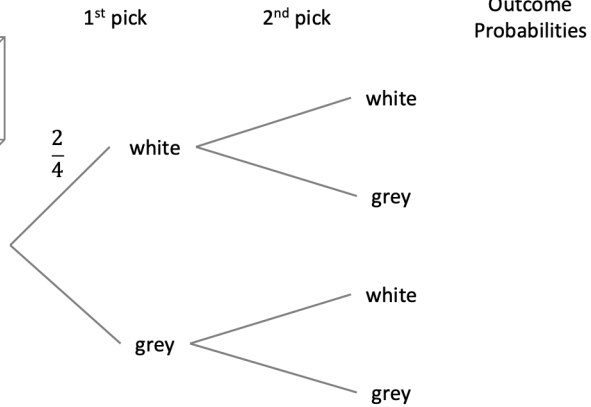
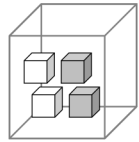
(a) Work out the probability that the test will show a positive result for a randomly selected tree.

(b) Given that the test on a tree sample has returned a positive result, work out the probability that the tree is diseased.

Fluency Practice

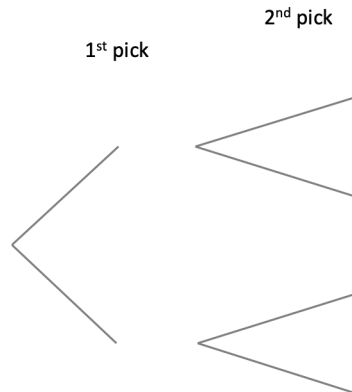
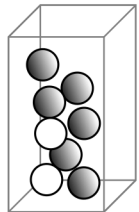
Probability Trees

1)



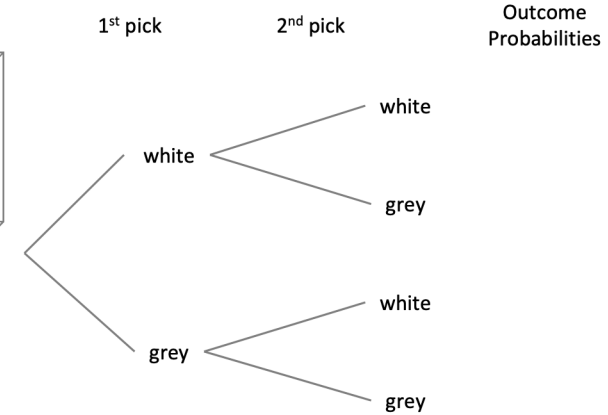
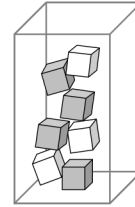
Two spheres are picked from the box.
What is P(2 white spheres)?
What is the probability at least one of the spheres is white?

3)



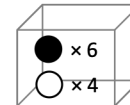
A cube is picked from the box, and **not replaced** before another is picked.
What is the probability 2 white cubes are picked?
What is the probability only 1 grey cube is picked?

2)



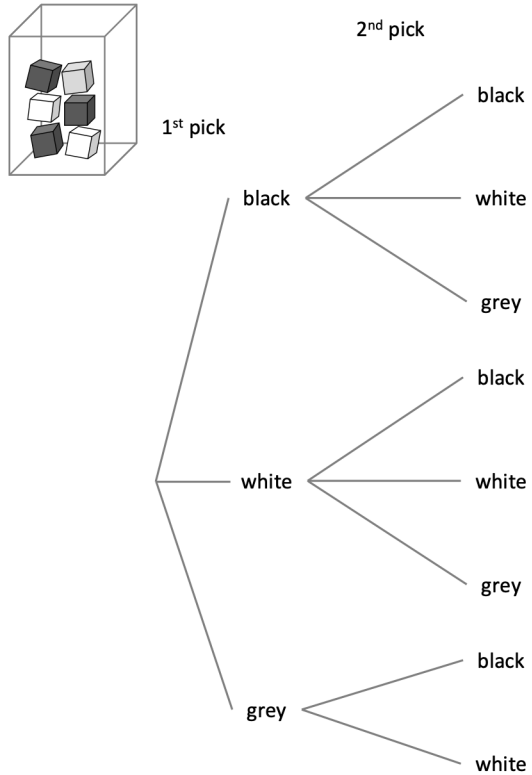
If two disks are picked from this box, what is P(different coloured disks)?
If three disks are picked from the box, what is P(3 white disks)?

4)



Fluency Practice

1) Two cubes are picked at random from the box.



Calculate:

P(two black cubes)

P(**not** two white cubes)

P(two cubes of different colours)

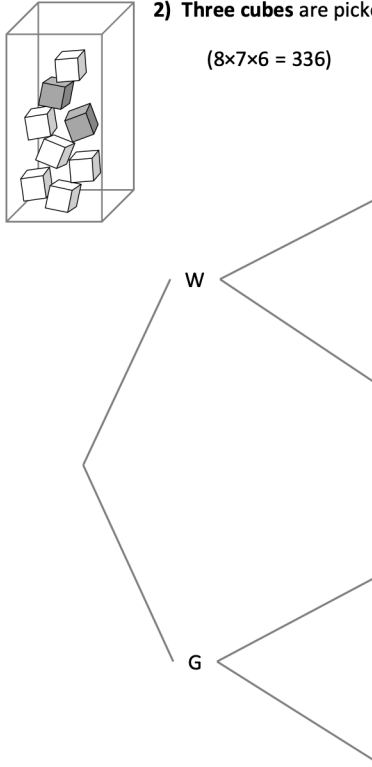
P(a grey cube & a black cube)

P(at least one black cube)

Outcome
Probabilities

2) Three cubes are picked at random from the box.

$$(8 \times 7 \times 6 = 336)$$



Calculate:

P(WWW)

P(2 or more white cubes)

P(2 grey cubes)

If 4 cubes are picked, what is P(WWWW)?

Fluency Practice

Q		Answers
1	There are 3 red and 5 blue counters in a bag. I take out one counter, record its colour, then put it back into the bag. I then take another counter. Work out the probability that both counters are blue.	
2	There are 3 red and 5 blue counters in a bag. I take out two counters. Work out the probability that both counters are blue.	
3	There are 5 red and 3 blue counters in a bag. I take out one counter, record its colour, then put it back into the bag. I then take another counter. Work out the probability that both counters are blue.	
4	There are 5 red and 3 blue counters in a bag. I take out two counters. Work out the probability that both counters are blue.	
5	There are 5 red and 3 blue counters in a bag. I take out one counter, record its colour, then put it back into the bag. I then take another counter. Work out the probability that the counters are different colours.	
6	There are 5 red and 3 blue counters in a bag. I take out two counters. Work out the probability that the counters are different colours.	
7	There are 2 red and 5 blue counters in a bag. I take out one counter, record its colour, then put it back into the bag. I take out another counter, record its colour, then put it back into the bag. Work out the probability that I take out only blue counters.	
8	There are 2 red and 5 blue counters in a bag. I take out three counters. Work out the probability that I take out only blue counters.	
9	There are 2 red and 5 blue counters in a bag. I take out one counter, record its colour, then put it back into the bag. I take out another counter, record its colour, then put it back into the bag. Work out the probability that I take out only red counters.	
10	There are 2 red and 5 blue counters in a bag. I take out three counters. Work out the probability that I take out only red counters.	

Purposeful Practice

- 1 A box contains 3 red discs, 5 blue discs and 6 green discs. I remove one disc at random, note its colour then return it to the box. I then remove a second disc at random.
 - (a) Find the probability that both discs are the same colour.
 - (b) Find the probability that exactly one disc is red.
 - (c) If I take a total of 5 discs, replacing the disc each time, find the probability that all 5 discs are red.
- 2 Another box contains 4 yellow discs and 7 black discs. I remove one at random, place it in my pocket then remove a second disc at random.
 - (a) Find the probability that the discs are different colours.
 - (b) If I take a total of three discs, find the probability that:
 - (i) all are yellow
 - (ii) all three are different colours.
- 3 A bag contains some red balls, some blue balls and some green balls. When a ball is removed at random the probability that it is red is $\frac{1}{3}$ and the probability that it is blue is $\frac{2}{9}$. There are more than 30 balls in the box. What is the fewest number of green balls?
- 4 Each morning the probability that Anna gets up late is $\frac{1}{5}$. If she gets up late the probability that she misses her bus is $\frac{5}{6}$. If she doesn't get up late the probability that she misses her bus is $\frac{1}{4}$.
 - (a) Find the probability that, on Monday morning, she misses her bus.
 - (b) Find the probability that she catches her bus every morning from Monday to Friday.
- 5 A box contains some red ball and some blue balls. There are four more blue balls than red balls. A ball is removed at random, replaced and a second ball randomly removed. The probability that the two balls are different colours is $\frac{21}{50}$. How many balls of each colour are in the box?
- 6 A box contains some white balls and some blue balls. There are 5 more blue balls than white balls. One ball is removed at random and not replaced. A second ball is then removed at random. The probability that the balls are different colours is $\frac{52}{105}$. Find the probability that both balls are white.
- 7 A prize must be randomly awarded to just one of a group of ten people. Which of the following methods is the fairest?

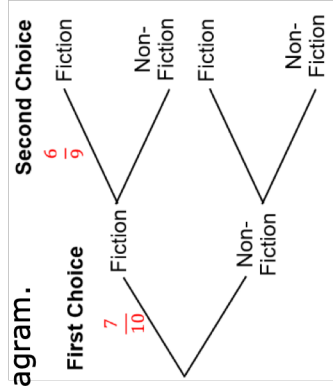
A: Fold 10 identical pieces of paper in half and put them in a bag. Nine have 'LOSE' written on them and one has 'WIN'. Line the people in alphabetical order. The first person randomly takes a piece of paper. If it says 'WIN' they are the winner and the game stops there. If it says 'LOSE' they do not replace the paper and the next person has a go.

B: Same as 'A' but this time the paper is put back in the bag before the next person has a go.

C: Same as 'B', that is each piece of paper is returned to the bag, but there are 999 'LOSE' papers and one 'WIN' paper.

Purposeful Practice

A bookshelf contains three non-fiction books and seven fiction books. Bob chooses two books at random. Complete the tree diagram.



Find the probability that Bob chooses two non-fiction books.

A drawer contains five red socks and three black socks. A sock is taken out at random and not replaced. A second sock is then taken out. Draw a tree diagram and calculate the probability that either a pair of red socks or a pair of black socks is chosen.

A bag contains 4 yellow balls and 6 green balls. A ball is taken from the bag and not replaced. A second ball is then taken. Draw a tree diagram and find the probability that the two balls are different colours.

Amir has a biscuit tin containing 3 bourbons and 5 custard creams. He chooses a biscuit at random, eats it, then chooses another and eats that too. Draw a tree diagram and use it to find the probability that Amir has eaten at least one custard cream.

In a zoo, there are four elephants, three tigers and two giraffes. The zookeeper wants to choose two animals at random to take part in a promotional photo. Draw a tree diagram and use it to find the probability that the zookeeper chooses two of the same animal.

Purposeful Practice

(a) A bag contains seven red balls and three black balls. Anton takes out a ball, notes the colour and replaces it. He does this three times. What is the probability that he takes out at least one red ball?

(b) Debbie has 5 tins of tomatoes and 7 tins of beans in the cupboard, but the labels have come off the tins. Debbie chooses two tins at random. Calculate the probability that Debbie chooses two tins of the same type.

(c) Ashok has six coins in his pocket. He has one 5 cent coin, two 10 cent coins and three 20 cent coins. He takes at random a coin from his pocket, records its value and puts the coin back into his pocket. He does this twice. Calculate the probability that the second coin he takes has a higher value than the first coin he takes.

(d) A bag contains 3 black beads, 5 red beads and 2 green beads. Gianna takes a bead at random from the bag, records its colour and replaces it. She does this two more times. Work out the probability that, of the three beads Gianna takes, exactly two are the same colour.

(e) Barney has a biased coin. When the coin is thrown once, the probability that the coin will land heads is 0.3. Barney throws the coin 4 times. Work out the probability that the coin will land heads at least once.

(f) Carolyn has 20 biscuits in a tin. She has 12 plain biscuits, 5 chocolate biscuits and 3 ginger biscuits. Carolyn takes at random two biscuits from the tin. Work out the probability that the two biscuits were **not** the same type.

Fluency Practice

Conditional Tree Diagrams



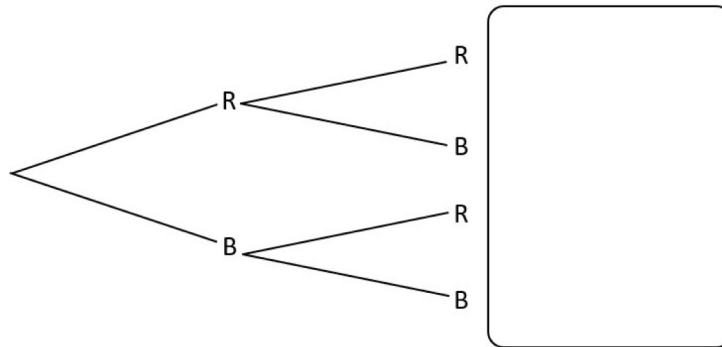
There are 5 red and 3 blue buttons in a sack.

