



KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS



KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2025

Level 2 Further Mathematics

2026

Unit 30 Tasks

DO NOT WRITE INSIDE

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1 Advanced Equations of Circles

Find the centre and radius of each of these circles:

- (a) $(x - 1)^2 + (y - 1)^2 = 9$
- (b) $(x - 1)^2 + (y - 1)^2 = 25$
- (c) $(x - 3)^2 + (y - 2)^2 = 25$
- (d) $(x + 3)^2 + (y + 2)^2 = 16$
- (e) $x^2 + (y + 2)^2 = 16$
- (f) $(x - 4)^2 + y^2 = 36$

Write down the equation of the circle with:

- (a) Centre (1, 2) and radius 8
- (b) Centre (7, 3) and radius 2
- (c) Centre (-2, 5) and radius 5
- (d) Centre (-5, -1) and radius 4
- (e) Centre (3, -6) and radius $\sqrt{7}$
- (f) Centre (0, -4) and radius $\sqrt{20}$

Find the centre and radius of the circle with equation:

- (a) $x^2 + y^2 - 2x + 8y - 8 = 0$
- (b) $x^2 + y^2 + 12x - 4y = 9$
- (c) $x^2 + y^2 - 22x - 6y + 40 = 0$
- (d) $x^2 + y^2 - 4x - 11 = 0$

(a) Show that the point (2, 10) lies on the circle with equation

$$(x - 2)^2 + (y - 7)^2 = 9$$

(b) A circle has centre (5, 10). The point (2, 14) lies on the circumference of the circle. Find the equation of the circle.

Question 1: Write down the (i) centre and the (ii) radius for each of these circles.

(a) $(x - 3)^2 + (y - 2)^2 = 100$ (b) $(x - 1)^2 + (y + 3)^2 = 16$

(c) $(x + 5)^2 + (y - 7)^2 = 4$ (d) $(x + 6)^2 + (y + 8)^2 = 1$

(e) $x^2 + y^2 = 25$ (f) $(x + 4)^2 + y^2 = 9$

(g) $(x + 2)^2 + (y - 7)^2 = 5$ (h) $(x - 6)^2 + (y - 1)^2 = 20$

Question 2: Write the equations of each of these circles.

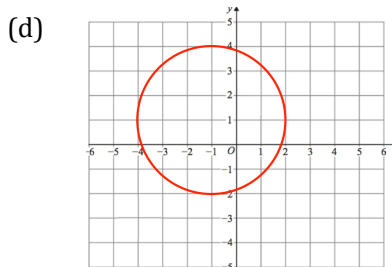
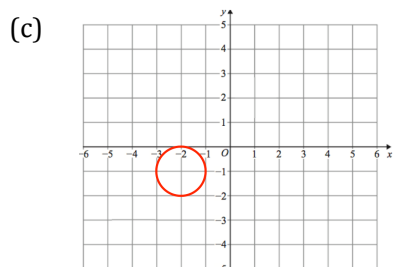
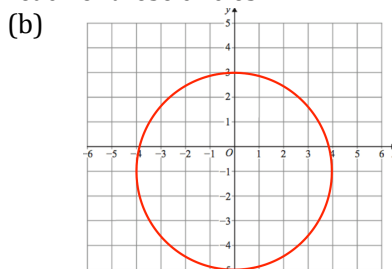
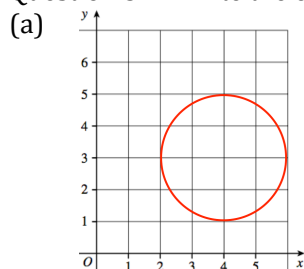
(a) Centre is (2, 7) and has a radius of 6. (b) Centre is (4, -2) and has a radius of 3.

(c) Centre is (-1, 3) and has a radius of 4. (d) Centre is (-1, -2) and has a radius of 9.

(e) Centre is (0, 5) and has a radius of 5. (f) Centre is the origin and has radius 8.

(g) Centre is (11, 0) and has a radius of $\sqrt{3}$. (h) Centre is (-3, -7) and has a radius of $2\sqrt{2}$.

Question 3: Write the equations of each of these circles.



