KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 11

2024 Mathematics 2025

Unit 22 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

KING EDWARD VI
HANDSWORTH GRAMMAR
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KING EDWARD VI
ACADEMY TRUST
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Unit 22 Booklet – Part 2

HGS Maths

Tasks

Dr Frost Course

Name: _____

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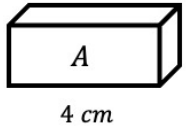
Contents

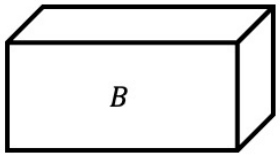
1 Similarity with Area and Volume

Worked Example

Cuboids A and B are similar.



4 cm



8 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

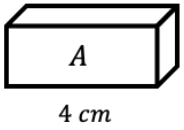
Surface Area $B \rightarrow A$

Volume $A \rightarrow B$

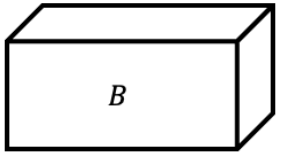
Volume $B \rightarrow A$

Your Turn

Cuboids A and B are similar.



4 cm



12 cm

Write down the scale factor for:

Length $A \rightarrow B$

Length $B \rightarrow A$

Surface Area $A \rightarrow B$

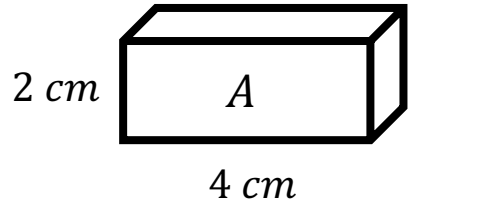
Surface Area $B \rightarrow A$

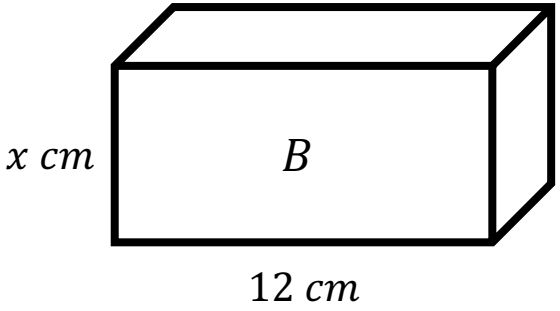
Volume $A \rightarrow B$

Volume $B \rightarrow A$

Worked Example

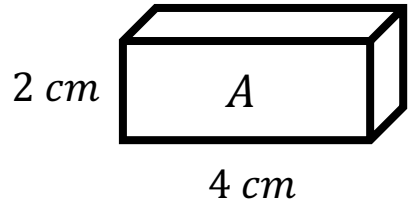
Cuboids A and B are similar.
Find x.

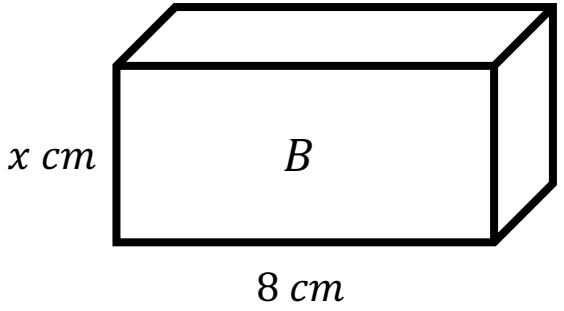




Your Turn

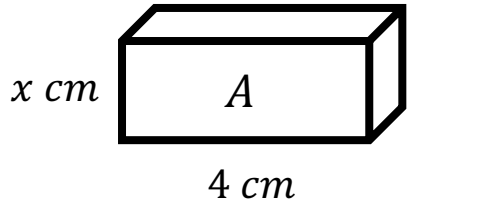
Cuboids A and B are similar.
Find x.

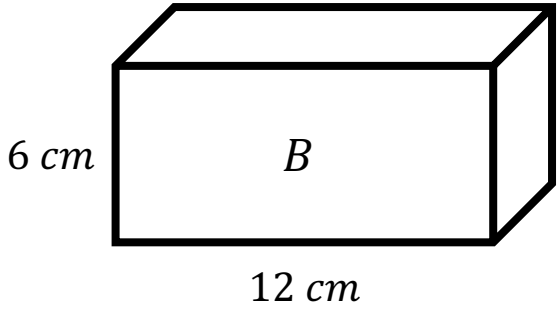




Worked Example

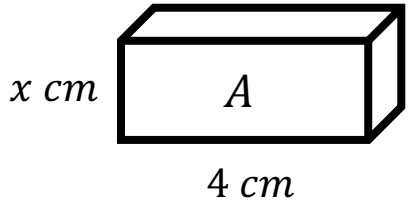
Cuboids A and B are similar.
Find x.

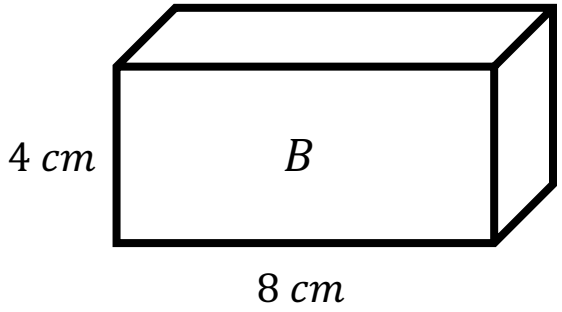




Your Turn

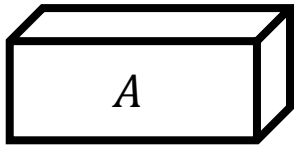
Cuboids A and B are similar.
Find x.



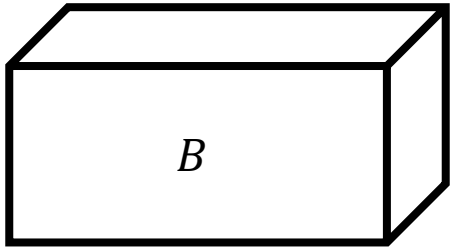


Worked Example

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



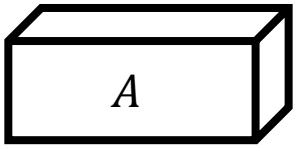
$4\ cm$



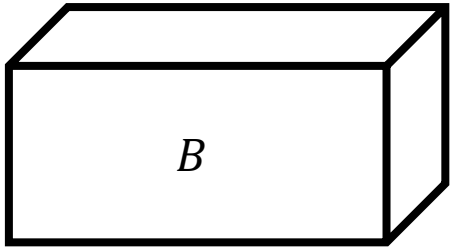
$12\ cm$

Your Turn

Cuboids A and B are similar.
The surface area of cuboid A is $72\ cm^2$. What is the surface area of cuboid B?



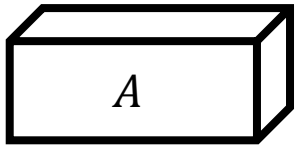
$4\ cm$



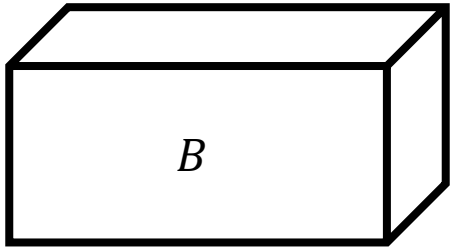
$8\ cm$

Worked Example

Cuboids A and B are similar.
The surface area of cuboid B is $72\ cm^2$. What is the surface area of cuboid A?



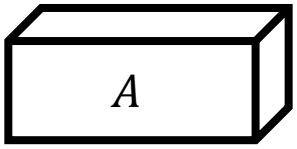
$4\ cm$



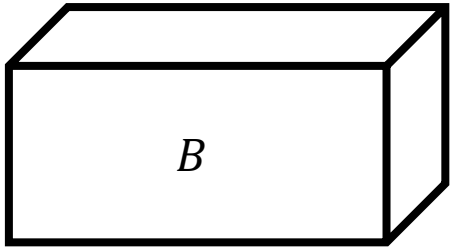
$12\ cm$

Your Turn

Cuboids A and B are similar.
The surface area of cuboid B is $72\ cm^2$. What is the surface area of cuboid A?



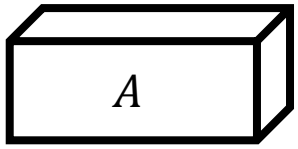
$4\ cm$



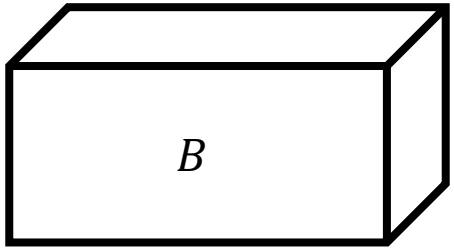
$8\ cm$

Worked Example

Cuboids A and B are similar.
The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



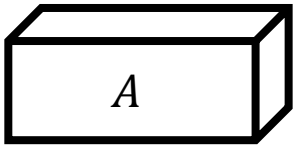
$4\ cm$



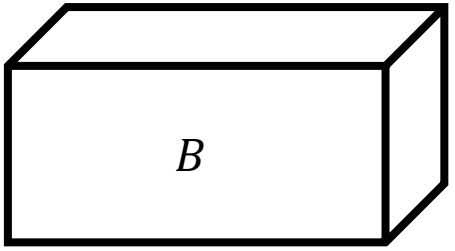
$12\ cm$

Your Turn

Cuboids A and B are similar.
The volume of cuboid A is $432\ cm^3$. What is volume of cuboid B?



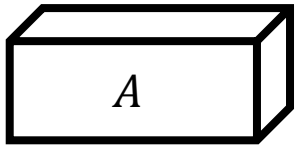
$4\ cm$



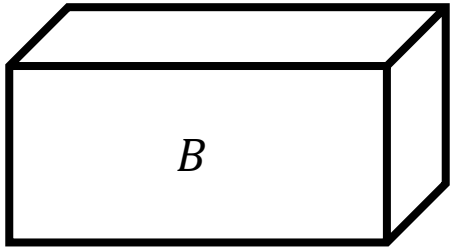
$8\ cm$

Worked Example

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



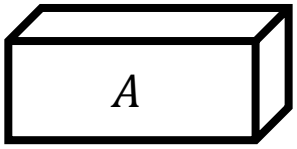
$4\ cm$



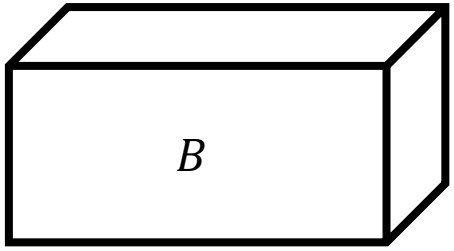
$12\ cm$

Your Turn

Cuboids A and B are similar.
The volume of cuboid B is $432\ cm^3$. What is volume of cuboid A?



$4\ cm$



$8\ cm$

Worked Example	Your Turn
A and B are mathematically similar solids. The surface area of A is $100\ cm^2$. The surface area of B is $64\ cm^2$. Work out the ratio of the volume of A to the volume of B.	A and B are mathematically similar solids. The surface area of A is $120\ cm^2$. The surface area of B is $480\ cm^2$. Work out the ratio of the volume of A to the volume of B.

Worked Example	Your Turn
A and B are mathematically similar solids. The volume of A is $500\ cm^3$. The volume of B is $256\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.	A and B are mathematically similar solids. The volume of A is $120\ cm^3$. The volume of B is $960\ cm^3$. Work out the ratio of the surface area of A to the surface area of B.

Fill in the Gaps

	Length	Area	Volume
Scale Factor	2	4	8
Shape A	5 cm	30 cm^2	100 cm^3
Shape B	10 cm	120 cm^2	

	Length	Area	Volume
Scale Factor	3	9	27
Shape A	4 cm	16 cm^2	50 cm^3
Shape B	12 cm		

	Length	Area	Volume
Scale Factor	2		
Shape A	15 mm	35 mm^2	64 mm^3
Shape B	30 mm		

	Length	Area	Volume
Scale Factor			
Shape A	2 m	1.5 m^2	16 m^3
Shape B	5 m		

	Length	Area	Volume
Scale Factor			
Shape A	4 cm		
Shape B	12 cm	72 cm^2	540 cm^3

	Length	Area	Volume
Scale Factor			
Shape A	2.4 m		
Shape B	1.2 m	4 m^2	5.6 m^3

	Length	Area	Volume
Scale Factor	1.5		
Shape A			
Shape B	7.5 cm	45 cm^2	202.5 cm^3

	Length	Area	Volume
Scale Factor	$\frac{4}{3}$		
Shape A	1.8 cm		54 cm^3
Shape B		80 cm^2	

Fill in the Gaps

	Length	Area	Volume
Scale Factor		4	8
Shape A	3 cm	10 cm^2	25 cm^3
Shape B			

	Length	Area	Volume
Scale Factor		9	
Shape A	4 cm	20 cm^2	70 cm^3
Shape B			

	Length	Area	Volume
Scale Factor			125
Shape A	0.5 m	2 m^2	5 m^3
Shape B			

	Length	Area	Volume
Scale Factor			
Shape A	4.5 mm	20 mm^2	35 mm^3
Shape B		180 mm^2	

	Length	Area	Volume
Scale Factor			
Shape A	2.5 cm	8 cm^2	20 cm^3
Shape B			67.5 cm^3

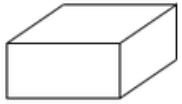
	Length	Area	Volume
Scale Factor			
Shape A	0.6 m	2.8 m^2	
Shape B		0.7 m^2	1.4 m^2

	Length	Area	Volume
Scale Factor			
Shape A		1.8 cm^2	
Shape B	2 cm	5 cm^2	25 cm^3

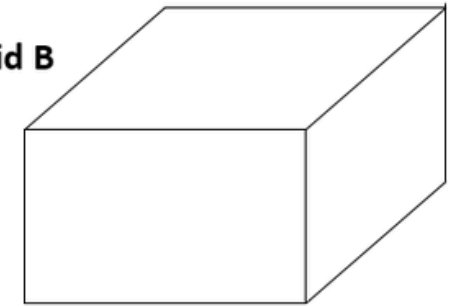
	Length	Area	Volume
Scale Factor			
Shape A	7.5 cm		135 cm^3
Shape B		22 cm^2	40 cm^3

Fill in the Gaps

Cuboid A



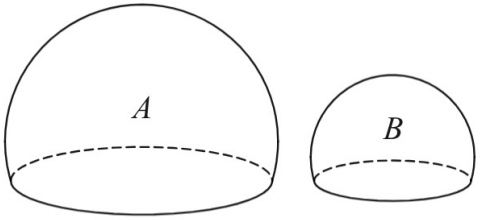
Cuboid B



Height of A (cm)	Height of B (cm)	Surface area of A (cm^2)	Surface area of B (cm^2)	Volume of A (cm^3)	Volume of B (cm^3)
15	45	120		60	
90	180	2000			12000
2	20		700		12000
18		70	1750	32	
	10	4.3	68.8	7.2	
6.8		12.5		6.6	178.2
	45		125	42	656.25
	11.36	15.2		54	221.184

Worked Example

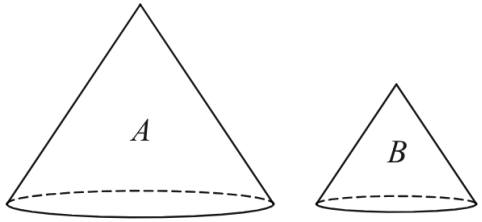
A and B are two solid shapes that are mathematically similar.
The shapes are made from the same material.



The total surface area of shape A is 62.5 cm^2.
The total surface area of shape B is 22.5 cm^2.
The mass of A is 375 g. Find the mass of B.

Your Turn

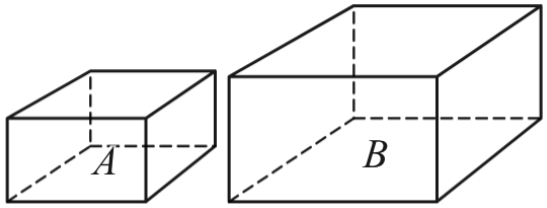
A and B are two solid cones that are mathematically similar.
The cones are made from the same material.



The total surface area of shape A is 75 cm^2.
The total surface area of shape B is 27 cm^2.
The mass of B is 81 g. Find the mass of A.