A button is taken out and not replaced. Then another button is taken out.
What is the probability both buttons are blue?

1st Event

2nd Event

Results



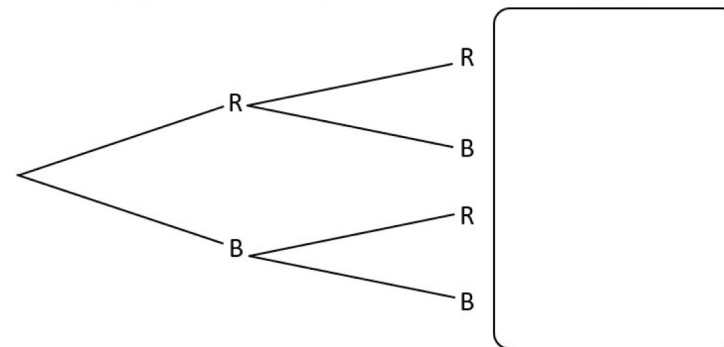
There are 2 red and 6 blue buttons in a sack.

A button is taken out and not replaced. Then another button is taken out.
What is the probability both buttons are red?
What is the probability of choosing at least one red button?

1st Event

2nd Event

Results



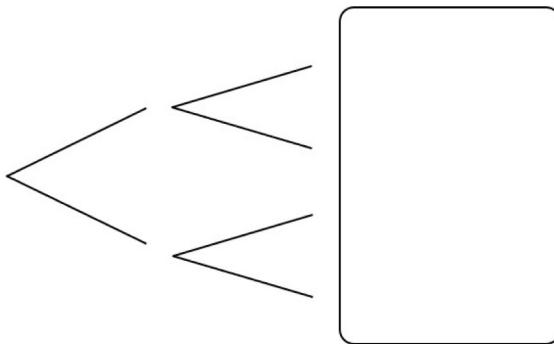
There are 4 red and 5 green buttons in a sack.

A button is taken out and not replaced. Then another button is taken out.
What is the probability of choosing at least one red button?
What is the probability of choosing only one red button?

1st Event

2nd Event

Results



d) A science assessment has a practical and written test. Students have an 80% chance of passing the written test. If they pass the written test there is a 60% chance they pass the practical test. If they fail the written test, there is a 30% chance they will pass the practical test.

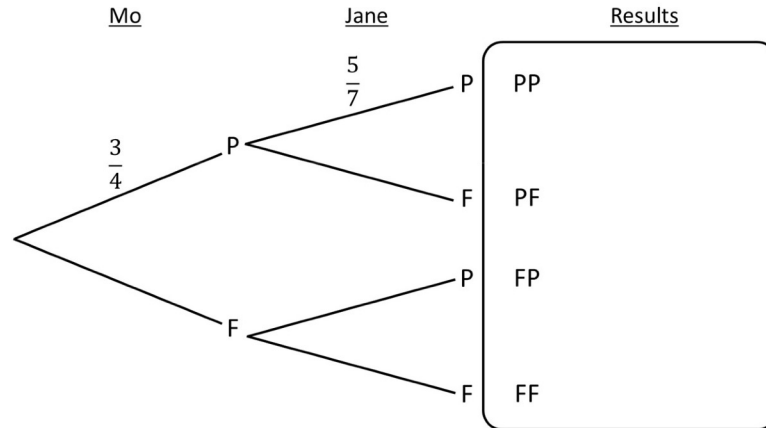
- 1) Draw a Probability Tree to show these events.
- 2) What is the probability a student passes both tests?
- 3) What is the probability a student passes only one of the tests?

Fluency Practice

Tree Diagrams Revision		
(a) Allie and Alex play two games of tennis. The probability of Allie winning the game is 0.45. (a) Complete the tree diagram. 1st Game 0.45 Allie wins Alex wins 2nd Game 0.45 Allie wins Alex wins (b) Find the probability that Alex wins both matches. (c) Find the probability that Allie and Alex win one match each.	(b) Yusuf spins two fair spinners, A and B. Spinner A can land on a 1, 2 or 3. Spinner B can land on a 2, 3 or 4. (a) Draw a tree diagram. (b) Find the probability that the total on the spinners is an odd number.	(c) There are 10 biscuits in a tin. 7 are digestives and 3 are bourbons. Temi takes a biscuit at random from the tin and eats it. She does this two more times. Calculate the probability that she has eaten at least two digestives.

Fluency Practice

The probability Mo passes his driving test is $\frac{3}{4}$. The probability Jane passes is $\frac{5}{7}$.
Complete the Tree Diagram and find the probability they both fail their tests.



There are 2 red & 7 blue buttons in a sack. ③

A button is taken out and not replaced. Then another button is taken out.

- What is the probability both buttons are red?
- What is the probability of choosing two buttons of the same colour?

①

The probability Latifa passes her Maths test is $\frac{7}{9}$.
The probability she passes her English test is $\frac{5}{6}$.

- Complete a Tree Diagram and find the probability she passes both tests.
- What is the probability she fails both tests?
- What is the probability she passes only one test?

②

A science assessment has a practical & a written test. Kara has a 70% chance of passing the written test. If she passes the written test there is a 80% chance she passes the practical test.
If she fails the written test, there is a 30% chance she will pass the practical test.

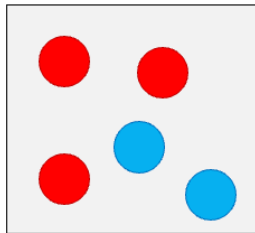
- Draw a Probability Tree to show these events.
- What is the probability Kara fails both tests?
- What is the probability Kara passes only one of the tests?

④

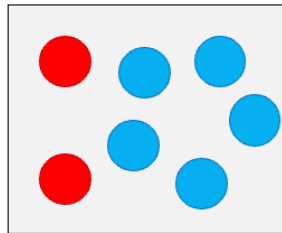
Problem Solving

Laura picks a marble from Box A and puts it into Box B.

She then picks a marble from Box B.



Box A

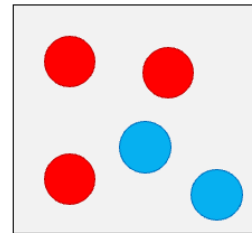


Box B

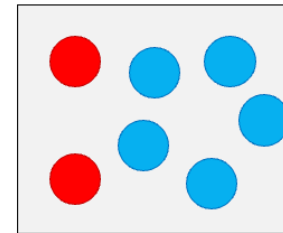
vs

Laura picks a marble from Box B and puts it into Box A.

She then picks a marble from Box A.



Box A



Box B

- Does it matter which box Laura chooses from first?
- Will the order affect the probabilities of the combined outcomes?

Problem Solving

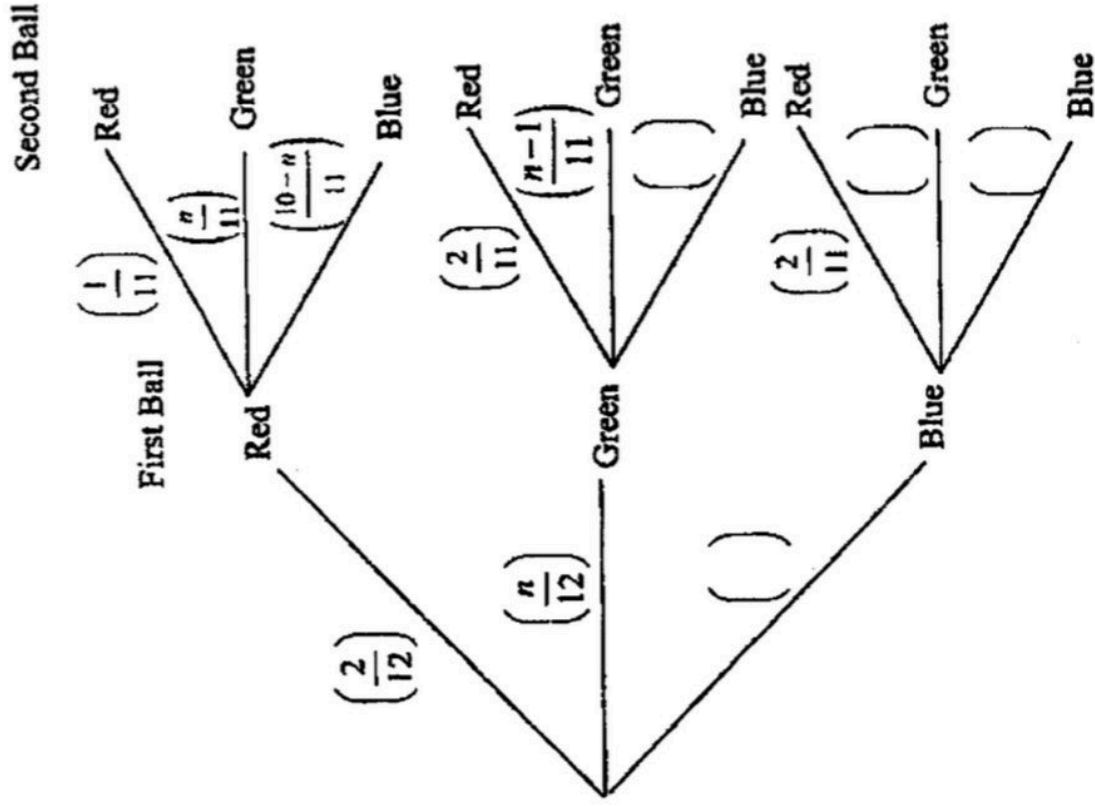
Donna and James each arrange 36 square tiles into a quadrilateral. They then calculate the area and perimeter of their quadrilaterals.

What is the probability that they both make quadrilaterals where the value of the perimeter is greater than the value of the area?

Purposeful Practice

A bag contains 12 balls.
2 of the balls are red, n are green balls and the rest are blue balls.
Eunice takes two balls from the bag, at random, without replacement.

- (a) Complete the tree diagram.



- (b) The probability that Eunice takes two blue balls is $\frac{5}{22}$.

Write down an equation to represent this information and show that it simplifies to

$$n^2 - 19n + 60 = 0.$$

Fluency Practice

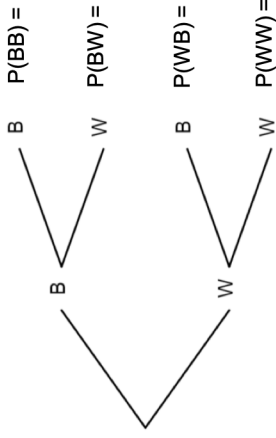
1. A fair spinner has ten equally likely sectors. n of the sectors are black and the rest are white.

(a) The spinner is to be spun once.

(b) Write expressions in terms of n for:

(i) $P(\text{black})$

(ii) $P(\text{white})$



(b) The spinner is to be spun twice.

Complete the tree diagram with expressions in terms of n for the probabilities.

(c) Given that when the spinner is spun twice, the probability that it lands on black both times is $\frac{16}{25}$, work out the value of n .

2. A fair spinner has eight equally sized sectors. n of the sectors are red and the rest are blue.

(a) The spinner is to be spun once. Write expressions in terms of n for:

(i) $P(\text{red})$

(ii) $P(\text{blue})$

(b) If the spinner is spun twice, the probability of it landing on red both times is $\frac{9}{16}$. Work out the value of n .

3. A fair six-sided die has n green faces. The rest of the faces are black.

(a) The die is to be rolled once. Write expressions in terms of n for:

(i) $P(\text{green})$

(ii) $P(\text{black})$

(b) If the die is rolled twice, the probability of it landing on two different colours is $\frac{1}{2}$. Work out the value of n .

4. There are 10 cards in a pack. x of the cards have a triangle and the rest have a star.

(a) A card is to be picked at random. Write expressions in terms of x for:

(i) $P(\text{triangle})$

(ii) $P(\text{star})$

(b) If a card is picked at random, replaced, and then a second card picked, the probability that the two counters have the same shape is $\frac{29}{50}$. Work out the two possible values of x .

5. A bag contains 15 counters. x of the counters are yellow.

(a) A counter is to be picked at random. Write expressions in terms of x for:

(i) $P(\text{yellow})$

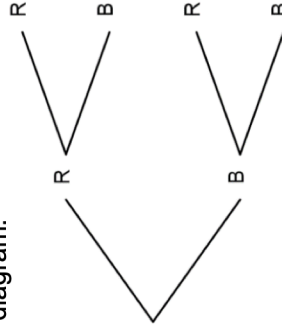
(ii) $P(\text{yellow'})$

(b) If a counter is picked from the bag at random, replaced, and then a second counter picked, the probability that at least one counter will be yellow is $\frac{24}{25}$. Work out the value of x .

Fluency Practice

6. A bag contains red and blue counters.
There are n red counters and twice as many blue counters.

(a) Two counters are picked at random without replacement.
Work out the probabilities (in terms of n) for the tree diagram.



The probability that the
counters are both red is $\frac{1}{10}$.

(b) Work out the value of n .

