KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2026 Mathematics 2027

Unit 21 Student

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents

1 Advanced Statistics

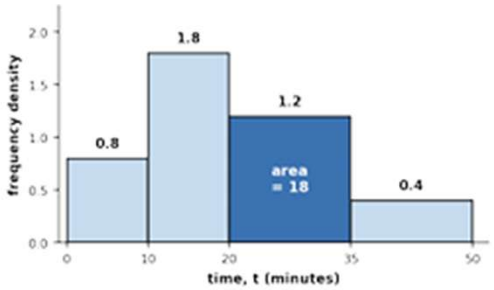
HISTOGRAMS

Area represents frequency

Knowledge Organiser • GCSE Edexcel Higher Mathematics

WHAT IS A HISTOGRAM?

A histogram shows **continuous grouped data** with **unequal class widths**. It is the **AREA** of each bar that represents the frequency — not the height. Plot **frequency density** on the y-axis.



Time spent by 50 shoppers — shaded bar: F = 1.2 × 15 = 18

KEY RULES (FORMULAE)

$$\text{FD} = \frac{\text{frequency}}{\text{class width}}$$

Frequency = FD × width (= area of bar)

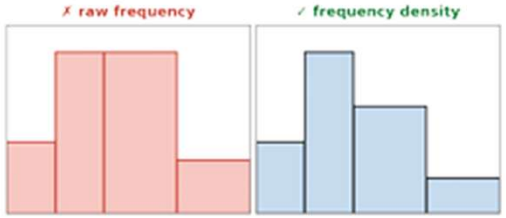
Σ areas = total frequency

FREQUENCY DENSITY — DRAWING BARS

Add **width** and **FD** columns to the table, then plot FD as the bar height.

$0 \le t < 10$, F=8: $\text{FD} = \frac{8}{10} = 0.8$

$20 \le t < 35$, F=18: $\text{FD} = \frac{18}{15} = 1.2$



FINDING A FREQUENCY — THE AREA

Frequency = FD × class width (the **area** of the bar).

```
FD = 1.2, w = 15 → F = 18
FD = 0.4, w = 15 → F = 6
FD = 1.8, w = 10 → F = 18
```

MISSING FREQUENCY

Missing F = total − sum of known frequencies; then find its FD.

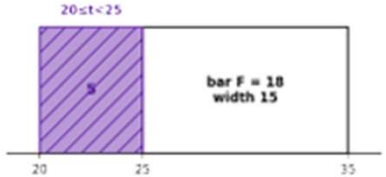
```
Total = 50, known F: 8, 18, 18
```

Missing F = 6: $\text{FD} = \frac{6}{15} = 0.4$

PARTIAL BARS — INTERPOLATION

When a boundary **cuts through** a bar, assume the data is spread evenly across it:

$$\text{Est. F} \approx \frac{\text{width needed}}{\text{bar width}} \times \text{bar's frequency}$$



$$F \approx \frac{5}{15} \times 18 = 6$$

ESTIMATING THE MEDIAN

Find the **n/2**-th position, locate its bar with cumulative frequency, interpolate along the width.

```
n=50 → 25th value; class 10≤t<20
cum. freq: 8 → 26
```

$$\text{median} \approx 10 + \frac{25 - 8}{18} \times 10 = 19.4$$

Quartiles: same method with the **n/4**-th (LQ) and **3n/4**-th (UQ) positions; IQR = UQ − LQ.

AREA KEYS (NO FD SCALE GIVEN)

Some questions give a **key**, e.g. "1 square represents 2 people". Count squares, then convert.

```
Bar covers 9 squares → F = 9 × 2 = 18
```

COMMON MISTAKES — EXAMINERS' REPORTS

- ✗ Plotting **raw frequency** as the bar height — always convert to frequency density first.
- ✗ Reading the **height** as the frequency — it is the **area** (FD × width).
- ✗ Treating an **interpolation estimate** as exact — data may not be spread evenly within a bar.
- ✗ Adding **frequency densities** to find a total — only frequencies (areas) sum to the total.
- ✗ Misreading boundaries: 10≤t<25 has width **15**, not 25.
- ✗ Leaving the y-axis unlabelled — it must say "frequency density".

Frequently examined on Edexcel 1MA1 GCSE Higher Paper 2 and Paper 3.

Fill in the Gaps

Group	Frequency	Group width	Frequency density
$8 \leq p < 12$	20		
$8 \leq p < 12$	10		
$8 \leq p <$	10		1.25
$\leq p < 28$	10	20	
$25 \leq p <$		20	0.6
$\leq p < 65$	12		0.24
$20 \leq p <$	120	50	
$200 \leq p < 250$			1.6
$\leq p < 250$		20	4
$\leq p < 175$	20	80	
$\leq p < 175$	20		0.333 …
$15 \leq p <$		60	0.3

Worked Example

Plot a histogram:

Height, x (cm)	Frequency
$140 < x \leq 155$	6
$155 < x \leq 175$	14
$175 < x \leq 185$	6
$185 < x \leq 190$	21

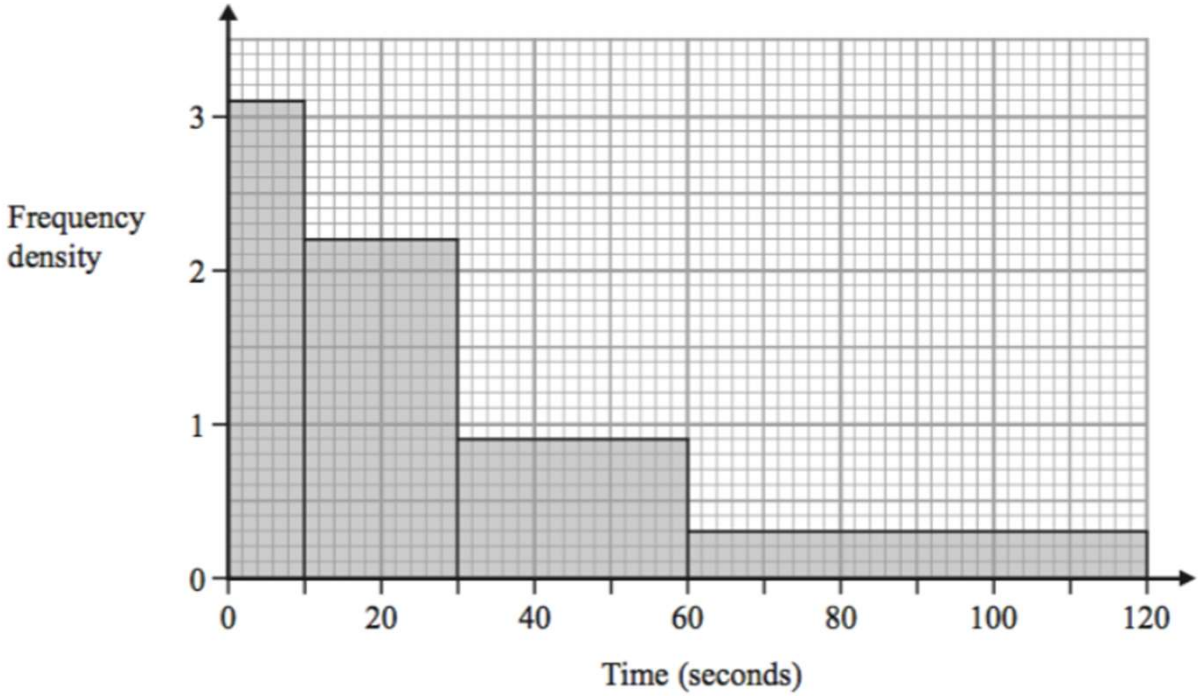
Your Turn

Plot a histogram:

Price, y (£)	Frequency
$0 < y \leq 10$	4
$10 < y \leq 20$	9
$20 < y \leq 25$	8
$25 < y \leq 35$	10
$35 < y \leq 50$	12

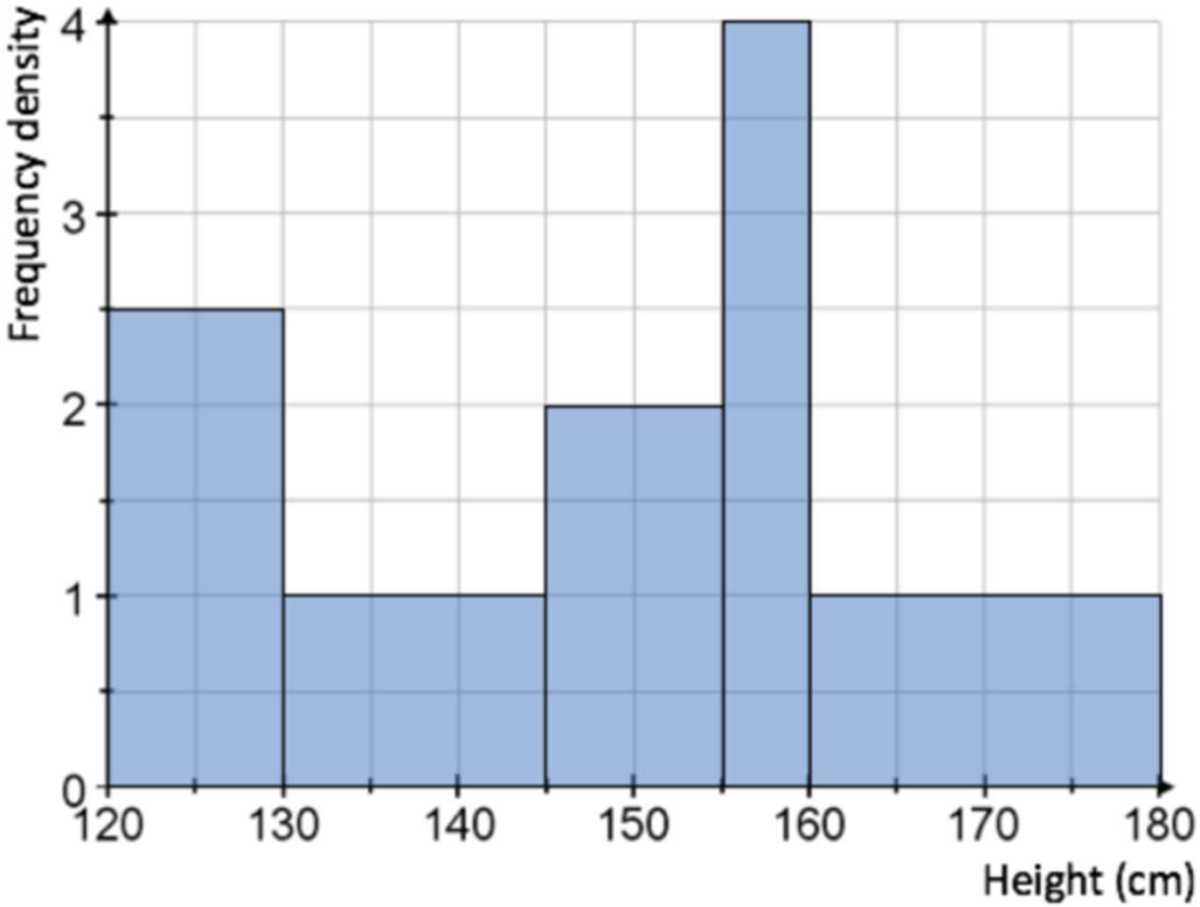
Worked Example

Draw a frequency table from the histogram:



Your Turn

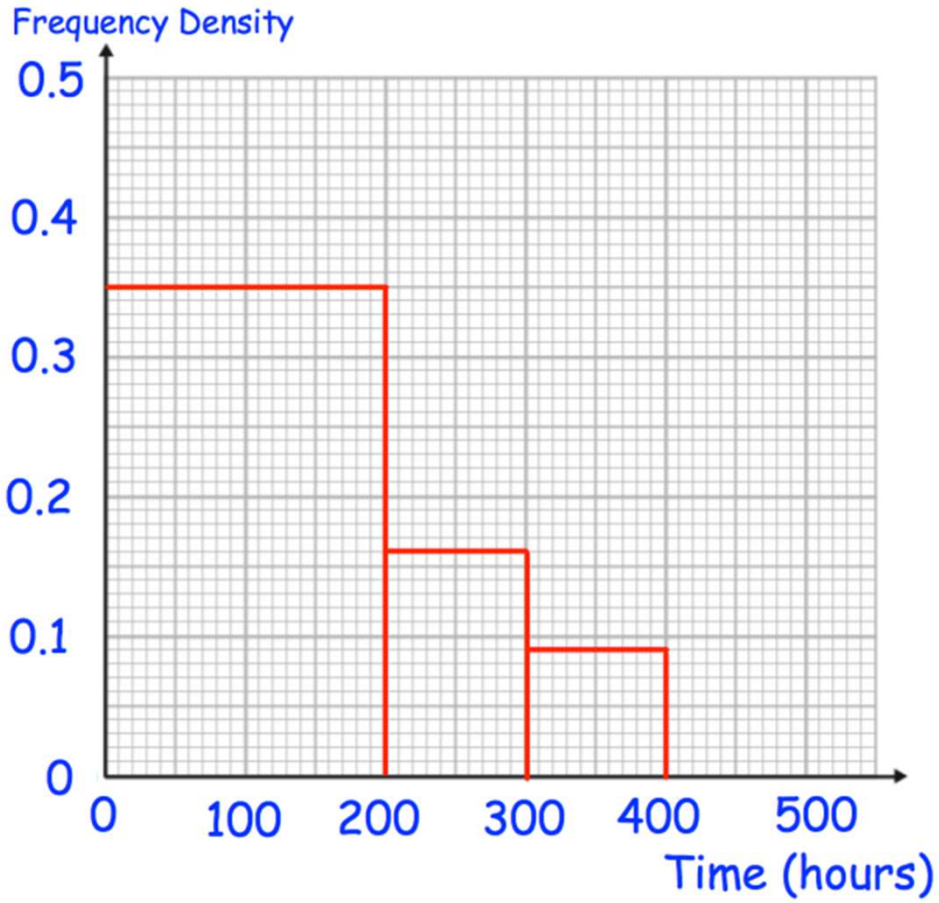
Draw a frequency table from the histogram:



Worked Example

a) Estimate the number of pilots who have flown under 350 hours.

b) Work out the percentage of pilots who have flown under 350 hours.

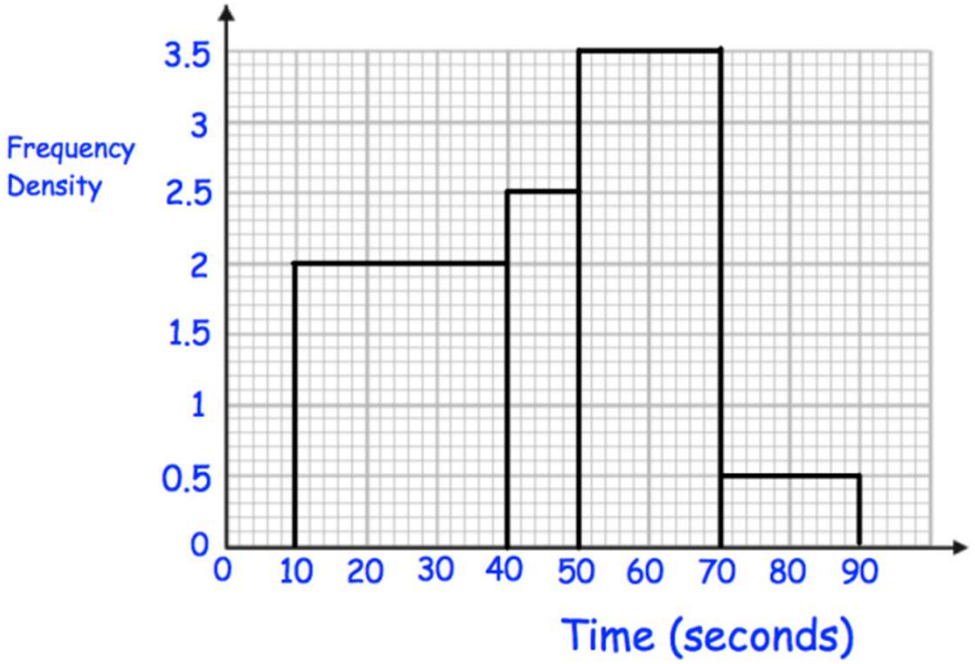


Dr Frost 401c and 401d

Your Turn

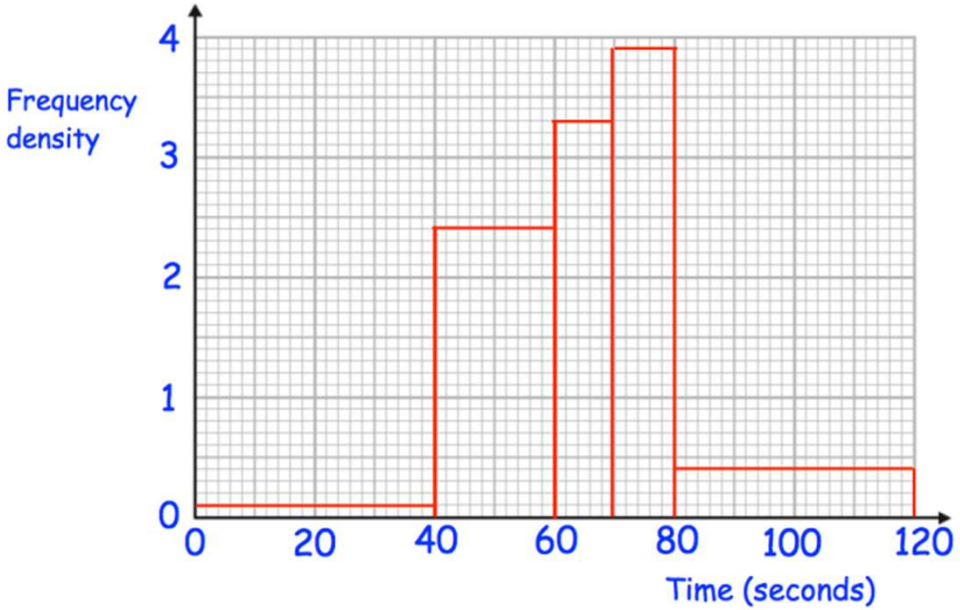
a) Estimate the number of students who took less than 60 seconds to complete the puzzle.

b) Work out the percentage of students who took less than 60 seconds to complete the puzzle.



Worked Example

Estimate the mean time.

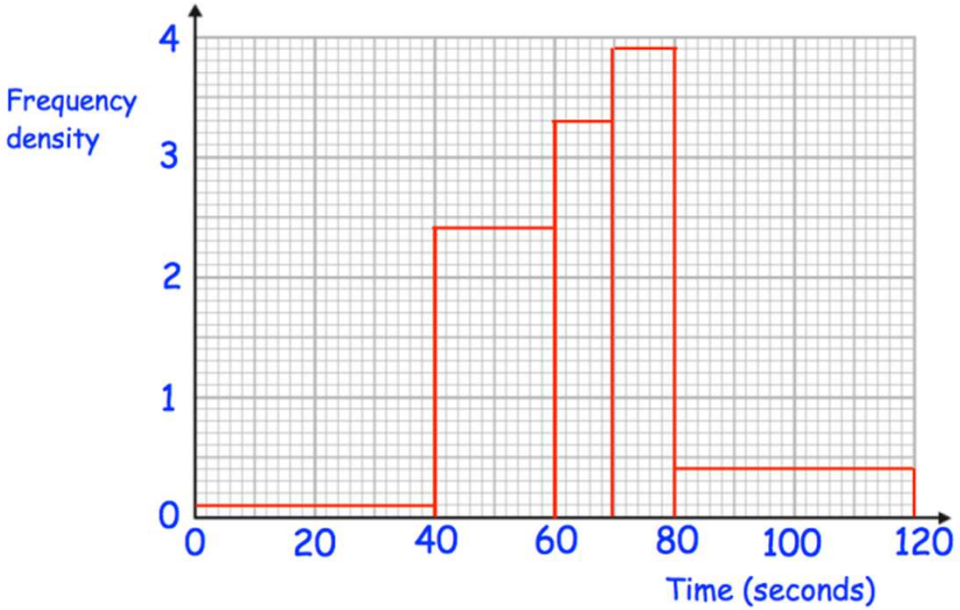


Your Turn

Estimate the mean weight.

Worked Example

Estimate the median time.

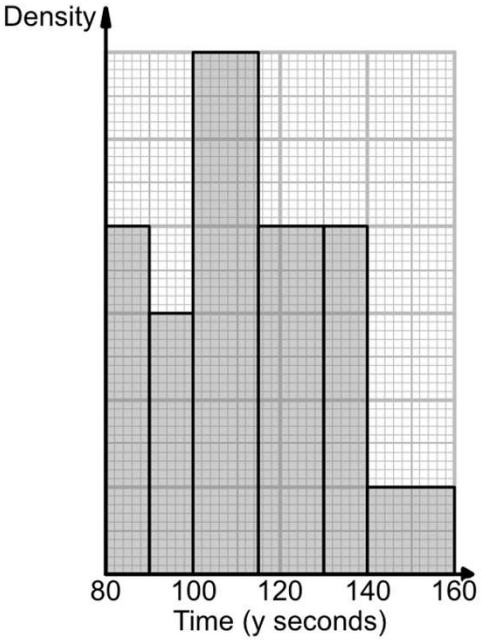


Your Turn

Estimate the median weight.

Worked Example

Lizzie collects the running times of some athletes and represents the data on the histogram drawn below.

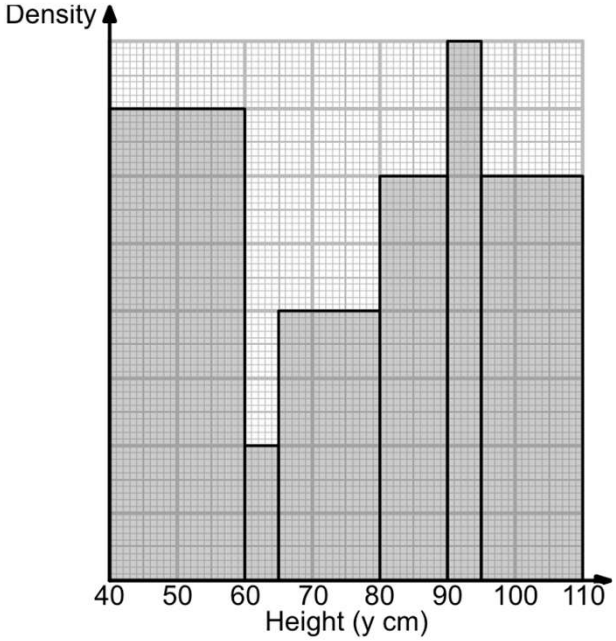


There are 10 athletes with a time between 140 and 160 seconds.
Estimate how many athletes have a time between 100 and 115 seconds.