Question 4: Find if the coordinates given lie on the circles below:

(a) Does $(4, 2)$ lie on the circle with equation $(x - 1)^2 + (y + 2)^2 = 25$

(b) Does $(3, 9)$ lie on the circle with equation $(x - 4)^2 + (y - 3)^2 = 36$

(c) Does $(-16, 4)$ lie on the circle with equation $(x + 4)^2 + (y - 9)^2 = 169$

(d) Does $(2, -2)$ lie on the circle with radius 8 and centre $(5, 5)$?

(e) Does $(0, 7)$ lie on the circle with radius $\sqrt{10}$ and centre $(-3, 8)$?

Question 5: Find where the circle $(x - 3)^2 + (y - 5)^2 = 106$ meets the x-axis.

Question 6: Find where the circle $(x - 1)^2 + (y + 2)^2 = 20$ meets the x-axis.

Question 7: Find where the circle $(x + 2)^2 + (y + 3)^2 = 45$ meets the x-axis.

Question 8: Find where the circle $(x - 2)^2 + (y - 1)^2 = 68$ meets the x-axis.

Question 9: Find where the circle $(x - 4)^2 + (y - 6)^2 = 32$ meets the x-axis.

Apply

Question 1: A circle has centre $(7, 2)$.
The point $(1, -6)$ lies on the circle.
Find the equation of the circle.

Question 2: A circle has centre A.
The points C $(0, 4)$ and D $(10, 4)$ lie on the diameter of the circle.

- (a) Find the coordinates of A.
- (b) Find the equation of the circle.

(a) A circle C has the equation $x^2 + (y + 3)^2 = 169$. The line $x = 5$ passes through the circle at points A and B. Find the length of the line AB.

(b) The points A $(-3, 5)$ and B $(7, 1)$ lie on circle C. The line AB is a diameter of the circle. Find the equation of the circle.

(a) The point P with coordinates $(1, 8)$ lies on the circle with equation $x^2 + y^2 + 4x - 6y - 21 = 0$. Point Q also lies on the circle, and PQ is a diameter of the circle. Find the coordinates of point Q.

(b) A circle C has centre $(-3, -1)$. Point P with coordinates $(3, 2)$ lies on circle C. Find the coordinates of the points where the circle crosses the y -axis.

(a) Determine whether the point $(4, 5)$ lies inside, outside or on the circle with equation $x^2 + y^2 + 4y - 49 = 0$.

(b) A circle has diameter AB where A is $(-5, -1)$ and B is $(0, -7)$. Find the equation of the tangent to the circle at point A, giving your answer in the form $ax + by + c = 0$, where a , b and c are integers to be found.

(a) A circle with equation $x^2 + y^2 = 25$ has centre O and passes through the point P with coordinates $(3, 4)$. Line L is the tangent to the circle at point P. Line L meets the x -axis at A and the y -axis at B. Find the area of the triangle OAB.

(b) The circle C has the equation $x^2 + y^2 - 12x + 4y - 24 = 0$. Find the two values of a for which the line $y = a$ is a tangent to circle C.

Sample Problems

- Find the coordinates of all points where the circle $x^2 + (y - 3)^2 = 50$ and the line $y = -2x + 8$ intersect each other.
- Find the points where the circles $(x + 4)^2 + (y + 1)^2 = 10$ and $(x + 1)^2 + (y - 5)^2 = 25$ intersect each other.
 - Find the points where the circles $(x + 2)^2 + (y + 2)^2 = 50$ and $(x - 2)^2 + (y - 1)^2 = 25$ intersect each other.
- Consider the circles $x^2 + y^2 = 36$ and $(x - 5)^2 + y^2 = 16$. Let C be the intersection of the two common tangent lines drawn to the circle.
 - Find the coordinates of C .
 - Consider one of the common tangent lines. Find the distance between the points of tangency.
- Find an equation of the circle that passes through the points $A(0, 2)$, $B(-2, -2)$, and $C(-8, -4)$.