Worked Example

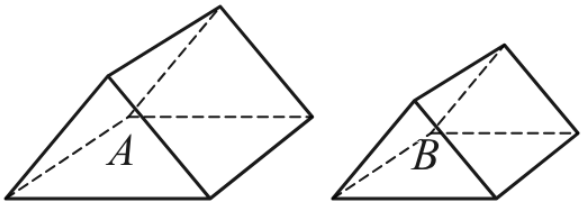
A and B are two solid cuboids that are mathematically similar.
The surface area scale factor from A to B is 2.25



Find the ratio of the volume of cuboid A to the volume of cuboid B

Your Turn

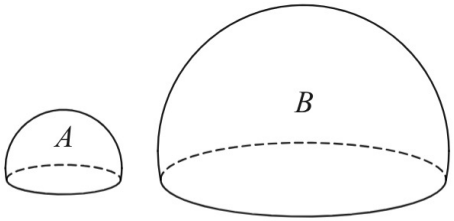
A and B are two solid prisms that are mathematically similar.
The surface area scale factor from A to B is 0.64



Find the ratio of the volume of prism A to the volume of prism B

Worked Example

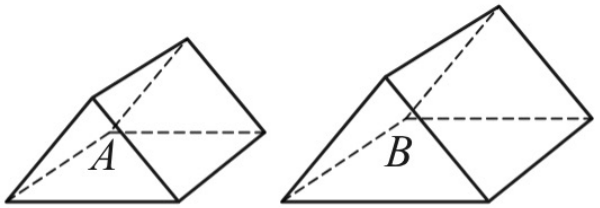
Shapes A and B are similar.



The volume of B is 1462.5% greater than the volume of A. Find the percentage increase from the surface area of shape A to the surface area of shape B.

Your Turn

Prisms A and B are similar.



The surface area of B is 44% greater than the surface area of A. Find the percentage increase from the volume of prism A to the volume of prism B.

Worked Example	Your Turn
The surface area of two mathematically similar solids are in the ratio $16:49$. The volume of the smaller solid is $128\ cm^3$. Work out the volume of the larger solid.	The surface area of two mathematically similar solids are in the ratio $9:25$. The volume area of the smaller solid is $108\ cm^3$. Work out the volume of the larger solid.

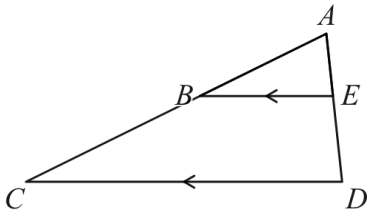
Worked Example	Your Turn
The volume of two mathematically similar solids are in the ratio $64: 343$. The surface area of the smaller solid is $32\ cm^2$. Work out the surface area of the larger solid.	The volume of two mathematically similar solids are in the ratio $27: 125$. The surface area of the smaller solid is $36\ cm^2$. Work out the surface area of the larger solid.

Worked Example	Your Turn
a) The scale of a map is $1 : 3000000$ On the map, the area of Toonhill Valley is 6.4 cm^2. Calculate the actual area of Toonhill Valley. Give your answer in square kilometres. b) The scale of a map is $1 : 400000$ The area of Lake Troycou is 646.4 km^2. Calculate the area of Lake Troycou on the map in cm^2.	a) The scale of a map is $1 : 70000$ On the map, the area of Fort Prisetomb is 50.6 cm^2. Calculate the actual area of Fort Prisetomb. Give your answer in square kilometres. b) The scale of a map is $1 : 60000$ The area of Saint Ralhay is 10.8 km^2. Calculate the area of Saint Ralhay on the map in cm^2.

Dr Frost 324n and 324o

Worked Example

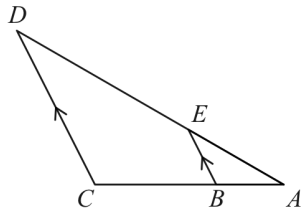
In the diagram ABC and AED are straight lines and BE is parallel to CD.



The length AB is 20 cm and the length BC is 27.5 cm.
The area of triangle ABE is 83.2 cm^2.
Work out the area of triangle ACD.

Your Turn

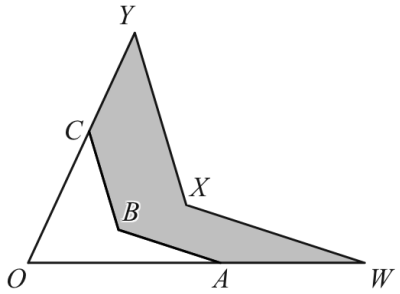
In the diagram ABC and AED are straight lines and BE is parallel to CD.



The length AB is 10.5 cm and the length BC is 18.9 cm.
The area of triangle ABE is 45 cm^2.
Work out the area of triangle ACD.

Worked Example

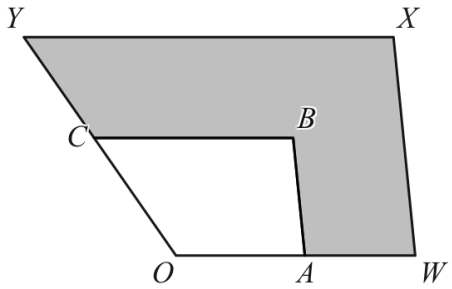
The diagram shows two similar quadrilaterals $OABC$ and $OWXY$.



The length of AB is 4 cm and the length of WX is 7 cm.
The area of quadrilateral $OABC$ is 9.6 cm^2.
Find the shaded area.

Your Turn

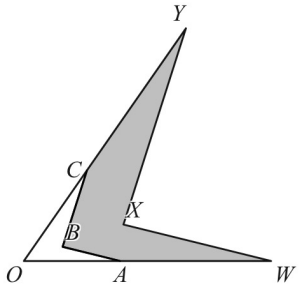
The diagram shows two similar trapeziums $OABC$ and $OWXY$.



The length of OA is 7 cm and the length of AW is 6 cm.
The area of trapezium $OABC$ is 27.44 cm^2.
Find the shaded area.

Worked Example

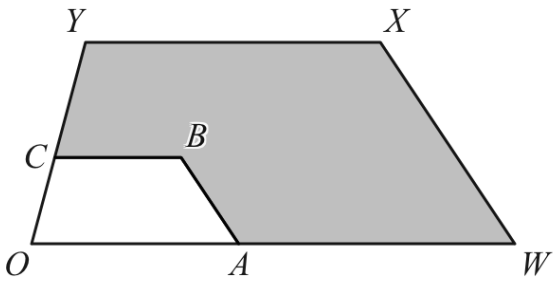
The diagram shows two similar quadrilaterals $OABC$ and $OWXY$.



The length of AB is 7 cm and the length of WX is 18 cm.
The area of quadrilateral $OWXY$ is 324 cm².
Find the shaded area.

Your Turn

The diagram shows two similar trapeziums $OABC$ and $OWXY$.



The length of OA is 6 cm and the length of AW is 8 cm.
The area of trapezium $OWXY$ is 235.2 cm².
Find the shaded area.

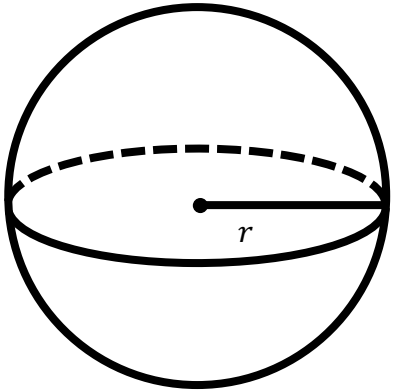
Extra Notes

2 Volume and Surface Area of Non-Prisms

Volume of Spheres

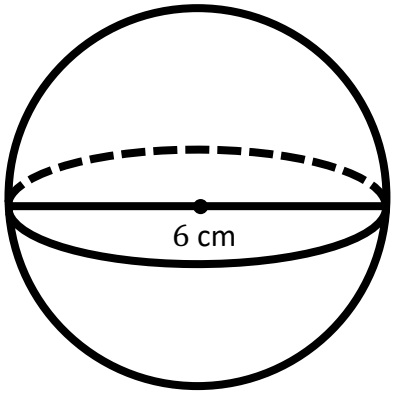
$$\text{Volume of Sphere} = \frac{4}{3} \times \pi \times \text{Radius}^3$$

$$V = \frac{4}{3}\pi r^3$$



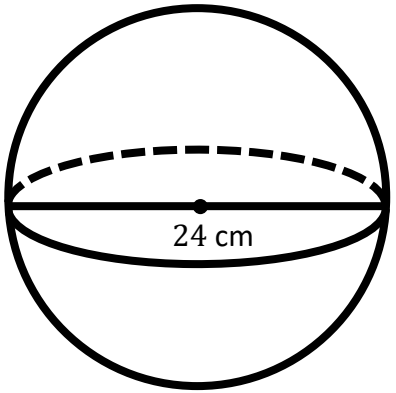
Worked Example

Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.



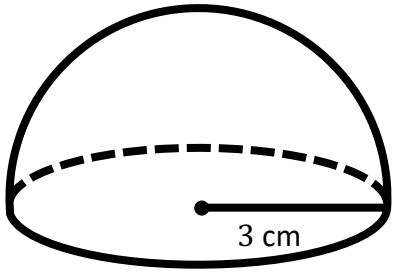
Your Turn

Calculate the volume of the following sphere. Give your answer in terms of π and to 1 decimal place.



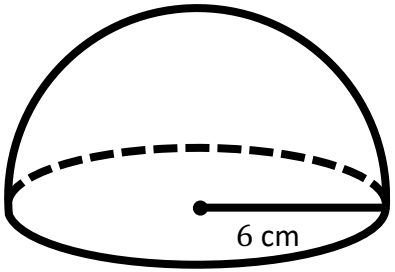
Worked Example

Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



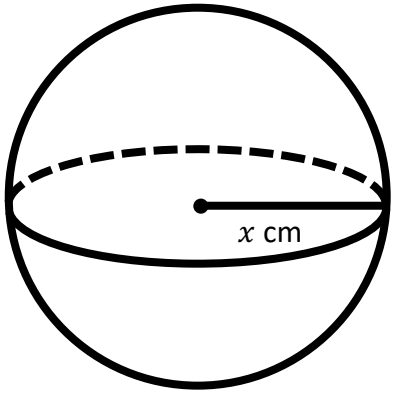
Your Turn

Calculate the volume of the following hemisphere. Give your answer in terms of π and to 1 decimal place.



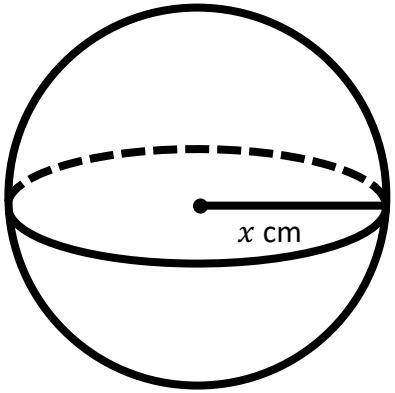
Worked Example

Find the radius, x, given that the volume of the following sphere is $113.1\ cm^3$. Give your answer to 1 decimal place.



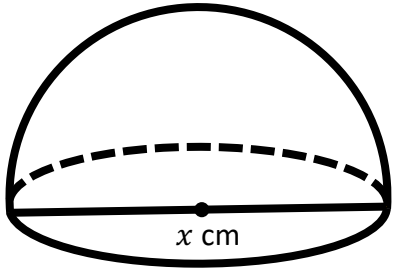
Your Turn

Find the radius, x, given that the volume of the following sphere is $904.8\ cm^3$. Give your answer to 1 decimal place.



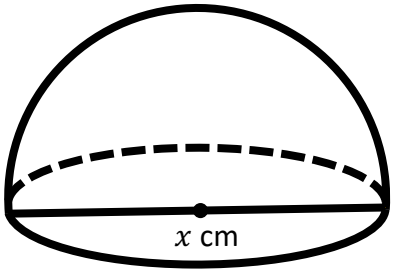
Worked Example

Find the diameter, x, given that the volume of the following hemisphere is $56.5\ cm^3$. Give your answer to 1 decimal place.



Your Turn

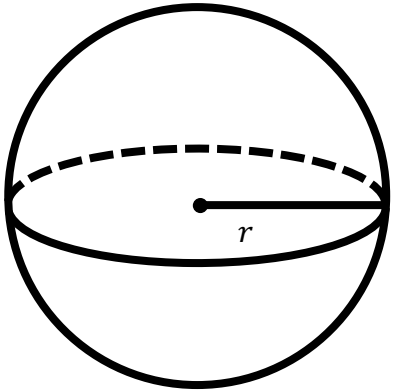
Find the diameter, x, given that the volume of the following hemisphere is $452.4\ cm^3$. Give your answer to 1 decimal place.



Surface Area of Spheres

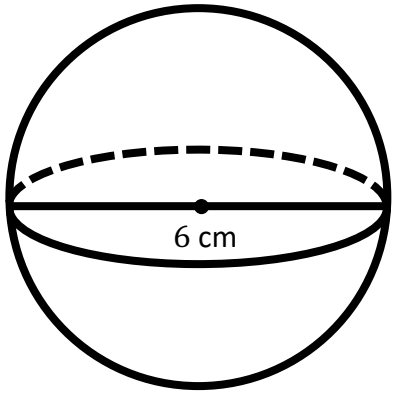
Surface Area of Sphere $= 4 \times \pi \times \text{Radius}^2$

$SA = 4\pi r^2$



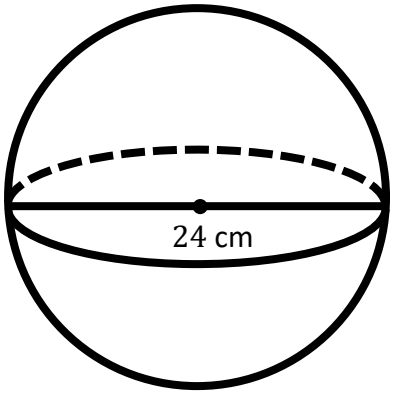
Worked Example

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



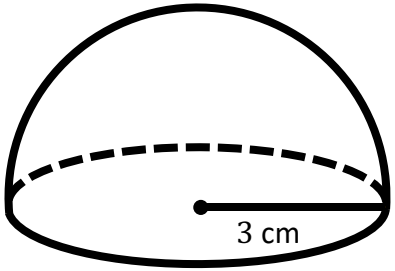
Your Turn

Calculate the surface area of the following sphere. Give your answer in terms of π and to 1 decimal place.



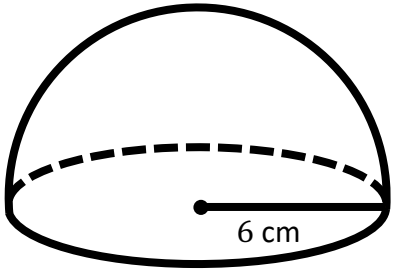
Worked Example

Calculate the total surface area of the following hemisphere.
Give your answer in terms of π and to 1 decimal place.



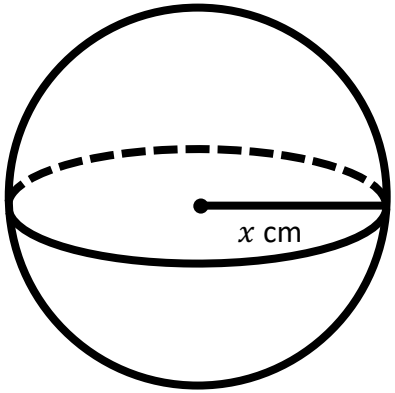
Your Turn

Calculate the total surface area of the following hemisphere.
Give your answer in terms of π and to 1 decimal place.



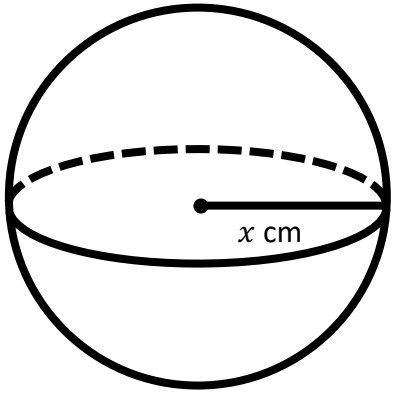
Worked Example

Find the radius, x, given that the surface area of the following sphere is $113.1\ cm^2$. Give your answer to 1 decimal place.



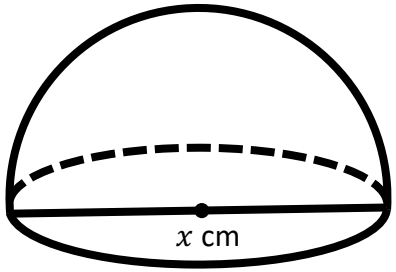
Your Turn

Find the radius, x, given that the surface area of the following sphere is $452.4\ cm^2$. Give your answer to 1 decimal place.



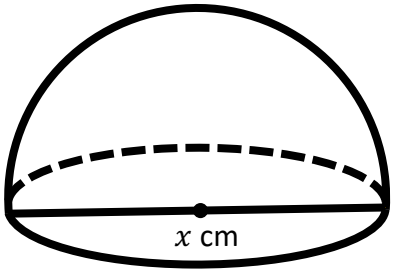
Worked Example

Find the diameter, x, given that the total surface area of the following hemisphere is $84.8\ cm^2$. Give your answer to 1 decimal place.



Your Turn

Find the diameter, x, given that the total surface area of the following hemisphere is $339.3\ cm^2$. Give your answer to 1 decimal place.