7. A bag contains yellow and red sweets in
the ratio 1 : 4.

If two counters are picked at random (without
replacement), the probability that they will
both be yellow is $\frac{1}{35}$.

Work out how many yellow sweets are in the
bag.

9. A bag contains n counters. 5 of the
counters are blue and the rest are pink.

If two counters are picked at random
(without replacement), the probability that
they will both be blue is $\frac{5}{18}$.

(a) Show that $n^2 - n - 72 = 0$.

(b) Work out the value of n .

11. A bag contains n red counters and 3
yellow counters.

If two counters are picked at random
(without replacement), the probability that
they will both be red is $\frac{5}{12}$.

(a) Show that $7n^2 - 37n - 30 = 0$.

(b) Work out the value of n .

8. A bag contains black and white tokens.

There are 3 times as many white tokens as
black discs.

If two tokens are picked at random (without
replacement), the probability that they will be
the same colour is $\frac{3}{5}$.

Work out how many black tokens are in the
bag.

10. A bag contains n counters. 8 of the
counters are green and the rest are yellow.

If two counters are picked at random (without
replacement), the probability that they will be
different colours is $\frac{16}{45}$.

(a) Show that $n^2 - 46n + 360 = 0$.

(b) Work out the two possible values of n .

12. A bag contains 5 gold counters and at least
one silver counter.

If two counters are picked at random (without
replacement), the probability that they will be
the same colour is $\frac{2}{3}$.

Work out the two possible numbers of silver
counters.

Fluency Practice

Question 1: A drawer contains 6 red socks and 2 green socks.



A red sock is taken out of the drawer and not replaced.

Dexter is going to pick a sock at random from the remaining socks in the drawer.

- (a) Write down the probability that Dexter picks a red sock?
- (b) Write down the probability that Dexter picks a green sock?

Question 2: There are 10 chocolates in a bag.
Four are milk chocolate and six are dark chocolate.
Two chocolates are taken out at random **without** replacement.

(a) Copy and complete the tree diagram

1st chocolate

2nd chocolate

outcome

probability

$\frac{4}{10}$

Milk

$\frac{6}{10}$

Dark

$\frac{3}{9}$

Milk

$\frac{6}{9}$

Dark

MM

MD

DM

DD

$\frac{4}{10} \times \frac{3}{9} = \frac{12}{90}$

(b) Find the probability of getting two dark chocolates.

(c) Find the probability of getting one chocolate of each flavour.

(d) Find the probability of getting at least one milk chocolate.

Question 3: 13 of the 20 students in Mr Davidson’s class are girls.
Two students are chosen at random.

(a) Copy and complete the tree diagram

1st student

2nd student

outcome

probability

boy

girl

boy

girl

BB

BG

GB

GG

$\frac{13}{20} \times \frac{12}{19} = \frac{156}{190}$

(b) Work out the probability of two boys being selected.

(c) Work out the probability of two girls being selected.

Fluency Practice

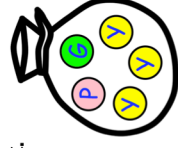
Question 4: There are 12 sweets in a bag
7 are lemon and 5 are orange.

Two sweets are taken out at random **without** replacement.

- (a) Work out the probability that the two sweets are both lemon
- (b) Work out the probability that the two sweets are both orange
- (c) Work out the probability that the two sweets are the same flavour
- (d) Work out the probability that the two sweets are different flavours

Question 5: There are five counters in a bag.

One counter is pink, one counter is green and three counters are yellow.
A counter is selected at random from the bag **without** replacement.
Then a second counter is taken at random.



- (a) Find the probability that both counters are yellow.
- (b) Find the probability of a pink counter then a yellow counter.

Question 6: In dry weather, the probability of a bus being late is $\frac{1}{10}$

In rainy weather, the probability of a bus being late is $\frac{1}{4}$

In snowy weather, the probability of a bus being late is $\frac{2}{3}$

The probability of dry weather is $\frac{3}{4}$

The probability of wet weather is $\frac{1}{5}$

The probability of snow is $\frac{1}{20}$

- (a) Show this information on a tree diagram
- (b) Calculate the probability that the weather is dry **and** the bus is on time.
- (c) Calculate the probability that the bus is late

Question 7: Catherine has two bags of counters.

Bag A contains 6 red counters and 2 black counters.

Bag B contains 1 red counters and 4 black counters

Catherine rolls a fair ordinary six-sided dice.

If the dice lands on a 1, she takes a counter at random from Bag B.

If the dice lands on any other number, Catherine takes a counter randomly from Bag A.

Calculate the probability of Catherine getting a black counter.

Fluency Practice

Question 8:

There are three flavours of crisps in a cafe.
There are 3 packets of salt and vinegar

5 packets of cheese and onion

1 packet of roast chicken

Bella takes two packets of crisps at random.

- (a) Work out the probability that she takes 2 packets of crisps that are the **same** flavour.
- (b) Work out the probability that she takes 2 packets of crisps that are **different** flavours.

Question 9:

Toby has 20 counters in a bag.

11 counters are yellow.

6 counters are red.

3 counters are white.

Toby takes two counters from the bag at random.

Work out the probability that the two counters are **not** the same colour.

Question 10:

There are 9 sweets in a bag.

Five sweets are purple, three sweets are white and one sweet is pink.

Three sweets are selected at random **without** replacement.

Calculate the probability that the sweets are **not** all the same colour.

Question 11: Florence has these letter tiles.

M A T H E M A T I C S

She picks three tiles at random without replacement.

- (a) Calculate the probability that all three times are vowels.
- (b) Calculate the probability that there are no vowels.
- (c) Calculate the probability that exactly one T is taken from the bag.

Question 12:

There are 11 lego blocks in a bag, with each block the same size.

Five are red and six are yellow.

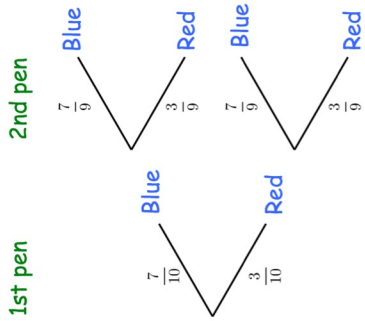
Four blocks are selected at random.

- (a) Calculate the probability that all the blocks are yellow
- (b) Calculate the probability that at least 3 of the blocks are red.

Purposeful Practice

Apply

Question 1: Samantha has 10 pens.
7 of the socks are blue and the rest are red.
Samantha takes a pen at random without replacement.
She then takes a second pen at random.

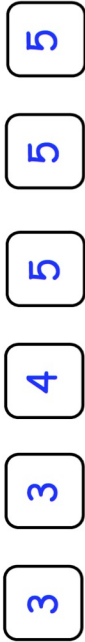


Samantha drew this tree diagram.

(a) Write down what is wrong with the probabilities in the tree diagram.

(b) Calculate the probability that Samantha picks at least one red pen.

Question 2: Here are six number tiles.



Charlie takes a tile at random without replacement.
Charlie then takes a second tile at random.

- (a) Work out the probability the both tiles have the number 5 on them
- (b) Work out the probability that the number on the second is less than the number on the first tile.

Question 3: There are 20 passengers on a coach.
70% of the passengers are going to Bristol.
The rest are going to Bath.
Four passengers are chosen at random to complete a survey.

Calculate the probability that all four passengers are going to Bath.

Question 4: Ethan has 12 coins.
There are three 10p coins and nine 20p coins.
Ethan chooses 3 coins at random.
Work out the probability that he takes exactly 50p.

Purposeful Practice

Question 5: There are 50 students in Year 11.
Each student studies one language.

	French	German
Female	11	12
Male	7	20

Two of these students are selected at random.

Calculate the probability that the chosen students will be a female studying French and any male.

Question 6: A box contains 40 pens.
30 of the pens are blue and the rest are green.
One pen is taken out at random and replaced by five of the other colour.
Another pen is taken out at random and replaced by five of the other colour.
A final pen is taken out at random.

- (a) Work out the probability that all three pens are green.
- (b) Work out the probability that all three pens are the same colour.
- (c) Work out the probability that all three pens are **not** the same colour.

Question 7: Jamie has some coins.



Jamie has to pay 60p for a car park ticket.
He selects 3 coins at random, without replacement, from his pocket.
Work out the probability that he has chosen the exact price of the ticket.

Question 8: Rebecca has 10 cards, each with a number on it.

2	2	2	3	4	5	6	7	7	9
---	---	---	---	---	---	---	---	---	---

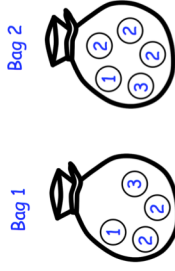
She picks three cards at random, without replacement.
Rebecca adds the three numbers together to get a score.
Calculate the probability that the score is an odd number.

Purposeful Practice

Question 9: Harrison has two bags, each containing discs.

Bag 1 contains one disc labelled one, two discs labelled two and one disc labelled three.
Bag 2 contains one disc labelled one, three discs labelled two and one disc labelled three.

Harrison chooses a disc at random from Bag 1.
If the disc is labelled 1, he puts the disc in Bag 2.
If the disc is not labelled 1, he does **not** put the disc in Bag 2.
Harrison then chooses a disc at random from Bag 2.



Harrison adds together the numbers from the two discs he selected to give his score.

Find the probability of Harrison scoring 4.

Question 10: 45 students were asked if they have visited Canada, Mexico or the USA.

11 students had been to Canada

1 student had visited all three countries

2 students had visited Canada and Mexico but not the USA.

3 students had visited Mexico and the USA.

12 students had not visited any of the countries.

6 out of the 19 students who had visited the USA, had been to at least one of the other countries.

Two of the 45 students are chosen at random.

Work out the probability that they both had only visited Mexico.

Question 11: There are x apples in a crate.
2 of the apples are bad.

Jesse chooses two apples from the crate, without replacement.

The probability that he selects two bad apples is $\frac{1}{28}$

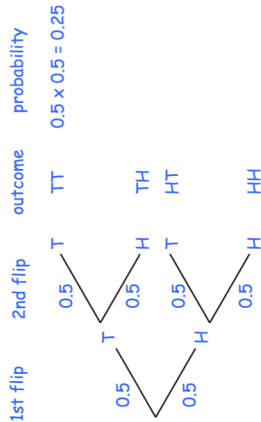
(a) Prove $x^2 - x - 56 = 0$

(b) Find x , the number of apples in the crate

Fluency Practice

Question 1: A fair coin is flipped twice.

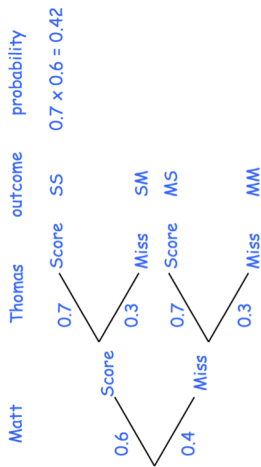
- (a) Find the probability of getting two heads
- (b) Find the probability of getting a head and then a tail
- (c) Find the probability of getting at least one head



Question 2: Matt and Thomas each take a penalty.

The probability that Matt scores is 0.6
The probability that Thomas scores is 0.7

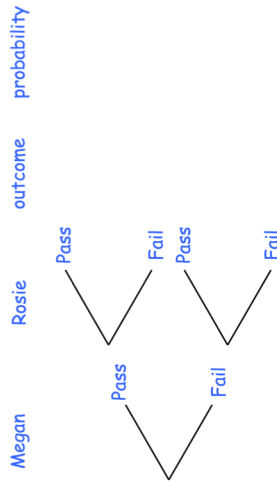
- (a) Find the probability of both boys missing
- (b) Find the probability of one boy scoring
- (c) Find the probability of at least one boy missing



Question 3: Megan and Rosie sit their driving tests.

The probability that Megan passes the test is 0.8
The probability the Rosie fails the test is 0.3

- (a) Copy and complete the tree diagram
- (b) Find the probability that both women pass
- (c) Find the probability that Megan fails and Rosie passes
- (d) Find the probability that at least one woman passes



Fluency Practice

Question 4: Harry goes to an arcade. He has one go on the Teddy Grabber and one go on the Penny Drop.

The probability that he wins on the Teddy Grabber is $\frac{1}{3}$

The probability that he wins on the Penny Drop is $\frac{2}{5}$

(a) Copy and complete the tree diagram

Teddy Grabber

Penny Drop

outcome

probability

$\frac{1}{3}$

Win

$\frac{2}{5}$

Win

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

$\frac{1}{3}$

Lose

$\frac{2}{5}$

Lose

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

(b) Work out the probability that Harry loses on the Teddy Grabber and he also loses on the Penny Drop

$\frac{1}{3}$

Win

$\frac{2}{5}$

Win

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

$\frac{1}{3}$

Lose

$\frac{2}{5}$

Lose

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

(c) Work out the probability that Harry wins on exactly one machine

$\frac{1}{3}$

Win

$\frac{2}{5}$

Win

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

$\frac{1}{3}$

Lose

$\frac{2}{5}$

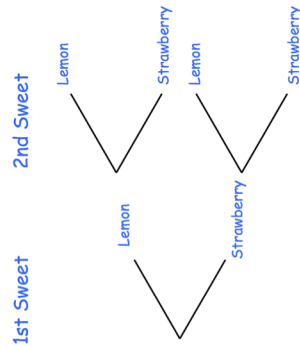
Lose

$\frac{1}{3} \times \frac{2}{5} = \frac{2}{15}$

Question 5: There are 5 lemon and 4 strawberry sweets in a bag.