Practice Problems

- Find the coordinates of all point(s) where the circle and the line intersect each other.
 - $(x + 5)^2 + (y - 7)^2 = 8$ and $x + y = 6$
 - $(x - 2)^2 + (y + 1)^2 = 50$ and $y = -x + 9$
 - $x^2 + (y + 4)^2 = 25$ and $y = x - 24$
 - $(x - 3)^2 + (y + 1)^2 = 25$ and $y = -x + 1$
- Find the coordinates of all points where the given circles intersect each other.
 - $(x - 1)^2 + (y - 2)^2 = 10$ and $(x - 10)^2 + (y - 5)^2 = 40$
 - $(x - 3)^2 + y^2 = 20$ and $x^2 + (y - 1)^2 = 50$
 - $x^2 + (y - 8)^2 = 26$ and $(x - 7)^2 + (y - 3)^2 = 4$
- Consider the circles $(x + 5)^2 + y^2 = 100$ and $x^2 + y^2 = 49$. Let C be the intersection of the two common tangent lines drawn to the circle.
 - Find the coordinates of C .
 - Consider one of the common tangent lines. Find the distance between the points of tangency.
- Let C_1 and C_2 be circles defined by $x^2 + (y + 7)^2 = 1$ and $x^2 + (y - 6)^2 = 16$, respectively. Consider one of the common tangent lines. Find the distance between the points of tangency.
- Given the three points, find an equation of the circle that contains all three points.
 - $P(0, 0)$, $Q(0, 8)$, and $R(-6, 0)$.
 - $A(-1, 6)$, $B(-7, 12)$, and $C(-3, 0)$.
 - $X(3, -7)$, $Y(5, -1)$, and $Z(-3, -5)$.

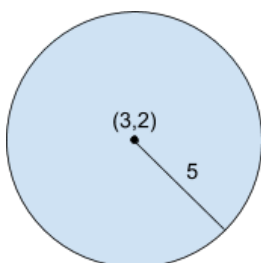
INCREASINGLY DIFFICULT CIRCLE EQUATIONS

1) Find the centre and radius of the circle with the equation $(x - 2)^2 + (y + 1)^2 = 9$.

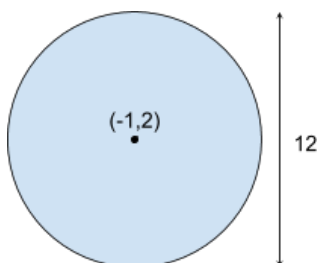
2) Find the centre and radius of the circle with the equation $(x + 2)^2 + y^2 = 40$.

3) Find the centre and radius of the circle with the equation $x^2 + y^2 - 4x + 2y - 4 = 0$.

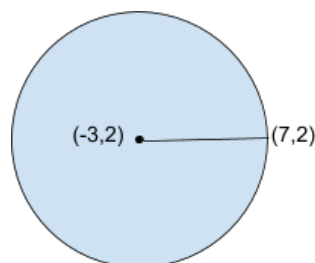
4) Find the equation of the circle:



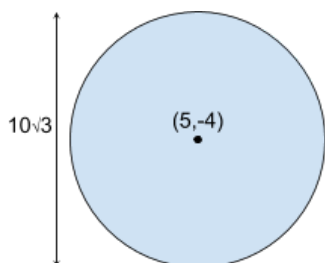
5) Find the equation of the circle:



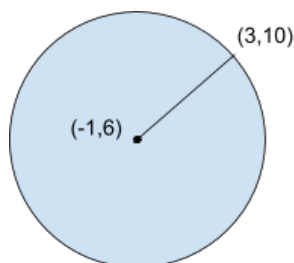
6) Find the equation of the circle:



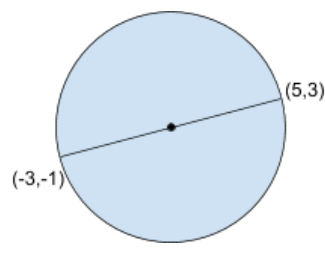
7) Find the equation of the circle:



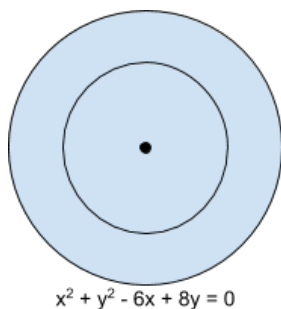
8) Find the equation of the circle:



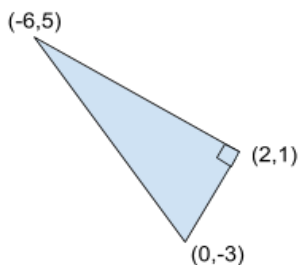
9) Find the equation of the circle:



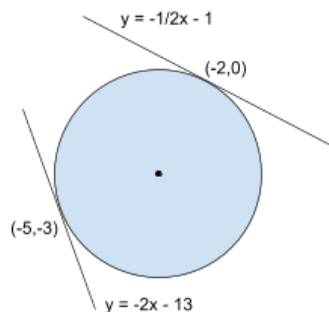
10) Find the equation of the circle which is concentric to $x^2 + y^2 - 6x + 8y = 0$ but is half as wide.