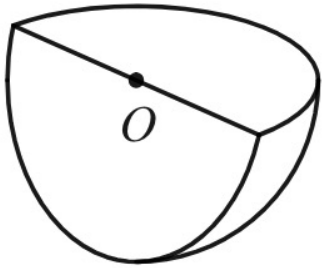


Worked Example	Your Turn
A sphere has a surface area of $36\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.	A sphere has a surface area of $144\pi\ cm^2$. Work out the volume of the sphere. Give your answer in terms of π and to 1 decimal place.

Worked Example	Your Turn
A sphere has a volume of $36\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.	A sphere has a volume of $288\pi\ cm^3$. Work out the surface area of the sphere. Give your answer in terms of π and to 1 decimal place.

Worked Example

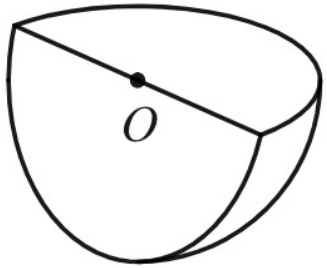
The diagram shows a solid quarter sphere, centre O.



The volume of the solid is 9750 cm^3.
Work out the surface area of the solid.
Give your answer to 1 decimal place.

Your Turn

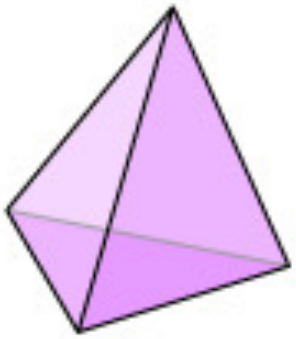
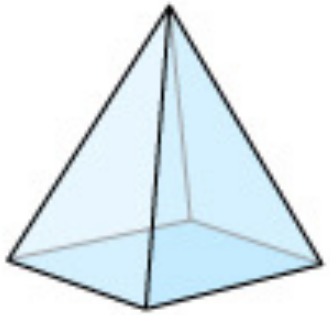
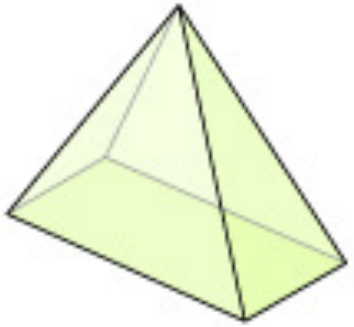
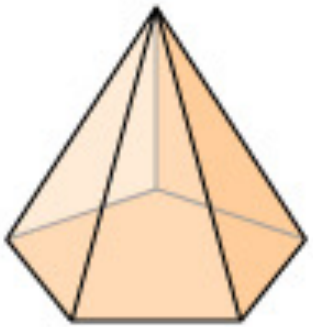
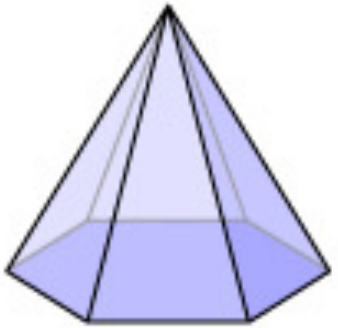
The diagram shows a solid quarter sphere, centre O.



The volume of the solid is 3500 cm^3.
Work out the surface area of the solid.
Give your answer to 1 decimal place.

Pyramids

A **pyramid** is a 3D shape with flat faces. The base of a pyramid is a polygon and is used to describe the pyramid (e.g. a square-based pyramid, triangle-based pyramid etc). Its sides are triangles which meet at the top.

Triangular	Square	Rectangular	Pentagonal	Hexagonal
				
4 Faces 4 Vertices 6 Edges	5 Faces 5 Vertices 8 Edges	5 Faces 5 Vertices 8 Edges	6 Faces 6 Vertices 10 Edges	7 Faces 7 Vertices 12 Edges

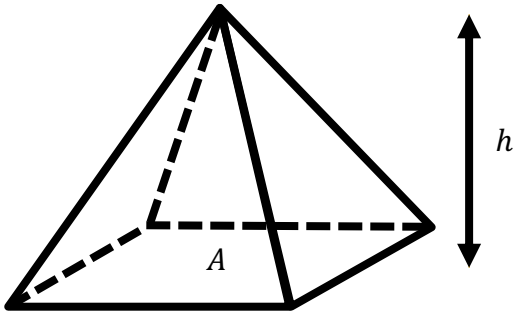
Frayer Model – Pyramid

Definition	**Characteristics**
Examples	**Non-Examples**

Volume of Pyramids

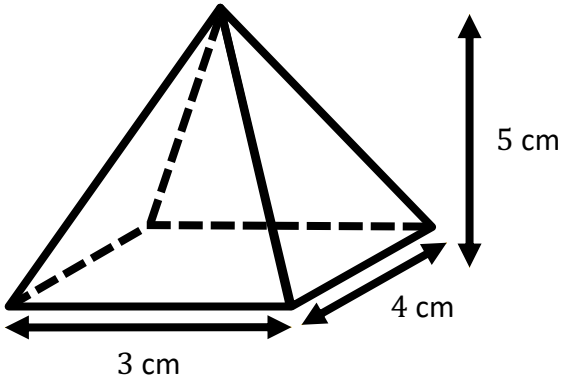
$$\text{Volume of Pyramid} = \frac{1}{3} \times \text{Area of Base} \times \text{Vertical Height}$$

$$V = \frac{1}{3}Ah$$



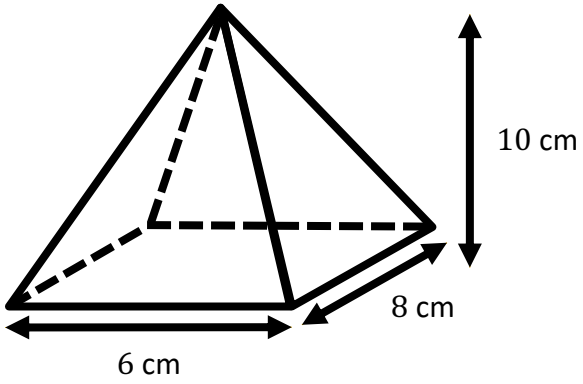
Worked Example

Calculate the volume of the following rectangular-based pyramid.



Your Turn

Calculate the volume of the following rectangular-based pyramid.



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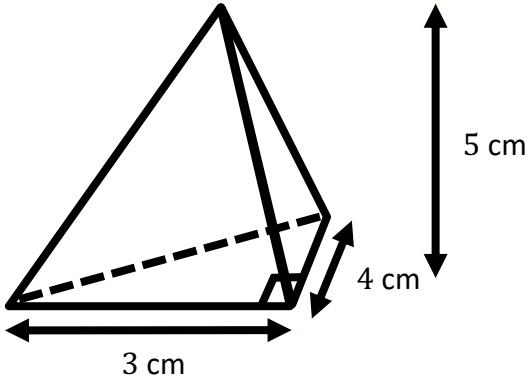
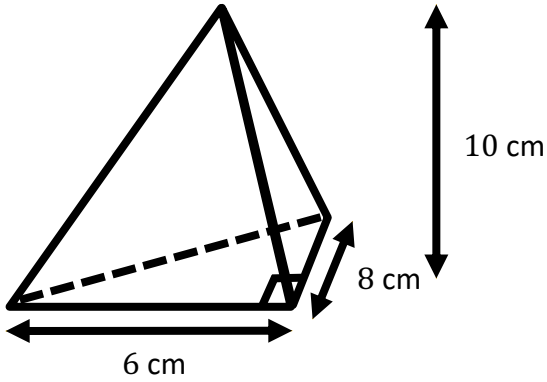
Worked Example

Calculate the volume of the following triangular-based pyramid.



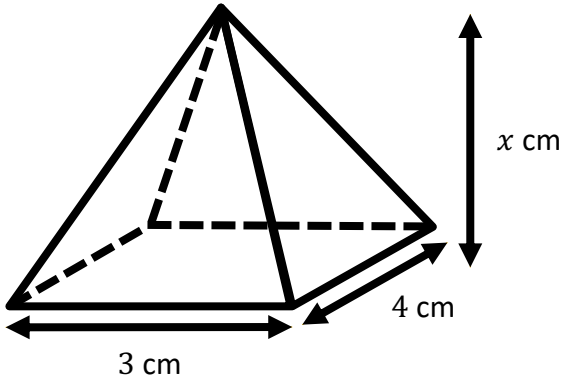
Your Turn

Calculate the volume of the following triangular-based pyramid.



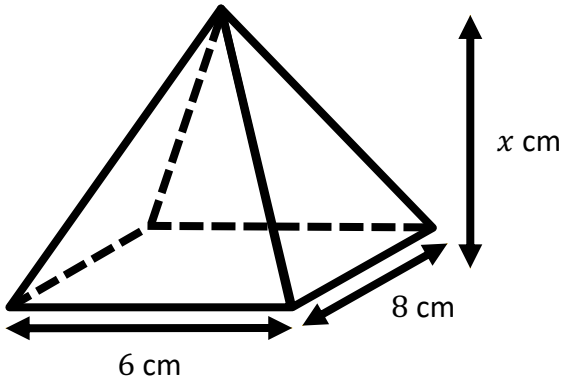
Worked Example

Find the height, x, given that the volume of the following rectangular-based pyramid is $20\ cm^3$.



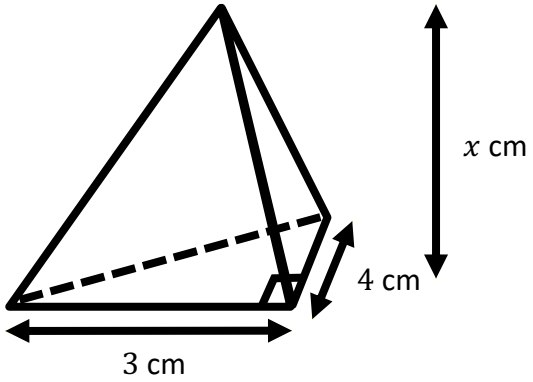
Your Turn

Find the height, x, given that the volume of the following rectangular-based pyramid is $160\ cm^3$.



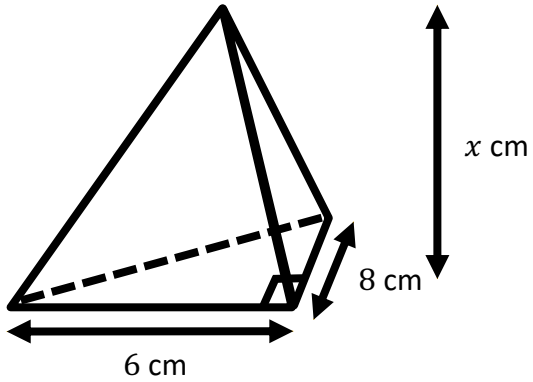
Worked Example

Find the height, x, given that the volume of the following triangular-based pyramid is $10\ cm^3$.



Your Turn

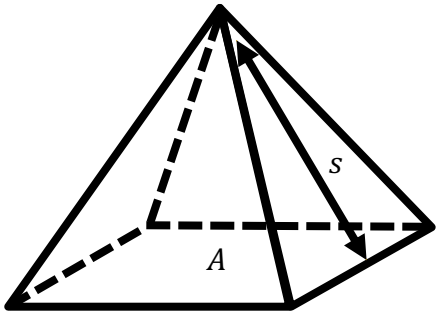
Find the height, x, given that the volume of the following triangular-based pyramid is $80\ cm^3$.



Surface Area of Pyramids

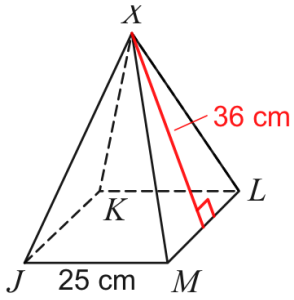
$$\text{Total Surface Area of Pyramid} = \frac{1}{2} \times \text{Perimeter of Base} \times \text{Slant Height} + \text{Area of Base}$$

$$\text{TSA} = \frac{1}{2}\text{Ps} + \text{A}$$



Worked Example

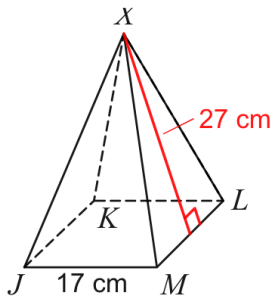
The diagram shows a square based pyramid $JKLMX$. The vertex, X, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 25 cm. The distance from X to the midpoint of each of the sides of the base is 36 cm. Find the total surface area of the pyramid.

Your Turn

The diagram shows a square based pyramid $JKLMX$. The vertex, X, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 17 cm. The distance from X to the midpoint of each of the sides of the base is 27 cm. Find the total surface area of the pyramid.

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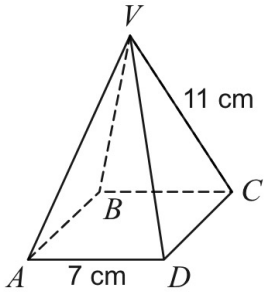
Worked Example

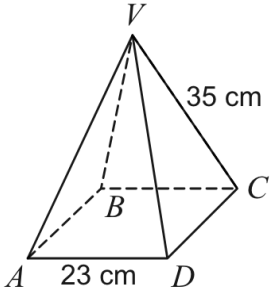
The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 7 cm. The length of each slanted edge is 11 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Your Turn

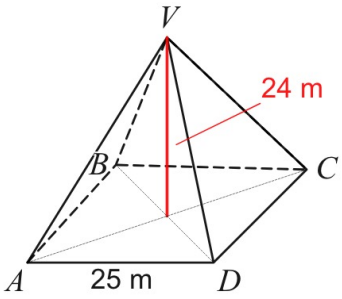
The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 23 cm. The length of each slanted edge is 35 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Worked Example

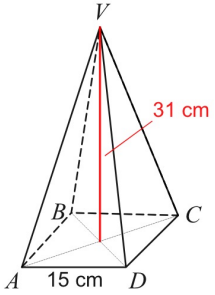
The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.



The length of each side of the base is 25 m. The height of the pyramid is 24 m. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Your Turn

The diagram shows a square based pyramid $ABCDV$. The vertex, V, of the pyramid is directly above the centre of the square base.

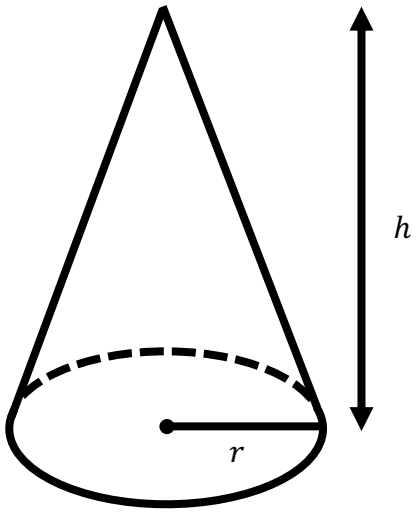


The length of each side of the base is 15 cm. The height of the pyramid is 31 cm. Find the total surface area of the pyramid. Give your answer to 1 decimal place.

Volume of Cones

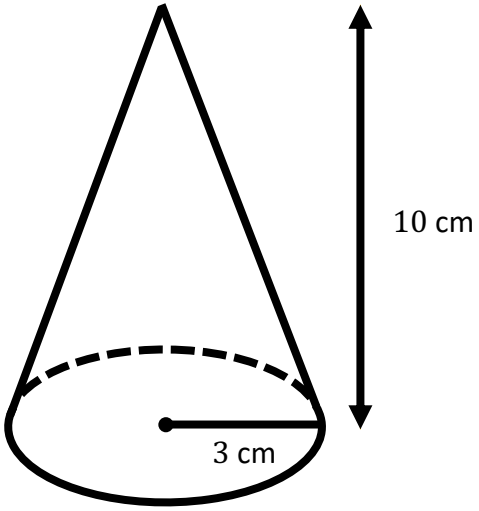
$$\text{Volume of Cone} = \frac{1}{3} \times \text{Area of Circle} \times \text{Height}$$

$$V = \frac{1}{3}\pi r^2 h$$



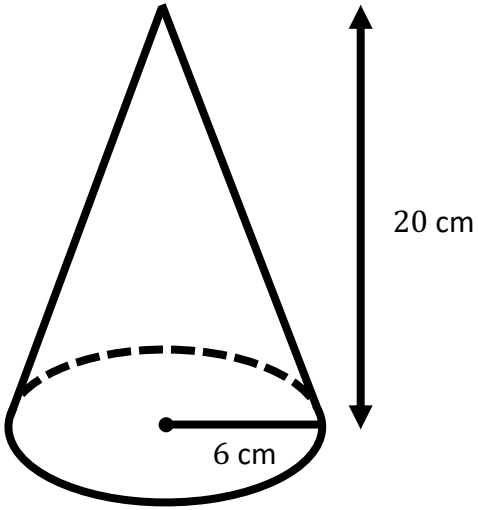
Worked Example

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



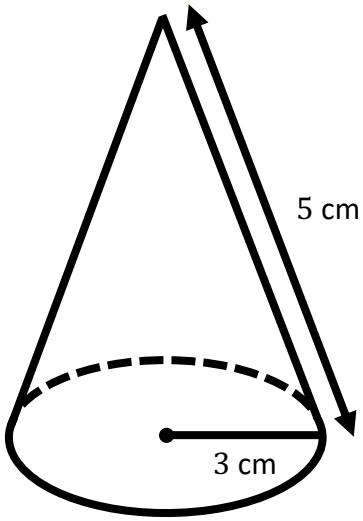
Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



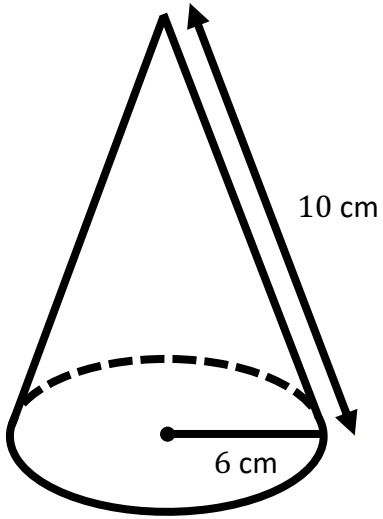
Worked Example

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



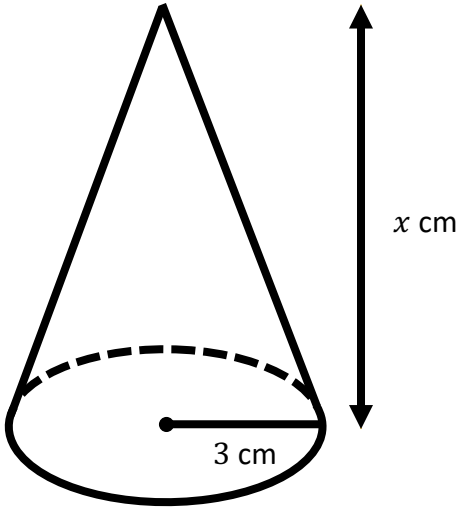
Your Turn

Calculate the volume of the following cone. Give your answer in terms of π and to 1 decimal place.