Hailey takes out a sweet at random, writes down its flavour and puts it back into the bag. Then Hailey takes out a second sweet, at random, and writes down its flavour.

- (a) Copy and complete the tree diagram
- (b) Find the probability of two lemon sweets
- (c) Find the probability of one of each flavour
- (d) Find the probability of no lemon sweets



Question 6: The probability that a bus arrives late is 0.1

Victor is travelling by bus on Monday and Tuesday.

- (a) Show this information on a tree diagram
- (b) Calculate the probability that the bus is on time both days.

Question 7: Each morning Martina attempts a crossword and a Sudoku

The probability that Martina successfully completes the crossword is 0.3
The probability that Martina successfully completes the Sudoku is 0.6

- (a) Show this information on a tree diagram
- (b) Work out the number of days that Martina successfully completes both the crossword and Sudoku over a period of 200 days.

Purposeful Practice

Apply

Question 1: Timothy is taking part in an archery competition.

The probability of windy weather is 0.2

If it is windy, the probability of Timothy hitting the target is 0.35
 If it is not windy, the probability of Timothy hitting the target is 0.8

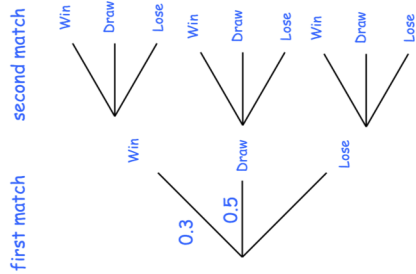
- (a) Draw a tree diagram to show this information
- (b) Find the probability of Timothy hitting the target.

Question 2: A football team has two matches to play.

The probability that the team wins is 0.3
 The probability that the team draws is 0.5

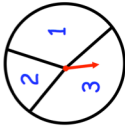
A win is worth 3 points, a draw 1 point and a loss 0 points.

Calculate the probability that the team will score at least 3 points over the two matches.



Question 3: Shown is a spinner.

The probability of a 1 is $3x$
 The probability of a 2 is x
 The probability of a 3 is $4x$



- (a) Calculate the value of x

The spinner is spun twice and the scores are multiplied together.

- (b) Work out the probability that the final score is odd.

Question 4: Freddie and Martha have dentist appointments.

The probability that Freddie is on time to his appointment is 0.9
 The probability that both Freddie and Martha are on time to their appointments is 0.72

- (a) Draw a tree diagram for this information
- (b) Find the probability that both people are late for their appointments

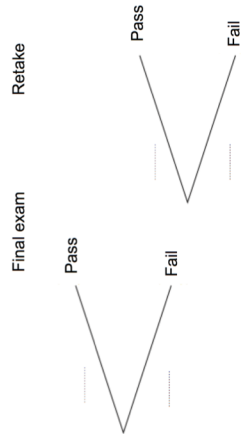
Purposeful Practice

Question 5: A college course consists of 8 weeks of teaching with a final exam at the end of the course

If a student fails the final exam, they have one opportunity to retake the exam.

The probability of a student passing the final exam is $\frac{7}{8}$

The probability of a student passing the retake is $\frac{2}{3}$

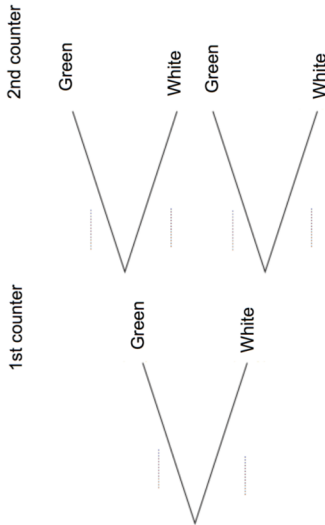


- (a) Complete the tree diagram
- If a student passes the final exam or retake, they receive a certificate.
- (b) Work out the probability that a student receives a certificate.

Question 6: There are 10 counters in a bag, 7 are green and the rest of white.

Erin takes out a counter at random and records its colour. Without replacement, Erin takes out another counter, at random.

- (a) Complete the tree diagram
- (b) Find the probability that both counters are different colours
- (c) Find the probability that both counters are the same colour



Question 7: Jenson is going to choose a ball at random from a bag and then flip a coin.

There are 5 balls in the bag, 2 white and 3 black.

A ball is picked at random from the bag and its colour is recorded.

If the ball is white, a fair coin is flipped.
If the ball is black, a biased coin is flipped, where heads has a probability of $\frac{7}{8}$

- (a) Draw a tree diagram to show this information
- Jenson selects a ball and flips the appropriate coin.
- (b) Find the probability that he obtains a tail.

Purposeful Practice

Question 8: There are x apples in a crate.
3 of the apples are bad.

Robert chooses two apples from the crate, without replacement
The probability that he selects two bad apples is $\frac{1}{12}$

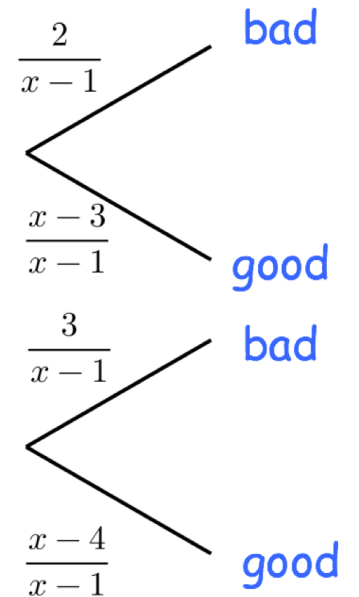
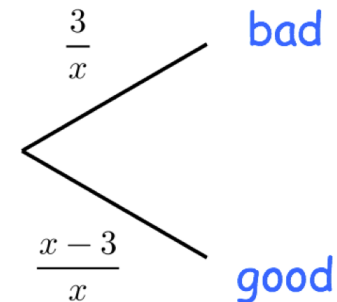
(a) Using the tree diagram, prove $x^2 - x - 72 = 0$

(b) Find the number of apples in the crate, x .

(c) Find the probability that both apples are good

1st Apple

2nd Apple



Fluency Practice

Question 1: A fair coin is flipped twice.



- (a) Find the probability that the coin lands on heads twice.
- (b) Find the probability that the coin lands on tails twice.
- (c) Find the probability that the coin lands on heads exactly once.

Question 2: Penelope is playing football.

When attempting to score a penalty, the probability she scores is $\frac{2}{3}$

During the game, Penelope takes two penalties.
Find the probability that Penelope scores both.

Question 3: Trevor is taking part in a quiz.

The probability that he answer a question correctly is $\frac{3}{5}$

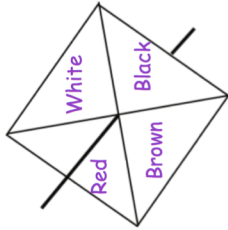
Trevor is asked two questions.

- (a) Calculate the probability that Trevor answers both questions correctly.
- (b) Calculate the probability that Trevor answers both questions incorrectly.

Question 4: Daisy has a biased spinner.

The probability of each colour is:

Colour	Red	White	Black	Brown
Probability	0.1	0.4	0.3	0.2



Daisy spins the spinner twice.

- (a) Find the probability of the spinner landing on white twice.
- (b) Find the probability of the spinner landing on black and then brown.
- (c) Find the probability of the spinner landing on the same colour in both spins.

Question 5: A fair six sided dice is rolled three times.

- (a) Find the probability of getting a two all three times.
- (b) Find the probability of getting no twos

Fluency Practice

Question 6:

Mark is playing darts.
The probability he hits the bullseye is 0.4
Mark throws two darts

- (a) Find the probability of Mark hitting the bullseye once.
- (b) Find the probability of Mark hitting the bullseye at least once.

Question 7:

A bag contains five yellow sweets, three green sweets and one purple sweet.
A sweet is taken out of the bag and replaced.
Another sweet is taken out.

- (a) Find the probability that both sweets are yellow.
- (b) Find the probability of neither sweet is green.
- (c) Find the probability that the two sweets are different colours.

Question 8:

The probability of a bus being on time is $\frac{3}{4}$

Archie catches the bus to work three times each week.

- (a) Work out the probability that the bus is late every time.
- (b) Work out the probability that the bus is on time every time.
- (c) Work out the probability that the bus is late exactly once.



Question 9: Jackson, Frederick and Kelvin each sit a test.

The probability Jackson passes is $\frac{9}{10}$

The probability Frederick passes is $\frac{2}{3}$

The probability Kelvin passes is $\frac{1}{2}$

- (a) Find the probability that Jackson and Kelvin pass, but Frederick fails.
- (b) Find the probability that Frederick passes, but Jackson and Kelvin fail.
- (c) Find the probability that at least two boys pass.

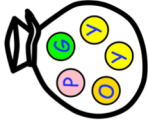
Question 10: The probability that Dylan reads at night is $\frac{4}{5}$

Calculate the probability that Dylan reads every night in one week.

Purposeful Practice

Apply

Question 1: Amelia is organising a game for a charity fête. She has put 1 orange, 1 pink, 1 green and 2 yellow counters into a bag.



To play, each person will pay £1 and take out a counter at random. They will then replace the counter and then take a second counter at random. The person will win £2.50 if both counters are the same colour.

Amelia expects 200 people to play the game.

How much money would Amelia expect to raise for charity?

Question 2: There are 12 tiles in a bag, each with a letter written on it.

C O R B E T T M A T H S

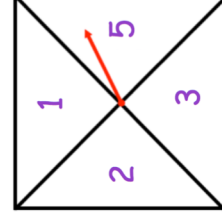
A tile is selected at random and then replaced.
Another tile is then selected.

Find the probability that both tiles have different letters on them.

Question 3: A fair spinner has four sections.

The spinner is spun three times.
The three numbers are added together to give a score.

- (a) Find the probability that the score is odd.
- (b) Find the probability that the score is greater than 3.



Question 4: Tom and Ben sit their driving test.
The probability Tom passes is 0.4
The probability that only one man passes is 0.56
Find the probability they both fail.

Fluency Practice

Q	Answers
1 There are $(n + 3)$ black buttons and $(n - 3)$ white buttons. Salma takes at random two buttons. The probability that both buttons are black is $\frac{6}{11}$. Work out the probability that both are white.	
2 There are $(n + 3)$ black buttons and $(n - 3)$ white buttons. Salma takes out a button, records its colour, then puts it back into the bag. The probability that both buttons are black is $\frac{9}{16}$. Work out the probability that both are white.	
3 There are $(2n + 3)$ black buttons and $(n - 1)$ white buttons. Salma takes at random two buttons. The probability that both buttons are black is $\frac{36}{55}$. Work out the probability that both are white.	
4 There are $(2n + 3)$ black buttons and $(n - 1)$ white buttons. Salma takes out a button, records its colour, then puts it back into the bag. The probability that both buttons are black is $\frac{9}{16}$. Work out the probability that both are white.	
5 There are $(2n + 3)$ black buttons and $(n - 1)$ white buttons. Salma takes at random two buttons. The probability that both buttons are black is $\frac{3}{4}$. Work out the probability that both are white.	
6 There are $(2n + 3)$ black buttons and $(n - 1)$ white buttons. Salma takes out a button, records its colour, then puts it back into the bag. The probability that both buttons are black is $\frac{49}{64}$. Work out the probability that both are white.	

Purposeful Practice

- (a) Jasmine throws a fair coin twice. Find the probability that it lands on heads both times.
- (b) Arthur rolls a fair dice twice. Find the probability that he rolls a 6 both times.
- (c) Lucy throws a fair coin once then rolls a fair dice one. Find the probability that the coin lands on tails and she rolls a 5 on the dice.
- (d) Mo rolls a fair dice twice. Find the probability that it lands on an odd number both times.

- (a) The probability that a biased coin lands on heads is $\frac{2}{3}$. Fatima throws the coin twice. Find the probability that it lands on heads both times.
- (b) The probability that a biased coin lands on tails is 0.75. Minahil throws the coin twice. Find the probability that it lands on heads on the first throw and tails on the second throw.

- (a) Oscar rolls a fair dice three times in a row. Find the probability that the dice lands on a 4 on all three rolls.
- (b) The probability that a biased coin lands on heads is 0.6. Alex throws the biased coin three times. Find the probability that the coin lands on heads for the first two throws and tails for the third throw.

- (a) Dumi throws a biased coin twice. The probability that it lands on heads twice is $\frac{49}{100}$. Find the probability that it lands on heads if the coin is thrown once.
- (b) Geraint throws a biased coin three times. The probability that it lands on heads all three times is $\frac{8}{125}$. Find the probability that it lands on tails all three times.

Purposeful Practice

(a) Arthur tosses a fair coin three times. Find the probability that the coin lands on tails exactly once.