Dr Frost 401c

Your Turn

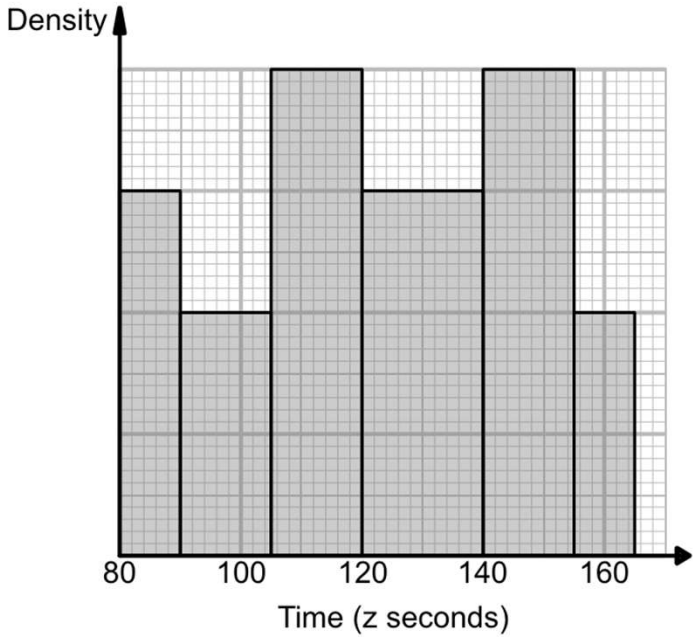
Lesley collects the heights of some plants and represents the data on the histogram drawn below.



There are 20 plants with a height between 90 and 95 cm.
Estimate how many plants have a height between 60 and 80 cm.

Worked Example

Lesley collects the running times of some athletes and represents the data on the histogram drawn below.

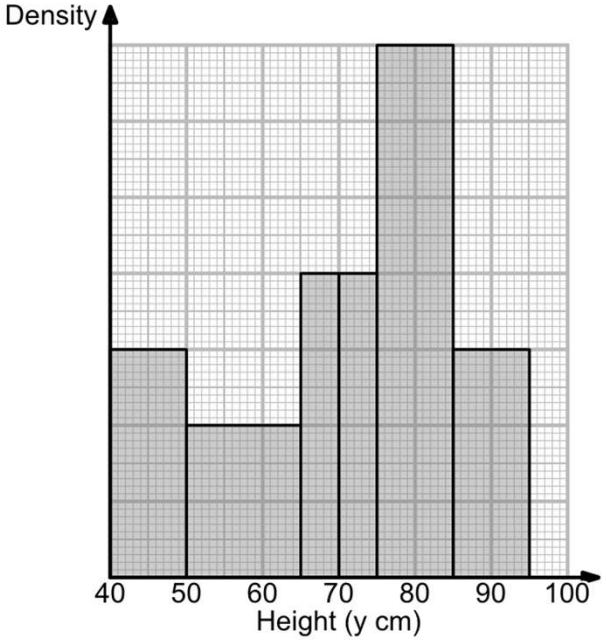


There are 15 athletes with a time between 80 and 90 seconds.
Find the total number of athletes included in Lesley's data.

Dr Frost 401d

Your Turn

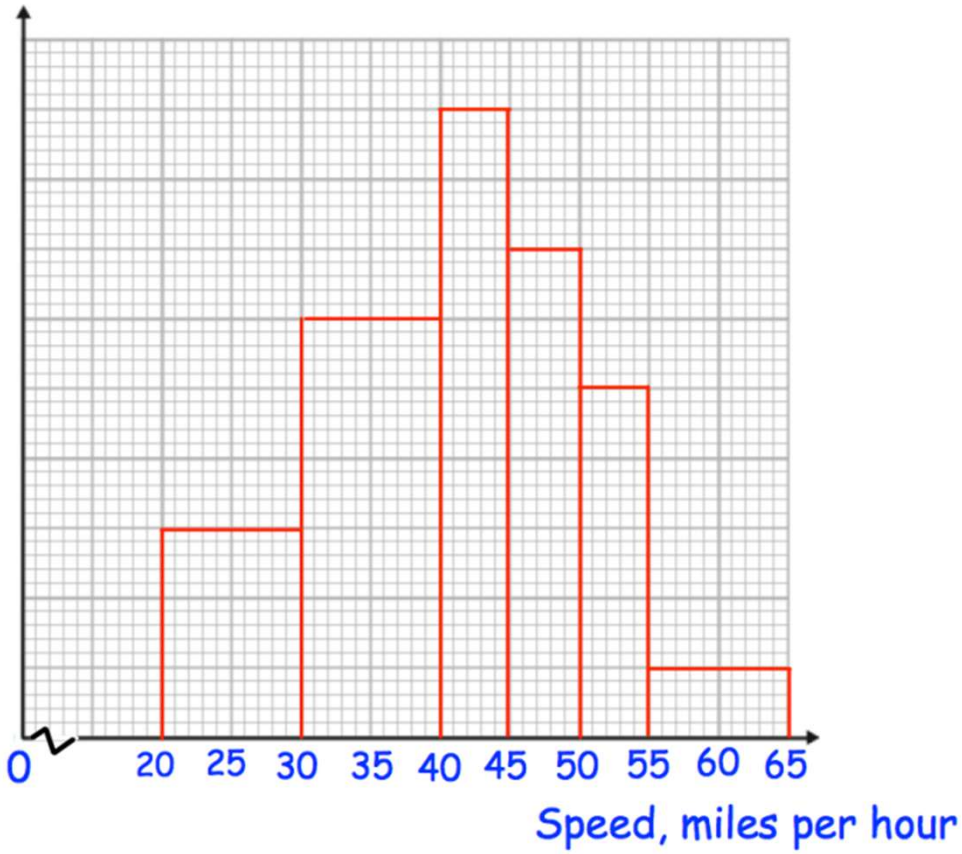
Lloyd collects the heights of some plants and represents the data on the histogram drawn below.



There are 15 plants with a height between 40 and 50 cm.
Find the total number of plants included in Lloyd's data.

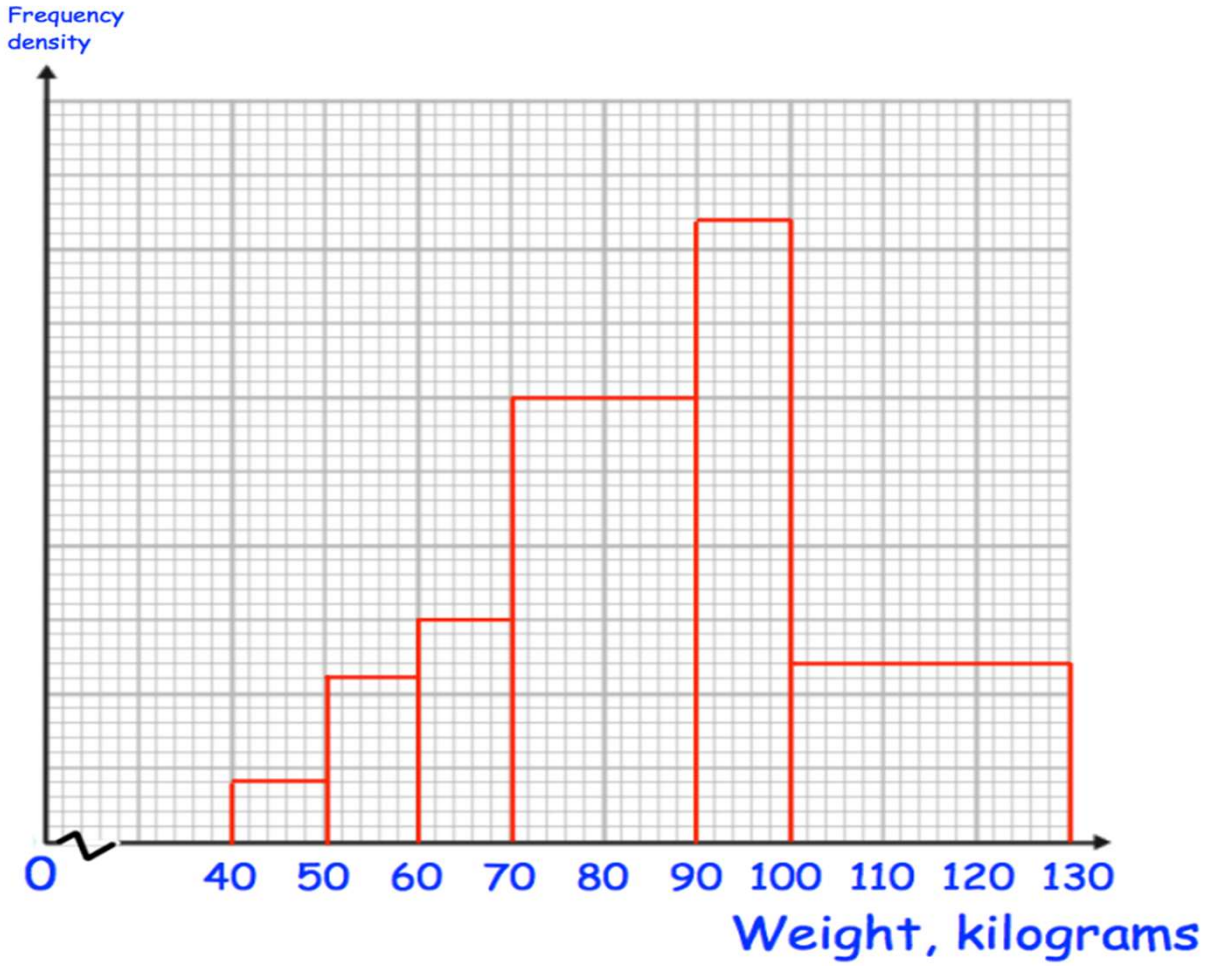
Worked Example

There were 82 cars on the road. 14 cars were travelling over 50 mph. Estimate the number of cars that were travelling between 40 and 49 mph.



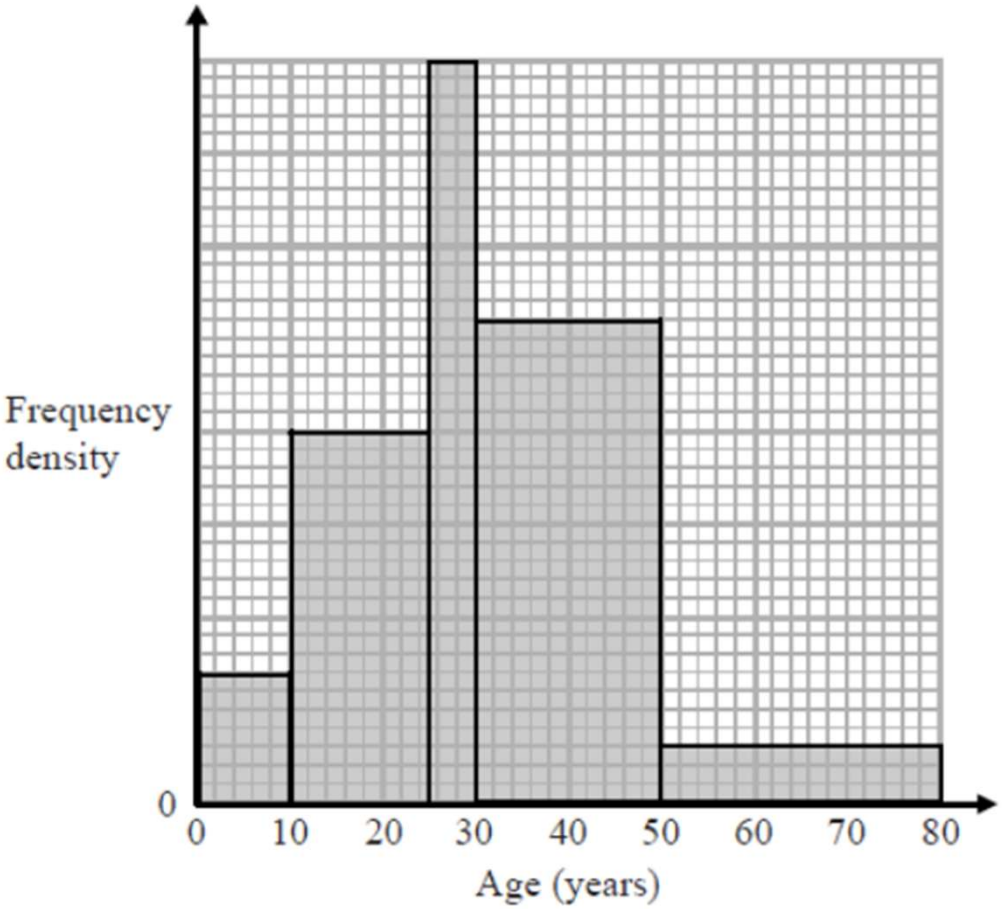
Your Turn

There were 504 athletes measured. 45 athletes weigh under 60 kg. Estimate the number of athletes between 70 and 95 kg.



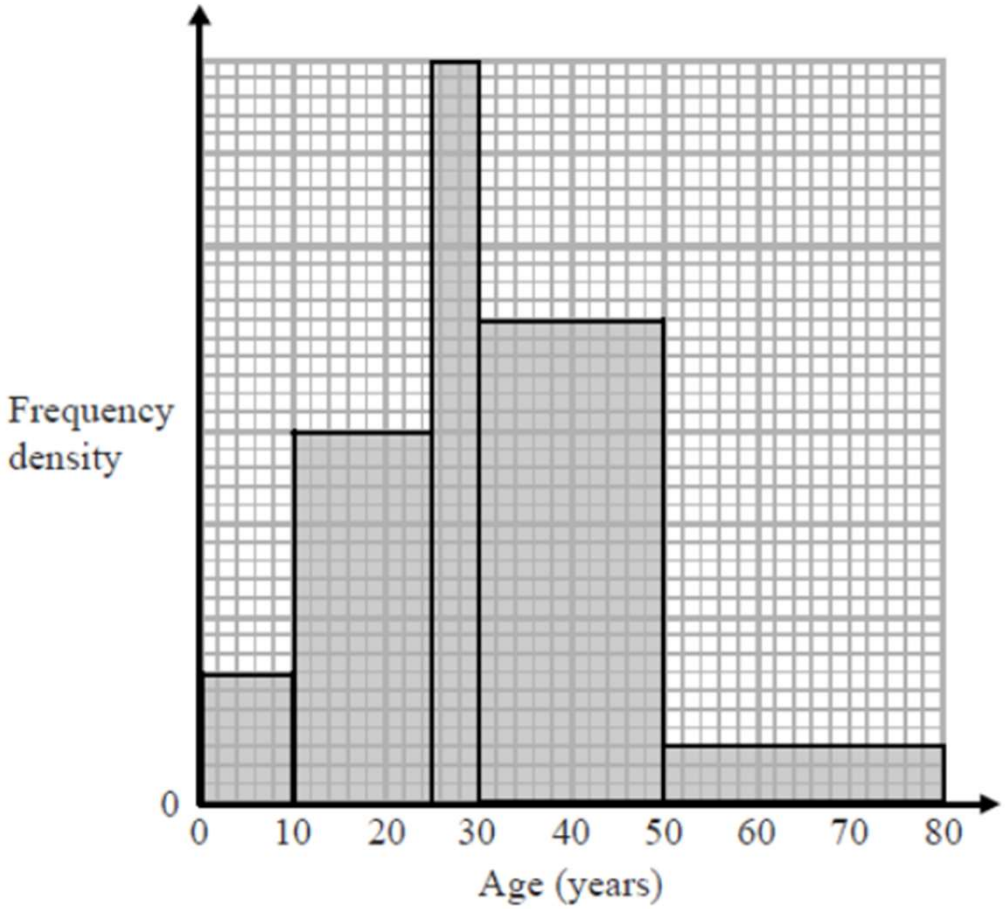
Worked Example

The histogram shows the ages of members of a club. There are 30 members aged between 10 and 25. A member of the club is chosen at random. What is the probability they are more than 25 years old?



Your Turn

The histogram shows the ages of members of a club. There are 20 members aged between 25 and 30. A member of the club is chosen at random. What is the probability they are more than 30 years old?



Question 1: Draw a histogram for each set of data below.

(a)

(b)

Length (cm)	Frequency
$0 \leq L < 20$	10
$20 \leq L < 30$	35
$30 \leq L < 40$	65
$40 \leq L < 80$	40

(c)

(d)

(e)

(f)

Apply

Question 1: Mr Smith has drawn a histogram to represent his classes' examination scores.

(a) Can you explain what Mr Smith has done wrong?
(b) Draw a correct histogram for Mr Smith

Question 2: The ages of the members of a snooker club are shown in the table below.

(a) Draw a histogram to represent the data.

Ronnie, the manager of the snooker club, says that the average age of the members of the snooker club is under 32.

(b) Work out an estimate of the mean age of the members

(c) Do you agree with Ronnie?
Explain your answer.

Question 3: Christine has drawn a histogram to show the value of some antiques.
She has made some mistakes.

(a) Can you spot all the mistakes?

(b) Draw a correct histogram to represent the data.

Question 4: Henry has 20 apples in a crate.
The masses of the apples are shown in the table.

(a) Work out an estimate of the mean mass of an apple.

(b) Draw a histogram to represent the data.

(c) What fraction of the apples are over 85g?

Henry takes two apples from the crate at random, without replacement.

(d) Work out the probability that both apples are over 90g.

Question 1: The histogram shows information about the time taken to travel to school by students.

Complete the frequency table.

Question 2: The histogram shows information about the temperatures in various locations.

Complete the frequency table.

Question 3: The histogram shows information about the salaries of workers in a company.

Complete the frequency table.

Question 4: The histogram shows information about ages of the people who live in a village.

Complete the frequency table.

Question 5: The histogram shows information about distances students live from a college.

(a) How many students attend the college?

(b) How many students live less than 10km from the college?

(c) How many students live between 15km and 20km from the college?

(d) Estimate how many students live more than 25km from the college.

(e) Estimate how many students live less than 5km from the college.

(f) Estimate how many students live between 5km and 12.5km from the college.

(g) Estimate how many students live between 12.5km and 17.5km from the college.

(h) Estimate how many students live between 10km and 14km from the college.

(i) Estimate how many students live between 8km and 16km from the college

(j) Estimate how many students live further than 16km from the college.

Question 1: An A-level course is marked out of 400 marks.
A teacher has created this histogram to represent the students' results in his college over the past 10 years.
The table shows the marks needed for each grade.

Work out an estimate of the number of students who achieved each grade.

Question 2: There are road works in Antrim, so all the buses this week were late.
The histogram shows information about all the buses.

(a) Estimate what fraction of buses were less than 3 minutes late.

The bus company offers a full refund if the bus is more than 10 minutes late.

(b) Estimate what percentage of passengers will receive a full refund.
Give your answer to two decimal places.

Question 3: 260 people sit a driving theory test.
Their results are shown in this histogram.

10% of the people scored less than x marks.

(a) Find x.

5% of people scored more than y marks.

(b) Find y

70% of people scored less than z marks.

(c) Find z

Question 4: The ages of the residents of a village are represented in this histogram

(a) How many people live in the village?

(b) Calculate an estimate of the mean age

Question 5: The heights of some sunflowers are represented in the histogram.

(a) Find an estimate of the median

(b) Find an estimate of the lower quartile

(c) Find an estimate of the upper quartile

(d) Find an estimate of the interquartile range

Question 6: The histogram shows the speed, in miles per hour, of cars on a road over 1 hour.
24 cars travelled faster than 40mph.

(a) How many cars travelled slower than 20mph?

(b) How many cars travelled between 20mph and 40mph?

(c) Estimate how many cars travelled between 15mph and 35mph.

Question 7: The histogram below shows the ages of rugby players.
There are 768 players that are under 26 years old.

Work out an estimate of how many players are over 32.

Question 8: The histogram below shows the monthly salaries of employees.
There are 216 people who have a monthly salary of between £1800 and £2100.

Work out an estimate of how many employees have a salary of between £2300 and £2900

Worked Example

Lenny collects the heights of 111 plants and records the data in the table below.

Height (y cm)	Frequency
$50 < y \leq 65$	15
$65 < y \leq 85$	28
$85 < y \leq 105$	26
$105 < y \leq 120$	15
$120 < y \leq 125$	27

A histogram was drawn and the class $50 < y \leq 65$ was represented by a rectangle of width 6 cm and height 9 cm.
Calculate the width and the height of the rectangle representing the class $85 < y \leq 105$.

Your Turn

Liam collects the heights of 98 plants and records the data in the table below.

Height (x cm)	Frequency
$0 < x \leq 20$	22
$20 < x \leq 30$	23
$30 < x \leq 45$	15
$45 < x \leq 55$	14
$55 < x \leq 60$	17
$60 < x \leq 80$	7

A histogram was drawn and the class $45 < x \leq 55$ was represented by a rectangle of width 1.5 cm and height 3.5 cm.
Calculate the width and the height of the rectangle representing the class $30 < x \leq 45$.

FREQUENCY POLYGONS

Seeing the shape of a distribution

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PLOT EVERY POINT AT (midpoint , frequency)

NOT the boundaries — e.g. class 20≤t<30 with F = 20 is plotted at (25, 20). Don't mix up with CF graphs, which plot (upper bound, CF)!

WHAT IS A FREQUENCY POLYGON?

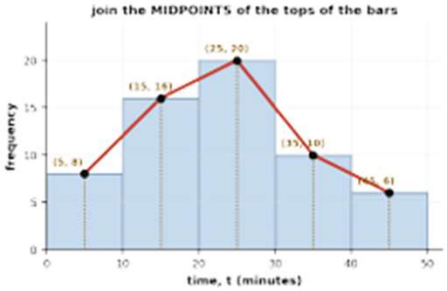
A frequency polygon shows **grouped continuous data** as points joined by **straight line segments** (use a ruler). We draw one because it makes the **shape of the distribution** much easier to see than a table — and two polygons can sit on the **same axes** to compare data sets.

$$\text{midpoint} = \frac{\text{lower bound} + \text{upper bound}}{2}$$

e.g. class 20≤t<30: $\frac{20 + 30}{2} = 25$

ON TOP OF A HISTOGRAM

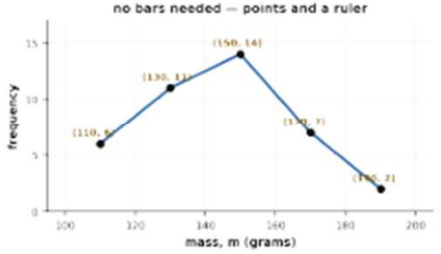
With **equal class widths**, join the **midpoints of the tops of the bars** with straight lines. The polygon and histogram show the same data.



STRAIGHT FROM A TABLE — NO HISTOGRAM

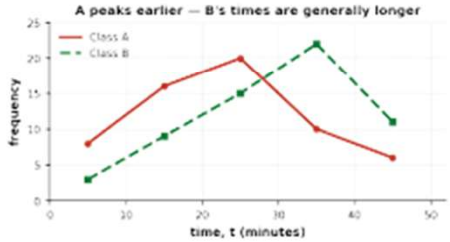
Add a **midpoint** column, then plot (midpoint, frequency) directly.

mass, m (g)	F	midpoint	plot
100≤m<120	6	**110**	(110, 6)
120≤m<140	11	**130**	(130, 11)
140≤m<160	14	**150**	(150, 14)
160≤m<180	7	**170**	(170, 7)
180≤m<200	2	**190**	(190, 2)



WHY DRAW ONE? SHAPE & COMPARISON

The polygon reveals the **distribution**: where the data peaks, whether it is symmetrical or skewed, and how spread out it is. Two polygons on the same axes make comparison easy:



"A peaks at 20–30 min, B at 30–40 min, so B generally took longer."

WHICH GRAPH PLOTS WHAT?