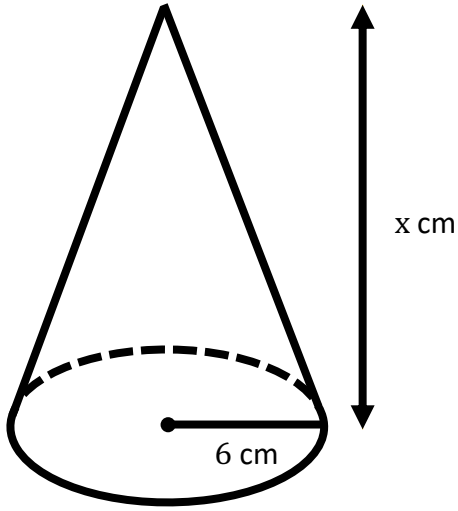
Worked Example

Find the height, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.



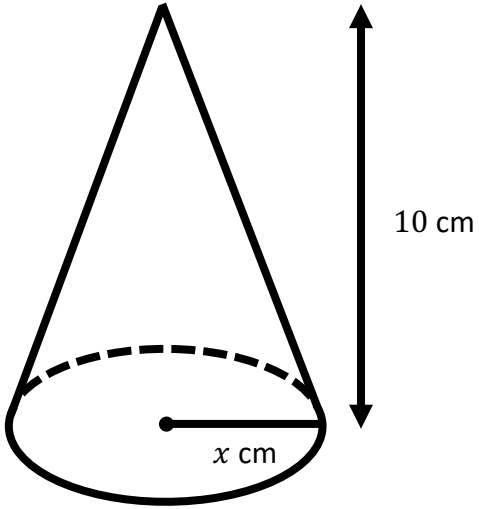
Your Turn

Find the height, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place.



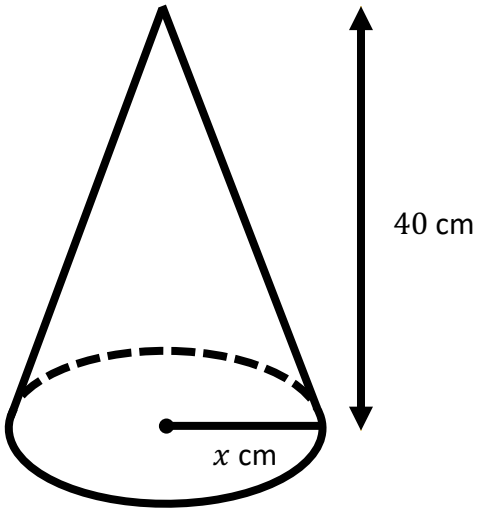
Worked Example

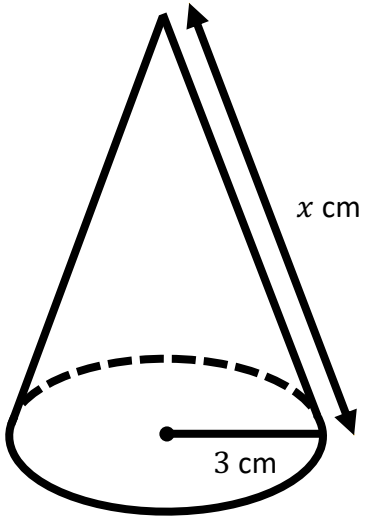
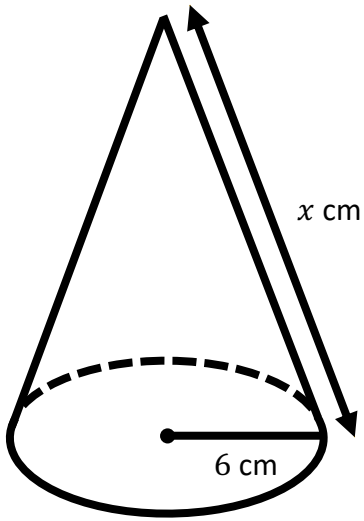
Find the radius, x, given that the volume of the following cone is $94.2\ cm^3$. Give your answer to 1 decimal place.

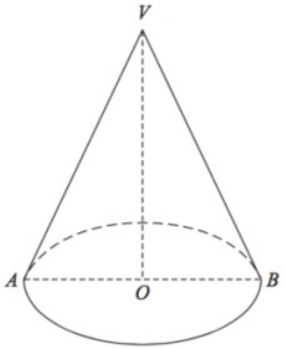
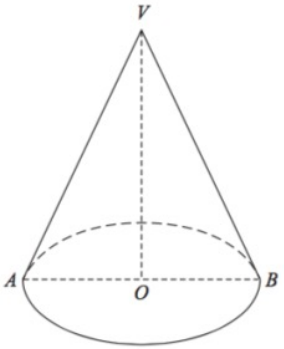


Your Turn

Find the radius, x, given that the volume of the following cone is $754.0\ cm^3$. Give your answer to 1 decimal place.



Worked Example	Your Turn
Find the slanted height, x, given that the volume of the following cone is $37.7\ cm^3$. Give your answer to 1 decimal place. 	Find the slanted height, x, given that the volume of the following cone is $301.6\ cm^3$. Give your answer to 1 decimal place. 

Worked Example	Your Turn
The diagram shows a solid cone. The base of the cone is a horizontal circle, centre O, with radius $9\ cm$. The curved surface area of the cone is $260\ cm^2$. Calculate the size of angle AVB. 	The diagram shows a solid cone. The base of the cone is a horizontal circle, centre O, with radius $4.5\ cm$. The curved surface area of the cone is $130\ cm^2$. Calculate the size of angle AVB. 

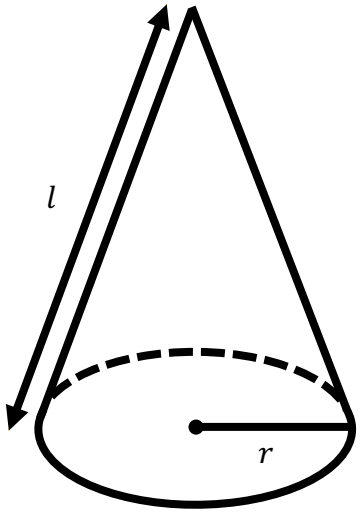
Surface Area of Cones

Curved Surface Area of Cone = π × Radius × Slanted Height

$CSA = \pi rl$

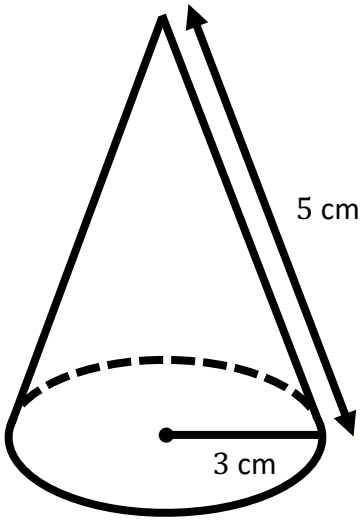
Total Surface Area of Cone = π × Radius × Slanted Height + π × Radius^2

$TSA = \pi rl + \pi r^2$



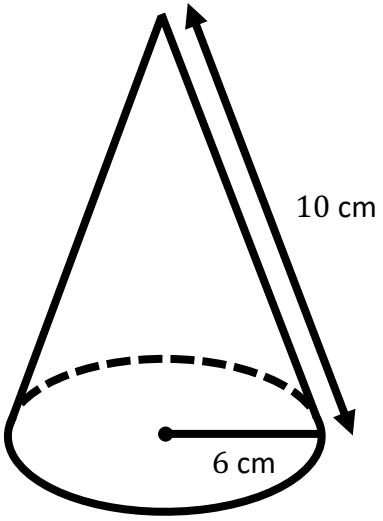
Worked Example

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



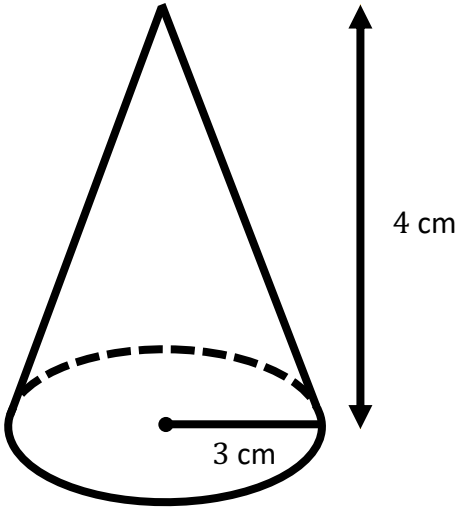
Your Turn

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



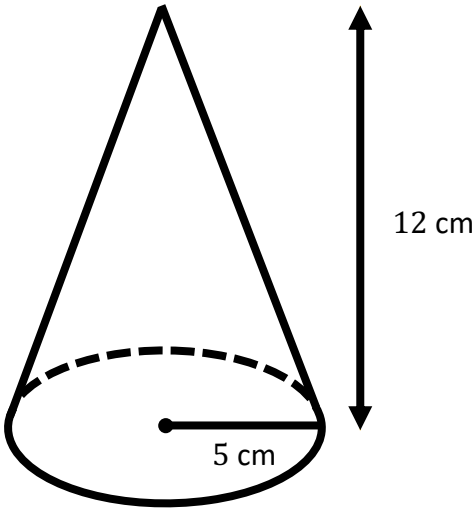
Worked Example

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



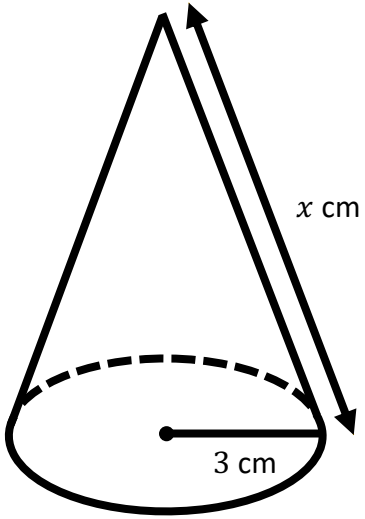
Your Turn

Calculate the total surface area of the following cone. Give your answer in terms of π and to 1 decimal place.



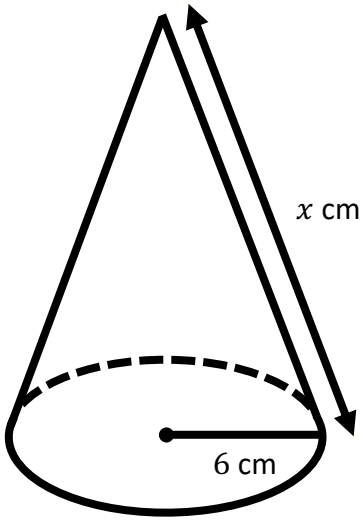
Worked Example

Find the slanted height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



Your Turn

Find the slanted height, x, given that the total surface area of the following cone is $301.6\ cm^2$. Give your answer to 1 decimal place.



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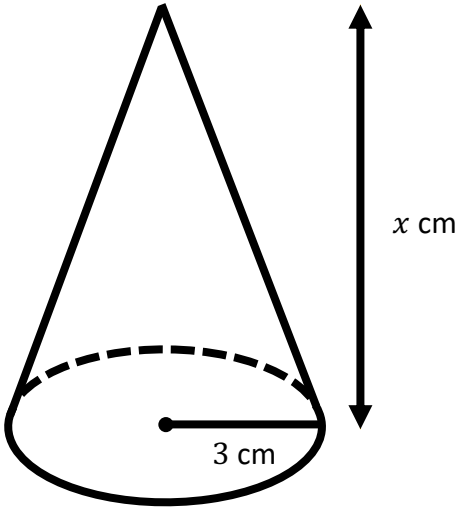
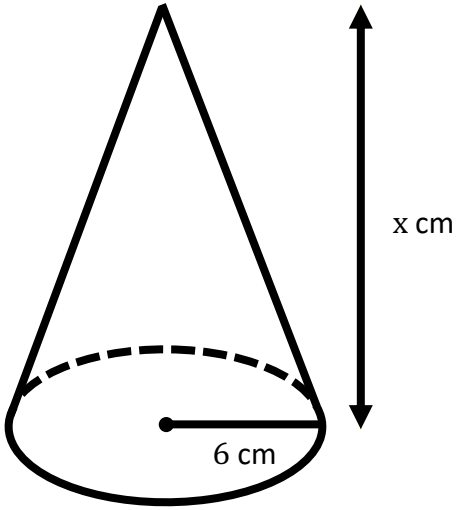
Worked Example

Find the perpendicular height, x, given that the total surface area of the following cone is $75.4\ cm^2$. Give your answer to 1 decimal place.



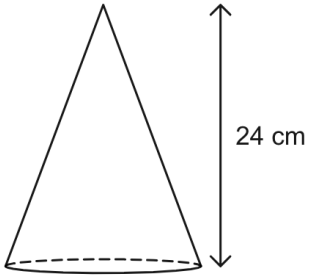
Your Turn

Find the perpendicular height, x, given that the total surface area of the following cone is $301.6\ cm^2$. Give your answer to 1 decimal place.



Worked Example

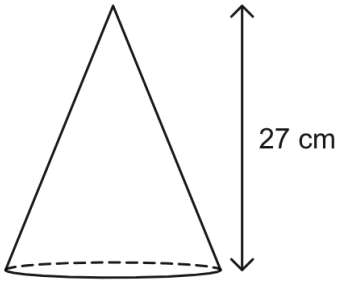
The diagram shows a cone with height 24 cm.



The volume of the cone is 2000 cm^3.
Find the total surface area of the cone.
Give your answer to one decimal place.

Your Turn

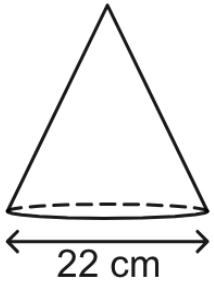
The diagram shows a cone with height 27 cm.



The volume of the cone is 3400 cm^3.
Find the total surface area of the cone.
Give your answer to one decimal place.

Worked Example

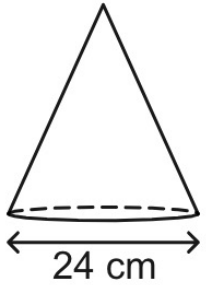
The diagram shows a cone with diameter 22 cm.



The total surface area of the cone is 1200 cm^2.
Find the volume of the cone.
Give your answer to one decimal place.

Your Turn

The diagram shows a cone with diameter 24 cm.



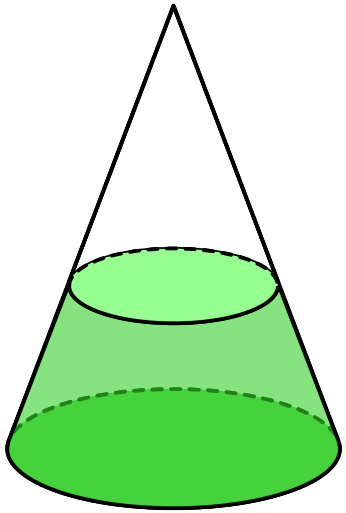
The total surface area of the cone is 1500 cm^2.
Find the volume of the cone.
Give your answer to one decimal place.