(b) Iris and Usma play three games of tennis. The probability that Iris wins a game of tennis is $\frac{3}{8}$. Find the probability that Usma wins at least one game of tennis.

(a) Erik throws a biased coin twice. The probability of the coin landing on tails twice is $\frac{25}{81}$. Find the probability that the coin lands on heads twice.

(b) A spinner has two unequal sections coloured black and white. When the spinner is spun three times, the probability that it lands on black three times is 0.064. Find the probability that when the spinner is spun three times, it lands on white exactly twice.

(a) There are 20 counters in a bag – some are red and the rest are blue. A counter is chosen at random from the bag, it is replaced and then a second counter is chosen. The probability that two red counters are chosen is 0.09. Find the number of red counters, the number of blue counters and the probability of choosing one counter of each colour.

(b) Peta and Paul play one game of tennis and one game of pool against each other. The probability that Peta wins the game of pool is 0.66. The probability that Paul wins both games is 0.272. Find the probability that they win exactly one game each.

A spinner has three unequal sections – white, grey and black. The grey section is twice the size of the black section. When the spinner is spun four times, the probability of it landing on white four times in a row is 0.2401. Find the probability of the spinner landing on grey four times in a row.

Purposeful Practice

Milo and Milly both sit their driving theory test. The probability of passing the test is higher than the probability of failing the test. The probability that Milo passes the test but Millie fails the test is $\frac{21}{100}$. Find the probability that both of them fail the test.

There are 9 yellow pencils and some green pencils in a pencil case. Two pencils are pulled out at random. The probability that both pencils are yellow is $\frac{6}{35}$. Find the number of green pencils in the pencil case.

There are x balls in a bag. 11 of the balls are white and the rest are orange. Two balls are chosen at random from the bag. The probability that the first ball chosen is white and the second ball chosen is orange is $\frac{77}{300}$. Find the value of x .

A jar contains only orange sweets and lemon sweets. There is one more orange sweet than there are lemon sweets. Two sweets are chosen at random from the jar. The probability of choosing two lemon sweets is $\frac{11}{50}$. Find the number of orange sweets and the number of lemon sweets in the jar.

There are black and white socks in a drawer. There are four more black socks than white socks. Finn chooses two socks at random from the drawer. The probability of him choosing one of each colour sock is $\frac{48}{95}$. Find the number of white socks and the number of black socks in the drawer.

Purposeful Practice

1. Mr Stone gets breakfast in the morning with probability $\frac{1}{3}$, else he has nothing. If he has breakfast there is a $\frac{3}{4}$ chance he will be in a good mood. If he has not had breakfast there is a $\frac{1}{5}$ chance he will be in a good mood. On a randomly chosen day what is the probability Mr Stone will be in a good mood?
2. A darts player is aiming for trebles. If his previous throw is a treble then the probability his next throw is a treble is $\frac{2}{3}$. If his previous throw is not a treble then the probability his next throw is not a treble is $\frac{3}{4}$. On his first throw of three he has a $\frac{1}{2}$ probability of hitting a treble. On a round of three throws, find the probability he hits exactly two trebles.
3. There are 4 red, 2 green and 5 black counters in a bag. Two are taken from the bag at once. Find the probability they are
 - (a) the same colour.
 - (b) different colours.What do you notice about your answers to parts (a) and (b)? Explain.
4. There are 3 red, 4 yellow and 5 green balls in a bag. Three balls are taken *without replacement*. Find the probability that the three balls contain exactly two of one colour and one of another.
5. There are 3 red, 4 yellow and 5 green balls in a bag. Three balls are taken one at a time *with replacement*. Find the probability that the three balls contain exactly two of one colour and one of another.
6. There are r red beads and 3 blue beads in a bag. Two beads are taken at once from the bag. The probability they are both blue is $\frac{3}{28}$. Find r .
7. I have 44 socks in my drawer, each sock either red or black. In the dark I randomly pick two socks. The probability they do not match is $\frac{192}{473}$. How many of the 44 socks are red? [Maclaurin 2011]
8. A bag contains 3 red, 4 blue and g green beads. Two balls are taken from the bag at once. The probability they are different colours is $\frac{13}{18}$. Find g .
9. There are n sweets in a bag. 6 of the sweets are orange. The rest are yellow. Hannah takes a sweet at random from the bag and eats it. Hannah then takes another sweet at random from the bag and eats it. The probability that she eats two orange sweets is $\frac{1}{3}$.
 - (a) Show that $n^2 - n - 90 = 0$.
 - (b) Find the number of yellow sweets in the bag.
10. A prisoner is brought in front of the Emperor. The Emperor tells the prisoner he must distribute 100 white beads and 100 black beads into two urns as he wishes (there must be at least one bead in each urn). He is told that he will then be blindfolded and he will then pick one of the urns at random and pick a bead at random from the selected urn. If the bead is white he will be set free. Otherwise he will have to listen to Justin Bieber songs for all eternity (i.e. a fate worse than death). Find the maximum probability that he is set free (if he is smart and comes up with the best possible solution).

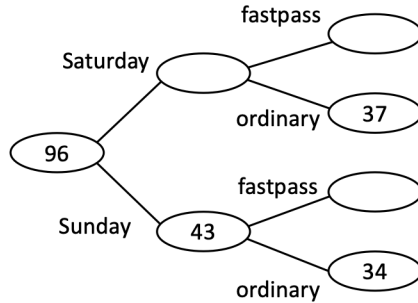
Purposeful Practice

11. A pack of seeds tells me that each seed has a $\frac{2}{3}$ chance of flowering (independent of each other).
- (a) If I plant three seeds, what is that probability that two of them flower?
 - (b) If I plant six seeds, what is the probability that four of them flower? [Here you don't want to draw the tree diagram, but visualise it and try to count the number of routes through the diagram with the desired properties.]
12. There are r red beads and b blue beads in a bag. Two beads are taken from the bag *with replacement*. The probability they are both different colours is $\frac{12}{25}$. Find $\frac{b}{r}$.

Fluency Practice

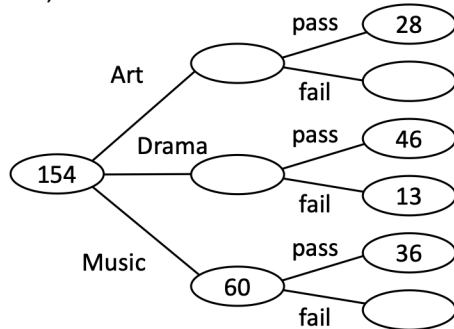
Frequency Trees

1A) A theme park recorded ticket sales on the weekend & whether it was upgraded to a fastpass.



$P(\text{A ticket was bought on Saturday}) =$
 $P(\text{A ticket on Sunday was a fastpass}) =$

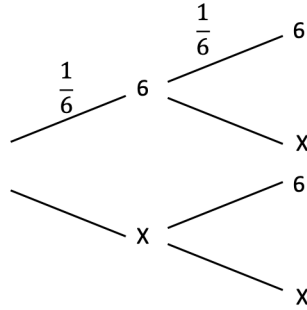
2A) Ms Doolittle recorded end-of-year results for Art, Drama & Music.



Did a greater proportion of Art students pass compared to Music students?
 $P(\text{A student failed}) =$
 $P(\text{A passing student took Drama}) =$

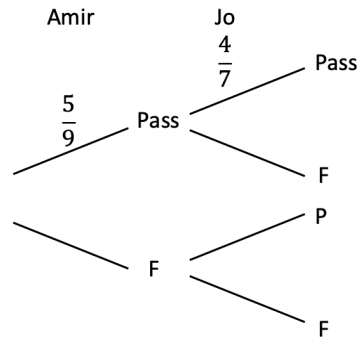
Probability Trees: Unconditional

1B) A dice is rolled twice.



$P(2 \text{ sixes}) =$
 $P(\text{Exactly one six}) =$
 $P(1 \text{ or more sixes}) =$

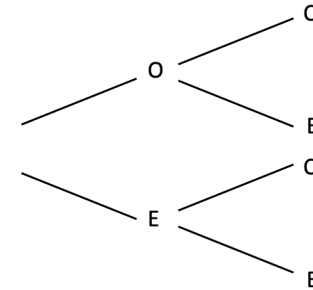
2B) Amir & Jo will both take a driving test.



$P(\text{They both pass}) =$
 $P(\text{Only one passes}) =$
 $P(\text{They get the same result}) =$

Probability Trees: Conditional

1C) 10 cards are numbered 1-10. Two cards are picked simultaneously.



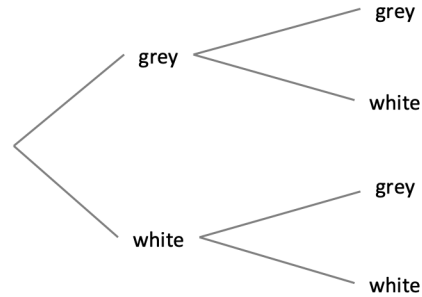
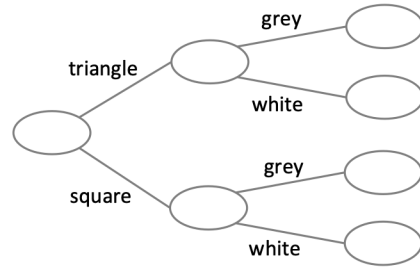
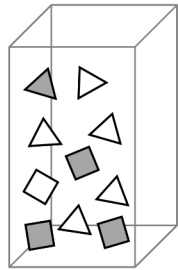
$P(\text{Both odd numbers}) =$
 $P(1 \text{ odd number \& 1 even number}) =$
 $P(\text{Both odd, or both even}) =$

2C) $P(\text{Sophie wins her first tennis match}) = 0.7$
 If she wins that match, the probability she wins her next match is 0.8 If she loses, that probability changes to 0.4

$P(\text{Wins then loses}) =$
 $P(\text{Wins or loses both}) =$
 $P(\text{Wins at least 1}) =$

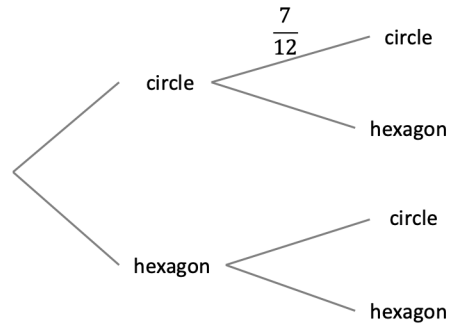
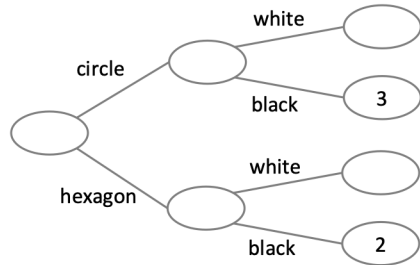
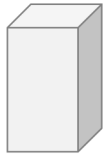
Fluency Practice

① Complete the frequency & tree diagrams (2 shapes are picked **with replacement**).



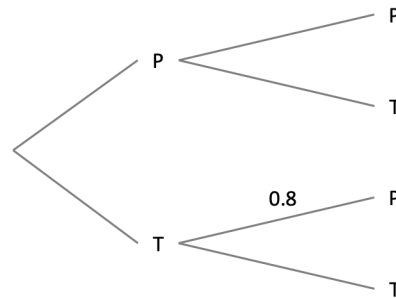
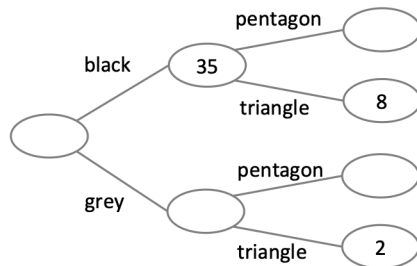
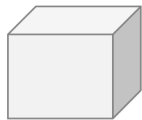
P(grey shape then grey shape) =

②



P(two different shapes) =

③

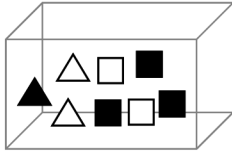


P(at least one pentagon) =

Fluency Practice

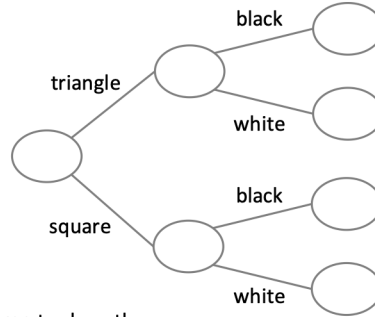
①

Here is a box of shapes.

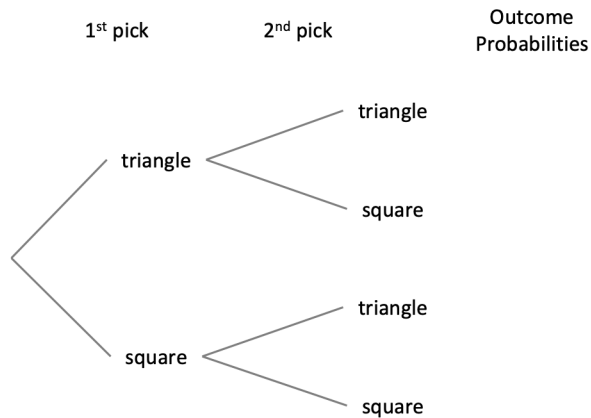


Mary picks a shape at random, records it & replaces it.

a) Complete the **frequency tree** for the group of shapes.



b) Complete the **probability tree** to show the probabilities if Mary picks a shape, replaces it, then picks another.