Frequency polygon	(**midpoint** , frequency)
CF graph	(**upper bound** , cumulative frequency)
Histogram	bars: height = frequency density

COMMON MISTAKES — EXAMINERS' REPORTS

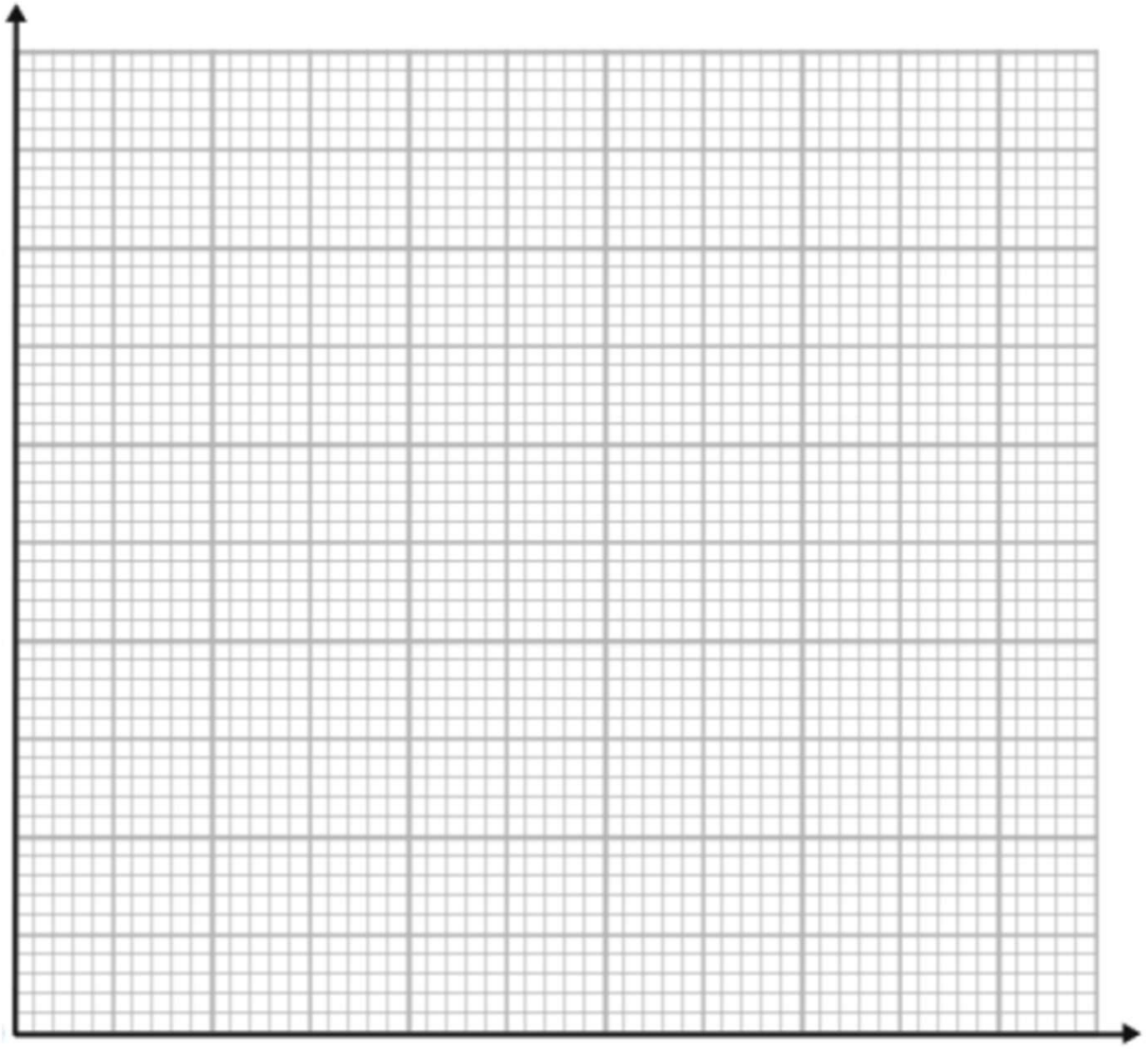
- ✗ Plotting at the **class boundaries** (or upper bounds) instead of the **midpoints**.
- ✗ Joining the first or last point **down to the x-axis** — do NOT close the polygon at GCSE.
- ✗ Joining the points with a **curve** — use a ruler; straight segments only.
- ✗ Plotting **cumulative frequency** instead of frequency — that's the CF graph, a different question.
- ✗ Comparing two polygons without **context** — refer to what the data measures, not just "B is higher".

Frequently examined on Edexcel 1MA1 GCSE Higher Paper 2 and Paper 3.

Worked Example

Draw a frequency polygon for the information:

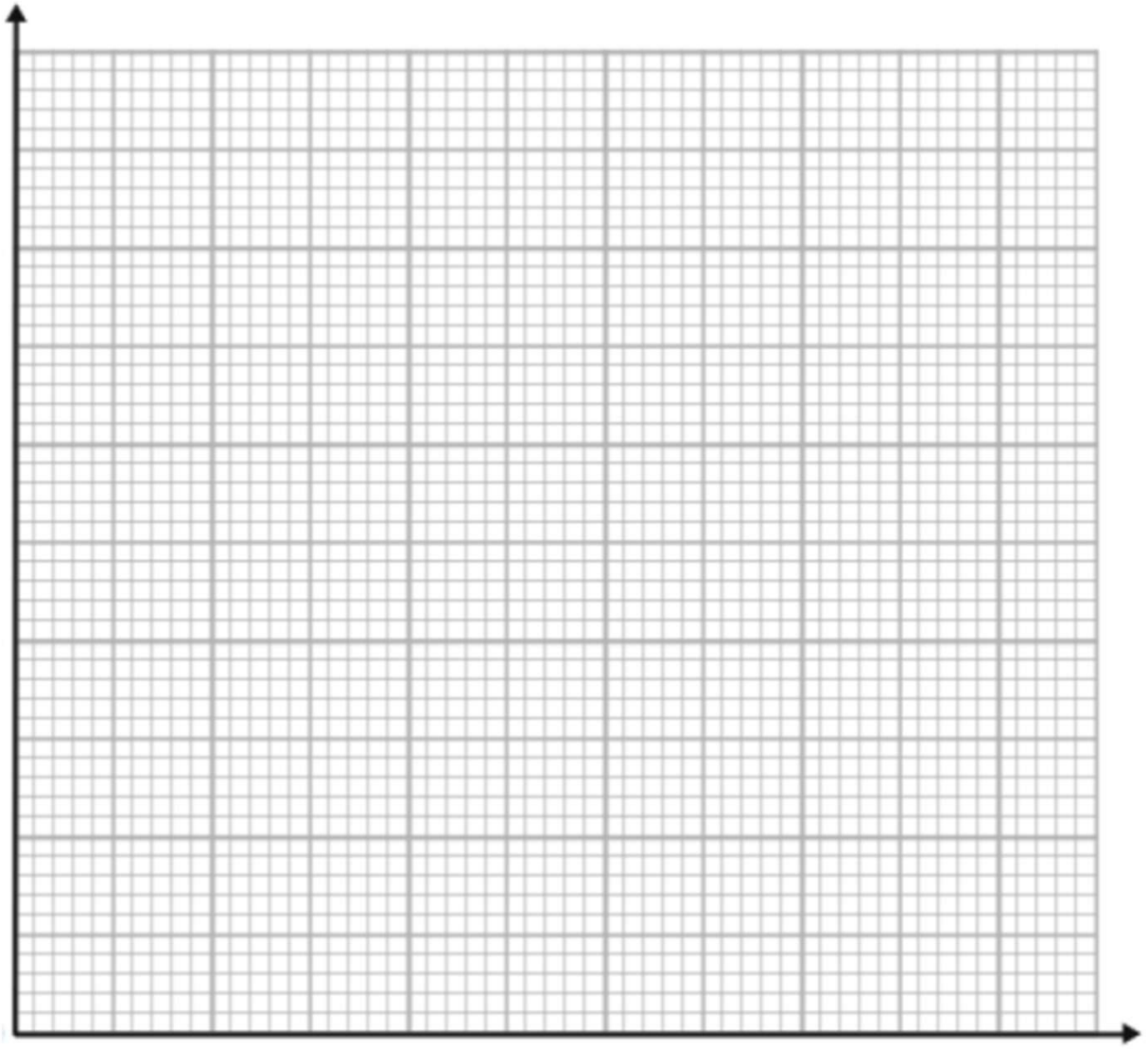
Lengths (cm)	Frequency
$0 < L \leq 0.5$	8
$0.5 < L \leq 1$	17
$1 < L \leq 1.5$	20
$1.5 < L \leq 2$	10
$2 < L \leq 2.5$	5



Your Turn

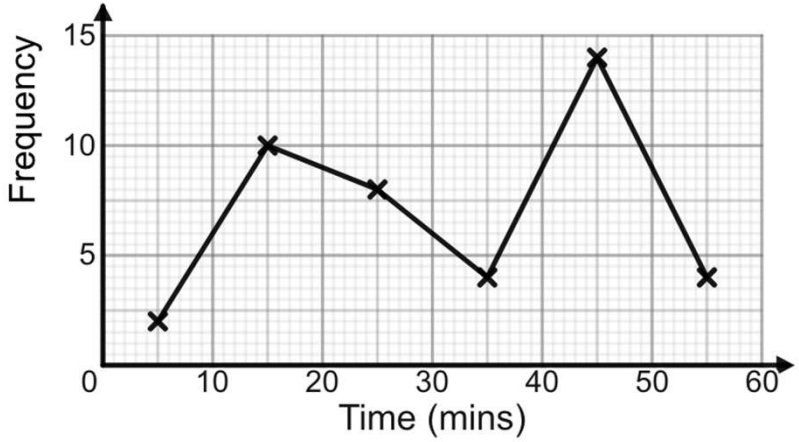
Draw a frequency polygon for the information:

Time (minutes)	Frequency
$0 < t \leq 10$	10
$10 < t \leq 20$	28
$20 < t \leq 30$	46
$30 < t \leq 40$	23
$40 < t \leq 50$	12



Worked Example

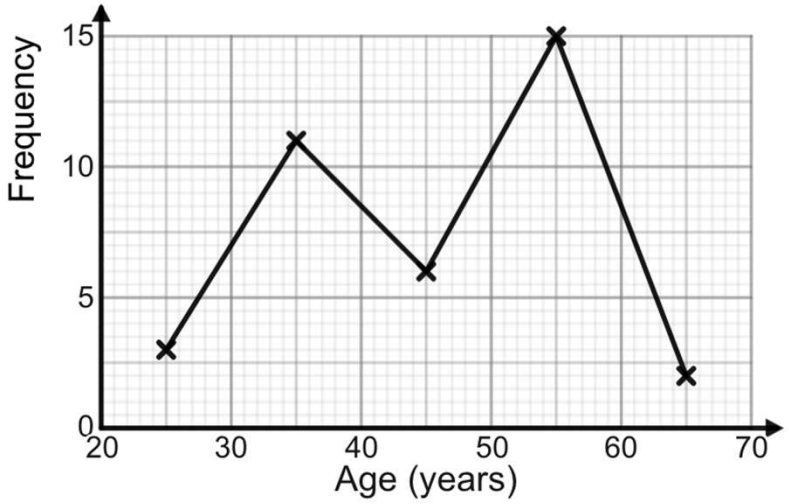
The frequency polygon shows the information for the amount of time spent by people in a gym.



Calculate how many people spent between 20 and 50 minutes in the gym.

Your Turn

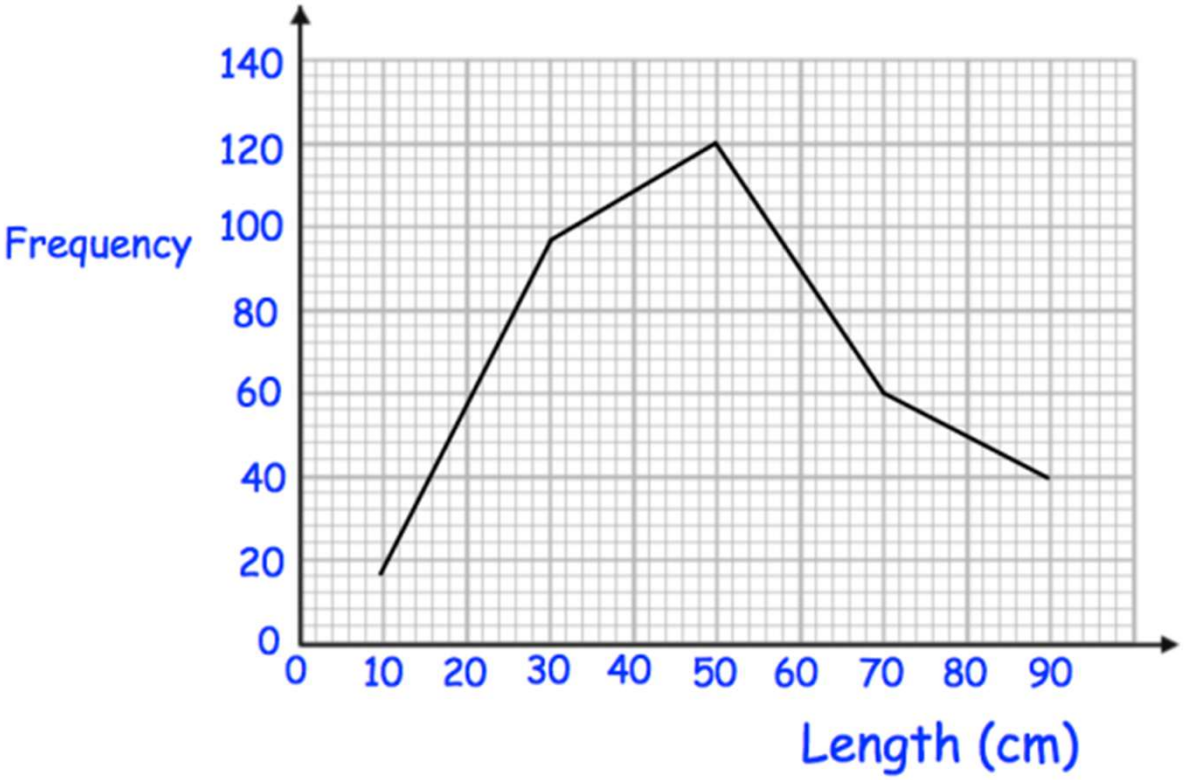
The frequency polygon shows the information for the age of people who are buying properties in a town.



Work out how many people are more than 50 years old in the sample.

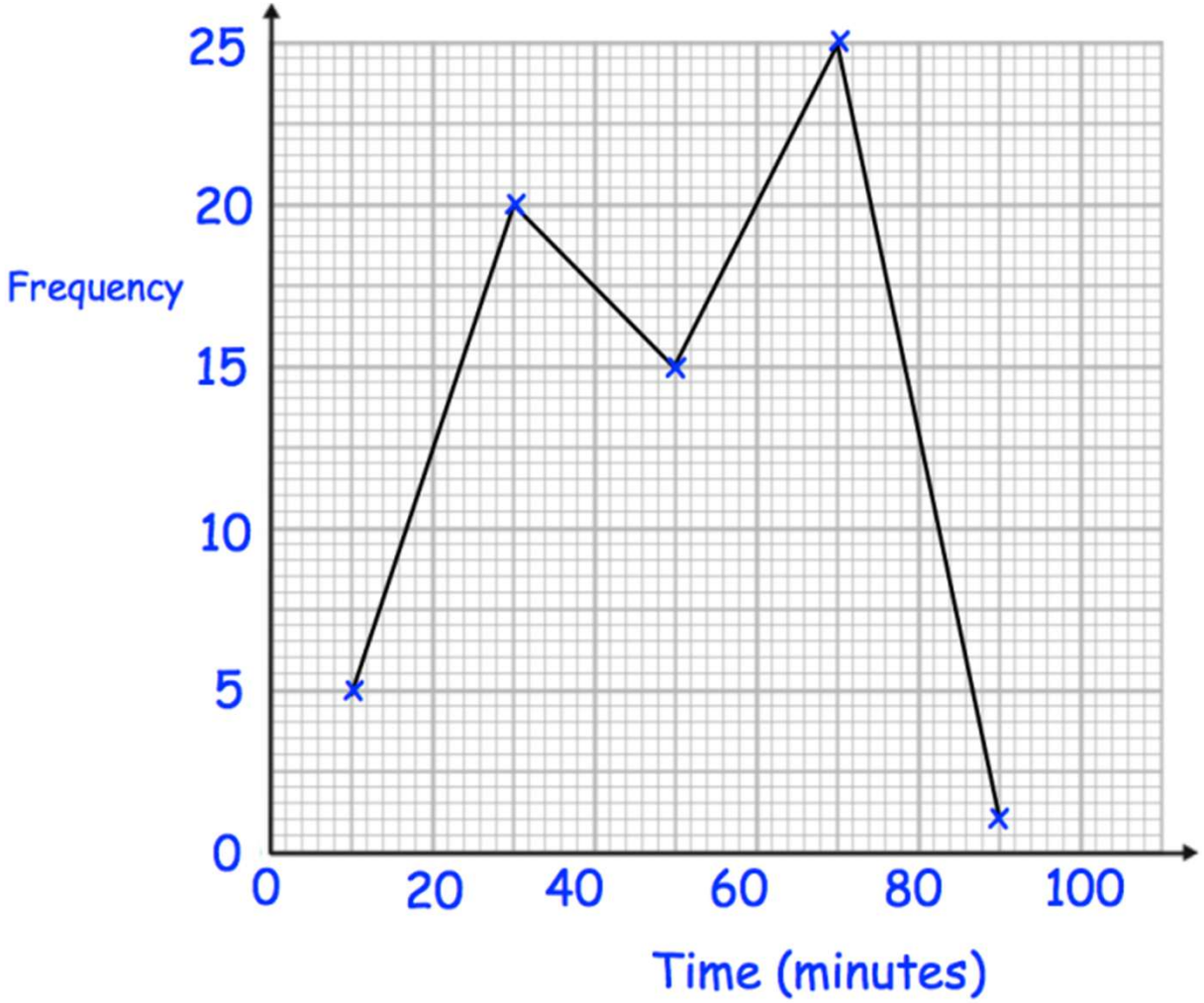
Worked Example

Estimate the mean length:



Your Turn

Estimate the mean length:



Worked Example

The frequency polygons show the information for the salary of the employees under 40 and over 40 in an office.

Work out how many more employees under 40 than over 40 earn between £$20,000$ and £$40,000$

Dr Frost 315e

Your Turn

The frequency polygons show the information for the amount of time spent by people in two clothing stores.

Find how many more people spent over 50 minutes in Clothing Store B than Clothing Store A

Question 1: Draw a frequency polygon for each table of information below.

(a)

Goals	Frequency
0	7
1	10
2	12
3	3
4	1

(b)

(c)

Question 2: Draw a frequency polygon for each table of information below.

(a)

Marks	Frequency
$0 < m \leq 10$	8
$10 < m \leq 20$	11
$20 < m \leq 30$	23
$30 < m \leq 40$	19
$40 < m \leq 50$	15

(b)

(c)

Question 3: For each frequency polygon, complete the frequency table.

(a)

TVs owned	Frequency
0	
1	
2	
3	
4	
5	

(b)

Amount Withdrawn	Frequency
£10	
£20	
£30	
£40	
£50	

Question 4: For each frequency polygon, complete the frequency table.

(a)

(b)

Apply

Question 1: Tia asked her friends how many times they visited the cinema last month. The frequency polygon shows her results.

(a) How many people visited the cinema twice?

(b) What is most popular number of times that her friends visited the cinema?

(c) What is the most number of times that somebody visited the cinema?

(d) How many people did Tia survey in total?

Question 2: Henry surveyed 50 people.
This frequency polygon shows their shoe sizes.

(a) What is the modal shoe size?

(b) What is the range of the shoe sizes?

(c) What fraction of the people surveyed have size 5 shoes?

(d) What percentage of the people surveyed have size 7 shoes?

(e) Henry picks somebody at random to win a prize.
Write down the probability that the winner has size 6 shoes.

Question 3: Roy is a striker for Rovers.
The frequency polygon shows the number of goals scored in each game over 20 games he has played.

(a) Work out the median number of goals scored per game.

(b) Work out the mean number of goals scored per game.

(c) A journalist asks him for the "average" number of goals scored per game.
Which average should he use?

Question 4: The frequency polygons show the amount of time that 30 students in year 7 and 30 students in year 11 spent on their last maths homework.

Compare the time spent on homework by the year 7s and the year 11s.

Question 5: 50 boys and 50 girls attend a primary school.
The table below shows how many clubs they attend.

(a) On the same grid, draw a frequency polygon for the boys and a frequency polygon for the girls.

(b) Compare the distributions.

Question 6: The frequency polygon shows information about the amount of time people spend in the gym.

Calculate an estimate of the mean time spent in the gym.

Question 1(a)

Question 1(b)

Templates

Question 1(c)

Question 2(a)

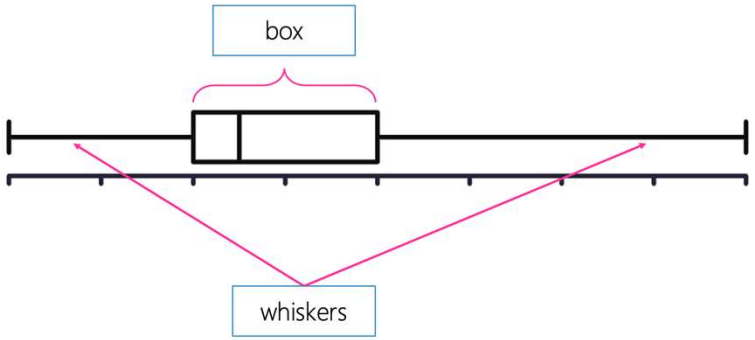
Templates

Question 2(b)

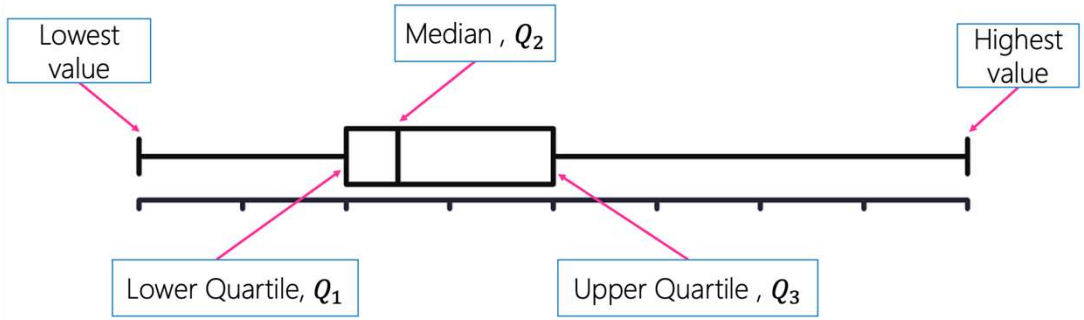
Question 2(c)

Box Plots

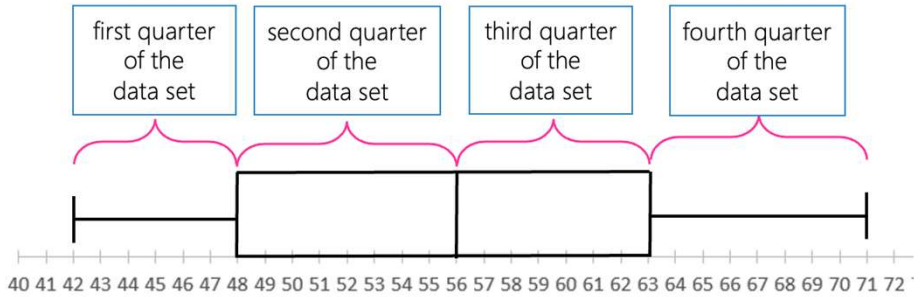
This is a box and whisker plot, or boxplot.