11) Find the equation of the circle of the circle passing through the vertices of the right angled triangle as shown:



12) Find the equation of the circle with a tangent $y = -2x - 13$ which touches the circle at point (-5,-3), and a tangent $y = -\frac{1}{2}x - 1$ which touches the circle at point (-2,0).



Purposeful Practice

- ① Write down the equations of these circles.
- (i) centre $(1, 2)$, radius 3
 - (ii) centre $(4, -3)$, radius 4
 - (iii) centre $(1, 0)$, radius 5
 - (iv) centre $(-2, -2)$, radius 2
 - (v) centre $(-4, 3)$, radius 1
- ② For each of the circles given below
- (a) state the coordinates of the centre
 - (b) state the radius
 - (c) sketch the circle, paying particular attention to its position in relation to the origin and the coordinate axes.
- (i) $x^2 + y^2 = 25$
 - (ii) $(x - 3)^2 + y^2 = 9$
 - (iii) $(x + 4)^2 + (y - 3)^2 = 25$
 - (iv) $(x + 1)^2 + (y + 6)^2 = 36$
 - (v) $(x - 4)^2 + (y - 4)^2 = 16$
- ③ Work out the equation of the circle with centre $(2, -3)$ which passes through $(1, -1)$.
- ④ A and B are $(4, -4)$ and $(2, 6)$ respectively. Work out
- (i) the midpoint C of AB
 - (ii) the distance AC
 - (iii) the equation of the circle that has AB as its diameter.
- ⑤ Show that the equation $x^2 + y^2 - 4x - 8y + 4 = 0$ represents a circle. Hence give the coordinates of the centre and the radius of the circle, and sketch the circle.
- ⑥ Why does the equation $x^2 - 4x + y^2 - 4y + 22 = 0$ not represent a circle?
- ⑦ A circle of radius 5 cm passes through the points $(0, 0)$ and $(0, 6)$. Sketch two possible positions of the circle and write down the equation in each case.
- ⑧ $(6, 3)$ is a point on the circle with centre $(11, 8)$.
- (i) Work out the radius of the circle.
 - (ii) Work out the equation of the circle.
 - (iii) Work out the coordinates of the other point where the diameter through $(6, 3)$ meets the circle.

(a) The point P $(6, -2)$ lies on the circle with equation $x^2 + y^2 = 40$. Find the gradient of the tangent to the circle at point P.

(b) The point Q $(2, 1)$ lies on the circle with equation $x^2 + y^2 = 5$. Find the gradient of the tangent to the circle at point Q.

(a) The point A $(2, -5)$ lies on the circle with equation $x^2 + y^2 = 29$. Find the equation of the tangent to the circle at point A.

(b) The point B $(-3, -2)$ lies on the circle with equation $x^2 + y^2 = 13$. Find the equation of the tangent to the circle at point A.

(a) The point R $(-2, -3)$ lies on the circle with centre $(1, 2)$ and radius $\sqrt{34}$. Find the equation of the tangent to the circle at point R.

(b) The point S $(7, 1)$ lies on the circle with centre $(x - 3)^2 + (y + 2)^2 = 25$. Find the equation of the tangent to the circle at point S.

(a) Point P $(-2, 8)$ lies on a circle with centre $(-1, 6)$. Point Q with coordinates $(a, 5)$ lies on the tangent to the circle at P. Find the value of a .

(b) Point A $(-1, -3)$ lies on the circle with equation $(x - 3)^2 + (y + 2)^2 = 17$. The line L is the tangent to the circle at point A. Find the coordinates of the point where line L crosses the x -axis.