Using the diagrams...

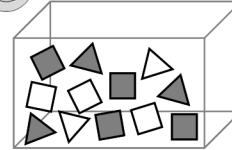
On her first pick, what is the probability Mary picks a white triangle?

What is the probability Mary picks a square, then picks another square?

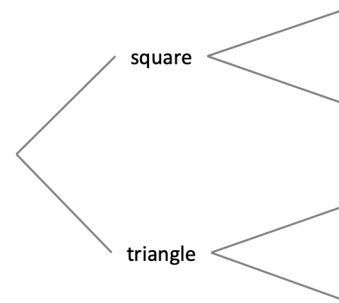
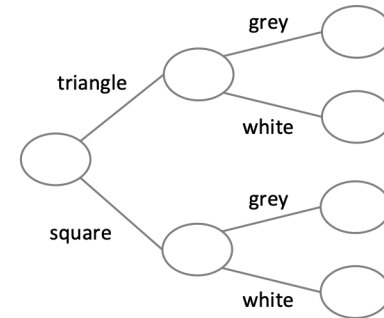
②



Tim picks & replaces shapes from this box.



Complete the frequency tree & probability tree diagram.



What is the probability Tim picks...

...a grey square?

...a white shape?

...two triangles in a row?

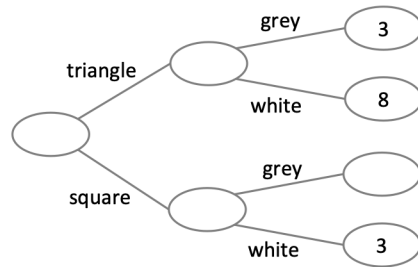
...two squares in a row?

...two of the same shape?

Fluency Practice

③

Complete this frequency tree for a group of 20 shapes.

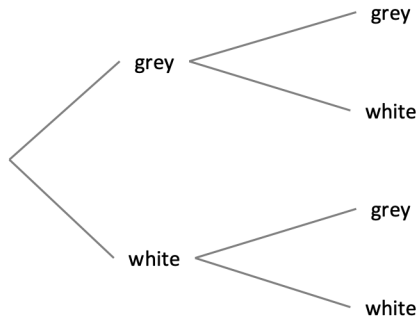


What is the % probability a grey square is picked randomly from the group?

What is $P(\text{a grey shape is picked})$?

A square has been picked, what is the probability it is grey?

A shape is picked from the group and replaced before another is picked. Complete this tree diagram.



What is the probability a grey shape is picked before a white shape is picked?

What is $P(2 \text{ white shapes are picked})$?

What is $P(2 \text{ shapes of the same colour})$?

What is the probability a grey square is picked, then a white triangle is picked?

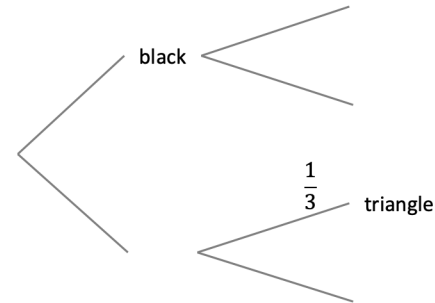
④



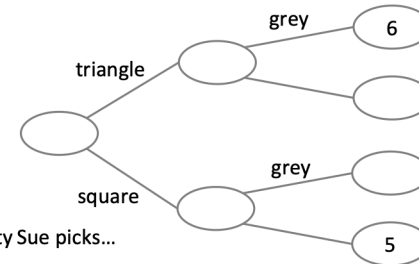
Sue randomly picks & replaces from a group of 30 shapes.

For her **first pick** she records the **colour**.

For her **second pick** she records the **type**.



Complete both tree diagrams.



What is the probability Sue picks...

...a black triangle?

...a black shape?

...a black shape then a square?

...a grey shape then a square?

...a square, given the shape picked is grey?

...a triangle given the shape picked before was black?

Fluency Practice

Multiple Event Probabilities: A

①

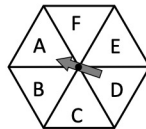


This is a fair spinner.

What is the percentage probability the arrow lands on 1 or on 4?

②

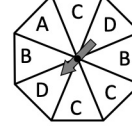
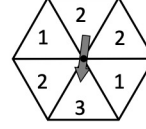
These are fair spinners.



If both spinners are spun, what is the probability the results are 2 & E?

③

These are fair spinners.



If both spinners are spun, what is...

P(1 & A)?

P(2 & C)?

④

On a fair dice, what is the probability two **sixes** are rolled in a row?

What is the probability two **ones** are rolled in a row?

What is the probability someone rolls two **sixes** or two **ones** in a row?

⑤

For a £4.49 'Meal Deal' you can choose 1 drink, 1 main & 1 desert.

Drink	Main	Desert
Soda	Pie	Ice Cream
Juice	Roll	Cake
	Curry	

A customer is chosen at random. By listing all the different possible 'Meal Deals' someone can order, what is the probability the customer has chosen 'Juice-Pie-Cake'?

⑥

A phone's security code is 4 digits long.

X	X	X	X
---	---	---	---

By choosing 4 digits randomly, what is the probability someone can guess the code in one try?

⑦



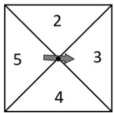
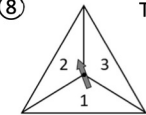
A car company gives each car an individual code using a letter and 2 digits.

For example, D-78

How many cars can be produced before the code needs to be expanded?

⑧

These two fair spinners are spun and their scores are **multiplied**.



2nd Spinner

1st Spinner

	1	2	3
2			6
3			
4			
5			

What is the probability the final score is 6?

What is...

P(even)?

P(above 7)?

⑨

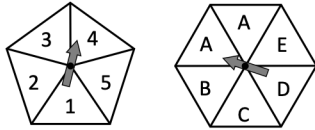
Kim has 2 sets of cards. The first set has the first 4 prime numbers on them. The second set has the first 3 positive square numbers on them. Kim takes one card from each set and sums the two numbers. What is the probability the total is prime? What is the probability the total is even or less than 6?



Fluency Practice

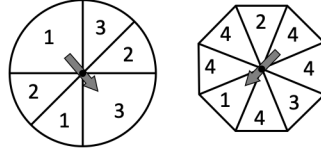
Multiple Event Probabilities: B

- ① These are fair spinners.



If both spinners are spun, what is the probability the results are 1 & A?

- ② These are fair spinners.



Both spinners are spun & their scores are multiplied. What is the probability the total is 12?

- ③ A shoe company offers:
5 fabric colours, 4 sole colours &
7 colours for the laces.
How many different shoe
colour combinations are there?

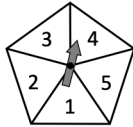
- ④ A credit card PIN is 4 digits long.

X	X	X	X
---	---	---	---

Bo knows the PIN is odd,
he knows it starts with
a positive even digit,
he knows it does not contain any zeros.

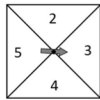
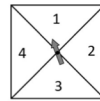
What is the probability Bo
guesses the PIN correctly?

- ⑤ This fair spinner is spun twice &
the two scores are added.



What is the probability
the score is exactly 8?

- ⑥ These two fair spinners are spun and
their scores are **added**.



		1 st Spinner			
		1	2	3	4
2 nd Spinner	2				
	3				
	4				
	5				

Complete the sample space.

What is the probability
the final score is 7?

What is...

P(prime)?

P(less than 6)?

- ⑦ A bag contains 4 red disks,
3 black disks & 3 white disks.

Three times a disk is chosen at random &
then replaced.

What is the probability...

....3 red disks are chosen?

....3 disks of the same colour are chosen?

- ⑧

A	B	C	D	E	F
---	---	---	---	---	---

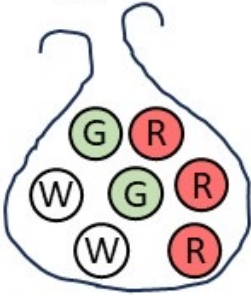
How many ways can we
arrange these 6 cards?

If we pick 3 cards at random,
what is the probability we pick the
cards showing A & B & C?

- ⑨ 8 players are on a basketball squad.
Only 5 players play at once.
How many different options does
the manager have when choosing the
first 5 players to play?

- ⑩ A bag contains 6 green disks, 5 blue disks & 3 pink disks.
2 disks are taken at random.
What is the probability at least one blue disk is picked?

Purposeful Practice

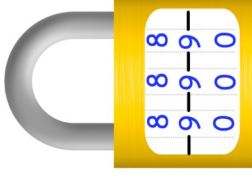
1 A fair coin is flipped, and a fair 6-sided die is rolled. Calculate: a $P(H \cap 6)$ b $P(T \cap \text{Even Number})$ c $P(T \cap 1')$	2 Calculate the probability of rolling three 1s on three fair 8-sided dice.	3 Charlie plays 2 games of chance, the probability he wins both games is 0.21. The probability of winning game A is 0.25. Calculate the probability he wins game B.	4 Chloe picks out two balls from the bag. Calculate the probability that both balls are red. 
5 A box contains pink, green and purple counters in the ratio 2:4:7. A counter is taken out of the box and replaced, a second is then taken out. Calculate the probability at least one of the counters is purple.	6 The following is a set meal at a restaurant. Starter: Tomato Soup Mozzarella Sticks Beef Nachos Main: Roast Chicken Salad Steak Pesto Pasta Dessert: Brownie Lava Cake Ice Cream a Calculate the number of combinations of the set menu. b Assuming the items of the set menu are chosen randomly, what is the probability a vegetarian meal set is chosen.	7 There are n balls in a bag of colours blue, red and green. A ball is picked randomly, the colour is recorded and returned to the bag. This is repeated a second time. $P(B \cap R) = 0.07$ $P(R \cap G) = 0.09$ $P(G \cap G) = 0.2025$ Calculate the least number of balls the bag could contain.	8 A set of 10 counters labelled 1-10 are in a bag. A counter is taken out of the bag, recorded and returned. Another counter is then taken. Ali wins if the sum of the numbers is even. Grace wins if the product of the numbers is odd. Calculate the probabilities of each player winning and determine whether this is a fair game.

3 Combinations and Permutations

Fluency Practice

Question 1: Shown is a 3-digit combination padlock.

Each dial can be set to 0, 1, 2, 3, 4, 5, 6, 7, 8, 9



(a) Work out the total number of different combinations that can be used.

(b) Work out the total number of different combinations that have three different digits that can be used.

Question 2: A restaurant has 4 starters and 6 main course on its menu. Hailey orders a starter and a main course.

How many different combinations of starters and main courses are there?

Question 3: A rugby coach is designing a new rugby strip.
She can choose from: 5 different pairs of socks
6 different pairs of shorts
and 14 different jerseys.

How many different strips are possible?

Question 4: Harry picks a 4 digit pin for his credit card.
Each digit is a number 0 to 9.
Harry can repeat digits.



(a) How many possible codes are there?

Harry chooses not to repeat any digits.

(b) How many possible codes are there now?

Question 5: Rosie picks a 4-digit **odd** number.

The first digit is 5.

The second digit is a 3 or a 4.

The third digit is prime.

How many different 4-digit numbers could Rosie pick?

Purposeful Practice

Question 6: Oliver picks a 4-digit **even** number that is greater than 3000. The second digit is a multiple of 4.

How many different numbers could Oliver pick?

Question 7: Sophia is creating a 6-digit code to lock her iPad.

She only uses digits greater than 2.

She only uses each digit once.

How many possible codes can Sophia create?

Question 8: In a class, there are 10 boys and 9 girls.

The teacher has been asked to pick one boy and one girl to win a prize.

How many possible pairs of students can the teacher pick?

Question 9: Jason picks a 5-digit number that is less than 80000.

The first digit is odd.

The fourth and fifth digits are equal.

How many different numbers can Jason pick?

Question 10: A headteacher wants to survey two Year 7 students.

There are 100 students in Year 7.

How many possible pairs of students can the headteacher pick?

Question 11: How many even numbers greater than 40000 can be created using these digits?

2	9	1	6	8
---	---	---	---	---

Apply

Question 1: On a school trip, students are given a packed lunch.

The students can choose one piece of fruit and one snack.

There are 8 different pieces of fruit and some different snacks.

Altogether there are 104 different ways to choose one piece of fruit and one snack

How many different snacks are there?

Purposeful Practice

Question 2: At a summer camp, children pick a morning, an afternoon and an evening activity.

There are 4 morning and 7 evening activities to pick from.

Altogether there are 224 different ways to choose their activities.

How many afternoon activities are there?

Question 3: In a gym there are

12 exercise classes on a Monday

13 exercise classes on a Wednesday

7 exercise classes on a Friday

Katie is going to attend either

a class on Monday and a class on Friday

or a class on Wednesday and a class on Friday

or a class on Monday, Wednesday and Friday

Work out how many different ways there are to pick which exercises classes Katie is going to attend.