This is a box and whisker plot, or boxplot.
We draw it with the median, quartiles and endpoints of the data set.



A boxplot shows the data in four parts.



25% of the set is less than 48 kg

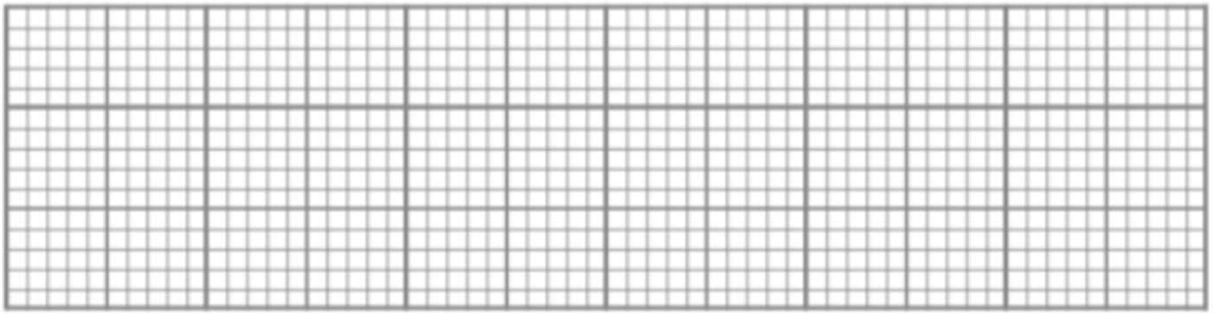
50% of the set is less than 56 kg

75% of the set is less than 63 kg

Worked Example

Draw a box plot to show this information:

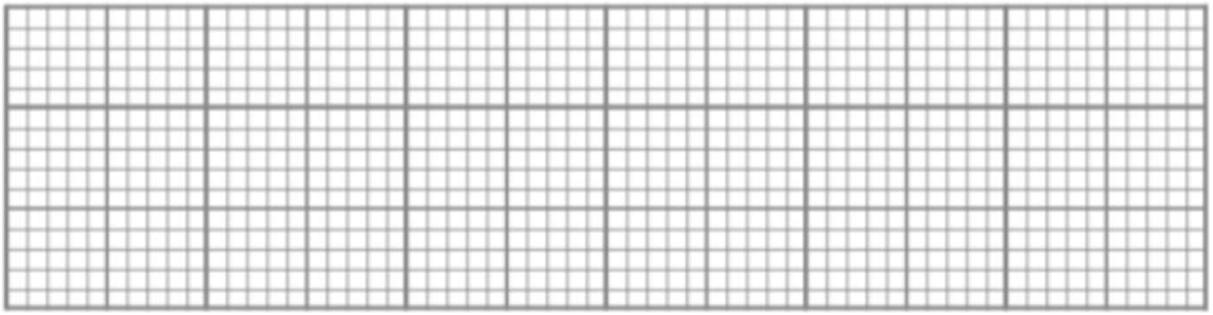
Lowest	51 kg
Lower Quartile	60 kg
Median	71 kg
Upper Quartile	74 kg
Highest	83 kg



Your Turn

Draw a box plot to show this information:

Lowest	68kg
Lower Quartile	74kg
Median	82kg
Upper Quartile	88kg
Highest	100kg

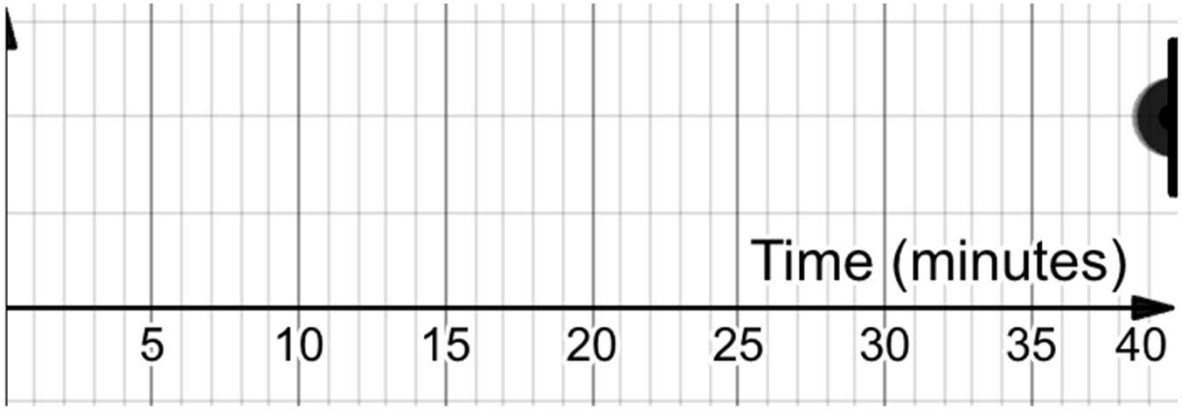


Worked Example

The times, in minutes, taken by 11 pupils to do a homework are listed in order below.

$5, 5, 6, 8, 10, 11, 22, 25, 31, 33, 35$

Draw a box plot for this data.

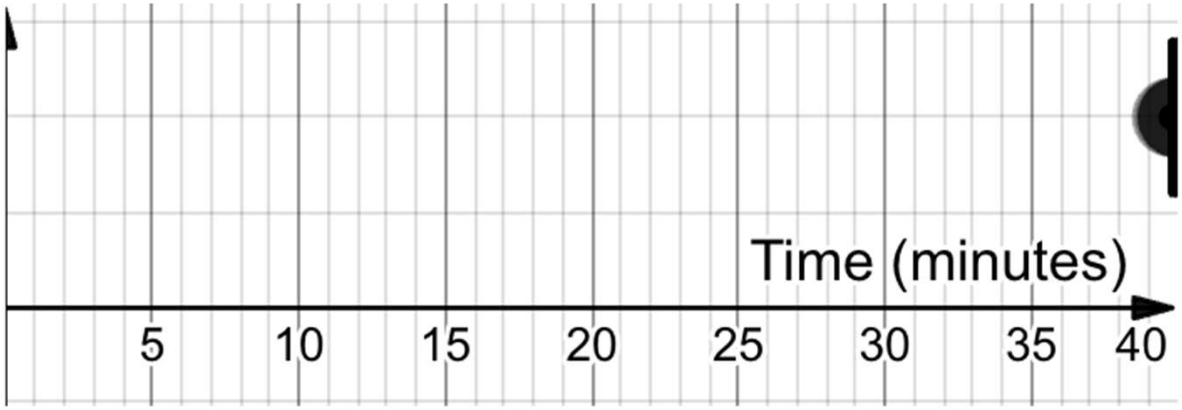


Your Turn

The times, in minutes, taken by 15 pupils to do a homework are listed in order below.

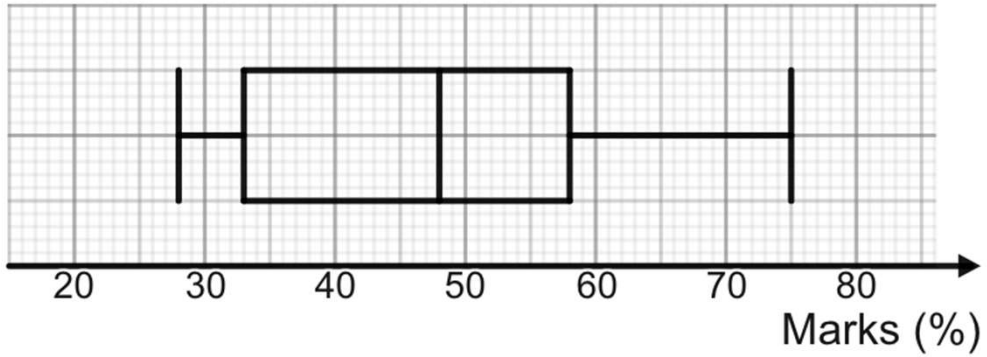
$5, 8, 12, 14, 15, 16, 18, 18, 20, 23, 25, 28, 29, 34, 34$

Draw a box plot for this data.



Worked Example

Selene recorded the marks some pupils achieved on a test and put this information on the box plot below.

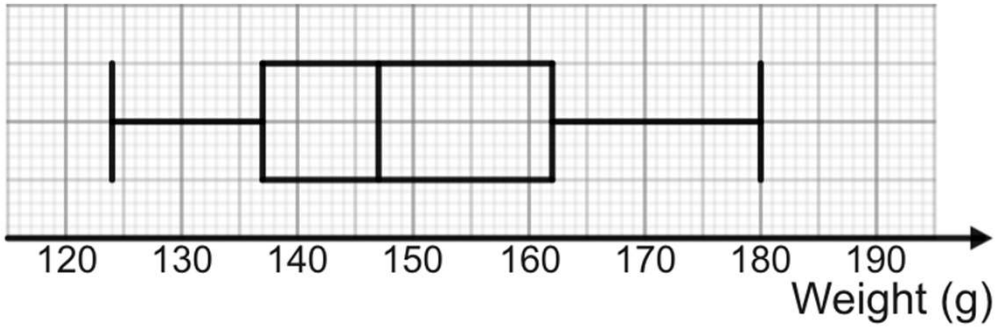


Find the lower quartile of this data set.

Dr Frost 399a

Your Turn

Kendra recorded the weights of some tomatoes.
The box plot below shows the distribution of the weights of the tomatoes.

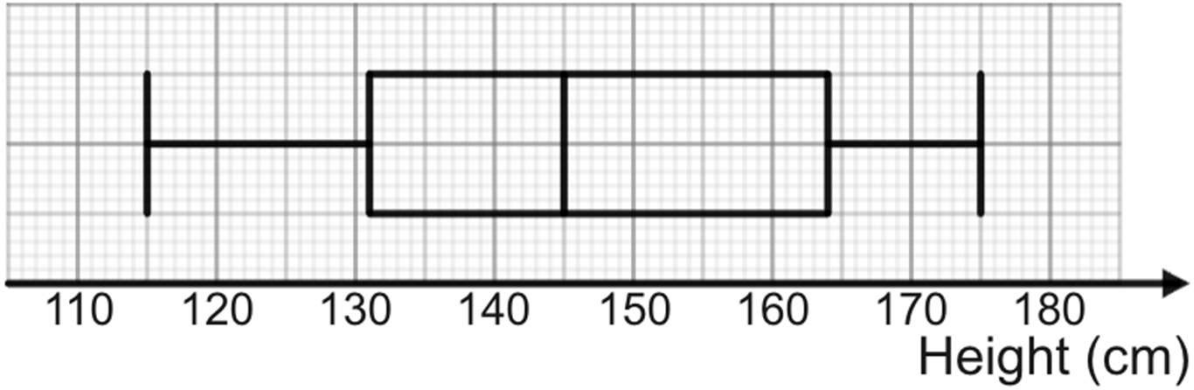


Find the upper quartile of this data set.

Worked Example

Hilda recorded the heights of 40 pupils.

The box plot below shows her results.



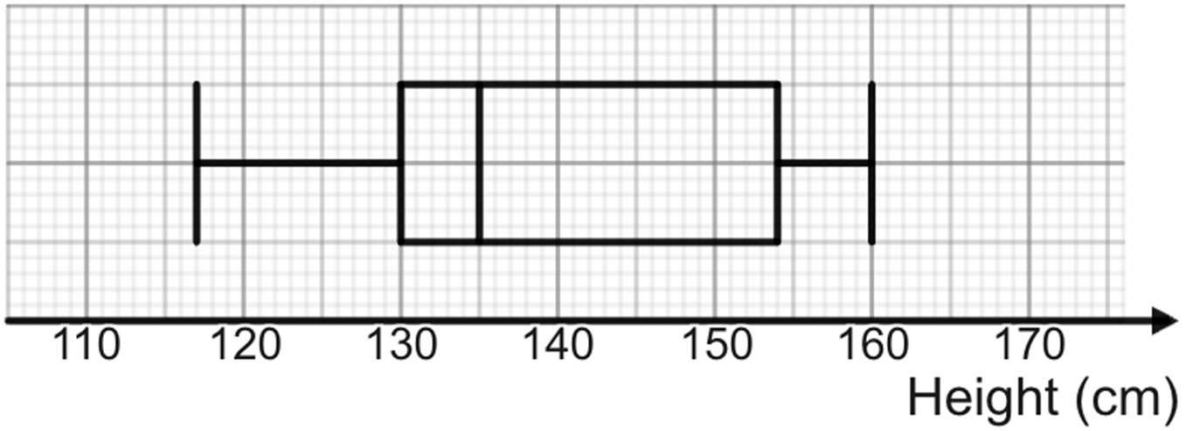
Find the number of pupils with a height more than 131 cm.

Dr Frost 399e

Your Turn

Noah recorded the heights of 60 pupils.

The box plot below shows her results.

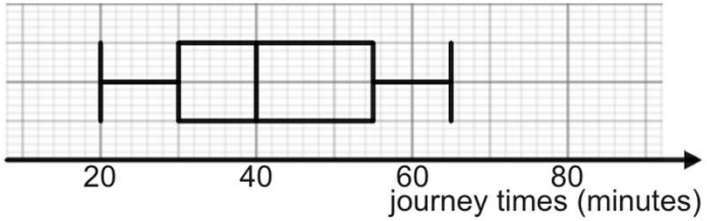


Find the percentage of pupils with a height less than 154 cm.

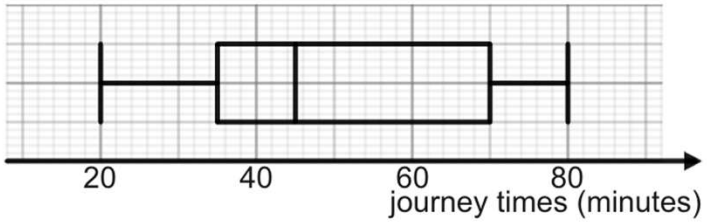
Worked Example

The journey times for Route A and Route B, in minutes, are shown in the box plots below.

Route A



Route B

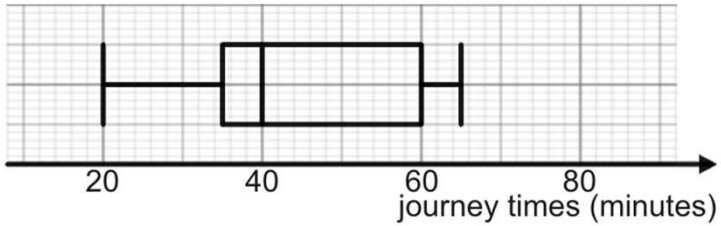


Compare the journey times for Route A and Route B

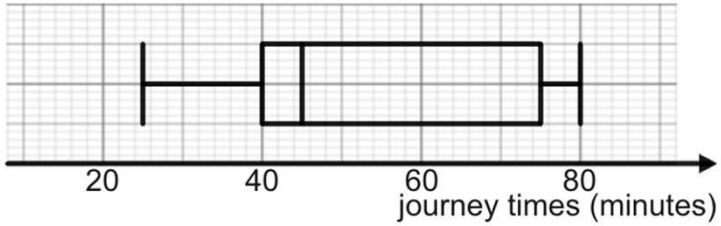
Your Turn

The journey times for Route A and Route B, in minutes, are shown in the box plots below.

Route A



Route B



Compare the journey times for Route A and Route B

Question 1: Draw a box plot for each of the following.

(a) (b) (c)

Question 2: For each box plot below, find the (i) median, (ii) interquartile range, (iii) range

(a) (b)

(c) (d)

Question 3: Draw a box plot for each of the following.

(a) (b) (c)

Question 4: Draw a box plot for each set of data

(a) 8, 10, 13, 14, 14, 15, 15, 16, 18, 19, 21, 22, 24, 29, 35

(b) 40, 80, 90, 90, 100, 120, 130

(c) 5.9, 7.3, 7.8, 8, 8.4, 8.7, 8.9, 8.9, 8.9, 9, 9, 9.1, 9.1, 9.3, 9.5, 9.6, 9.9, 10.5, 10.9

Question 5: Compare the distributions of each pair of box plots below.

(a)

(b)

(c)

(d)

Apply

Question 1: Gareth and Wayne are two footballers.
The table shows information about the number of passes they make in each game over a season.

(a) Find the missing values from the table

(b) Using the same scale, draw box plots to represent the data.

(c) Compare and contrast the two box plots

Question 2: Rosie is going on holiday to an island.
The box plots below show information about the daily average rainfall in May and June on the island.

(a) What was the median rainfall in May?

(b) What was the highest rainfall in June?

(c) What percentage of days in June had over 2.5mm of rain?

(d) What percentage of days in May had over 2.5mm of rain?

(e) What percentage of days in May had under 1.2mm of rain?

(f) When would you recommend Rosie visits the island?
Explain your answer.

Question 3: Mr Jones is an estate agent on the Isle of Man.
He has created this table to show information about the prices of houses he has sold.

Explain how you know he has made a mistake.

Question 4: The box plot show information about the masses of apples in a crate.

Jack is going to select apples at random from the crate.
After selecting each apple, he records its mass and returns it to the crate before picking another.
Work out the probability that:

(a) Jack picks two apples, both under 75g

(b) Jack picks two apples, both over 90g

(c) Jack picks two apples, both over 105g

(d) Jack picks two apples, one under 90g and one over 105g

(e) Jack picks three apples, all over 105g

(f) Jack picks three apples, two over 105g and one under 75g.

CUMULATIVE FREQUENCY & BOX PLOTS

Running totals, quartiles and comparing distributions

Knowledge Organiser • GCSE Edexcel Higher Mathematics

PLOT EVERY POINT AT `( upper class boundary , cumulative frequency )`

NOT the midpoint, NOT the lower bound — e.g. class 20≤t<30 with CF 44 is plotted at (30, 44)

WHAT IS CUMULATIVE FREQUENCY?

Cumulative frequency (CF) is a **running total** of the frequencies. Technically the graph is a smooth S-shaped curve, but at GCSE it is **easier and fully acceptable to join the points with straight line segments** using a ruler.

time, t (min)	F	CF (plot at)
0≤t<10	8	8 (at 10)
10≤t<20	16	24 (at 20)
20≤t<30	20	44 (at 30)
30≤t<40	10	54 (at 40)
40≤t<50	6	60 (at 50)

The final CF must equal the **total**, n = 60.

KEY RULES (POSITIONS)

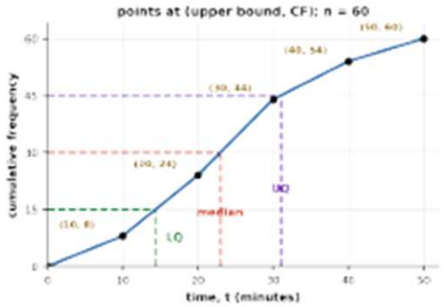
median position $= \frac{n}{2}$ **LQ** $= \frac{n}{4}$ **UQ** $= \frac{3n}{4}$

IQR = UQ − LQ

n = 60: $\frac{60}{2}$ = 30th, $\frac{60}{4}$ = 15th, $\frac{3 \times 60}{4}$ = 45th

READING THE GRAPH

Go **across** from the CF axis at the position, **down** to the x-axis, and read off the estimate.



`median ≈ 23, LQ ≈ 14.4, UQ ≈ 31`

IQR ≈ 31 − 14.4 = 16.6 minutes

"HOW MANY MORE THAN…?"

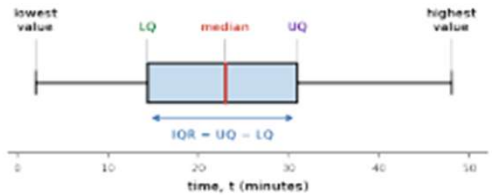
Go **up** from the value on the x-axis to the curve, read the CF, then **subtract from n** if asked for "more than".

```
More than 35 min: CF at 35 ≈ 49
60 − 49 = 11 students
```

These are **estimates** — the data is grouped.

BOX PLOTS — THE FIVE VALUES

A box plot displays: **lowest value, LQ, median, UQ, highest value** on a scaled axis.



The **box** spans LQ to UQ (the middle 50% of the data); **whiskers** reach the lowest and highest values.

FROM CF CURVE TO BOX PLOT

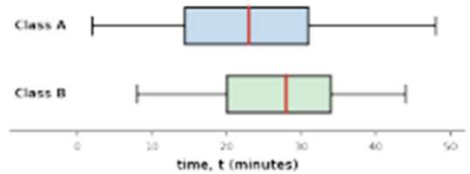
Read the median, LQ and UQ from the curve, take the lowest and highest values from the question or table, then draw the box plot on the **same scale**.

`min 2, LQ 14.4, med 23, UQ 31, max 48`

COMPARING DISTRIBUTIONS

Always compare **two** things, **in context**:

1 — an average: the medians.
2 — the spread: the IQRs (or ranges).




```
"B's median (28) > A's (23): B slower on
average.
A's IQR (16.6) > B's (14): A more varied."
```

COMMON MISTAKES — EXAMINERS' REPORTS

- ✗ Reading **n/2** as the answer — it is the **position**; the estimate is read off the x-axis.
- ✗ Giving UQ or LQ alone when the **IQR** (UQ − LQ) is asked for.
- ✗ Comparing box plots with numbers only — you must interpret **in context** (average *and* spread).
- ✗ Whiskers drawn to the LQ/UQ instead of the **lowest/highest values**.
- ✗ Forgetting to start the graph at **(lowest boundary, 0)** — here (0, 0).

Frequently examined on Edexcel 1MA1 GCSE Higher Paper 2 and Paper 3.

Worked Example

Ethan collects the lengths of 100 animals and records the data in the table below.

Length (y cm)	Frequency
$40 < y \leq 45$	7
$45 < y \leq 50$	10
$50 < y \leq 55$	63
$55 < y \leq 60$	11
$60 < y \leq 65$	9

Complete the cumulative frequency table.

Length (y cm)	Cumulative Frequency
$40 < y \leq 45$	
$40 < y \leq 50$	
$40 < y \leq 55$	
$40 < y \leq 60$	
$40 < y \leq 65$	

Your Turn

Meriem collects the running times of 80 athletes and records the data in the table below.

Time (y seconds)	Frequency
$30 < y \leq 40$	7
$40 < y \leq 50$	8
$50 < y \leq 60$	26
$60 < y \leq 70$	13
$70 < y \leq 80$	9
$80 < y \leq 90$	17

Complete the cumulative frequency table.

Time (y seconds)	Cumulative Frequency
$30 < y \leq 40$	
$30 < y \leq 50$	
$30 < y \leq 60$	
$30 < y \leq 70$	
$30 < y \leq 80$	
$30 < y \leq 90$	

Worked Example

Kaitlyn collects the running times of 50 athletes and records the data in the table below.

Time (t seconds)	Cumulative Frequency
$20 < t \leq 25$	3
$25 < t \leq 30$	10
$30 < t \leq 35$	26
$35 < t \leq 40$	34
$40 < t \leq 45$	41
$45 < t \leq 50$	50

Find the number of athletes whose times were between 25 and 40 seconds.