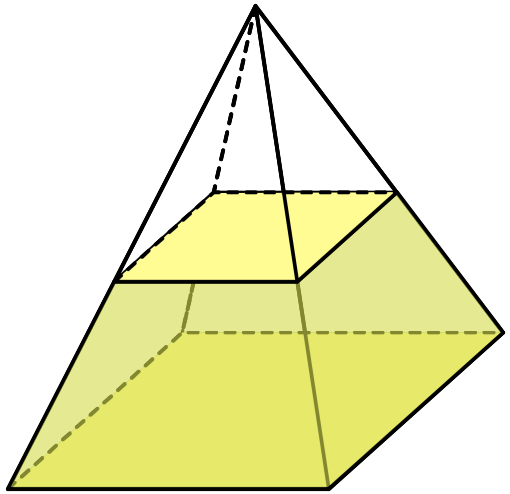
Fill in the Gaps

Radius r	Vertical Height h	Slanted Height l	Volume in terms of π	Volume to 3 s.f.	Curved Surface Area in terms of π	Total Surface Area in terms of π	Volume : Total Surface Area
$5\ cm$	$12\ cm$	$13\ cm$	$100\pi\ cm^3$			$90\pi\ cm^2$	$10 : 9$
$6\ cm$	$8\ cm$	$10\ cm$			$60\pi\ cm^2$		
	$30\ mm$	$34\ mm$		$8040\ mm^3$			
$0.7\ m$	$2.4\ m$						
$9\ cm$		$15\ cm$					
$2\ m$			$\frac{14}{5}\pi\ cm^3$				
		$20\ mm$			$240\pi\ mm^2$		
					$15\pi\ cm^2$	$24\pi\ cm^2$	
		$17\ cm$	$320\pi\ cm^3$				$8 : 5$

Frustums

A **frustum** is a pyramid/cone with part of the top chopped off.

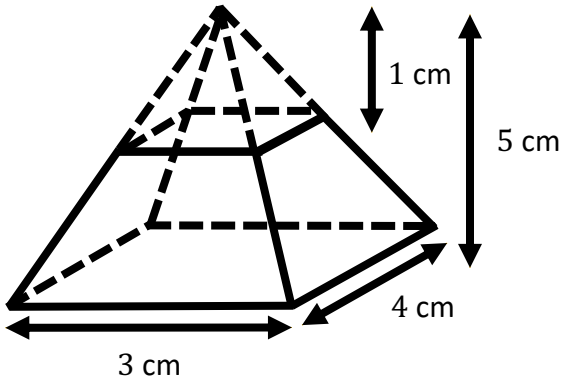




Volume of Frustums

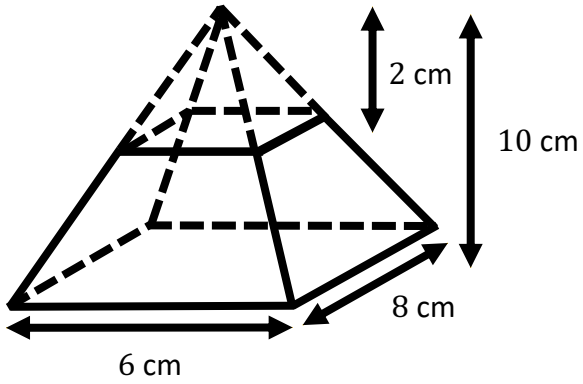
Worked Example

Calculate the volume of the following frustum.



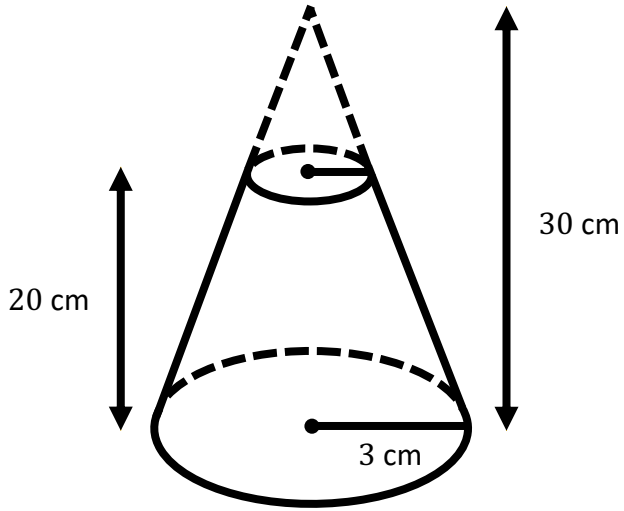
Your Turn

Calculate the volume of the following frustum.



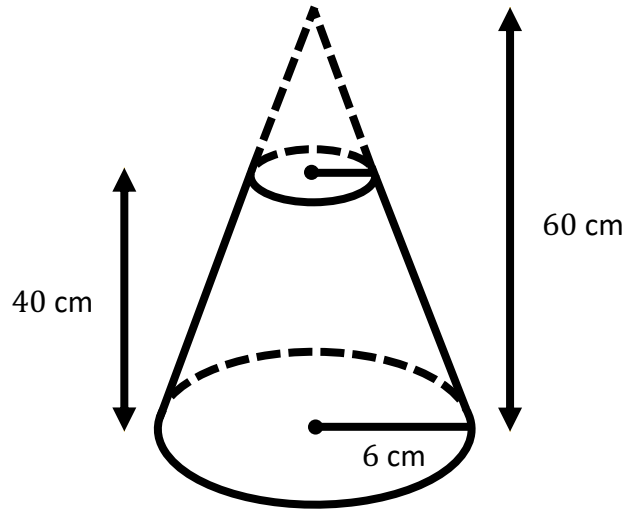
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Your Turn

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



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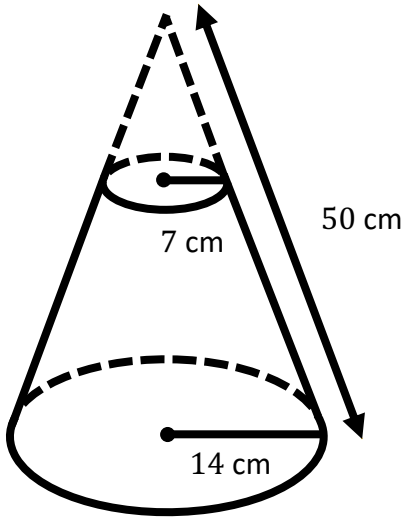
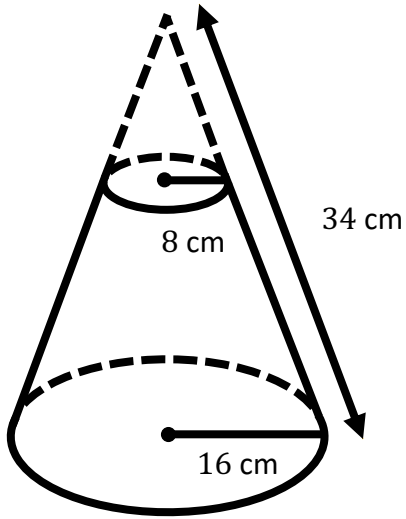
Worked Example

Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Your Turn

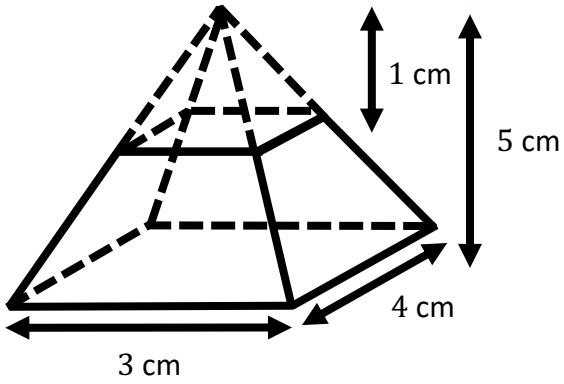
Calculate the volume of the following frustum. Give your answer in terms of π and to 1 decimal place.



Surface Area of Frustums

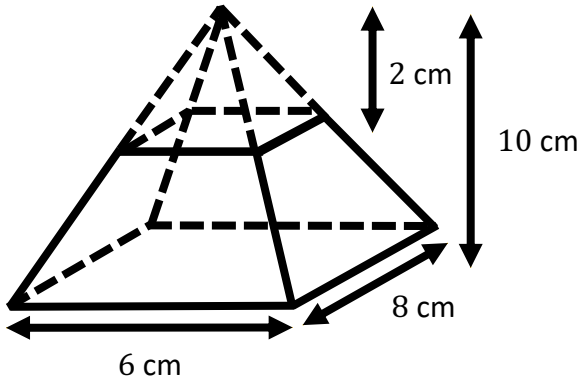
Worked Example

Calculate the total surface area of the following frustum.



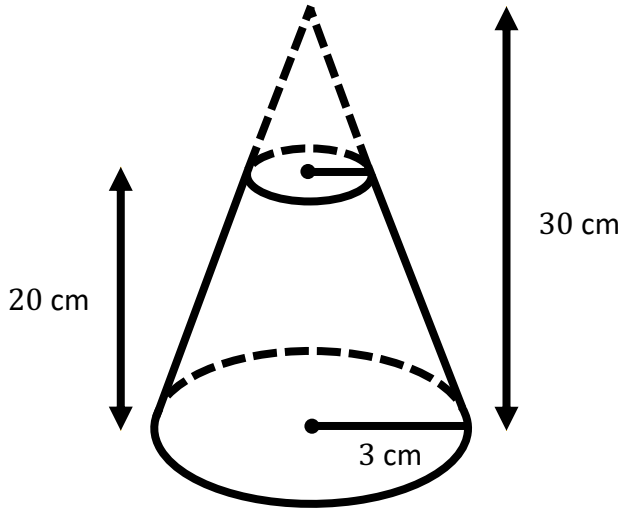
Your Turn

Calculate the total surface area of the following frustum.



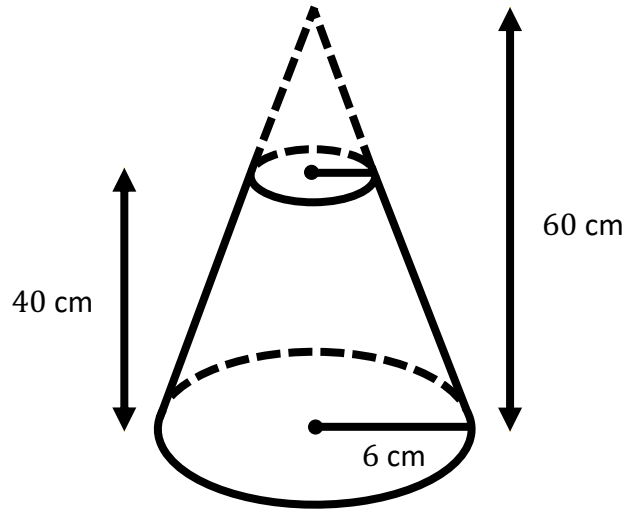
Worked Example

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



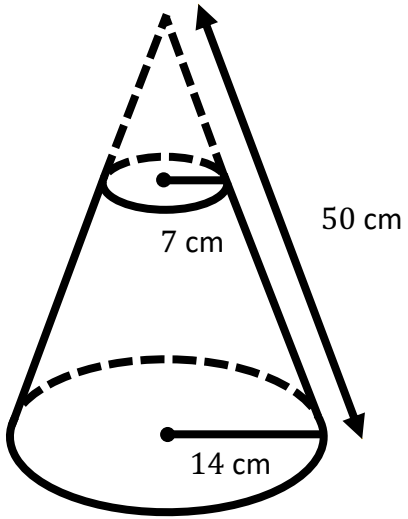
Your Turn

Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



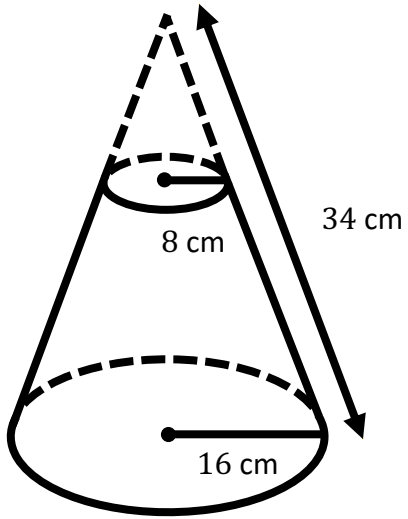
Worked Example

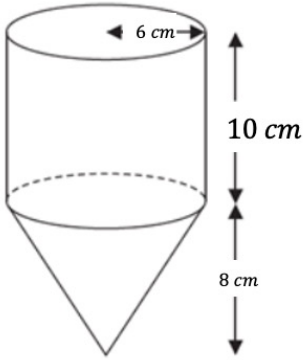
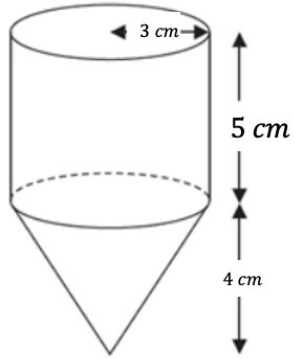
Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



Your Turn

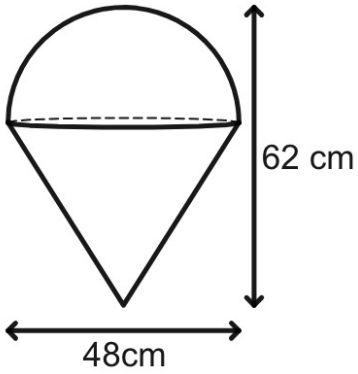
Calculate the total surface area of the following frustum. Give your answer in terms of π and to 1 decimal place.



Worked Example	Your Turn
Calculate the volume of this composite shape.  	Calculate the volume of this composite shape. 

Worked Example

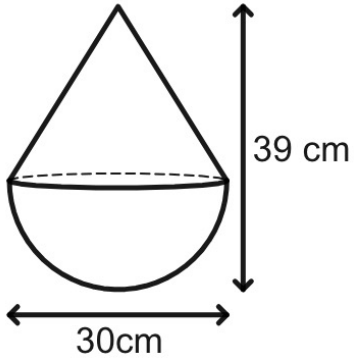
A child's toy is in the shape of a cone with a hemisphere on top, as shown below. The toy is 48 cm wide and 62 cm high.



Calculate the volume of the toy. Give your answer correct to two significant figures.

Your Turn

A child's toy is in the shape of a hemisphere with a cone on top, as shown below. The toy is 30 cm wide and 39 cm high.



Calculate the volume of the toy. Give your answer correct to two significant figures.

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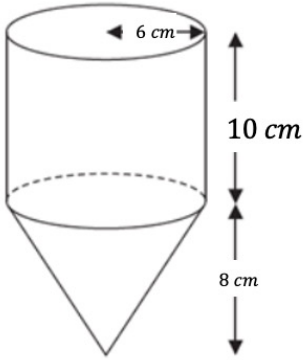
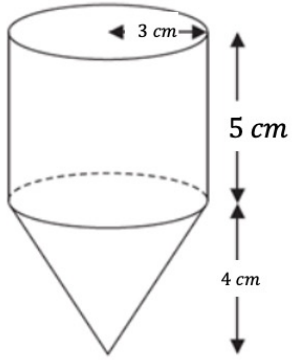
Worked Example

Calculate the total surface area of this composite shape.



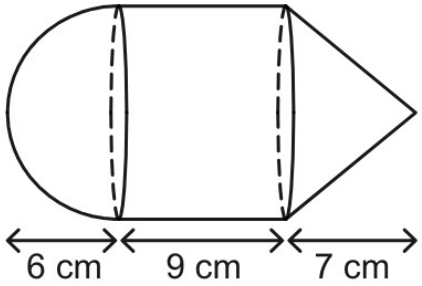
Your Turn

Calculate the total surface area of this composite shape.



Worked Example

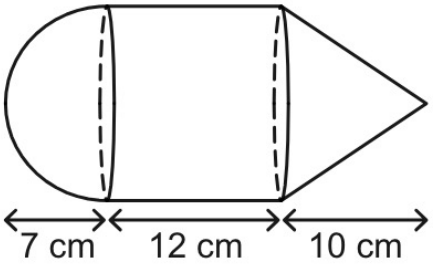
The diagram shows a solid formed from a hemisphere, a cylinder and a cone.



Find the total surface area of the solid.
Give your answer to one decimal place.

Your Turn

The diagram shows a solid formed from a hemisphere, a cylinder and a cone.



Find the total surface area of the solid.
Give your answer to one decimal place.