Question 4: A group of 10 people enter a room.

Each person shakes hands, once, with all the other people in the room.

How many handshakes are there in total?



Question 5: A pizza parlour sells 9 different toppings.

Michael orders a pizza with 2 different toppings.

(a) How many different pizzas can he choose from?

Beth orders a pizza with 3 different toppings.

(b) How many different pizzas can she choose from?

John orders a pizza with 4 different toppings.

(c) How many different pizzas can he choose from?

Purposeful Practice

A. There are 8 runners in a race. In how many different ways could the three medals gold, silver and bronze, be allocated?		B. Car registration plates in Great Britain include a random sequence of three letters. Any letters other than I or Q can be used and the same letter can appear more than once. How many different three letter codes could be used?	
C. 12 players take part in a singles table tennis competition. Each player must play each other player once. How many matches will there be in the competition?		D. On a school sports day, there are 6 different team events and 9 different individual events. Pupils must participate in two team events and one individual event. How many different ways are there of doing this?	
E. In a class there are 8 boys and 11 girls. (a) Two students are to be selected at random from the class. How many different pairs can be chosen? (b) One boy and one girl are to be selected at random from the class. In how many ways can this be done?		F. Two cards are chosen from a standard deck of 52 cards. How many different possible pairs of cards are there?	
G. Three cards are picked at random from this set. (a) In how many different ways could the digits picked be in the order <i>odd-even-odd</i>? (b) In how many different ways could the digits either be in the order <i>odd-even-odd</i> or <i>even-odd-even</i>?	123456789		
H. A form group has 32 members. A form captain, a sports captain and a charity representative are to be chosen. No one can be in more than one of these positions. How many different combinations of results are possible?		I. A team of three is to be chosen from a group of 8 boys and 7 girls. There must be at least one boy and at least one girl in the team. How many possibilities are there?	

Purposeful Practice

A. Jake has 5 different pairs of trousers and 8 different shirts. How many different combinations of these could he wear?		B. A badminton club has 23 members, 8 of whom are female. In how many ways can the club select a mixed doubles team consisting of one male and one female?	
C. UK banknotes are issued in denominations of £50, £20, £10 and £5. UK coins are issued in denominations of £2, £1, 50p, 20p, 10p, 5p, 2p and 1p. One coin and one banknote are to be selected at random. (a) In how many different ways could this be done? (b) In how many of the ways is the total at least £10.50?			
D. On a restaurant's menu, there are 5 different starters, 6 main courses and 4 desserts. For a set price, diners can choose either a starter and a main course or a main course and a dessert. How many different combinations are there?		E. In a class there are n boys. There are twice as many girls as boys. One boy and one girl are to be chosen at random. Write a simplified expression for the number of possible combinations.	
F. How many different combinations are possible in this supermarket lunch deal?		LUNCH DEAL Sandwich (5 options) + Snack (8 options) + Drink (7 options) = £3	
G. At a school, students are required to choose one language, one humanities subject and one technology subject to study. There are 4 languages to choose from, 3 humanities and 3 technologies. How many different combinations of subjects could be chosen?		H. An examination paper has three sections, each with six questions. The instructions read: 'Answer one question from each section.' How many different combinations of questions could be answered?	

Purposeful Practice

- (a) How many ways are there of arranging the word MATHS?
- (b) Ishaq wants to make a PIN code from the digits 1, 3, 5 and 7, using each digit once only. How many different PIN codes can he make?
- (c) How many different numbers can be made from the digits 2, 3, 4, 5 and 6?

- (a) 10 people are called for an interview. How many ways are there of arranging the order of the interviews?
- (b) 8 flags are to be flown outside a building hosting a world leaders' conference. How many ways are there for arranging the flags?
- (c) At a dog show, there is a gold, silver and bronze certificate for three top dogs. If 20 dogs enter, how many ways are there of awarding the certificates?

- (a) Using the digits 5, 6, 7 and 8 once each, how many possible numbers can be made that are multiples of 5?
- (b) Using the numbers 2, 3, 5, 7 and 9 once each, how many even numbers is it possible to make?

- Dr Austin randomly chooses 2 of her 24 students to compete in a quiz against Mrs Barber's class.
- (a) How many ways are there of Dr Austin selecting her 2 students?
- Mrs Barber has 26 students in her class and must also select 2 students at random.
- (b) How many different possible selections of the 4 students competing in the quiz are there?

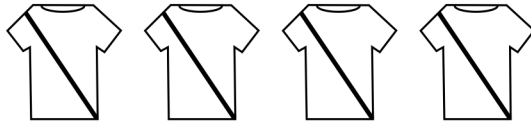
Purposeful Practice

T-Shirt Designing (Permutations)

Jess designs T-shirts.

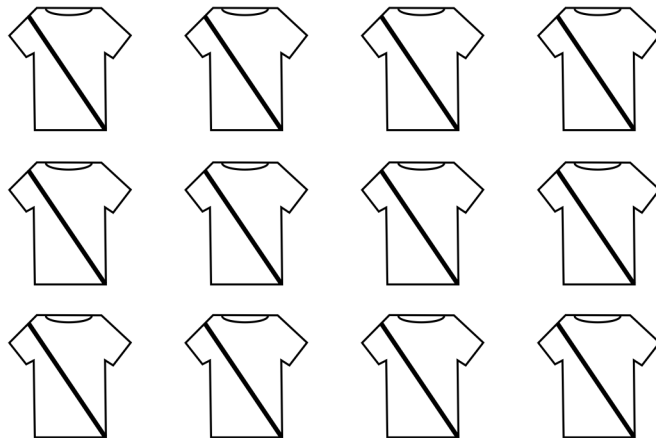
If Jess only uses red & yellow, she can make 4 different T-shirt designs.

Colour the T-shirts to show the different designs.



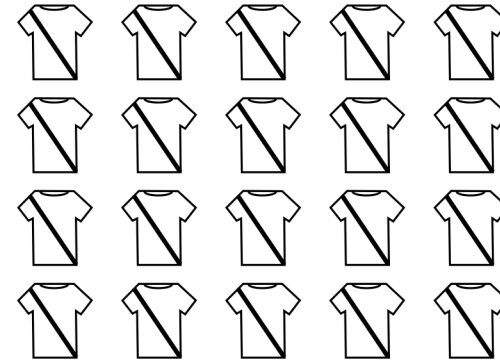
If Jess uses red, yellow & blue, how many different designs can she make?

(hint: it's less than 12)



Jess adds green.

How many different designs can she now make with 4 colours?



Jess uses the 4 colours – but she doesn't make any t-shirts with the same 2 colours (like red & red).

How many different designs can she make?

How many designs could Jess make if she added pink?

How many designs could Jess make with 7 colours, but she can only use a colour once?

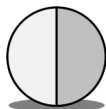


This design is made from 3 pieces of fabric.

If we use only red & green, how many different designs can we make? (including all the same colour).

If we use 3 colours, how many different designs can we make? (including all the same colour).

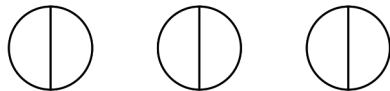
Purposeful Practice



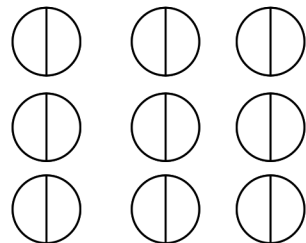
A company makes beachballs using 2 pieces of plastic.

If we use red & yellow plastic pieces, we can make 3 different designs. Colour those designs.

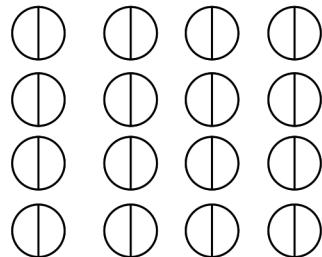
Why is it only 3 unique designs, not 4?



If we use red, yellow & blue pieces of plastic, how many **unique** designs can we make?



If we use red, yellow, blue & white pieces of plastic, how many **unique** designs can we make?



Choosing Combinations

The company adds green.

Complete the table to show all the possibilities for 5 colours.

		Left Side				
		Red	Yellow	Blue	White	Green
Right Side	Red	RR				
	Yellow	YR		YB		
	Blue					
	White					
	Green					

How many unique designs are there?

How many unique designs are there when a colour can't be used twice?

An ice cream shop has 6 flavours. For £1.99 you can get 2 scoops.

How many different 2-scoop combinations are there?



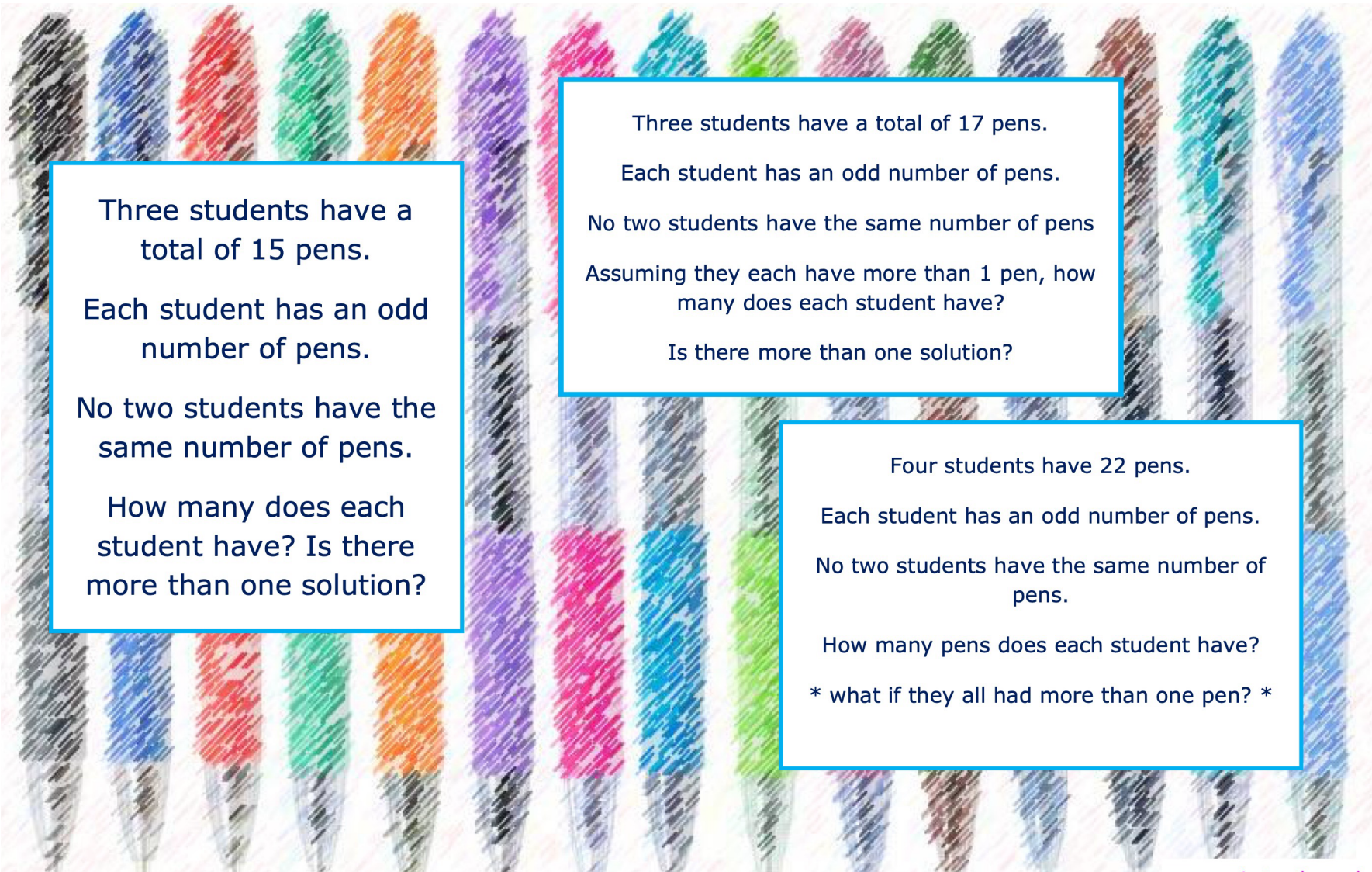
A pizza shop has 7 different types of topping.

You can choose two – but they must be different.

How many different types of 2-topping pizza are available?



Problem Solving



Three students have a total of 15 pens.

Each student has an odd number of pens.

No two students have the same number of pens.

How many does each student have? Is there more than one solution?

Three students have a total of 17 pens.

Each student has an odd number of pens.

No two students have the same number of pens

Assuming they each have more than 1 pen, how many does each student have?

Is there more than one solution?

Four students have 22 pens.

Each student has an odd number of pens.

No two students have the same number of pens.

How many pens does each student have?

* what if they all had more than one pen? *

Purposeful Practice

Product Rule for Counting

1 8

How many 2-digit numbers can you make with these 2 cards?

1 8 7

How does this change if we add a third card?