Your Turn

Kai collects the lengths of 100 animals and records the data in the table below.

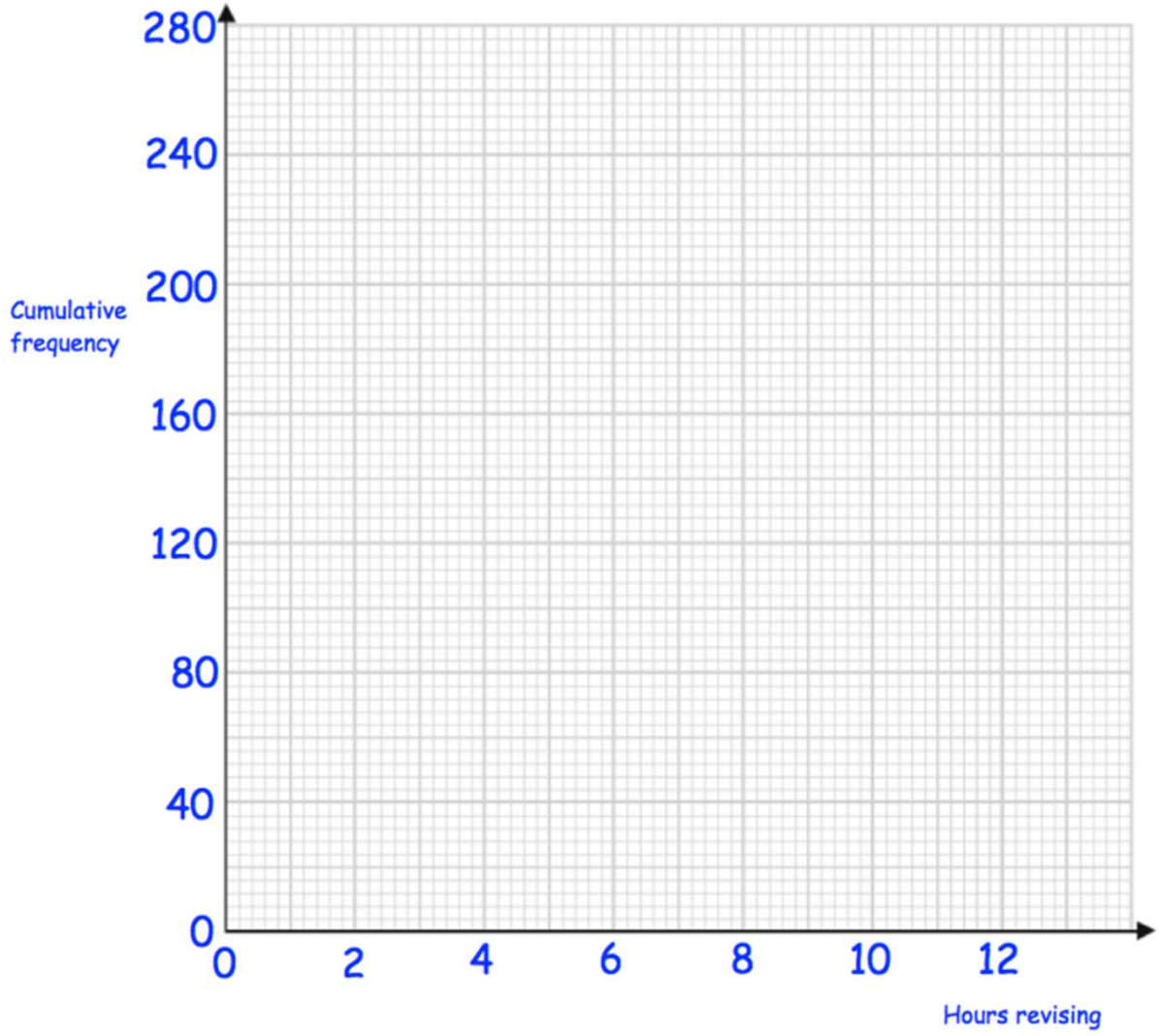
Length (x cm)	Cumulative Frequency
$20 < x \leq 30$	6
$30 < x \leq 40$	17
$40 < x \leq 50$	80
$50 < x \leq 60$	93
$60 < x \leq 70$	100

Determine how many animals were between 40 and 60 cm long.

Worked Example

Plot a cumulative frequency graph:

Number of hours (h)	Frequency
$0<h\leq 2$	20
$2<h\leq 4$	32
$4<h\leq 6$	48
$6<h\leq 8$	120
$8<h\leq 10$	24
$10<h\leq 12$	16

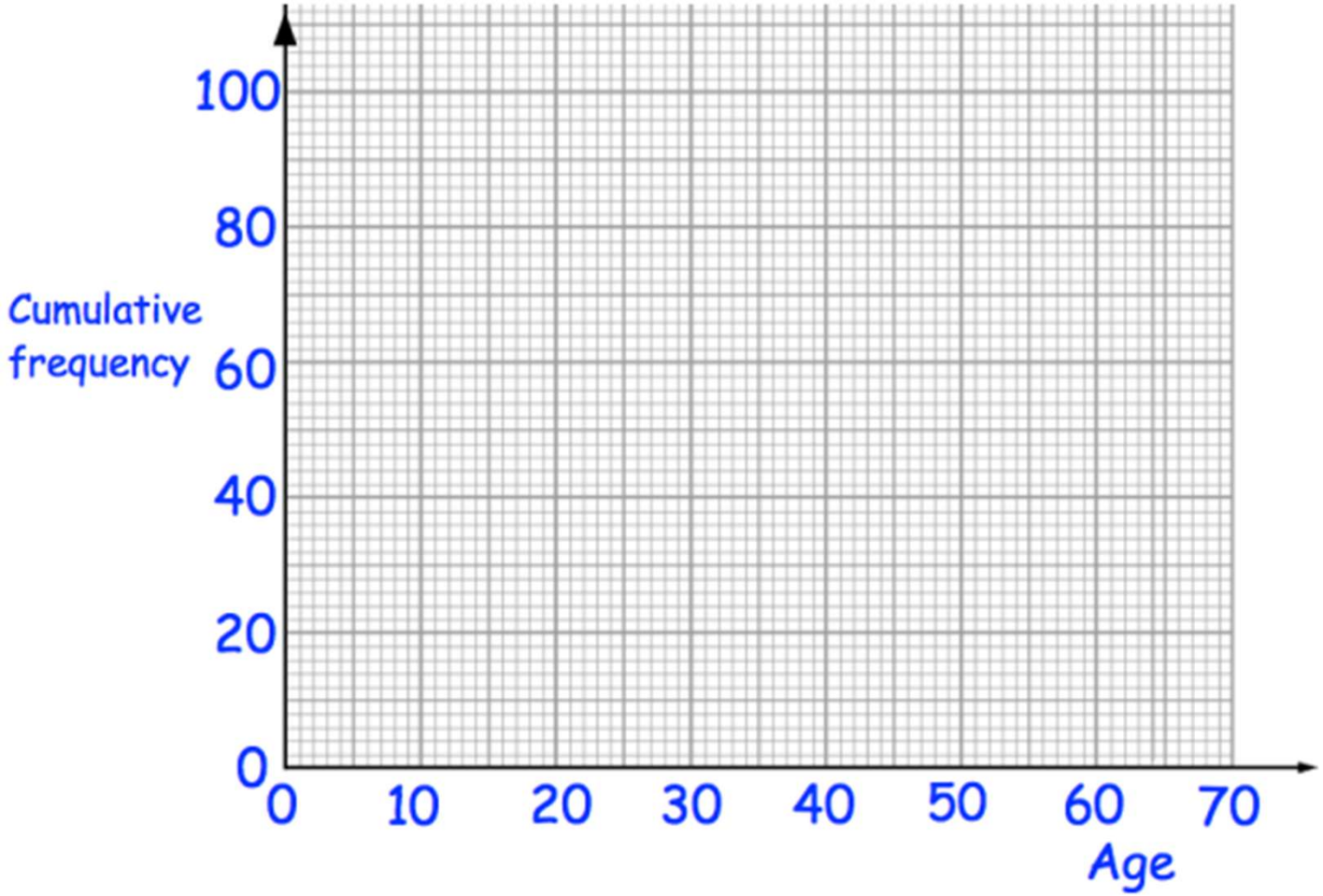


Dr Frost 398d and 398c

Your Turn

Plot a cumulative frequency graph:

Age, x years	Frequency
$20 < x \leq 30$	12
$30 < x \leq 40$	30
$40 < x \leq 50$	28
$50 < x \leq 60$	22
$60 < x \leq 70$	8

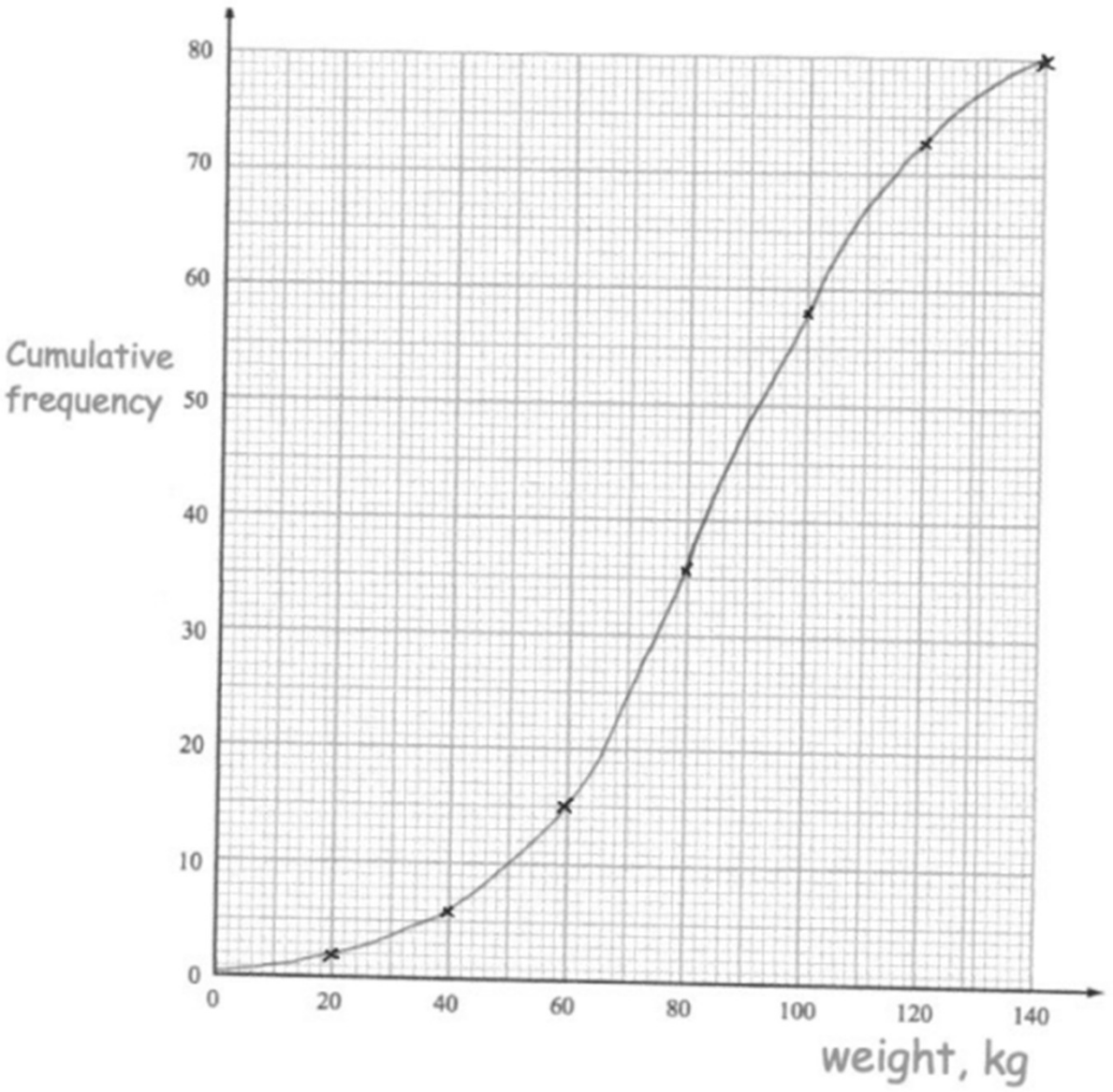


Worked Example

Using the cumulative frequency graph, estimate the:

a) Number of people who weight less than 40 kg

b) Number of people who weigh more than 100 kg



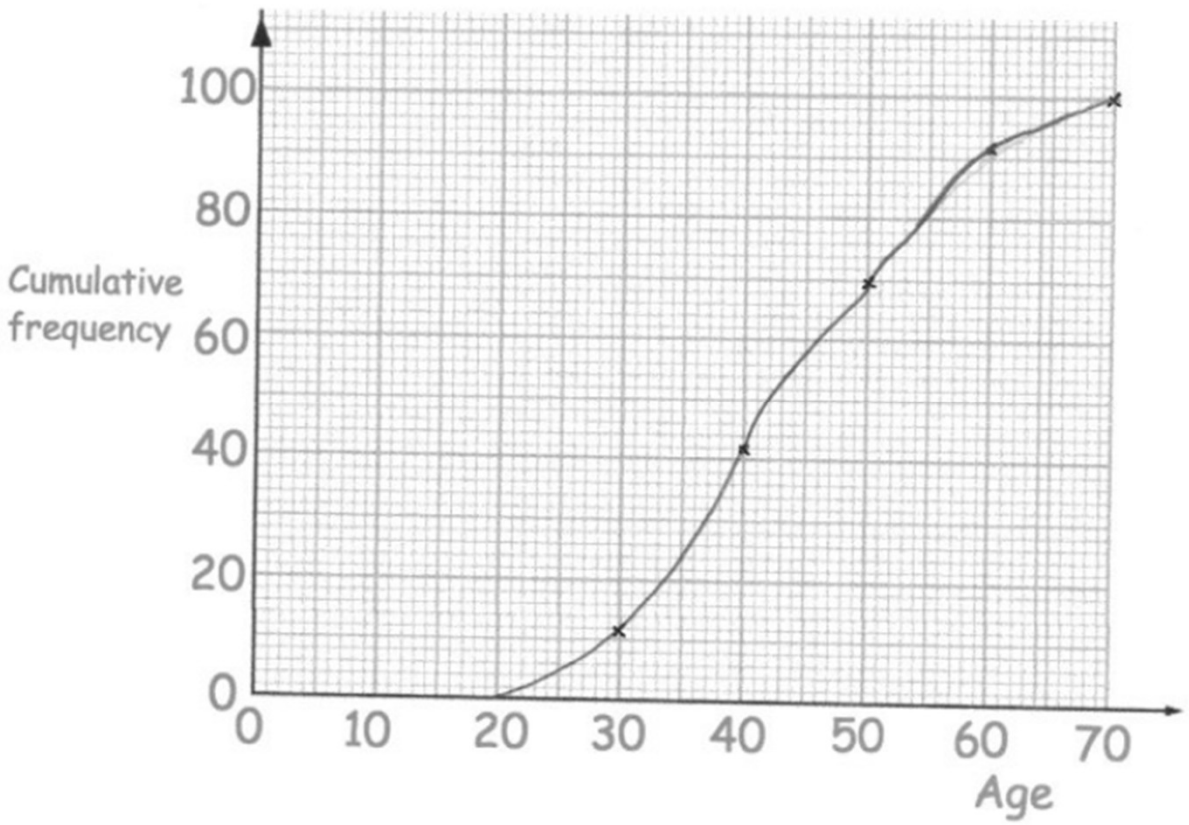
Dr Frost 398g

Your Turn

Using the cumulative frequency graph, estimate the:

a) Number of people younger than 35

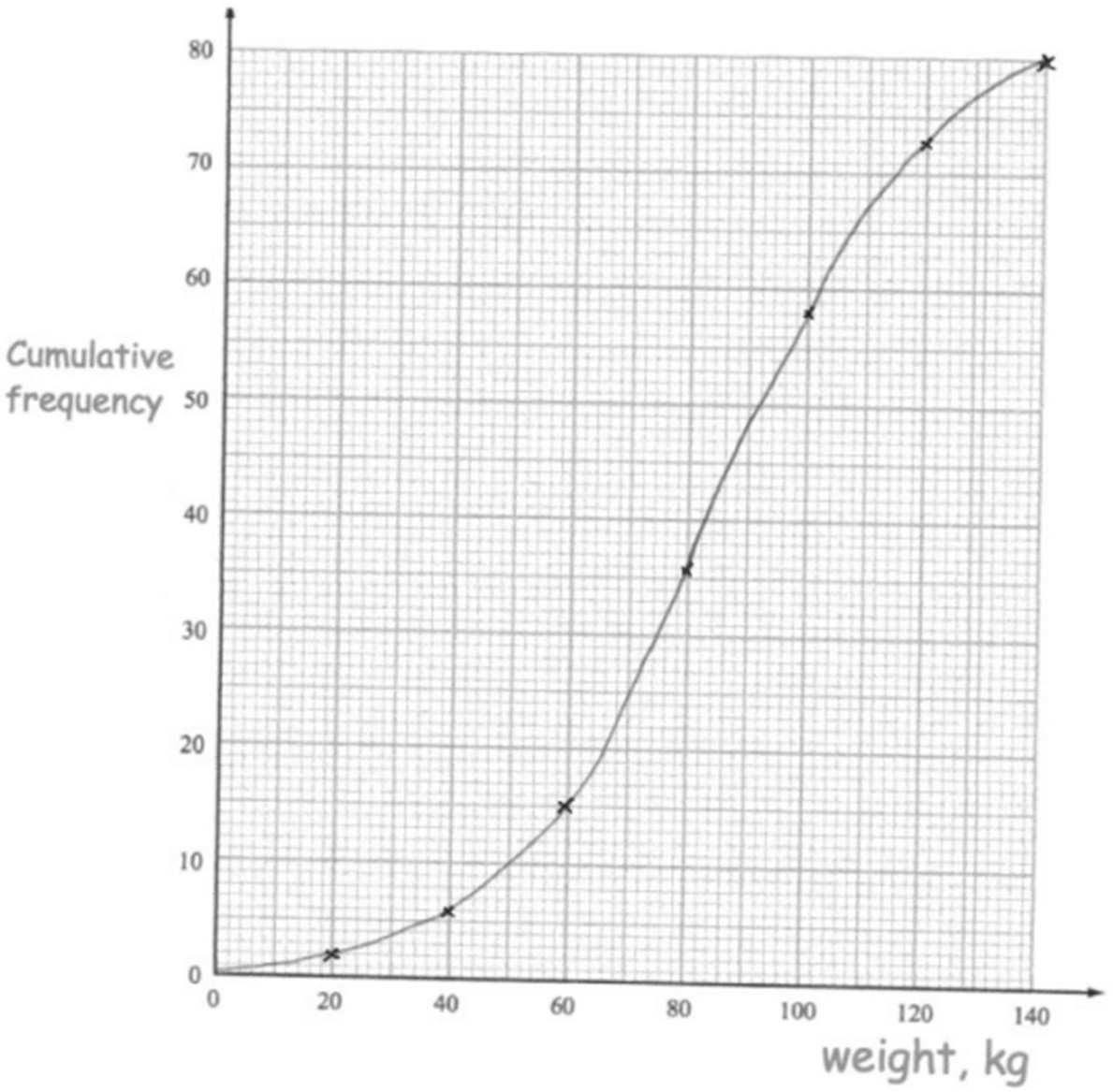
b) Number of people older than 62



Worked Example

Using the cumulative frequency graph, estimate the:

a) Median weight
b) Lower quartile weight
c) Upper quartile weight
d) Interquartile range of the weights

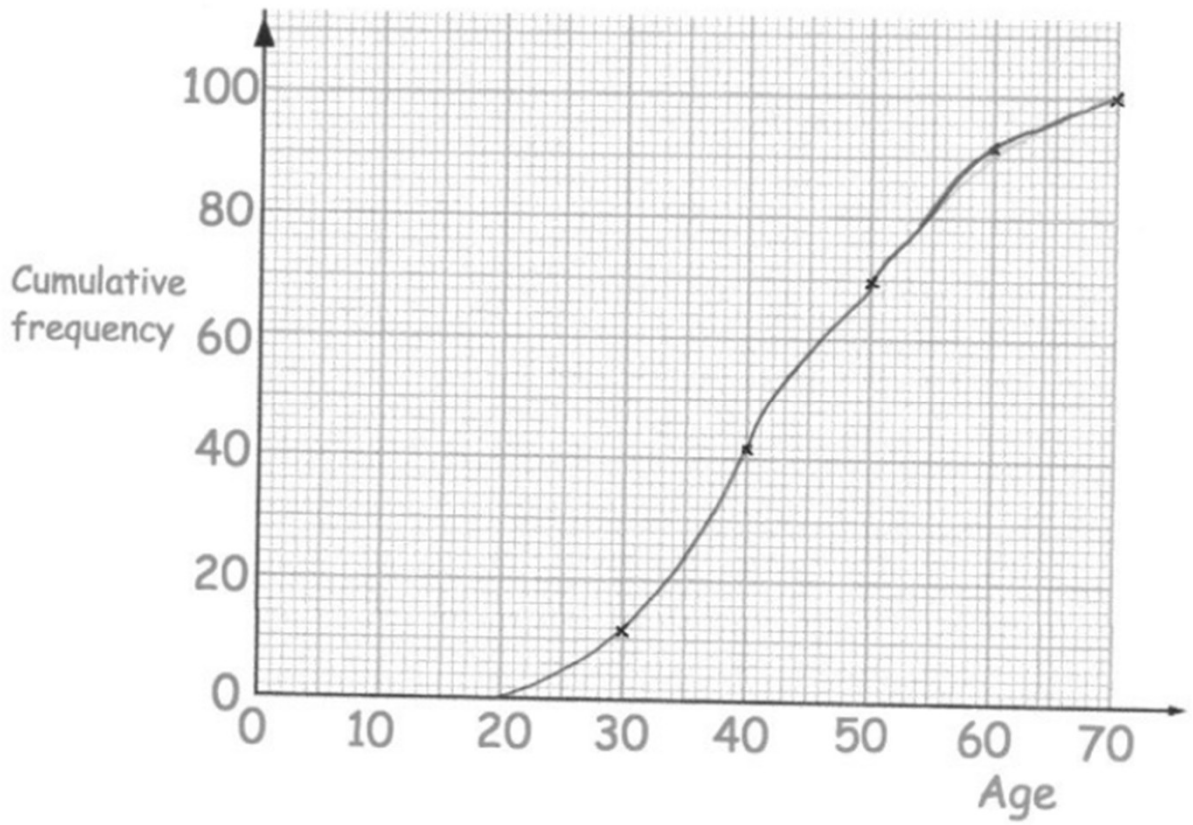


Dr Frost 398e and 398f

Your Turn

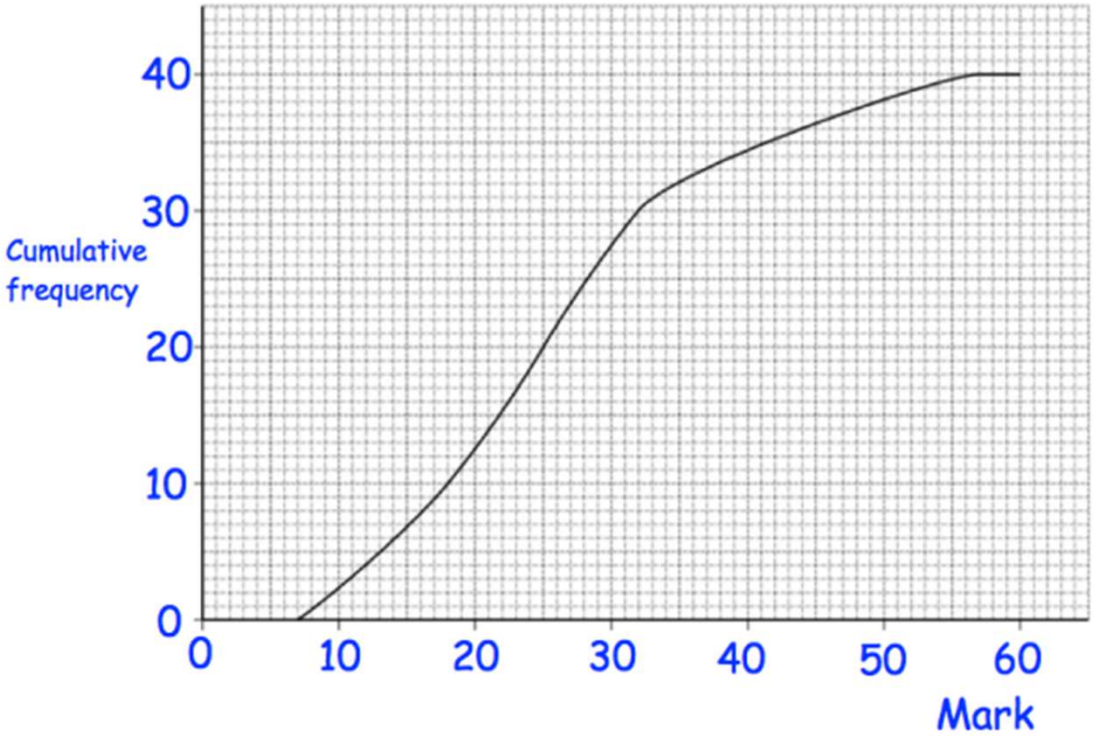
Using the cumulative frequency graph, estimate the:

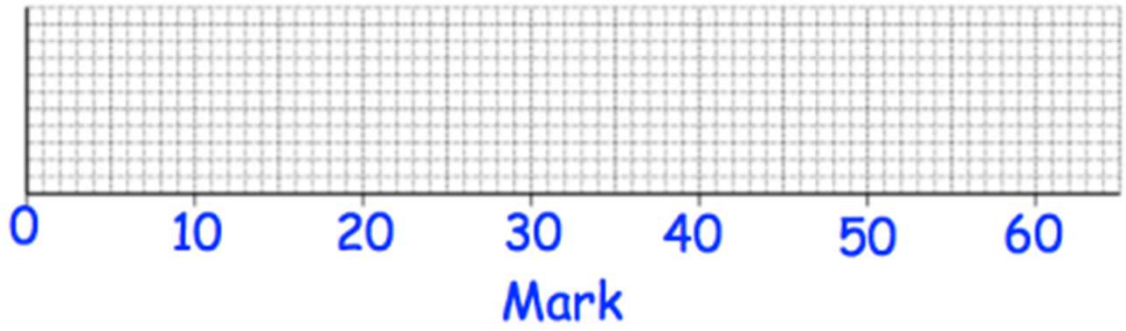
a) Median age
b) Lower quartile age
c) Upper quartile age
d) Interquartile range of the ages



Worked Example

Using the cumulative frequency graph, draw a box plot:

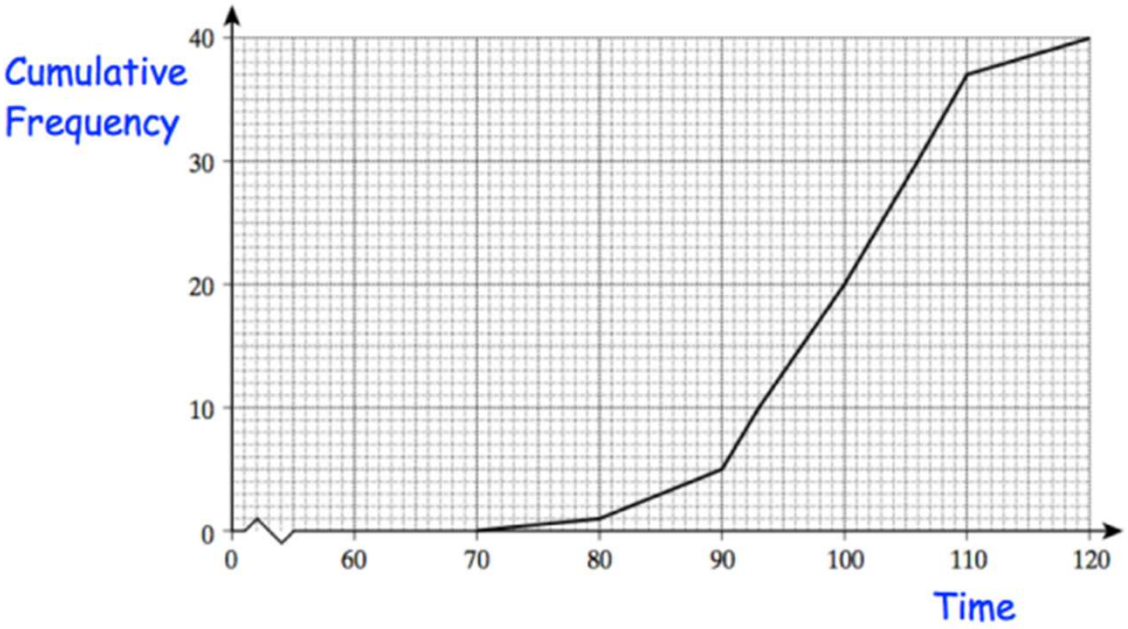


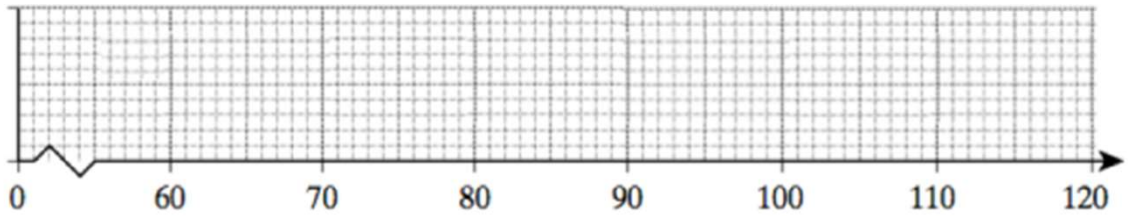


Dr Frost 399d

Your Turn

Using the cumulative frequency graph, draw a box plot:





Question 1: The table shows information about the lengths of a type of fish caught in a lake

(a) Complete the cumulative frequency table
(b) Draw a cumulative frequency graph for your table.

$0 < c \leq 5$
$0 < c \leq 10$
$0 < c \leq 15$
$0 < c \leq 20$
$0 < c \leq 25$

Question 2: The table shows information about the time taken to complete a puzzle

(a) Complete the cumulative frequency table
(b) Draw a cumulative frequency graph for your table.

$0 < t \leq 10$
$0 < t \leq 20$
$0 < t \leq 30$
$0 < t \leq 40$
$0 < t \leq 50$
$0 < t \leq 60$

Question 3: The table shows information about the speed of vehicles on a road.

(a) Complete the cumulative frequency table
(b) Draw a cumulative frequency graph for your table.

Speed, s mph	Frequency
$0 < s \leq 10$	2
$10 < s \leq 20$	4
$20 < s \leq 30$	14
$30 < s \leq 40$	21
$40 < s \leq 50$	9

Speed, s mph	Cumulative Frequency
$0 < s \leq 10$	
$0 < s \leq 20$	
$0 < s \leq 30$	
$0 < s \leq 40$	
$0 < s \leq 50$	

Question 4: Use each cumulative frequency graph to find an estimate for the median.

(a)

(b)

(c)

(d)

Question 5: For each table below (i) draw a cumulative frequency graph and
(ii) use your graph to find an estimate of the median

(a)

(b)

Question 6: Use each cumulative frequency graph to find the (i) lower quartile
(ii) upper quartile
(iii) interquartile range

(a) (b)

(c)

(d)

Question 7: The graph shows information about the time taken to solve a puzzle.

(a) How many people took less than 30 seconds?

(b) How many people took less than 10 seconds?

(c) How many people took longer than 25 seconds?

(d) How many people took longer than 35 seconds?

(e) The fastest 10 people completed the puzzle in under how many seconds?

(f) The slowest 2 people completed the puzzle in longer than how many seconds?

Question 8: The graph shows information about the speed of cars on a road.

(a) How many cars travelled under 50km/h?

(b) How many cars travelled over 110km/h?

(c) 42 cars were exceeding the speed limit. What is the speed limit?

(d) Mr Rodgers says 18% of the cars were travelling too slowly on this road. Below what speed does he feel is too slow?

Apply

Question 1: Some students complete a quiz.
The cumulative frequency graph shows their results

(a) How many students completed the quiz?

(b) Complete the frequency table below.

(c) What percentage of the students scored above 20 marks?

Question 2: The cumulative frequency graph below shows the salaries of 80 teachers.
The lowest salary is £4,000 and the highest salary is £39,000.

A teacher is picked at random to answer a survey.

(a) Find the probability that the teacher selected is paid less that £15,000.

(b) Find the probability that the teacher selected is paid over £25,000.

(c) Draw a box plot to represent the salaries.

Question 3: The table shows information about the members of Abbeyville Cricket Club

The youngest member is 9 and the oldest member is 58.

(a) Draw a cumulative frequency graph to represent this information.

(b) Draw a box plot to represent this information

(c) Work out the interquartile of the ages of the members of Abbeyville Cricket Club.

The box plot below shows information about Barry Town Cricket Club

(d) Write down the median age of the members of Barry Town Cricket Club

(e) Compare the distributions of the ages of the members of Abbeyville Cricket Club to the ages of the members of Barry Town Cricket Club.

Workout Question 1

Workout Question 2

Workout Question 3:

Workout Question 5
(a)

Workout
Question 5(b)

Apply Question 2(c)

Templates

Apply Question 3(a)

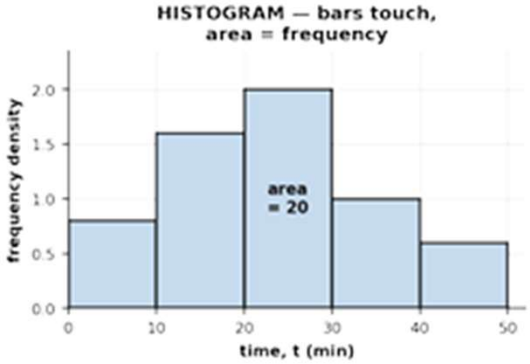
Apply Question 3(b)

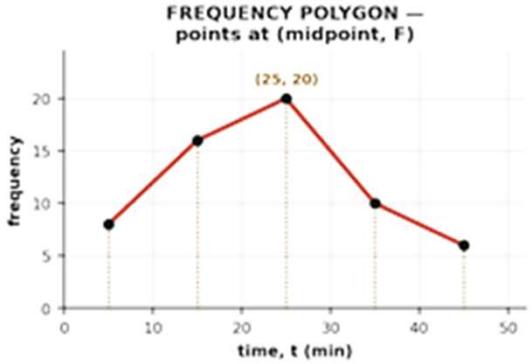
HISTOGRAM vs FREQUENCY POLYGON vs CF GRAPH

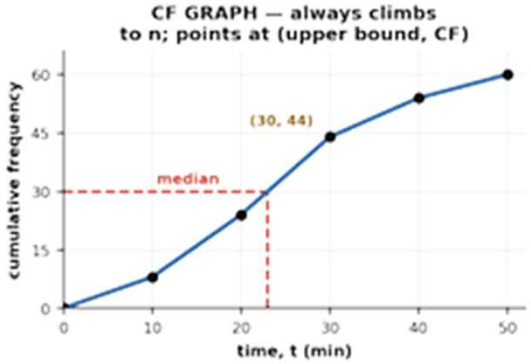
The same data, three ways — and how to tell the diagrams apart

Knowledge Organiser • GCSE Edexcel Higher Mathematics

HISTOGRAM	FREQUENCY POLYGON	CF GRAPH
`bar height = frequency density`	`( midpoint , frequency )`	`( upper bound , cumulative F )`







All three diagrams above show exactly the same data: 60 shoppers' times, classes of width 10, frequencies 8, 16, 20, 10, 6.

	HISTOGRAM	FREQUENCY POLYGON	CF GRAPH
y-axis says	**frequency density**	**frequency**	**cumulative frequency**
Where frequency lives	**AREA** of bar = FD × width	height of each point	**difference** between two CF readings
Drawn as	touching bars, no gaps	points + straight ruler segments; NOT closed to the x-axis	rising points joined by segments (or smooth curve), starting at (lowest bound, 0)
Shape to expect	blocks of different heights	zig-zag line with a peak	always climbs, finishing at n
Best for	**unequal class widths**	seeing & comparing the **shape** of distributions	estimating **median, quartiles, IQR**

WHICH DIAGRAM DOES THE QUESTION WANT?

- → Table with **unequal class widths**, or y-axis labelled frequency density → **histogram**.
- → "Estimate the **median / IQR**" or "how many more than…" → **CF graph** (read at $\frac{n}{2}$, $\frac{n}{4}$, $\frac{3n}{4}$).
- → "**Compare** the distributions" on one grid, equal widths → **frequency polygon**.
- → A graph that **never decreases** → the CF graph.

MIX-UPS EXAMINERS SEE EVERY YEAR

- ✗ Reading a histogram bar's **height** as its frequency — multiply by the width: frequency = area.
- ✗ Plotting a polygon at **boundaries**, or a CF graph at **midpoints** — the plotting rules are opposite ways round.
- ✗ Putting plain **frequency** on a histogram's y-axis when widths are unequal.
- ✗ A "CF graph" that goes **down** — a running total can never decrease.
- ✗ Closing a polygon down to the x-axis, or forgetting a CF graph starts at **(lowest bound, 0)**.

Frequently examined on Edexcel 1MA1 GCSE Higher Paper 2 and Paper 3.

2 Advanced Probability

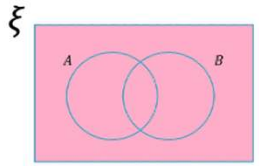
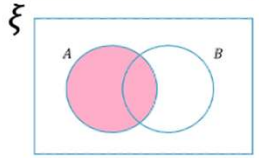
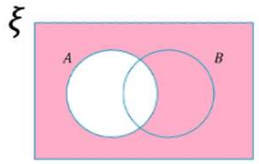
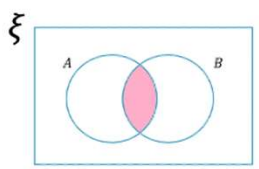
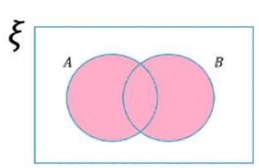
- Use of Venn Diagrams
- Without replacement problems (*total/denominator reduces)*
- Understanding the following associations:

NOT – think $1 - ANS$

OR – think $add\ probabilities$

AND – think $multiply\ probabilities$

Venn Diagrams

ξ	The universal set. All elements being considered.	
$A = \{\dots\}$	Set A.	
A'	The complement of A. Elements not in A.	
$A \cap B$	The intersection of A and B. Elements in both A AND B.	
$A \cup B$	The union of A and B. Elements in either A OR B.	

Fluency Practice

Complement: ‘

The <u>opposite</u> of a set.

B’ = everywhere not in B

Intersection: ∩

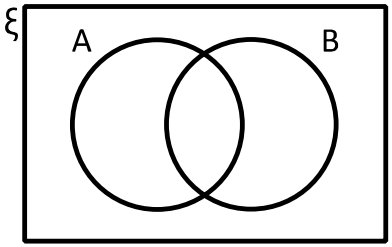
The <u>overlap</u> of regions.

A ∩ **B** = everywhere A and B overlap

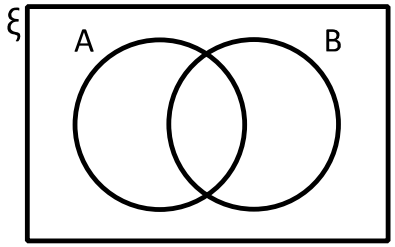
Union: ∪

The <u>sum</u> of regions.

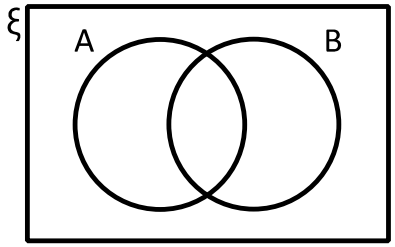
A ∪ **B** = A added to B



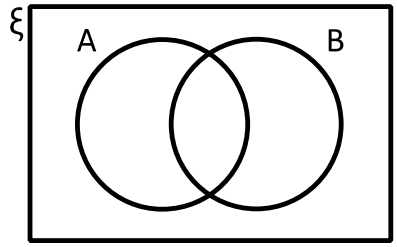
A



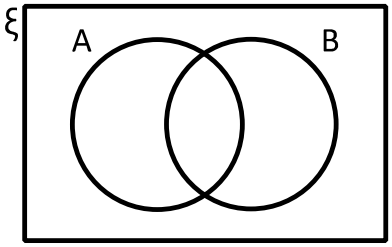
B



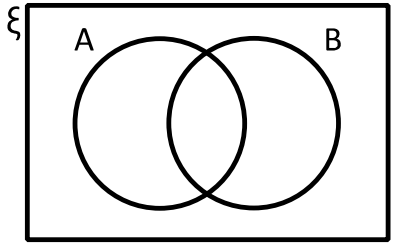
A’



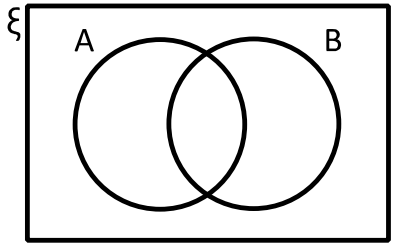
B’



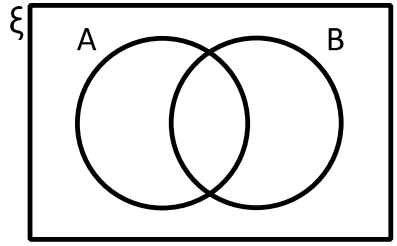
A ∪ B



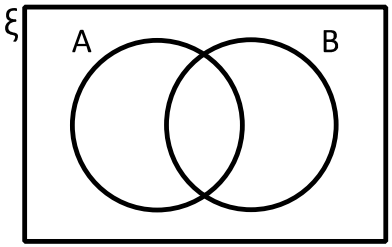
A ∪ B’



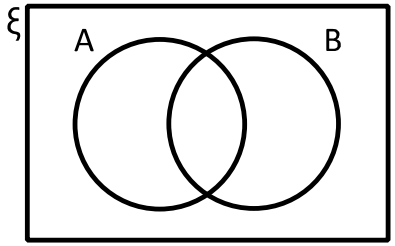
A’ ∪ B



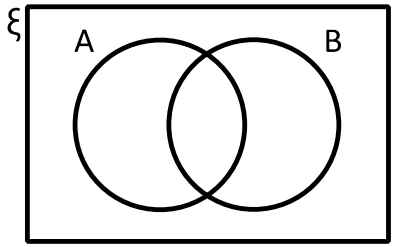
A’ ∪ B’



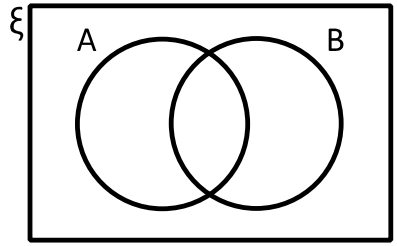
A ∩ B



A ∩ B’



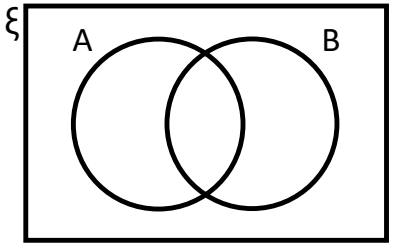
A’ ∩ B



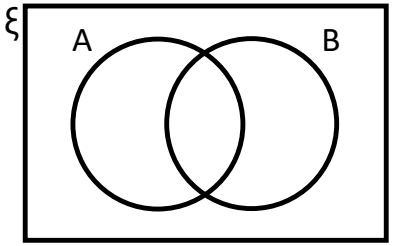
A’ ∩ B’

Fluency Practice

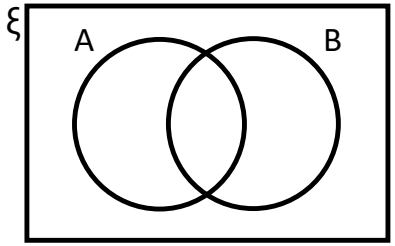
Shade the Venn Diagrams according to the notation. Remember! ∪ = sum, ∩ = overlap



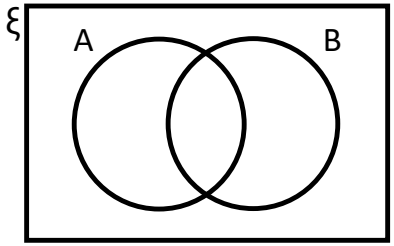
$A \cup B$



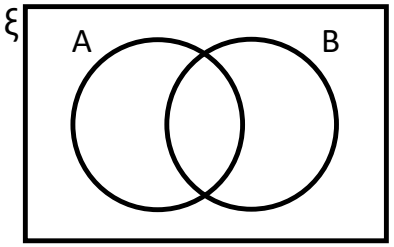
$A \cap B$



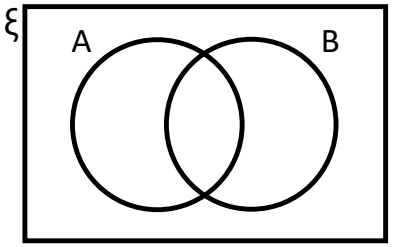
$A' \cup B$



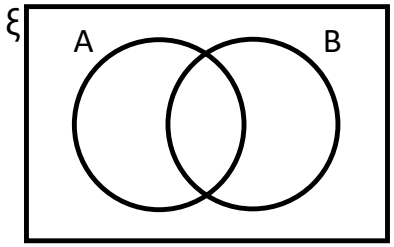
$A \cap B'$



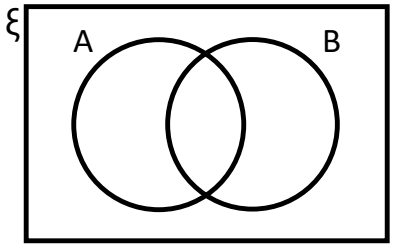
$A \cup B'$



$A' \cap B$



$A' \cup B'$



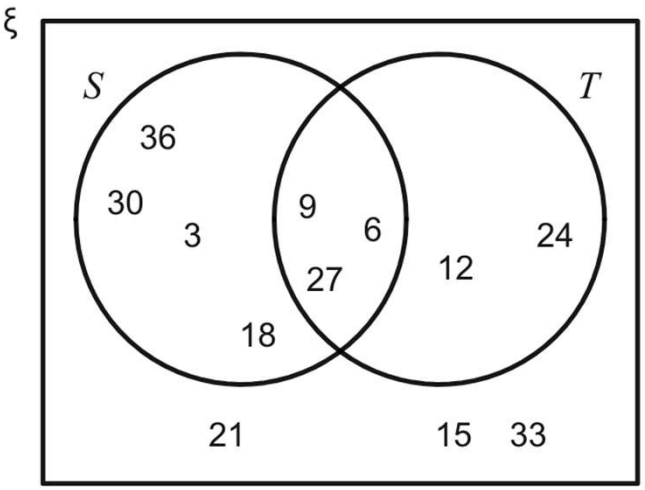
$A' \cap B'$

Dr Frost 243a and 243c

Worked Example

The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 3 between 1 and 37}\}$
$S = \{3, 6, 9, 18, 27, 30, 36\}$
$T = \{6, 9, 12, 24, 27\}$



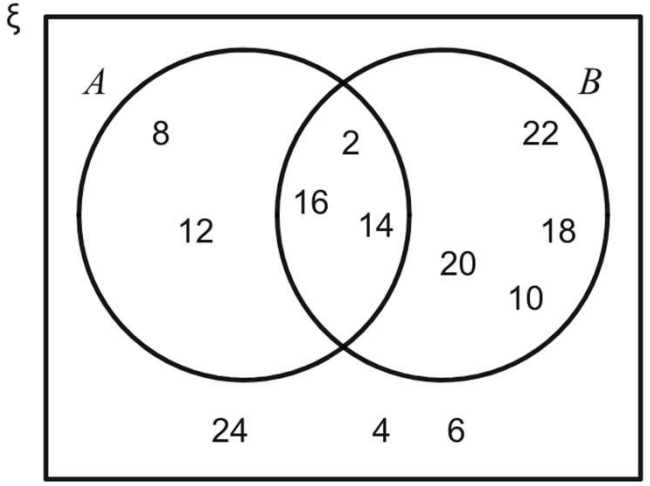
Find:

a) $P(S \cap T)$
b) $P(S')$
c) $P(S \cup T)$
d) $P(T)$

Your Turn

The Venn diagram represents the information from the following sets.