Extra Notes

3 Arcs, Sectors and Segments

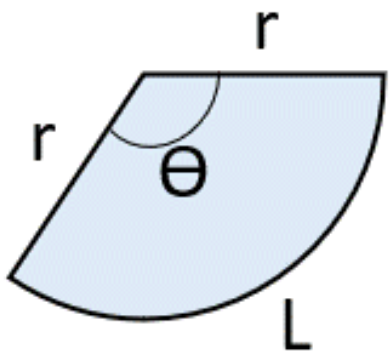
Arc Length and Perimeter of Sectors and Segments

Arc Length of a Sector $= \frac{\text{Angle}}{360} \times \pi \times \text{Diameter}$

$$\mathbf{L = \frac{\theta}{360} \times \pi d}$$

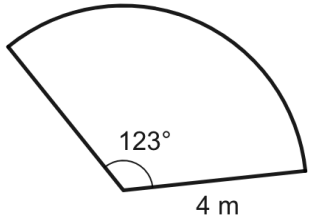
Perimeter of a Sector $= \frac{\text{Angle}}{360} \times \pi \times \text{Diameter} + 2 \times \text{Radius}$

$$\mathbf{P = \frac{\theta}{360} \times \pi d + 2r}$$



Worked Example

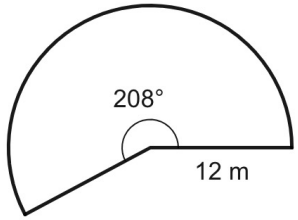
Find the arc length of the sector.



Give your answer correct to 1 decimal place.

Your Turn

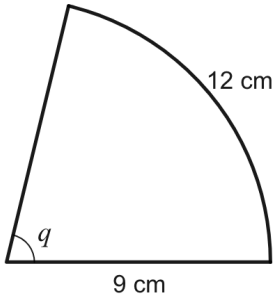
Find the arc length of the sector.



Give your answer correct to 1 decimal place.

Worked Example

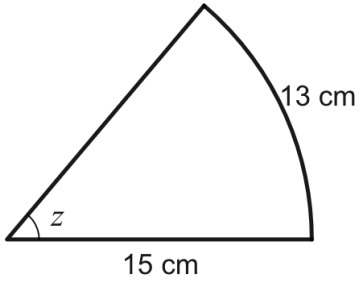
Given that the arc length of sector below is 12 cm, work out its angle, marked q on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

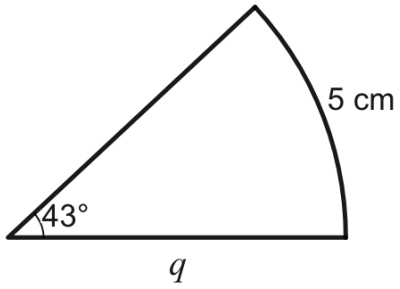
Given that the arc length of sector below is 13 cm, work out its angle, marked z on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

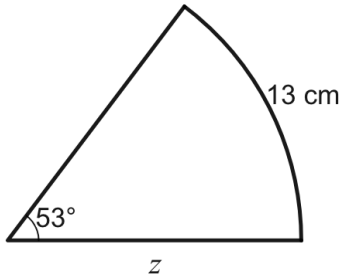
Given that the arc length of sector below is 5 cm, work out its radius, marked q on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

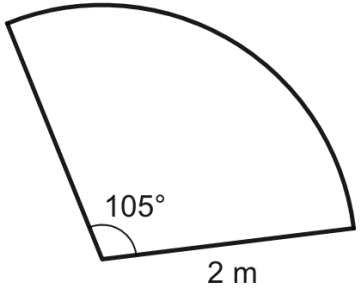
Given that the arc length of sector below is 13 cm, work out its radius, marked z on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

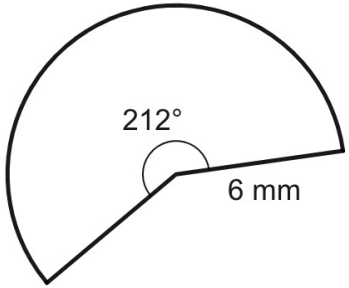
Work out the perimeter of the sector.



Give your answer correct to 1 decimal place.

Your Turn

Work out the perimeter of the sector.

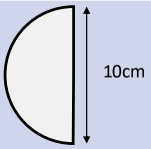
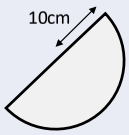


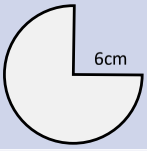
Give your answer correct to 1 decimal place.

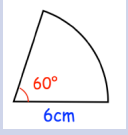
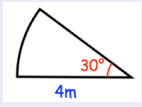
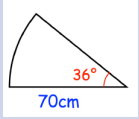
Fill in the Gaps

Radius	Angle	Arc Length	Perimeter
$8\ cm$	$90°$	$\frac{90}{360} \times \pi \times 2 \times 8 = 12.6\ cm$	$28.6\ cm$
$7\ cm$	$45°$	$\frac{45}{360} \times \pi \times 2 \times 7 = 5.5\ cm$	
$15\ mm$	$60°$	$\frac{60}{360} \times \pi \times 2 \times 15 = 15.7\ mm$	
$4\ cm$	$75°$		
$1.8\ m$	$130°$		
$82\ mm$	$335°$		
$11\ cm$	$275°$		
$9\ mm$	$32°$		
$10\ cm$		$\frac{\square}{360} \times \pi \times 2 \times 10 = 13.96\ cm$	
$25\ mm$		$\frac{\square}{360} \times \pi \times 2 \times 25 = 93.81\ mm$	
$2\ m$		$\frac{\square}{360} \times \pi \times 2 \times 2 = \square\ m$	$5.05\ m$
$8.9\ cm$		$\frac{\square}{360} \times \pi \times 2 \times 8.9 = \square\ cm$	$35.2\ m$
		$\frac{\square}{360} \times \pi \times 2 \times \square = 4.61\ cm$	$15.61\ cm$
		$\frac{\square}{360} \times \pi \times 2 \times \square = 55.29\ mm$	$99.29\ mm$

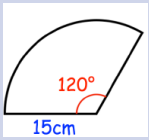
Fill in the Gaps

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
	$5cm$	$10cm$	$\frac{1}{2}$	$5\pi = 15.7cm$	$10cm$	$(5\pi + 10)\ cm$	$25.7cm$
							
							

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
		$4cm$	$\frac{1}{4}$				
	$3cm$		$\frac{1}{2}$				

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
							
							

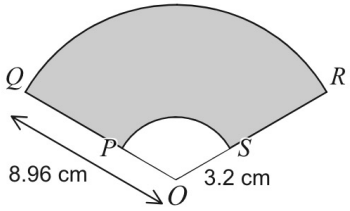
Fill in the Gaps

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
							
		$10cm$	$\frac{1}{3}$				
	$10cm$		$\frac{1}{8}$				

Sector.	Radius.	Diameter.	Fraction of the whole circle.	Arc length.	Length of straight sides.	Perimeter in terms of pi.	Perimeter to 1 decimal place.
			$\frac{1}{2}$		$7cm$		
			$\frac{1}{3}$			$(4\pi + 12)\ cm$	
			$\frac{1}{10}$		$40cm$		$46.3cm$

Worked Example

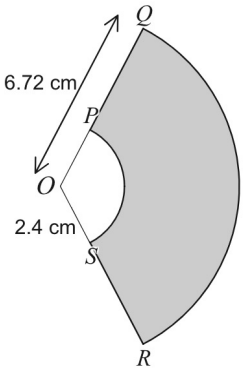
A shape is formed from the sectors of two circles with centre O.
OPQ and OSR are straight lines.



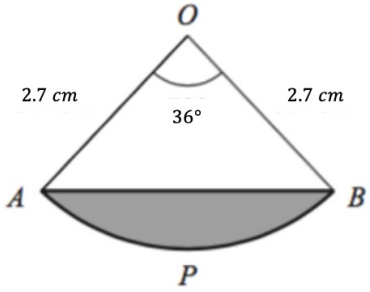
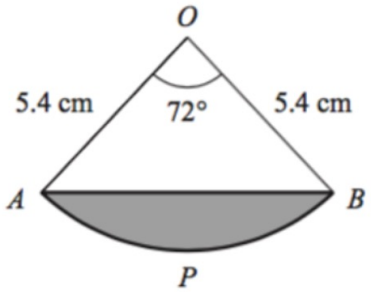
The length of OS is 3.2 cm.
The length of OQ is 8.96 cm.
Angle POS is $120°$.
Calculate the perimeter of the shaded region.
Give your answer to 1 decimal place.

Your Turn

A shape is formed from the sectors of two circles with centre O.
OPQ and OSR are straight lines.



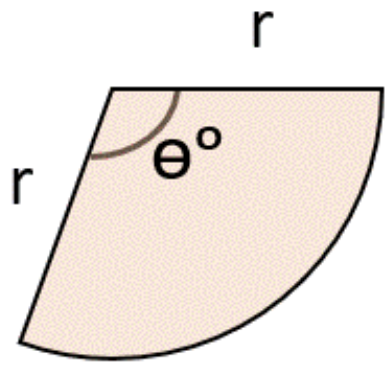
The length of OS is 2.4 cm.
The length of OQ is 6.72 cm.
Angle POS is $125°$.
Calculate the perimeter of the shaded region.
Give your answer to 1 decimal place.

Worked Example	Your Turn
Calculate the perimeter of the shaded segment APB.  	Calculate the perimeter of the shaded segment APB. 

Area of Sectors and Segments

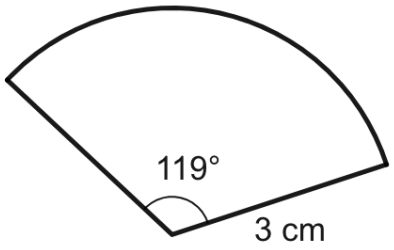
$$\text{Area of a Sector} = \frac{\text{Angle}}{360} \times \pi \times \text{Radius}^2$$

$$\mathbf{A = \frac{\theta}{360} \times \pi r^2}$$



Worked Example

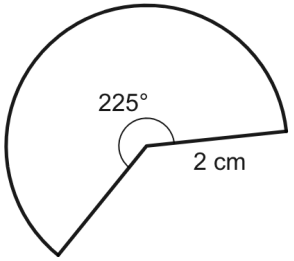
Find the area of the sector.



Give your answer correct to 1 decimal place.

Your Turn

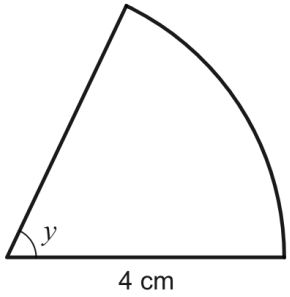
Find the area of the sector.



Give your answer correct to 1 decimal place.

Worked Example

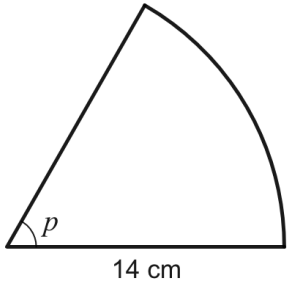
Given that the area of sector below is 9 cm^2, work out its angle, marked y on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

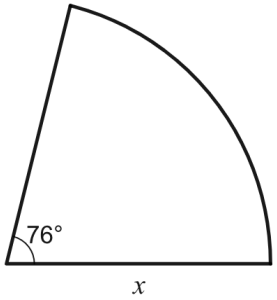
Given that the area of sector below is 103 cm^2, work out its angle, marked p on the diagram.



Give your answer correct to 1 decimal place.

Worked Example

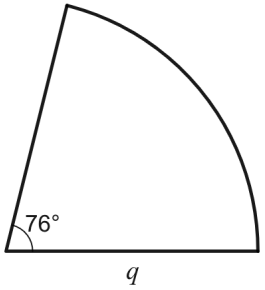
Given that the area of sector below is 17 cm^2, work out its radius, marked x on the diagram.



Give your answer correct to 1 decimal place.

Your Turn

Given that the area of sector below is 97 cm^2, work out its radius, marked q on the diagram.



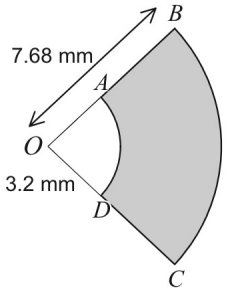
Give your answer correct to 1 decimal place.

Fill in the Gaps

Radius	Angle	Fraction	Area
$8\ cm$	$90°$	$\frac{90}{360} = \frac{1}{4}$	$\frac{90}{360} \times \pi \times 8^2 = 50.3\ cm^2$
$7\ cm$	$45°$	$\frac{45}{360} = \frac{1}{8}$	
$15\ mm$	$60°$		
$4\ cm$	$75°$		
$1.8\ m$	$130°$		
$82\ mm$	$335°$		
$11\ cm$	$275°$		
$9\ mm$		$\frac{\square}{360} = \frac{5}{36}$	
$10\ cm$		$\frac{\square}{360} = \frac{7}{9}$	
$25\ mm$			$\frac{\square}{360} \times \pi \times 25^2 = 327.2\ mm^2$
$2\ m$			$\frac{\square}{360} \times \pi \times 2^2 = 4.712\ m^2$
	$35°$		$\frac{35}{360} \times \pi \times \square^2 = 2.75\ cm^2$
	$315°$		$\frac{315}{360} \times \pi \times \square^2 = 464.6\ mm^2$
	$58°$		$\frac{58}{360} \times \pi \times \square^2 = 50.61\ cm^2$

Worked Example

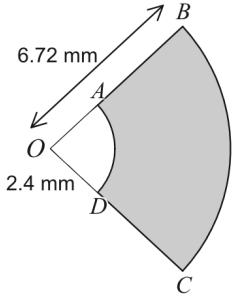
A metal component for a machine is formed from the sectors of two circles with centre O. The area of sector OAD is 7.7 mm^2.



OAB and ODC are straight lines.
The length of OD is 3.2 mm.
The length of OB is 7.68 mm.
Calculate the area of the shaded region.

Your Turn

A metal component for a machine is formed from the sectors of two circles with centre O. The area of sector OAD is 4.3 mm^2.

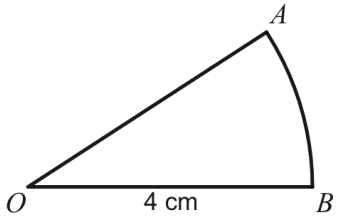


OAB and ODC are straight lines.
The length of OD is 2.4 mm.
The length of OB is 6.72 mm.
Calculate the area of the shaded region.

Dr Frost 319d

Worked Example

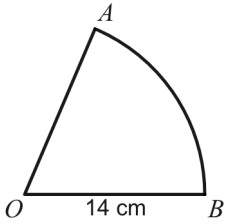
AOB is a sector of a circle, centre O and radius 4 cm.



The perimeter of the sector is 10 cm.
Work out the area of the sector.

Your Turn

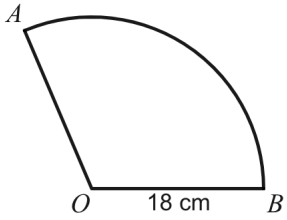
AOB is a sector of a circle, centre O and radius 14 cm.



The perimeter of the sector is 44 cm.
Work out the area of the sector.

Worked Example

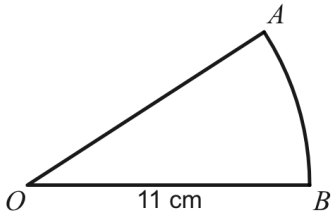
AOB is a sector of a circle, centre O and radius 18 cm.



The area of the sector is 320 cm^2.
Work out the perimeter of the sector.
Give your answer correct to 2 decimal places.

Your Turn

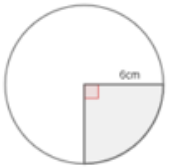
AOB is a sector of a circle, centre O and radius 11 cm.



The area of the sector is 35 cm^2.
Work out the perimeter of the sector.
Give your answer correct to 2 decimal places.

Dr Frost 319f

Fill in the Gaps

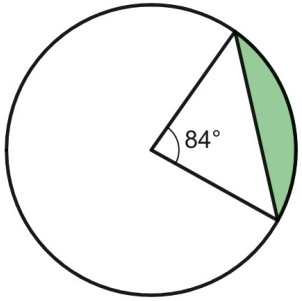
	Sketch	*Radius*	*Angle*	$\frac{Angle}{360}$ *(simplified)*	*Circumference of whole circle*	*Arc length*	*Area of whole circle*	*Area of shaded sector*
1		6	180^o	$\frac{1}{2}$	12π	6π		
2								
3			45^o		12π			
4		6		$\frac{3}{4}$				
5		6				2π		

Fill in the Gaps

	Sketch	*Radius*	*Angle*	$\frac{Angle}{360}$ *(simplified)*	*Circumference of whole circle*	*Arc length*	*Area of whole circle*	*Area of shaded sector*
6							36π	12π
7			330^o		12π			
8			330^o		24π			
9					24π	2π		
10						6π		36π

Worked Example

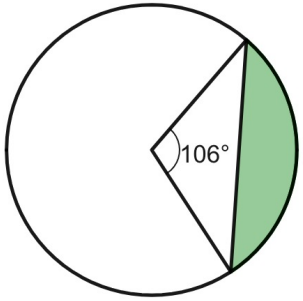
The radius of the circle is 6 cm.