Purposeful Practice

- 1) The circle C has equation $(x - 3)^2 + (y - 7)^2 = 100$.
- Verify the point $P(11,1)$ lies on C .
 - Find an equation of the tangent to C at the point P , giving your answer in the form $ax + by + c = 0$

- 2) A circle C has equation $(x - 4)^2 + (y + 4)^2 = 10$
The line l is a tangent to the circle and has gradient -3 .
Find two possible equations for l , giving your answers in the form $y = mx + c$.

- 3) A circle has centre $C(3,5)$, and goes through the point $P(6,9)$. Find the equation of the tangent of the circle at the point P , giving your equation in the form $ax + by + c = 0$ where a, b, c are integers..

Purposeful Practice

If a diagram is not given, drawing a sketch may help.

- ① AB is a diameter of a circle. P is a point on the circumference of the circle.

A is (2, 8) and P is (4, -2).

Work out the gradient of BP.

- ② A circle has centre C.

RS is a chord of the circle and R is (-1, 6).

Y(2, 3) is a point on RS such that angle CYR = 90°.

Work out the coordinates of S.

- ③ Look at Figure 5.30.

AB is a diameter of the circle.

A is (k, 5), P is (3, 8) and B is (7, 2).

Work out the value of k.

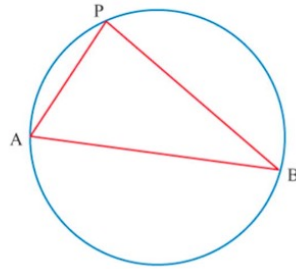


Figure 5.30

- ④ Figure 5.31 shows a circle, centre C.

AB is a chord of a circle.

D is a point on AB such that angle ADC is 90°.

Work out the equation of the line that passes through A and C.

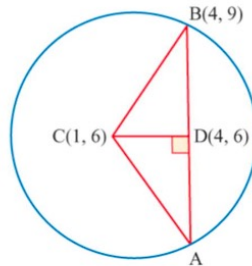


Figure 5.31

- ⑤ Look at Figure 5.32. T(3, -4) is a point on the circumference of the circle $x^2 + y^2 = 25$

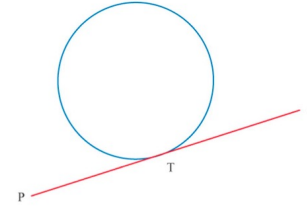


Figure 5.32

Work out the equation of the tangent PTQ.

Give your answer in the form $y = mx + c$.

- ⑥ Figure 5.33 shows a circle that intersects the x-axis at (-2, 0) and (6, 0).

The centre of the circle is (a, 3).

Work out the equation of the circle.

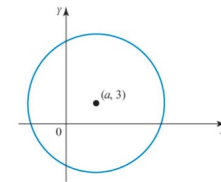


Figure 5.33

- ⑦ Figure 5.34 shows a circle with centre C(2, 1) and radius 5 units together with the tangent at the point A(5, -3).

(i) Write down the equation of the circle.

(ii) Work out the equation of the tangent and verify that the point D(-3, -9) lies on the tangent.

(iii) Write down the equation of the other tangent through the point D(-3, -9) and state the coordinates of the point B where this touches the circle.

(iv) Work out the lengths of the two tangents from D to the circle.

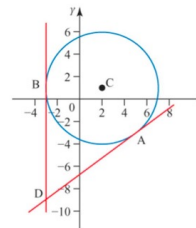


Figure 5.34

- ⑧ Figure 5.35 shows a circle $x^2 + y^2 = 25$ together with the tangent to the circle at the point A(3, -4).

(i) Show that the tangent at A has equation $3x - 4y - 25 = 0$

(ii) Write down the equation of the tangent to the circle at the point B(-5, 0).

(iii) Work out the coordinates of the point of intersection, C, of these tangents.

(iv) Show, by calculation, that CA = CB.