1 8 7 3

A) How many 4-digit numbers can you make using these cards?

* How many choices do you have for the 1st card?

* After you choose a 1st card,
how many choices do you have for the 2nd card, 3rd card & 4th card?

4 8 3 6 2

B) How many odd 5-digit numbers can you make with these cards?

* One number must be last, how does this affect our choices?

7 9 1 5 3

C) How many numbers greater than 40,000 can you make?

* How many choices are there for the 1st card?

5 6 8 3 4 2

D) How many odd 6-digit numbers can you make?

* How many choices are there for the last card?

7 1 6 5 2 9

E) How many numbers greater than 300,000 can you make?

1 6 5 2 7

F) How many 5-digit numbers that are a multiple of 2 can you make?

7 2 4 3 8

G) How many odd numbers greater than 50,000 can you make?

* Sum the choices for each starting card.

6 1 9 7 3 2

H) How many odd numbers greater than 500,000 can you make?

8 2 5 1 7 4

H) How many even numbers smaller than 600,000 can you make?

9 6 7 1

I) How many 2-digit numbers can we make using these cards?

J) How many 3-digit numbers can we make using these cards?

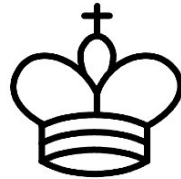
2 8 1 5 6

K) How many ways can we make 3 numbers from these cards?

L) The number 156 is made. How many different numbers use these same digits?

M) If we don't care about their order (the number they make):
how many ways can we pick 3 digits from the 5 cards?

Chess960 is a variant of chess in which some of the pieces are placed randomly at the start of the game.



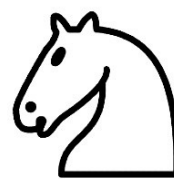
King



Queen



Bishop
(×2)



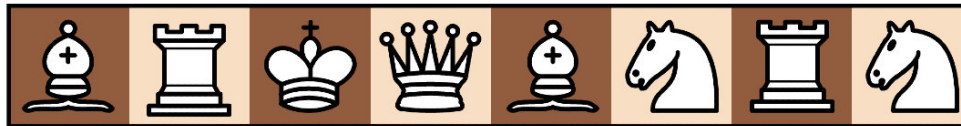
Knight
(×2)



Rook
(×2)

All eight pieces above are placed in a row following two rules:

-  The bishops must be placed on opposite-colour squares.
-  The king must be placed on a square between the rooks.



Can you work out why it is called Chess960?

4 Capture-Recapture

CHECK IT

Estimating population size using the capture-recapture method

- 1) In a wildlife park, 9 deer are caught, marked and released. A week later 10 deer are caught and 1 is found to be marked. Estimate the number of deer in the park
- 2) In a wildlife park, 28 fish are caught, marked and released. A week later 50 fish are caught and 2 are found to be marked. Estimate the number of fish in the park
- 3) In a wildlife park, 15 fish are caught, marked and released. A week later 30 fish are caught and 3 are found to be marked. Estimate the number of fish in the park
- 4) In a wildlife park, 16 deer are caught, marked and released. A week later 25 deer are caught and 2 are found to be marked. Estimate the number of deer in the park
- 5) In a wildlife park, 120 badgers are caught, marked and released. A week later 60 badgers are caught and 8 are found to be marked. Estimate the number of badgers in the park
- 6) In a wildlife park, 10 deer are caught, marked and released. A week later 20 deer are caught and 5 are found to be marked. Estimate the number of deer in the park
- 7) In a wildlife park, 25 fish are caught, marked and released. A week later 40 fish are caught and 1 is found to be marked. Estimate the number of fish in the park
- 8) In a wildlife park, 32 badgers are caught, marked and released. A week later 25 badgers are caught and 2 are found to be marked. Estimate the number of badgers in the park
- 9) In a wildlife park, 27 pheasants are caught, marked and released. A week later 50 pheasants are caught and 3 are found to be marked. Estimate the number of pheasants in the park
- 10) In a wildlife park, 14 pheasants are caught, marked and released. A week later 20 pheasants are caught and 2 are found to be marked. Estimate the number of pheasants in the park
- 11) In a wildlife park, 31 rabbits are caught, marked and released. A week later 30 rabbits are caught and 1 is found to be marked. Estimate the number of rabbits in the park
- 12) In a wildlife park, 13 pheasants are caught, marked and released. A week later 20 pheasants are caught and 1 is found to be marked. Estimate the number of pheasants in the park
- 13) In a wildlife park, 6 fish are caught, marked and released. A week later 10 fish are caught and 2 are found to be marked. Estimate the number of fish in the park
- 14) In a wildlife park, 20 badgers are caught, marked and released. A week later 20 badgers are caught and 4 are found to be marked. Estimate the number of badgers in the park
- 15) In a wildlife park, 100 deer are caught, marked and released. A week later 40 deer are caught and 5 are found to be marked. Estimate the number of deer in the park
- 16) In a wildlife park, 20 fish are caught, marked and released. A week later 20 fish are caught and 5 are found to be marked. Estimate the number of fish in the park
- 17) In a wildlife park, 60 deer are caught, marked and released. A week later 50 deer are caught and 4 are found to be marked. Estimate the number of deer in the park
- 18) In a wildlife park, 63 badgers are caught, marked and released. A week later 40 badgers are caught and 3 are found to be marked. Estimate the number of badgers in the park
- 19) In a wildlife park, 12 rabbits are caught, marked and released. A week later 30 rabbits are caught and 3 are found to be marked. Estimate the number of rabbits in the park
- 20) In a wildlife park, 11 badgers are caught, marked and released. A week later 20 badgers are caught and 1 is found to be marked. Estimate the number of badgers in the park
- 21) In a wildlife park, 15 badgers are caught, marked and released. A week later 10 badgers are caught and 3 are found to be marked. Estimate the number of badgers in the park
- 22) In a wildlife park, 30 deer are caught, marked and released. A week later 30 deer are caught and 3 are found to be marked. Estimate the number of deer in the park
- 23) In a wildlife park, 15 rabbits are caught, marked and released. A week later 20 rabbits are caught and 5 are found to be marked. Estimate the number of rabbits in the park
- 24) In a wildlife park, 50 rabbits are caught, marked and released. A week later 20 rabbits are caught and 2 are found to be marked. Estimate the number of rabbits in the park
- 25) In a wildlife park, 30 pheasants are caught, marked and released. A week later 40 pheasants are caught and 2 are found to be marked. Estimate the number of pheasants in the park
- 26) In a wildlife park, 7 pheasants are caught, marked and released. A week later 10 pheasants are caught and 1 is found to be marked. Estimate the number of pheasants in the park
- 27) In a wildlife park, 7 fish are caught, marked and released. A week later 40 fish are caught and 1 is found to be marked. Estimate the number of fish in the park
- 28) In a wildlife park, 33 rabbits are caught, marked and released. A week later 50 rabbits are caught and 3 are found to be marked. Estimate the number of rabbits in the park
- 29) In a wildlife park, 21 fish are caught, marked and released. A week later 50 fish are caught and 3 are found to be marked. Estimate the number of fish in the park
- 30) In a wildlife park, 12 rabbits are caught, marked and released. A week later 20 rabbits are caught and 1 is found to be marked. Estimate the number of rabbits in the park

Fluency Practice

Purposeful Practice

1. John is trying to work out the total number of fish in a lake. One day he captures 10 fish, marks them, and then returns them. On the next day, he captures 24 fish and finds that 3 of them are marked. Work out an estimate for the total number of fish in the lake.	2. Sophie is trying to work out the total number of fish in a lake. One day she captures 30 fish, marks them, and then returns them. On the next day, she captures 40 fish and finds that 8 of them are marked. Work out an estimate for the total number of fish in the lake.
3. Preet is trying to work out the total number of fish in a lake. One day she captures 12 fish, marks them, and then returns them. On the next day, she captures 24 fish and finds that 6 of them are marked. Work out an estimate for the total number of fish in the lake.	4. Graham is trying to work out the total number of fish in a lake. One day he captures 18 fish, marks them, and then returns them. On the next day, he captures another sample of fish and finds that 40% of them are marked. Work out an estimate for the total number of fish in the lake.
5. A park has a population of ducks. 20 of the ducks are marked with tags. A week later, 24 ducks are caught. 5 of these ducks have tags on. a) Work out an estimate for the number of ducks in the park. b) Some of the tags may have fallen off during the week. Explain the effect this would have on your estimate.	

Purposeful Practice



Capture & Recapture

A) A fisherman catches, tags 30 fish & releases them back into a pond. The next day, the fisherman catches 20 fish & 6 of them are tagged. Estimate the fish population in the lake.

B) Ralph waits 2 months between tagging some fish in a pond and then taking a recapture sample. Why might this affect the accuracy of any population estimate?

C) In a national park, Dr Howe catches, tags & releases 40 eagles. The next day, Dr Howe captures 20 eagles. 4 of those eagles are tagged.

- 1) Estimate the hawk population in the park.
- 2) What assumptions do we need to make to estimate the population?

D) A scientist tags & releases 50 salamander into a river. In a recapture sample, the scientist finds 15 out of 40 salamanders are tagged. Estimate the salamander population in the river.

E) A marine biologist tags fish using a shiny metal tag on one fin. Why might this affect the accuracy of a capture-recapture population estimate?

F) A scientist tags & releases 200 bees back into a hive. In a recapture sample, the scientist finds 7% of the bees are tagged. Estimate the bee population in the hive.

G) In a recapture sample, Dr Williams finds 3 out of 90 deer are tagged. She uses this to estimate a deer population of 300. How many deer were originally tagged?

H) If we tag 20 individuals from a population of 180, how many individuals do we expect to be tagged in a recapture sample of 50?

Fluency Practice

Question 1: Hannah wants to estimate the number of eels in a lake.

She catches and rings 50 eels.

She returns the 50 eels to the lake.

The next day Hannah catches 400 eels.

Of these 400 eels, 10 are ringed.



Work out an estimate for the total number of eels in the lake.

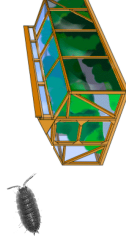
Question 2: Tom wants to estimate the number of woodlice in a greenhouse.

He catches and marks 32 woodlice.

The woodlice are then returned to the greenhouse.

The next day Tom catches 20 woodlice.

8 of these are marked.



Work out an estimate for the total number of woodlice in the greenhouse.

Question 3:

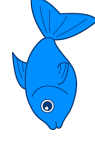
A scientist wants to estimate the total number of fish in a pond.

On Thursday, she catches 180 fish.

These fish are marked and returned to the pond.

On Friday, the scientist catches 305 fish.

45 of these fish are marked.



(a) Work out an estimate for the total number of fish in the pond

(b) What assumptions have you made?

Question 4:

Darren wants to estimate how many grasshoppers live in a field.

He catches and marks 24 grasshoppers.

He then releases the grasshoppers.

The next day, Darren returns to the same field and captures 51 grasshoppers.

7 of these have been marked.



Work out an estimate for the total number of grasshoppers in the field.

Purposeful Practice

Question 5: Heather has a large jar of jelly beans.

Heather wants to find an estimate for the total number of jelly beans in the jar. She takes out all the jelly beans and marks 200 of them. Heather mixes the jelly beans and puts them back into the jar.



Heather then takes 140 jelly beans from the jar. 3 of the jelly beans are marked. Heather then puts all the jelly beans back into the jar.

- (a) Work out an estimate for the number of jelly beans in the jar.
- (b) What assumptions have you made?

Apply

Question 1: Charlotte wants to work out an estimate of the number of fish living in a pond. She captures X fish and tags them. Charlotte returns the fish to the pond. The next day Charlotte catches 50 fish. Of these 50 fish, 32 are tagged.

Charlotte's estimate of the number of fish in the pond is 125.

Work out how many fish Charlotte tagged, X .

Question 2: Ronan wants to estimate the number of honey bees in a beehive.

On Wednesday, Ronan catches 660 honey bees from the beehive. He marks the honey bees and then releases them.

On Thursday, Ronan catches 400 honey bees and notes how many were marked.

Ronan then calculates his estimate as 22,000 honey bees in the beehive.

How many of the 400 honey bees caught on Thursday were marked?



Purposeful Practice

Question 3:

Rhys has a large tub of yellow counters.
Alex has a large tub of blue counters.

40 yellow counters are taken from Rhys' tub and placed into Alex's tub.
40 blue counters are taken from Alex's tub and placed into Rhys' tub.

Rhys randomly selects 100 counters from his tub.
8 of the 100 counters are blue.

Alex randomly selects 50 counters from his tub.
48 of the 50 counters are blue.

All the counters are then placed into one tub.

Work out an estimate for the ratio of yellow to blue counters in the tub.

Question 4:

A scientist wants to estimate the number of lions living in a region.

On Thursday, he locates and tags some lions.

On Friday he returns and locates 10 less lions than he had on Thursday.
He notices that 4 of the lions are tagged.

The scientist works out an estimate for the total number of lions living in the region.

He notices that the number of lions that he had caught on Thursday, was a fifth of the total number of lions.

How many lions live in the region?