$\xi = \{\text{Multiples of 2 between 1 and 25}\}$
$S = \{2, 8, 12, 14, 16\}$
$T = \{2, 10, 14, 16, 18, 20, 22\}$



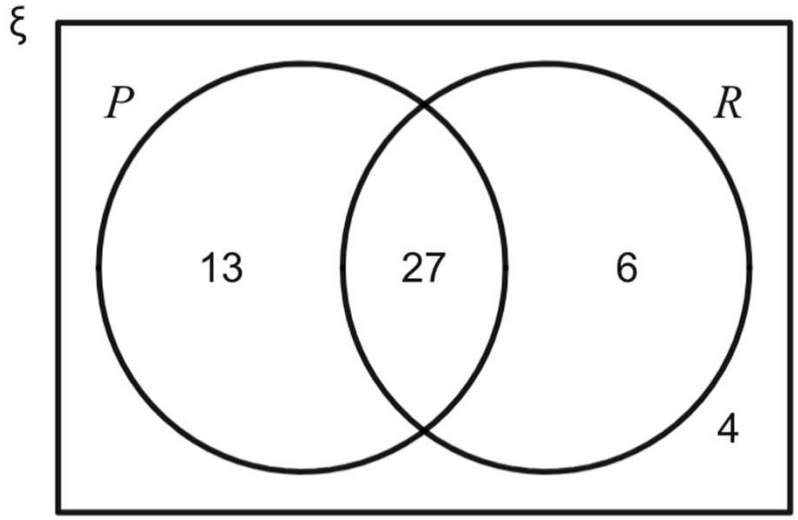
Find:

a) $P(A \cup B)$
b) $P(A)$
c) $P(A \cap B)$
d) $P(B')$

Worked Example

There are 50 pupils in a group.

The Venn diagram shows which languages they study from Portuguese and Russian.



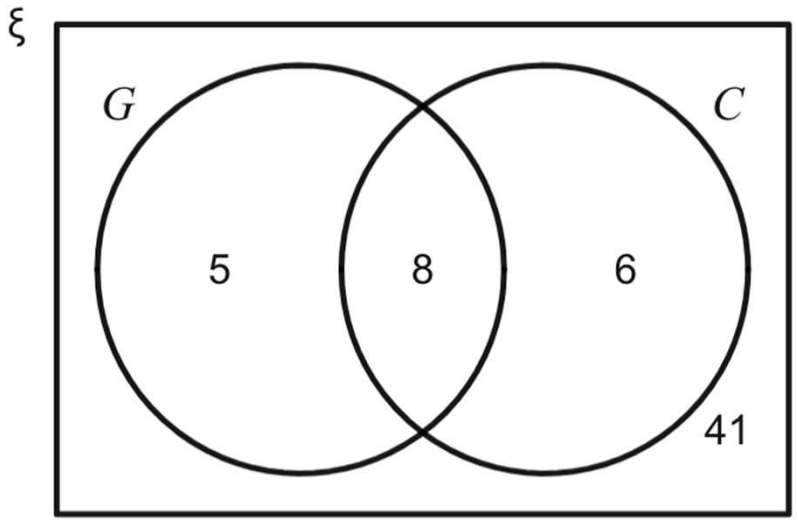
A pupil is selected at random.

a) Find the probability that this pupil studies Russian but not Portuguese.
b) Find the probability that this pupil studies Russian

Your Turn

There are 60 pupils in a group.

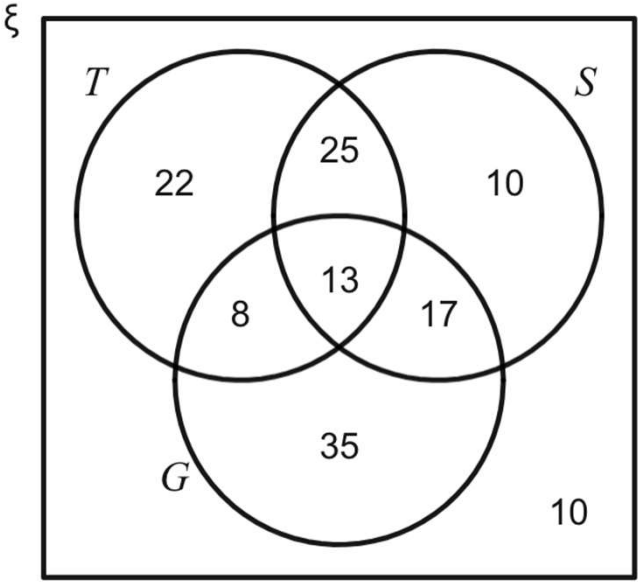
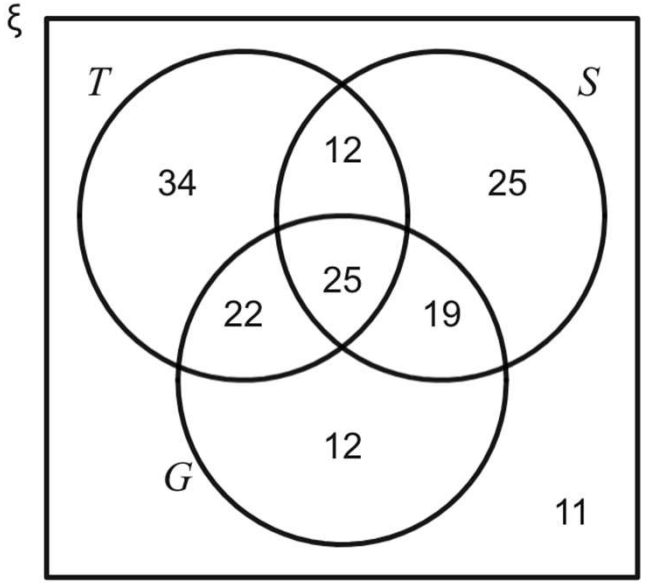
The Venn diagram shows which languages they study from German and Chinese.



A pupil is selected at random.

a) Find the probability that this pupil studies German but not Chinese.
b) Find the probability that this pupil studies Chinese.

Dr Frost 355a, 355b and 355d

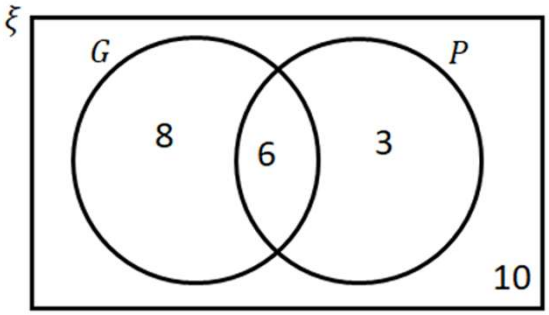
Worked Example	Your Turn
140 pupils in a sports centre are surveyed. The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T) The Venn diagram shows the information about what facilities they used.  A pupil is selected at random. Find the probability that this pupil uses both the swimming pool and the gym, but not the tennis courts.	160 pupils in a sports centre are surveyed. The pupils can only use the swimming pool (S), the gym (G) and the tennis courts (T) The Venn diagram shows the information about what facilities they used.  A pupil is selected at random. Find the probability that this pupil uses both the tennis courts and the swimming pool, but not the gym.

Dr Frost 355c and 355e

Conditional Probability

Worked Example

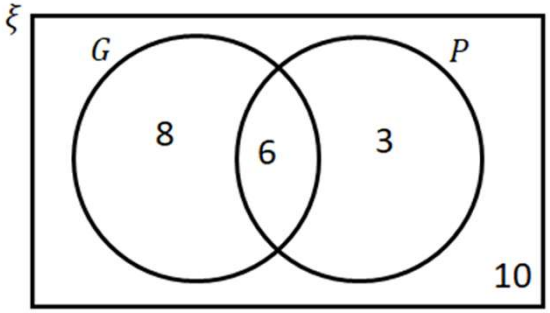
The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano.

Your Turn

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).

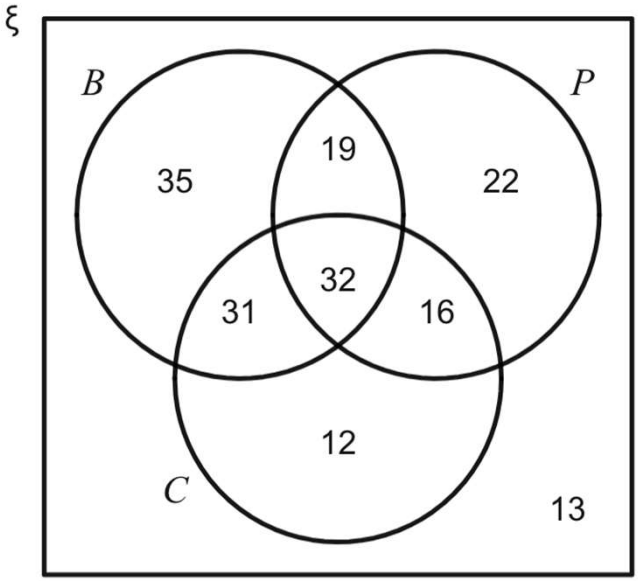


Find the probability that a student plays the piano, given that they play the guitar.

Worked Example

There are 180 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



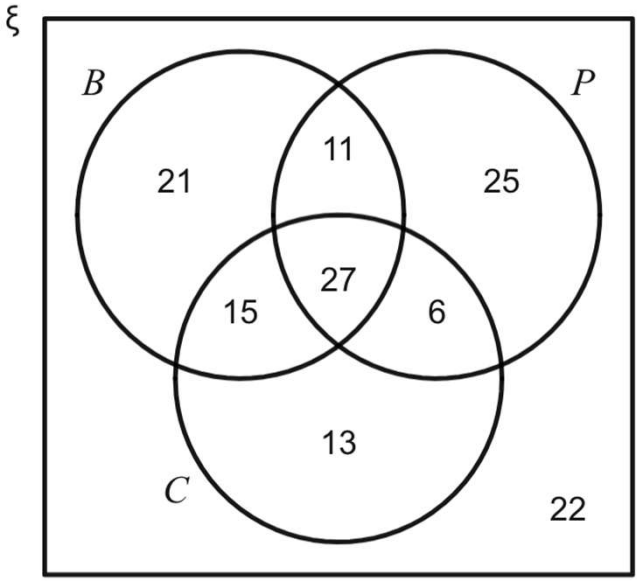
One of the pupils who passed Biology is selected at random.

Find the probability that this pupil also passed Chemistry but not Physics.

Your Turn

There are 140 pupils in a group who recently took three science tests.

The Venn diagram shows which tests they passed out of Physics, Chemistry and Biology.



One of the pupils who passed Chemistry is selected at random.

Find the probability that this pupil also passed Physics and Biology.

Worked Example

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Draw a Venn Diagram, and hence answer the following questions:

a) $\mathrm{P}(\text{owns dog only})$

b) $\mathrm{P}(\text{does not own tropical fish})$

c) $\mathrm{P}(\text{does not own dogs, cats, or tropical fish})$

d) Given that a randomly chosen person owns a cat, what's the probability they own a dog?

Your Turn

The following shows the results of a survey on the types of exercise taken by a group of 100 people.
65 run, 48 swim, 60 cycle, 40 run and swim, 30 swim and cycle, 35 run and cycle and 25 do all three.

a) Draw a Venn Diagram to represent these data.

Find the probability that a randomly selected person from the survey

b) takes none of these types of exercise,

c) swims but does not run,

d) takes at least two of these types of exercise.

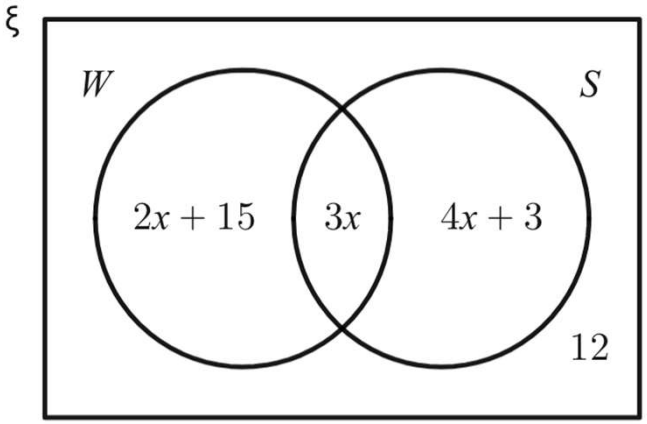
Jason is one of the above group. Given that Jason runs,

e) find the probability that he swims but does not cycle.

Worked Example

Yasmine has a collection of medals.

The Venn diagram gives information about the number of windsurfing (W) medals and silver (S) medals in her collection.



Yasmine takes a windsurfing medal from her collection at random.

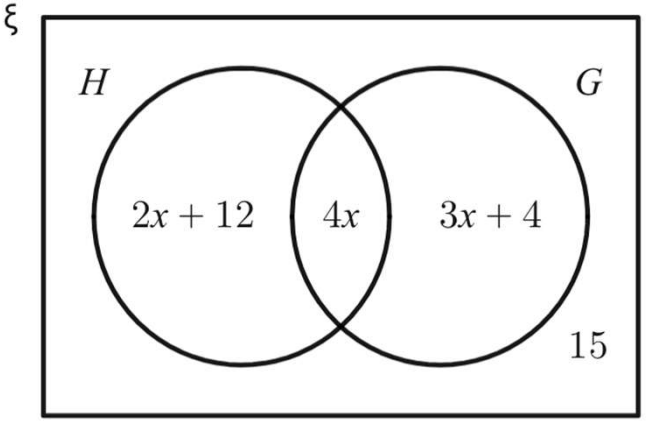
The probability that the medal is not a silver medal is $\frac{5}{8}$

Work out the total number of medals in Yasmine's collection.

Your Turn

Arthur has a collection of medals.

The Venn diagram gives information about the number of hockey (H) medals and gold (G) medals in her collection.



Arthur takes a hockey medal from his collection at random.

The probability that the medal is not a gold medal is $\frac{5}{9}$

Work out the total number of medals in Arthur's collection.

Frequency Trees

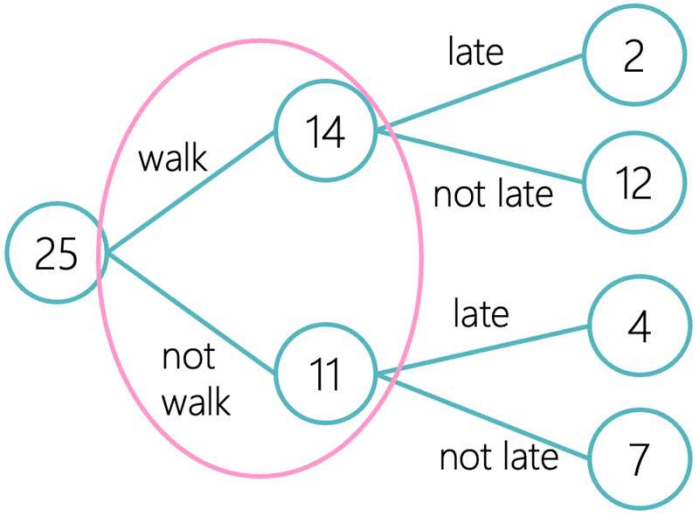
Frequency trees are a good way to organise information.
This frequency tree gives information about how pupils get to school and whether or not they were late.

25 pupils were asked.

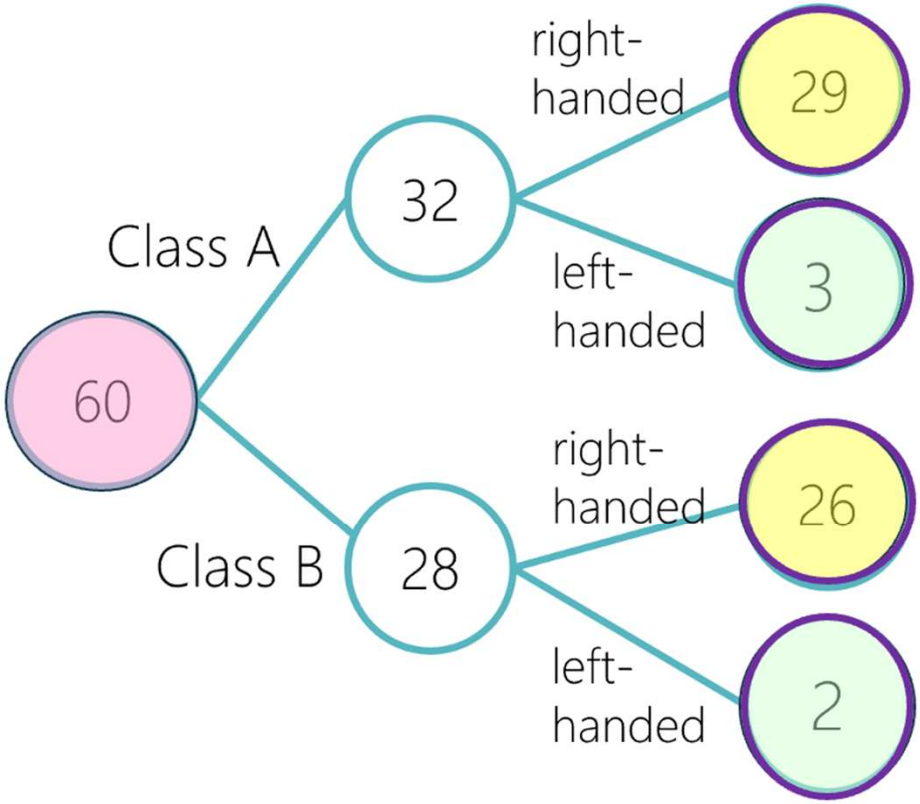
14 of those 25 pupils walked.
The other 11 did not walk.

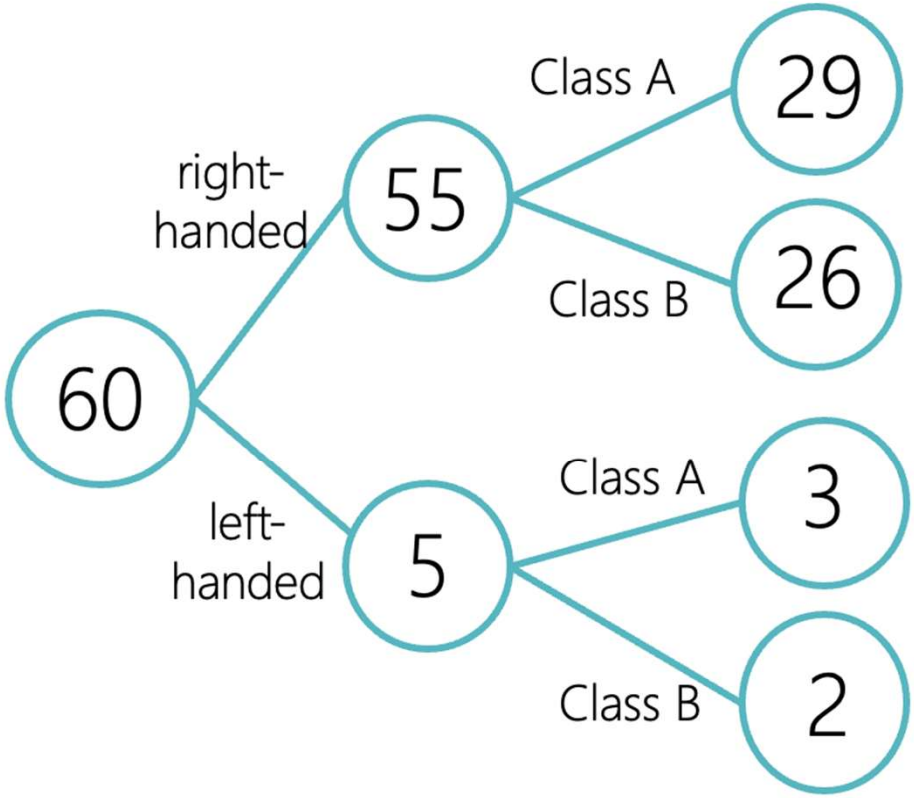
Walk and not walk
are mutually exclusive.

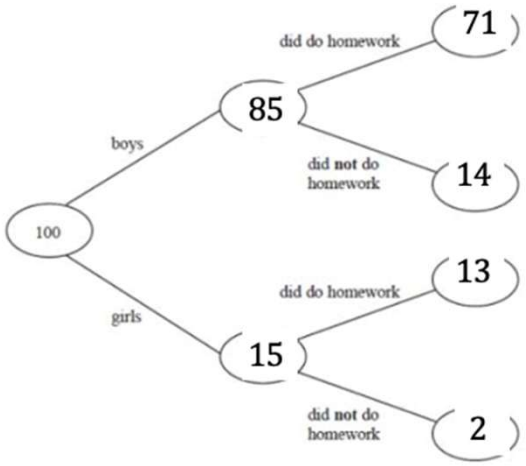
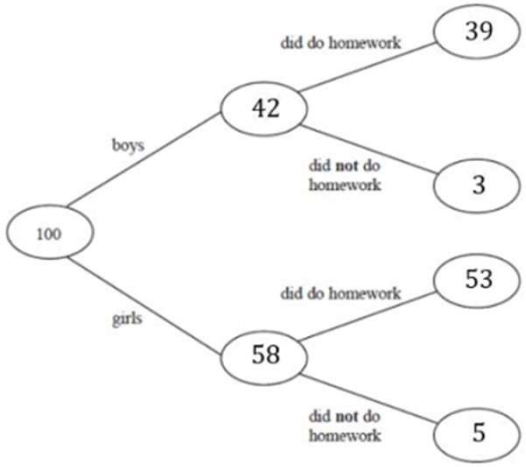
The frequencies sum to the total, 25.