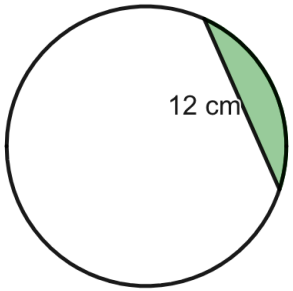
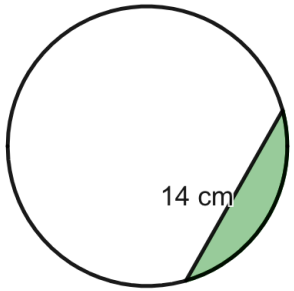
Find the area of the shaded region.

Your Turn

The radius of the circle is 15 cm.

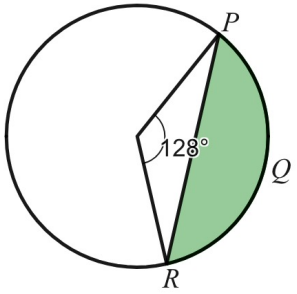


Find the area of the shaded region.

Worked Example	Your Turn
The radius of the circle is 9 cm.  Find the area of the shaded region.	The radius of the circle is 10 cm.  Find the area of the shaded region.

Worked Example

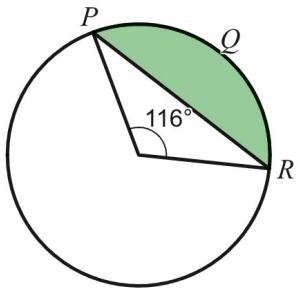
The area of the shaded region is 289 cm^2.



Find the length of the arc PQR.

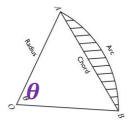
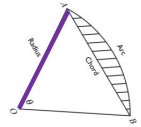
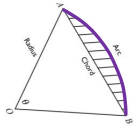
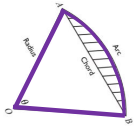
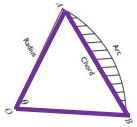
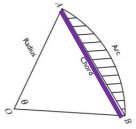
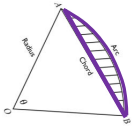
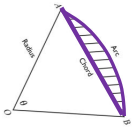
Your Turn

The area of the shaded region is 225 cm^2.

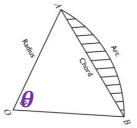
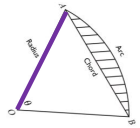
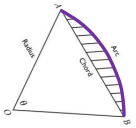
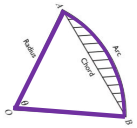
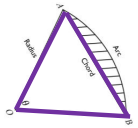
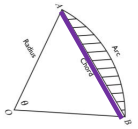
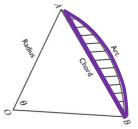
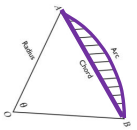


Find the length of the arc PQR.

Fill in the Gaps

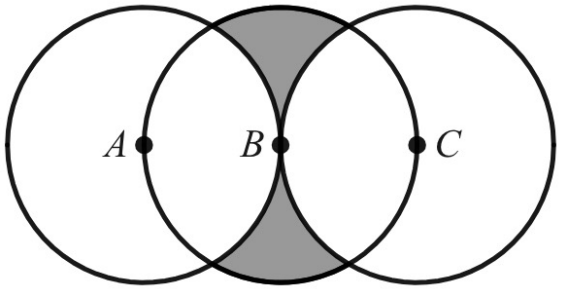
 Angle θ^c	 Radius $r\ cm$	 Arc Length	 Area of Sector	 Area of Triangle	 Length of Chord	 Area of Segment	 Perimeter of Segment
$\theta = 1.5^c$	$r = 10cm$	Arc Length $= r\theta$ $\rightarrow (10)(1.5)$ $= 15cm$	Sector Area $= \frac{1}{2}r^2\theta$ $\rightarrow \frac{1}{2}(10)^2(1.5)$ $= 75cm^2$	Triangle Area $= \frac{1}{2}ab\sin C$ $\rightarrow \frac{1}{2}(10)(10)\sin(1.5)$ $= 49.87cm^2$	Cosine Rule $a^2 = b^2 + c^2 - 2bc\cos A$ $a^2 = 10^2 + 10^2 - 2(10)(10)\cos(1.5)$ $a^2 = 185.853\ldots$ $a = 13.63cm$	Area = Sector – Triangle $\rightarrow 75 - 49.87$ $= 25.13cm^2$	Perimeter = Chord + Arc $\rightarrow 13.63 + 15$ $= 28.63cm$
1) 0.8^c	$7cm$						
2)	$20cm$	$60cm$					
3) 1.2^c			$21.6cm^2$				
4)		$6\pi\ cm$	$72\pi\ cm^2$				

Fill in the Gaps

 Angle θ^c	 Radius $r\ cm$	 Arc Length	 Area of Sector	 Area of Triangle	 Length of Chord	 Area of Segment	 Perimeter of Segment
$\theta = 1.5^c$	$r = 10cm$	**Arc Length** $= r\theta$ $\rightarrow (10)(1.5)$ $= 15cm$	**Sector Area** $= \frac{1}{2}r^2\theta$ $\rightarrow \frac{1}{2}(10)^2(1.5)$ $= 75cm^2$	**Triangle Area** $= \frac{1}{2}ab\sin C$ $\rightarrow \frac{1}{2}(10)(10)\sin(1.5)$ $= 49.87cm^2$	**Cosine Rule** $a^2 = b^2 + c^2 - 2bc\cos A$ $a^2 = 10^2 + 10^2 - 2(10)(10)\cos(1.5)$ $a^2 = 185.853\ldots$ $a = 13.63cm$	Area = Sector – Triangle $\rightarrow 75 - 49.87$ $= 25.13cm^2$	Perimeter = Chord + Arc $\rightarrow 13.63 + 15$ $= 28.63cm$
5)	$8cm$				$8cm$		
6)	$15cm$			$94.67cm^2$			
7)	$11cm$				$5.694cm$		
8) 2^c						$6.681cm^2$	

Worked Example

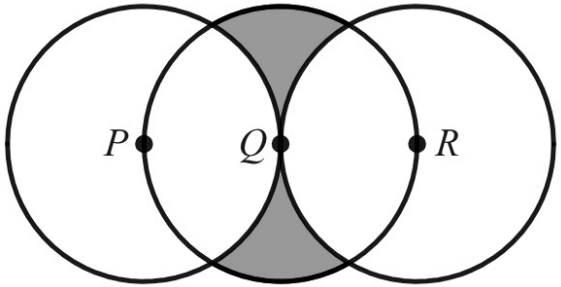
The diagram shows three circles, each of radius 9 cm.
The centres of the circles are A, B and C such that ABC is a straight line and $AB = BC = 9$ cm.



Work out the total area of the two shaded regions.
Give your answer in terms of π.

Your Turn

The diagram shows three circles, each of radius 6 cm.
The centres of the circles are P, Q and R such that PQR is a straight line and $PQ = QR = 6$ cm.



Work out the total area of the two shaded regions.
Give your answer in terms of π.

Dr Frost 467q

Extra Notes

4 Advanced Probability

Venn Diagrams

Sets

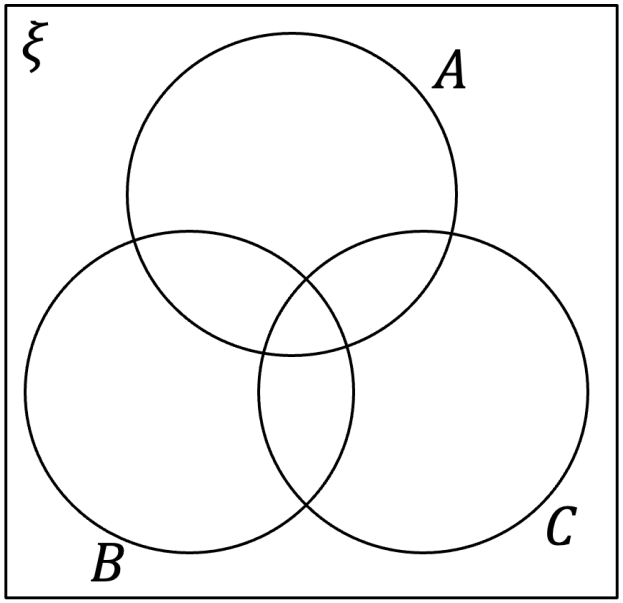
Worked Example

$\boldsymbol{\xi}$ = whole numbers from 1 to 15 inclusive

$\boldsymbol{A}$ = set of all prime numbers

$\boldsymbol{B}$ = set of all numbers one less than a power of 2

$\boldsymbol{C}$ = set of all square numbers



Your Turn

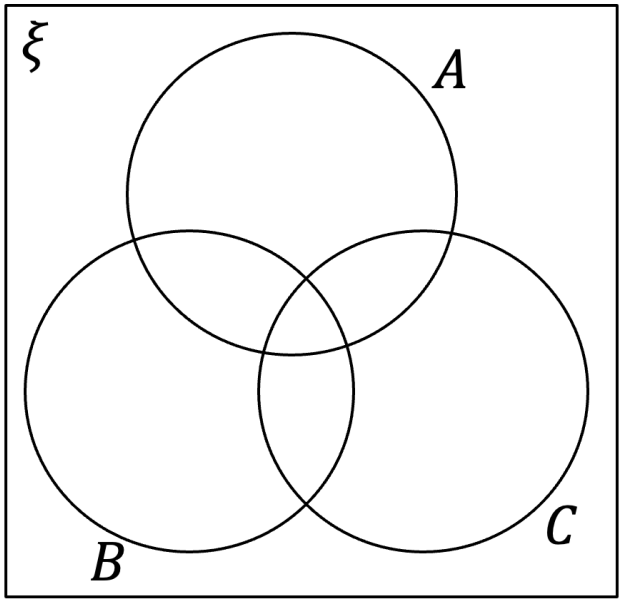
$\boldsymbol{\xi}$ = whole numbers from 1 to 10 inclusive

$\boldsymbol{A}$ = set of all cube numbers

$\boldsymbol{B}$ = set of all odd numbers

$\boldsymbol{C}$ = set of all multiples of 3

Bonus: If we extended ξ to include more positive integers, what is the smallest number that would appear in all three of A, B, C?



Worked Example

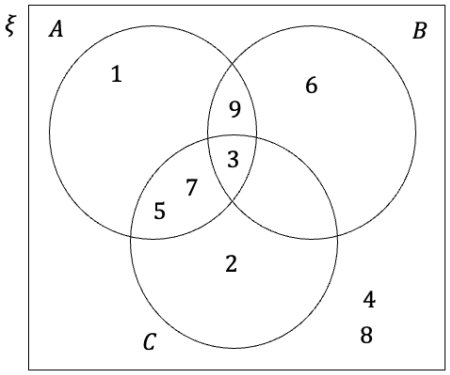
From the Venn diagram below, describe the sets:

$\xi =$

$A =$

$B =$

$C =$



Your Turn

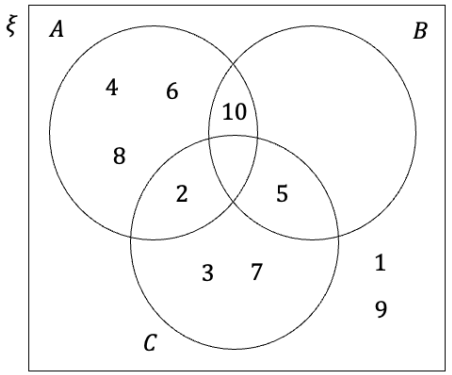
From the Venn diagram below, describe the sets:

$\xi =$

$A =$

$B =$

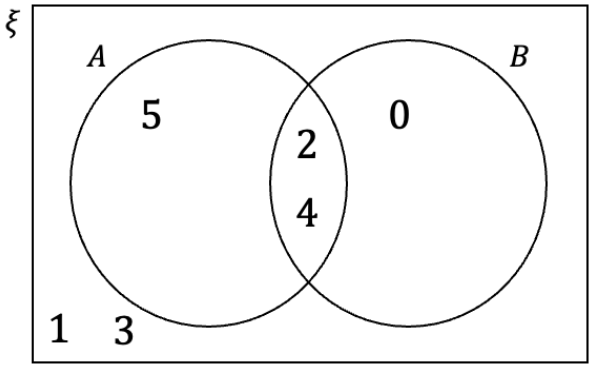
$C =$



Worked Example

A number is picked from the Venn diagram. Calculate the probability of picking:

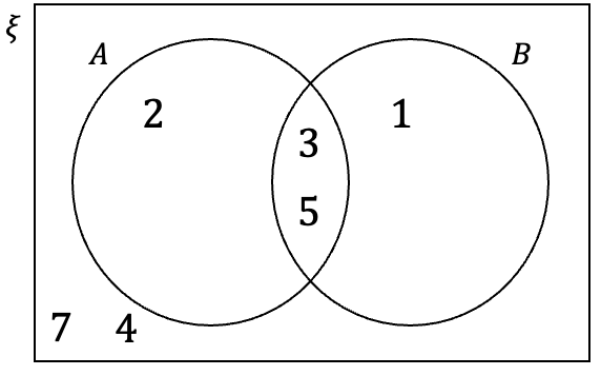
a) A prime number
b) An even number
c) A number greater than 5
d) A number less than or equal to 5



Your Turn

A number is picked from the Venn diagram. Calculate the probability of picking:

a) A prime number
b) An even number
c) A number greater than 5
d) A number less than or equal to 5



Worked Example	Your Turn
$\xi = \{1, 2, 3, 4, 5, 6\}$ $A = \{2, 3, 4\}$ $B = \{4, 5\}$ Construct a Venn Diagram to show these sets.	$\xi = \{2, 4, 6, 8, 10, 12, 14\}$ $A = \{4, 6, 8\}$ $B = \{8, 10, 12, 14\}$ Construct a Venn Diagram to show these sets.

Worked Example	Your Turn
$\xi = \{1, 2, 3, 4, 5, 6\}$ $A = \{1, 2, 3\}$ $B = \{1\}$ Construct a Venn Diagram to show these sets.	$\xi = \{2, 4, 6, 8, 10, 12, 14\}$ $A = \{4, 6, 8\}$ $B = \{6, 8\}$ Construct a Venn Diagram to show these sets.

Worked Example	Your Turn
There are 150 pupils. The examinations available are: English, Maths and Science. • 15 pupils are sitting English and Maths but not science. • 20 pupils are sitting Science and Maths but not English. • 18 pupils are sitting Science and English but not Maths. • 8 pupils are sitting all three exams. • 55 are sitting English in total. • 72 are sitting Maths in total. • 65 are sitting Science in total. A pupil is chosen at random. What is the probability that they are sitting no exams?	There are 130 pupils. The examinations available are: English, Maths and Science. • 10 pupils are sitting English and Maths but not science. • 20 pupils are sitting Science and Maths but not English. • 9 pupils are sitting Science and English but not Maths. • 13 pupils are sitting all three exams. • 49 are sitting English in total. • 83 are sitting Maths in total. • 62 are sitting Science in total. A pupil is chosen at random. What is the probability that they are sitting no exams?

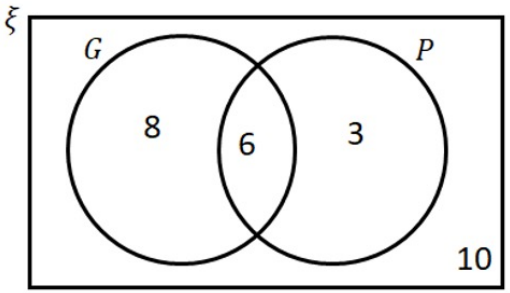
Worked Example	Your Turn
There are 150 pupils. The examinations available are: English, Maths and Science. • 15 pupils are sitting English and Maths. • 20 pupils are sitting Science and Maths. • 18 pupils are sitting Science and English. • 8 pupils are sitting all three exams. • 55 are sitting English in total. • 72 are sitting Maths in total. • 65 are sitting Science in total. A pupil is chosen at random. What is the probability that they are sitting no exams?	There are 170 pupils. The examinations available are: English, Maths and Science. • 10 pupils are sitting English and Maths. • 20 pupils are sitting Science and Maths. • 9 pupils are sitting Science and English. • 3 pupils are sitting all three exams. • 49 are sitting English in total. • 83 are sitting Maths in total. • 62 are sitting Science in total. A pupil is chosen at random. What is the probability that they are sitting no exams?