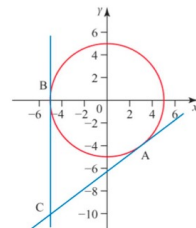


Figure 5.35

2 Limiting Values of Sequences

Intelligent Practice

As $n \rightarrow \infty$:

$$\frac{1}{n} \rightarrow$$

$$\frac{7n - 9}{5n - 1} \rightarrow$$

$$\frac{3n + 5}{5n - 9} \rightarrow$$

$$\frac{2}{3n} \rightarrow$$

$$\frac{9 - 7n}{1 - 5n} \rightarrow$$

$$\frac{n^2 + 3n + 5}{5n - 9} \rightarrow$$

$$\frac{2n}{3n} \rightarrow$$

$$\frac{7n^2 - 9}{1 + 5n} \rightarrow$$

$$\frac{6n - 8}{3 + 4n} \rightarrow$$

$$\frac{2n^2}{3n} \rightarrow$$

$$\frac{9 - 7n^2}{1 + 5n} \rightarrow$$

$$2 + \frac{6n - 8}{3 + 4n} \rightarrow$$

$$\frac{7n}{5n + 1} \rightarrow$$

$$\frac{9 - 7n^2}{1 - 5n^2} \rightarrow$$

$$2 - \frac{6n - 8}{3 + 4n} \rightarrow$$

$$\frac{7n + 8}{5n - 1} \rightarrow$$

$$\frac{9 - 7n^2}{1 - 5n^3} \rightarrow$$

$$\frac{2n^2 + 3n - 4}{3n^2 - 4n + 5} \rightarrow$$

Find the limits of these sequences as

$n \rightarrow \infty$

(a) $\frac{3n}{9n-4}$ (b) $\frac{n}{2n-7}$

(c) $\frac{4n}{2n+1}$ (d) $\frac{8n+1}{2n-3}$

(e) $\frac{3n+2}{6n-4}$ (f) $\frac{2-9n}{15n-4}$

Find the limits of these sequences as

$n \rightarrow \infty$

(a) $\frac{n^2}{n^2-1}$ (b) $\frac{4n^2}{n^2+1}$

(c) $\frac{5n^2+1}{10n^2-2}$ (d) $\frac{2n^2-1}{5n^2}$

(a) A sequence starts $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \dots$

Find the n th term for this sequence and the limiting value as $n \rightarrow \infty$.

(b) A sequence starts $\frac{1}{5}, \frac{4}{9}, \frac{7}{13}, \frac{10}{17}, \dots$

Find the n th term for this sequence and the limiting value as $n \rightarrow \infty$.

(a) A sequence with n th term $\frac{an+5}{5n-1}$ has a limiting value of $\frac{2}{5}$ as $n \rightarrow \infty$. Work out the value of a .

(b) A sequence with n th term $\frac{10-bn}{3n+2}$ has a limiting value of -3 as $n \rightarrow \infty$. Work out the value of b .

Fluency Practice

- ① The n th term of a sequence is $\frac{n+1}{2n+1}$.
- (i) Work out the first three terms of the sequence.
 - (ii) Work out the position of the term that has value 0.52.
- ② The n th term of a sequence is $\frac{4n-1}{2n-5}$.
- (i) Work out the position of the term that has value 2.36.
 - (ii) Show that 1 is not a term in the sequence.

- ③ The n th terms of sequences are shown.
Work out the limiting value of each sequence as $n \rightarrow \infty$.

(i) $\frac{2n}{n+1}$	(ii) $\frac{n+2}{n+3}$	(iii) $\frac{n}{3n-1}$
(iv) $\frac{2n-1}{4n+1}$	(v) $\frac{3n}{4n+1}$	(vi) $\frac{1-n}{n+4}$
(vii) $\frac{2n}{3-4n}$	(viii) $\frac{2-6n}{5-2n}$	

- ④ The n th term of a sequence is $\frac{5n+1}{2n+1}$.
Prove that the limiting value of the sequence as $n \rightarrow \infty$ is $\frac{5}{2}$.
- ⑤ The n th term of a sequence is $\frac{10-6n}{8n-3}$.
Prove that the limiting value of the sequence as $n \rightarrow \infty$ is -0.75
- PS** ⑥ The n th term of a sequence is $2 + \frac{7}{an+b}$, $a \neq 0$
Work out its limiting value.
- PS** ⑦ The n th term of a sequence is $\frac{an+3}{cn-1}$.
The 1st term of the sequence is 11, and its limiting value as $n \rightarrow \infty$ is 4
Work out the value of the 2nd term of the sequence.
- PS** ⑧ The n th term of a sequence is $\frac{2n^2+3n-4}{7n^2-n+2}$.
Work out its limiting value.
(Hint: divide the numerator and denominator by n^2 .)