We can show the same information in different ways.



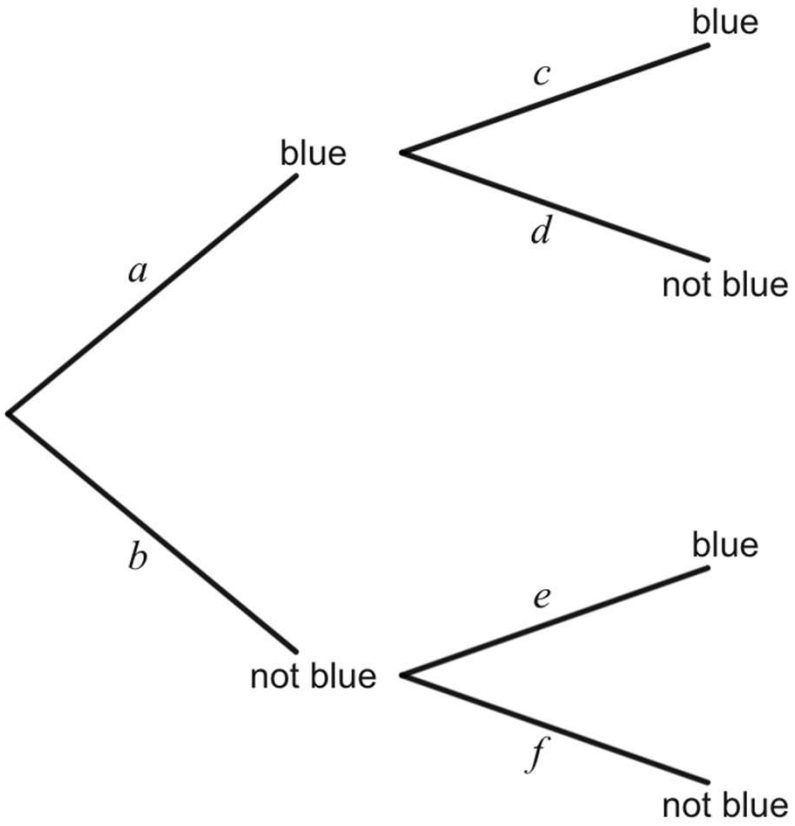


Worked Example	Your Turn
100 students had some homework. The frequency tree shows the number of students who did the homework.  A boy is chosen at random. Write down the probability that they did not do their homework.	100 students had some homework. The frequency tree shows the number of students who did the homework.  A girl is chosen at random. Write down the probability that they did do their homework.

Worked Example

Ruby has 2 blue balls, 2 purple balls and 1 black ball in a box.

Ruby takes one ball at random from the box, puts it back, and takes another ball from the box.

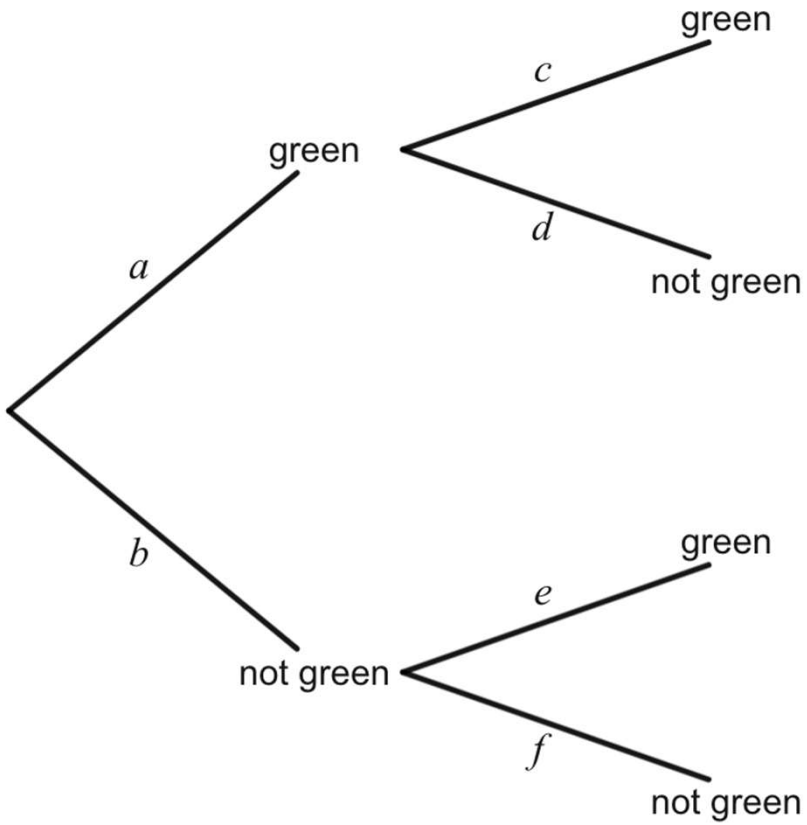


Complete the tree diagram.

Your Turn

Raphaël has 3 green socks, 1 blue sock and 1 yellow sock in a bag.

Raphaël takes one sock at random from the bag, puts it back, and takes another sock from the bag.



Complete the tree diagram.

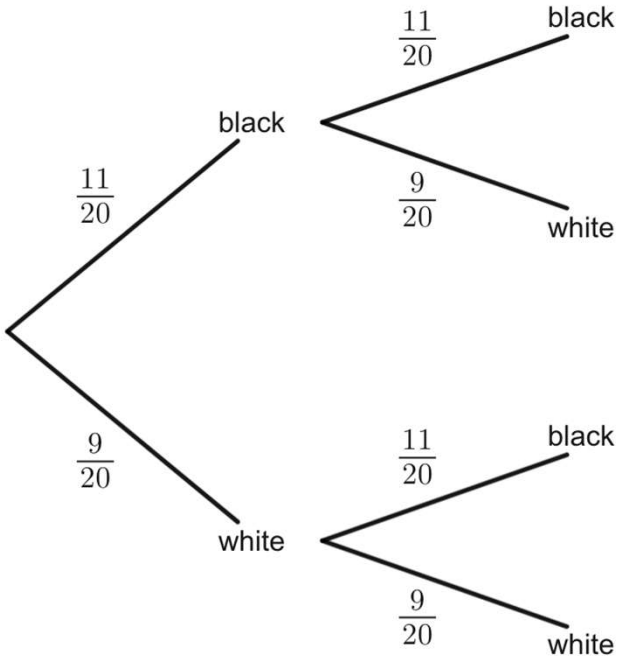
Dr Frost 353a

Worked Example

Zion has 11 black counters and 9 white counters in a box.

Zion takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



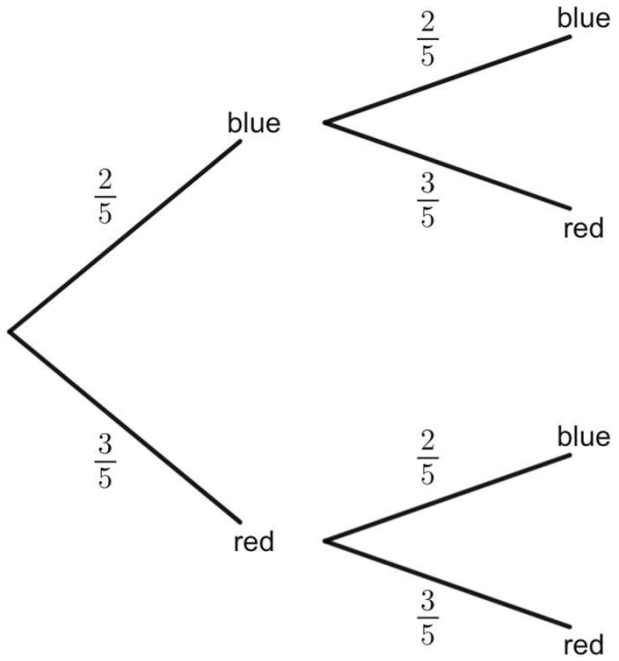
Find the probability that Zion takes two black counters.

Your Turn

Pierre has 2 blue balls and 3 red balls in a bag.

Pierre takes one ball at random from the bag, puts it back, and takes another ball from the bag.

A tree diagram is drawn to represent this information.



Find the probability that Pierre takes two balls that are not blue.

Dr Frost 353b

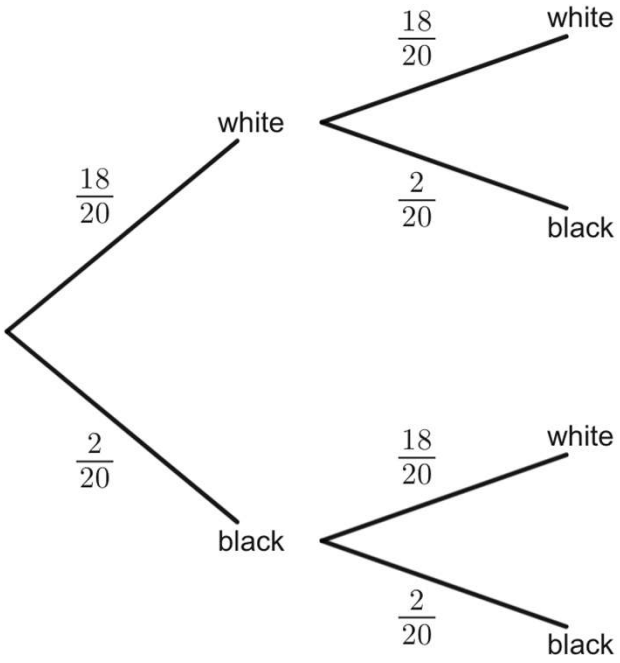
Worked Example	Your Turn
Diana has 3 red socks, 1 purple sock and 6 black socks in a bag. Diana takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Diana takes two socks that are not red.	Talula has 1 white counter, 4 yellow counters and 5 blue counters in a box. Talula takes one counter at random from the box, puts it back, and takes another counter from the box. Find the probability that Talula takes two counters that are not white.

Worked Example

Sophia has 18 white counters and 2 black counters in a box.

Sophia takes one counter at random from the box, puts it back, and takes another counter from the box.

A tree diagram is drawn to represent this information.



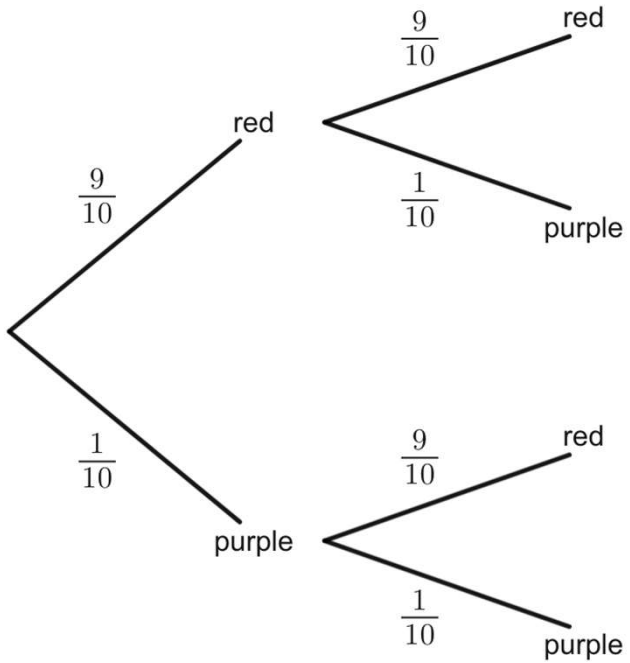
Find the probability that Sophia takes at least one white counter.

Your Turn

Alessia has 9 red balls and 1 purple ball in a box.

Alessia takes one ball at random from the box, puts it back, and takes another ball from the box.

A tree diagram is drawn to represent this information.



Find the probability that Alessia takes at least one red ball.

Worked Example	Your Turn
Holly has 18 red socks, 1 yellow sock and 1 white sock in a bag. Holly takes one sock at random from the bag, puts it back, and takes another sock from the bag. Find the probability that Holly takes at least one red sock.	Vamika has 4 purple socks, 1 red sock and 5 black socks in a box. Vamika takes one sock at random from the box, puts it back, and takes another sock from the box. Find the probability that Vamika takes at least one purple sock.

Dr Frost 353e

Worked Example

Astrid has a box of 30 nails.

The table shows the length of each nail.

Length of nail (cm)	3	7	8	9
Number of nails	9	10	6	5

She takes at random one of the nails and records its length.

She replaces the nail in the box.

She then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails she takes have a total length of 16 cm.

Your Turn

Xavier has a box of 20 nails.

The table shows the length of each nail.

Length of nail (cm)	3	4	5	7	8	9
Number of nails	1	1	4	7	3	4

He takes at random one of the nails and records its length.
He replaces the nail in the box.

He then takes at random a second nail from the box and records its length.

Calculate the probability that the two nails he takes have a total length of 14 cm.

Worked Example

Imani has a bag of 30 coloured counters.

The table shows how many of each colour she has.

green	black	blue	purple
9	6	7	8

She picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Imani does not pick a black counter until her second pick.

Your Turn

Étienne has a bag of 20 coloured counters.

The table shows how many of each colour he has.

blue	purple	green	red
8	4	3	5

He picks at random one of the counters and records its colour and then puts it back in the bag.

Calculate the probability that Étienne does not pick a purple counter until his third pick.

Dr Frost 353g

Worked Example	Your Turn
Alessia is going to play one game of Othello and one game of Tennis. The probability that she will win the game of Othello is 0.3 The probability that she will win both games is 0.15 Find the probability that she will win just one game.	Zara is going to play one game of Hearts and one game of Chess. The probability that she will win the game of Hearts is 0.7 The probability that she will win both games is 0.35 Find the probability that she will win just one game.

Worked Example	Your Turn
Kadiatou has a bag containing a large number of counters, some of which are red and the rest are green. She draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag. The probability of Kadiatou getting a red counter on any one draw is $\frac{4}{5}$ Find the probability that Kadiatou will draw exactly 3 green counters **before** a red counter is drawn.	Mohammed has a bag containing a large number of counters, some of which are purple and the rest are orange. He draws counters from the bag one at a time, notes the colour of the counter and returns it to the bag. The probability of Mohammed getting a purple counter on any one draw is $\frac{3}{10}$ Find the probability that Mohammed will draw exactly 4 orange counters **before** a purple counter is drawn.

Worked Example

Flynn has a box of 20 silk ribbons.

The table shows the length of each silk ribbon.

Length of silk ribbon (cm)	10	11	12	13	14	16
Number of silk ribbons	6	1	4	3	4	2

He takes at random two of the silk ribbons from the box, without replacement.

Calculate the probability that the two silk ribbons he takes have a total length of 24 cm.

Your Turn

Hugo has a deck of 40 cards each with different numbers on them.

The table shows the information about the cards.

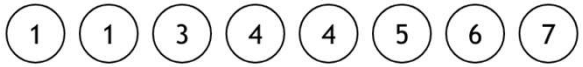
Number on card	2	3	4	5	6	7	8	9
Frequency	5	5	5	8	3	5	5	4

He takes at random two of the cards from the deck, without replacement.

Calculate the probability that the two cards he takes can be multiplied together to give a product of 16

Worked Example

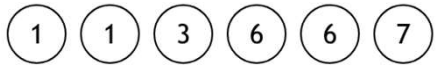
There are 8 counters in a bag.
There is a number on each counter.



Martha takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **odd** number.

Your Turn

There are 6 counters in a bag.
There is a number on each counter.



Emily takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **even** number.

Worked Example

There are 9 counters in a bag.
There is a number on each counter.

Zoe takes at random three of the counters.
She works out the **sum** of the numbers on the three counters.
Work out the probability that the sum is an **odd** number.

Your Turn

There are 7 tiles in a sack.
There is a number on each tile.

Jules takes at random three of the tiles.
She **adds** together the numbers on the three counters to get a total.
Find the probability that her total is an **even** number.

Dr Frost 354f

Review

...with replacement:
The item is returned before another is chosen. The probability of each event on each trial is fixed.

...without replacement:
The item is not returned.
- Total balls decreases by 1 each time.
- Number of items of this type decreases by 1.

Note that if the question doesn't specify which, e.g. "You pick two balls from a bag", then PRESUME WITHOUT REPLACEMENT.

Worked Example	Your Turn
A person throws a biased coin two times. The probability of flipping a heads on any throw is p a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails	A person throws a biased coin two times. The probability of flipping a heads on any throw is q a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails

Worked Example	Your Turn
There are c balloons in trolley A, of which 4 are green and the rest are purple. Trolley B contains 4 green balloons and 10 purple balloons. Noah picks one balloon from each trolley. Given that the probability that he picks two balloons of the same colour is $\frac{4}{7}$, find the value of c	There are v blue and 10 black balloons in box A. Box B contains 5 blue balloons and 10 black balloons. Piotr picks one balloon from each box. Given that the probability that he picks two balloons of different colours is $\frac{3}{7}$, find the value of v

Dr Frost 463d

Worked Example	Your Turn
Neha has n sweets, of which 6 are red. The remainder of the sweets are blue. Neha eats a sweet, does not regurgitate it, and then eats another sweet. The probability that she eats two red sweets is $\frac{15}{28}$ Show that $n^2 + an + b = 0$, where a and b are constants to be found.	Neha has k marbles, of which 7 are green. The remainder of the marbles are red. Neha takes a marble, does not replace it, and then takes another marble. The probability that she takes two green marbles is $\frac{3}{4}$ Show that $k^2 + ak + b = 0$, where a and b are constants to be found.

Dr Frost 463e

Worked Example	Your Turn
There are n sweets in a bag. 4 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{2}{5}$. Find n	There are n sweets in a bag. 6 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{1}{3}$. Find n

Worked Example	Your Turn
There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{90}$ Work out the number of blue counters in the bag at the start.	There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{80}$ Work out the number of blue counters in the bag at the start.

Worked Example	Your Turn
There are only green pens and blue pens in a box. There are three more blue pens than green pens in the box. There are more than 12 pens in the box. Simon is going to take at random two pens from the box. The probability that Simon will take two pens of the same colour is $\frac{27}{55}$ Work out the number of green pens in the box.	A bag contains red and blue balls. There is 2 more blue balls than red balls. The total number of balls is at least 7. Sophie takes two balls from the bag. The probability that they are different colours is $\frac{8}{15}$ Determine the number of balls in the bag.

Extra Notes

Recap

You have to pick a team of one boy and one girl. The boys are Adam, Bob and Charles. The girls are Diane and Elaine.
List out all the possible choices of team. (You may use 'A' for Adam' and so on)

There were 3 boys and 2 girls to choose from, and we needed one of each.
Without listing, we previously saw there were 3×2 total possibilities.

✎ If there are m choices for one thing and n for the other, there are $m \times n$ possibilities when we choose from both.

Quickfire Examples

1. I have 5 choices for a starter, 4 choices for the main course and 3 for the pudding. How many possible meal selections are there?

$$\mathbf{5 \times 4 \times 3 = 60}$$

2. For each move in a path, I can move North, South, East or West. I make 3 moves. How many possible paths are there?

$$\mathbf{4 \times 4 \times 4 = 64}$$

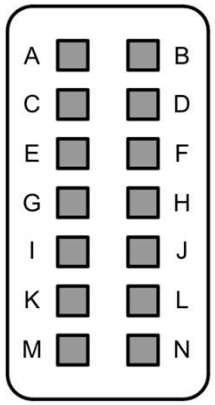
Worked Example	Your Turn
In a football academy there are 20 different players. Two players are going to picked to go to a training camp. Work out how many different combinations of footballers they can pick.	In a choir there are 27 different singers. Two are going to be picked to sing a duet. Work out how many different ways they can pick two different singers.

Dr Frost 381h

Worked Example	Your Turn
Bruno makes 4 digit numbers using all of the cards below. $3, 5, 6, 7$ Work out how many different odd numbers Bruno can make greater than 5000	Bruno makes 4 digit numbers using all of the cards below. $2, 5, 7, 9$ Work out how many different odd numbers Bruno can make less than 7000

Worked Example

A warehouse has a security lock.
To open the warehouse you must press the correct four buttons in the correct order.

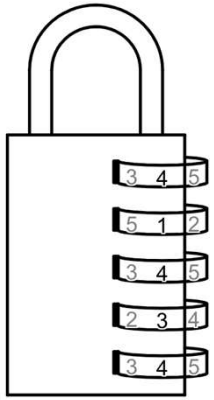


For example: $MBIM$

Work out what fraction of all the possible codes contain 4 different letters.

Your Turn

There are five dials on a combination lock.
Each dial can be set to any number between 1 and 5



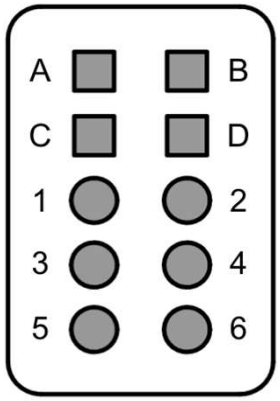
For example: 41434 is one way the dials can be set.

Work out what fraction off all possible codes involve 5 different numbers.

Dr Frost 381g

Worked Example

A bike shed has a security lock.
To open the bike shed you must press the correct buttons.



Determine the probability that a code chosen at random containing any 3 characters is in one of the following orders:

Your Turn

Erik is going stargazing with his telescope. He can potentially look at 5 planets and 11 moons.

He is going to pick three different celestial bodies at random to observe.

Determine the probability that the three objects he views will be in one of the following orders:

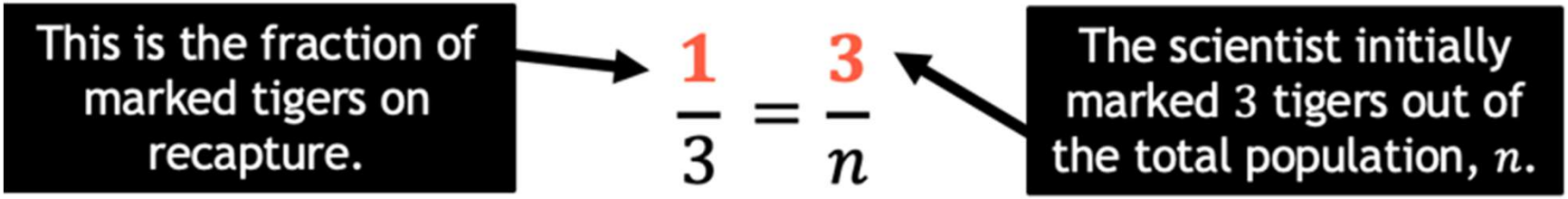
planet moon planet	OR	moon planet moon

Extra Notes

4 Capture-Recapture

Capture: Scientists catch a sample of animals from the population.
They **mark** these animals and release them back into the wild.

Recapture: After some time, they return and catch another sample of animals.
Some of these animals will have **marks** and some will be unmarked.



Worked Example	Your Turn
I catch 12 fish in a pond and mark them with a red dot. The next day I catch 8 fish, of which, 3 have red dots on them. Estimate the population of fish in the pond.	I catch 18 fish in a pond and mark them with a red dot. The next day I catch 7 fish, of which, 2 have red dots on them. Estimate the population of fish in the pond.