Worked Example	Your Turn
In a group of 28 scientists: • 20 have degrees in Physics. • 18 have degrees in Chemistry. • Some have degrees in both. • 4 scientists have degrees which are neither Physics nor Chemistry. A scientist is chosen at random. Find the probability that the scientist has a degree in: a) Physics b) Chemistry c) Both Physics and Chemistry d) Neither Physics nor Chemistry	In a group of 30 mathematicians: • 15 have studied Calculus. • 22 have studied Topology. • Some have studied both. • 3 mathematicians have not yet studied either Calculus or topology. A mathematician is chosen at random. Find the probability that the mathematician has studied: a) Calculus b) Topology c) Both Calculus and Topology d) Neither Calculus nor topology

Conditional Probability

Worked Example

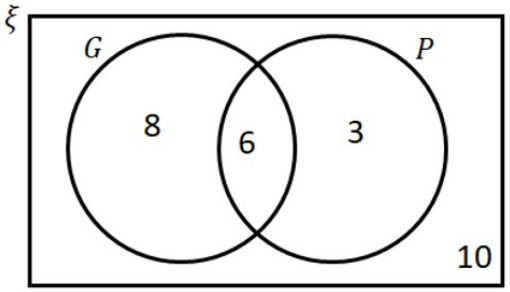
The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the guitar, given that they play the piano.

Your Turn

The Venn diagram shows a sample of people who play the guitar (G) or piano (P).



Find the probability that a student plays the piano, given that they play the guitar.

Worked Example

A vet surveys 100 of her clients. She finds that
25 own dogs, 15 own dogs and cats, 11 own dogs and tropical fish, 53 own cats, 10 own cats and tropical fish, 7 own dogs, cats and tropical fish, 40 own tropical fish.

Draw a Venn Diagram, and hence answer the following questions:

a) $\mathrm{P}(\text{owns dog only})$
b) $\mathrm{P}(\text{does not own tropical fish})$
c) $\mathrm{P}(\text{does not own dogs, cats, or tropical fish})$
d) Given that a randomly chosen person owns a cat, what's the probability they own a dog?

Your Turn

The following shows the results of a survey on the types of exercise taken by a group of 100 people.
65 run, 48 swim, 60 cycle, 40 run and swim, 30 swim and cycle, 35 run and cycle and 25 do all three.

a) Draw a Venn Diagram to represent these data.

Find the probability that a randomly selected person from the survey

b) takes none of these types of exercise,
c) swims but does not run,
d) takes at least two of these types of exercise.

Jason is one of the above group. Given that Jason runs,

e) find the probability that he swims but does not cycle.

Notation

Fluency Practice

Complement: ‘

The opposite of a set.

B’ = everywhere not in B

Intersection: ∩

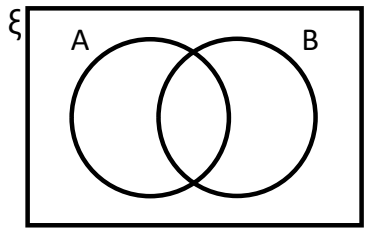
The overlap of regions.

A ∩ B = everywhere A and B overlap

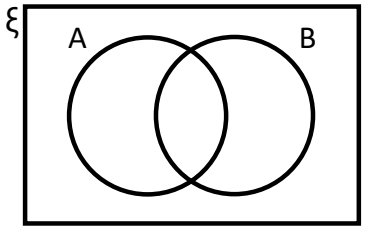
Union: ∪

The sum of regions.

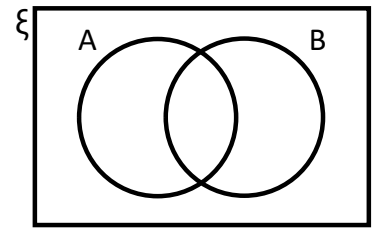
A ∪ B = A added to B



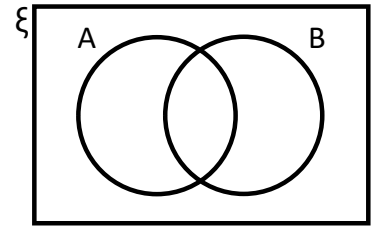
A



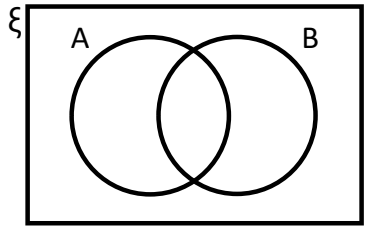
B



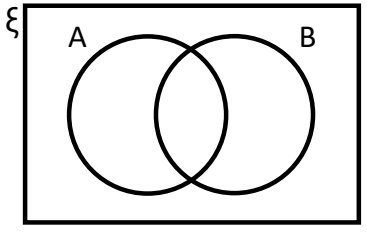
A’



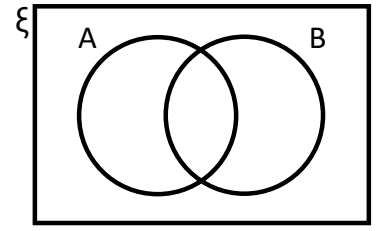
B’



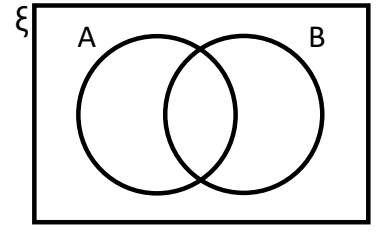
A ∪ B



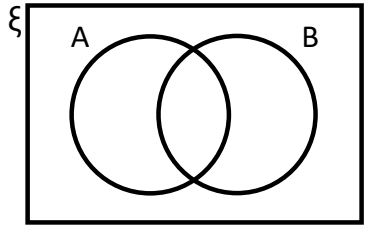
A ∪ B’



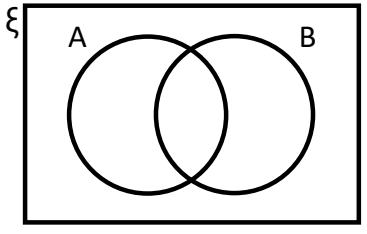
A’ ∪ B



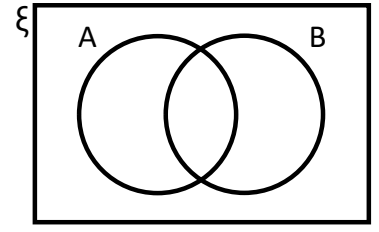
A’ ∪ B’



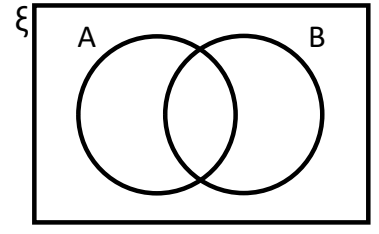
A ∩ B



A ∩ B’



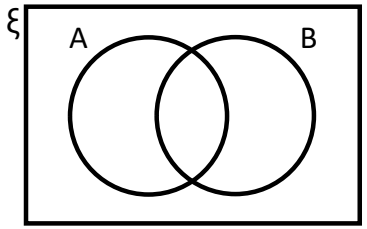
A’ ∩ B



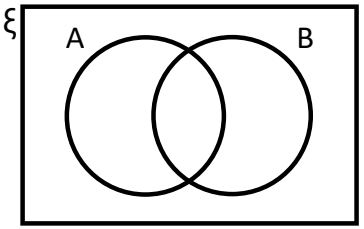
A’ ∩ B’

Fluency Practice

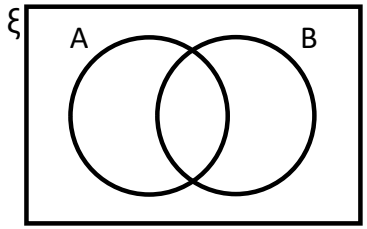
Shade the Venn Diagrams according to the notation. Remember! ∪ = sum, ∩ = overlap



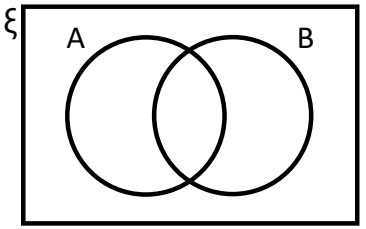
A ∪ B



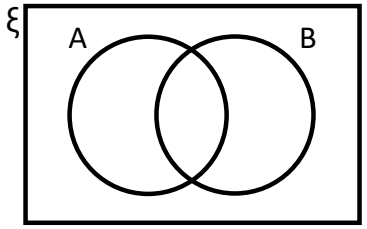
A ∩ B



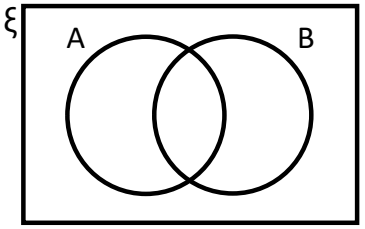
A’ ∪ B



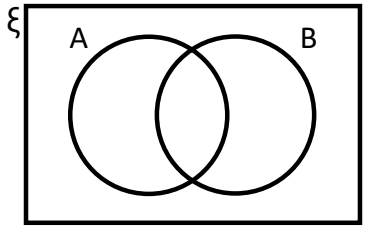
A ∩ B’



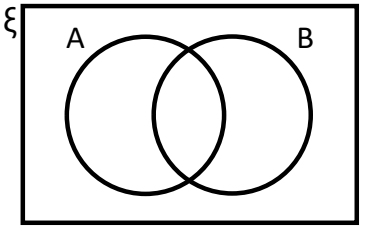
A ∪ B’



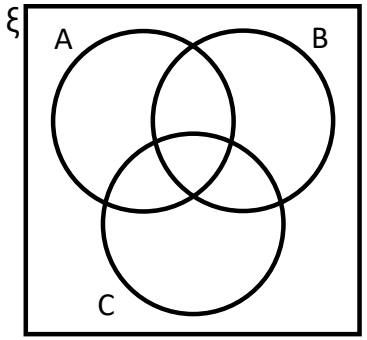
A’ ∩ B



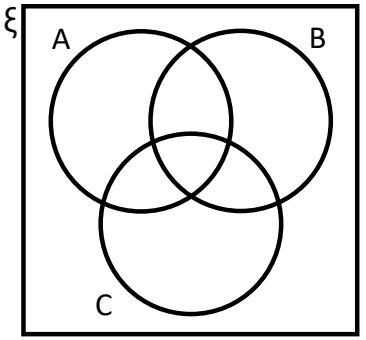
A’ ∪ B’



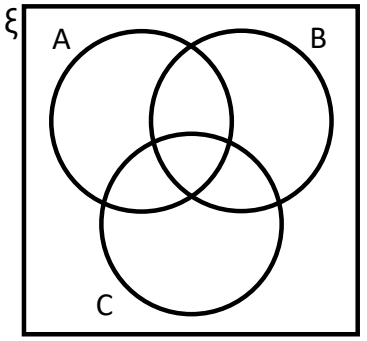
A’ ∩ B’



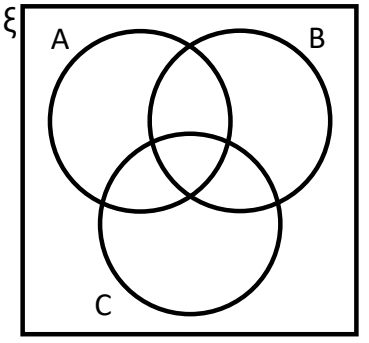
B ∩ C



A’ ∩ B ∩ C



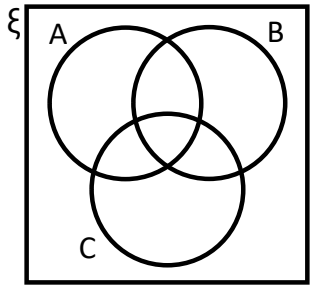
B ∪ A’ ∪ C



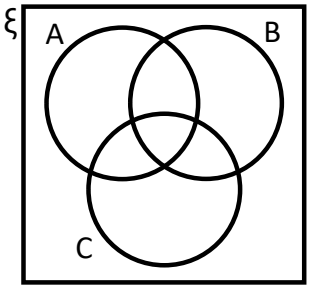
C’ ∪ A’ ∪ B

Fluency Practice

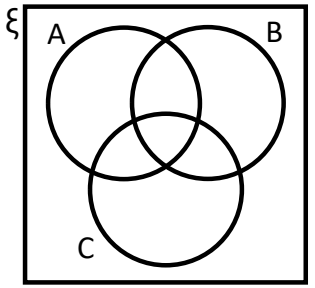
Shade the Venn Diagrams according to the notation. Remember! ∪ = sum, ∩ = overlap



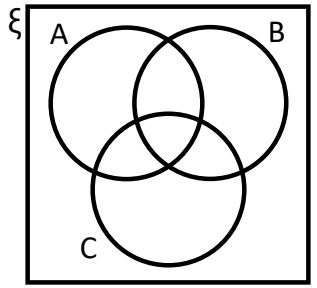
$A \cap B \cap C$



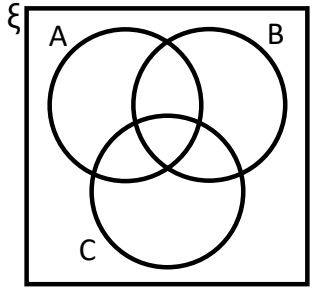
$A \cap B \cap C'$



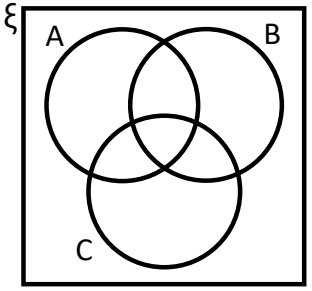
$A \cup B \cup C$



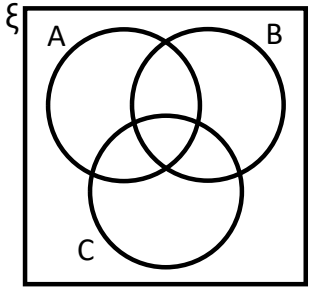
$A \cup B' \cup C$



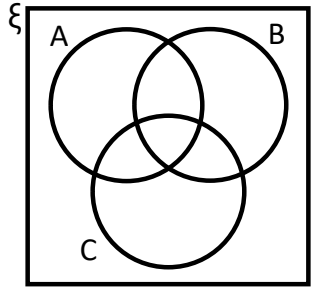
$A' \cup B' \cup C$



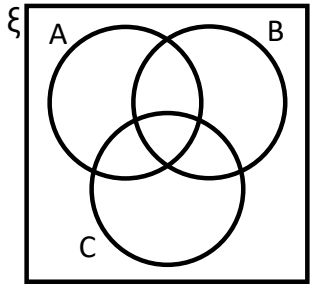
$A \cap B' \cap C'$



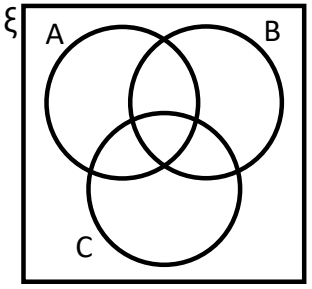
$(A \cap B \cap C)'$



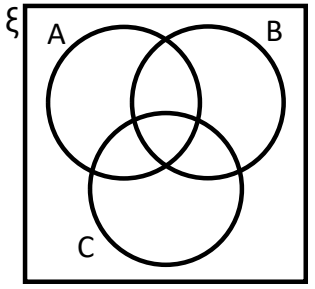
$(A' \cup B \cup C)'$



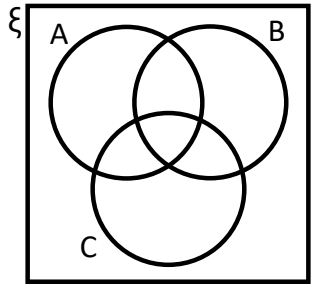
$(A \cap B)' \cap C$



$(A \cup B)' \cup C$

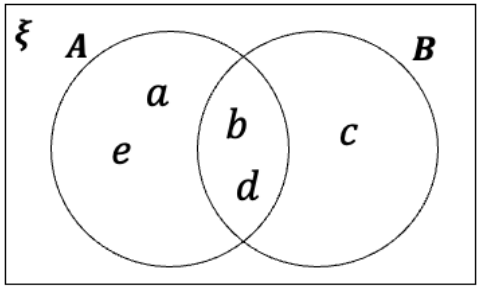


$A' \cap (B \cap C)'$



$(A' \cap B)' \cup C'$

Worked Example



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

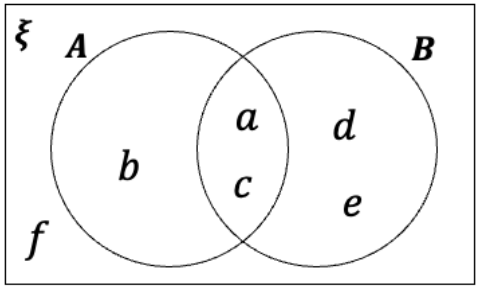
d) $B' =$

e) $A \cap B' =$

f) $A' \cap B =$

g) $A' \cap B' =$

Your Turn



a) $A \cap B =$

b) $A \cup B =$

c) $A' =$

d) $B' =$

e) $A \cap B' =$

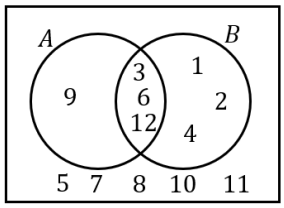
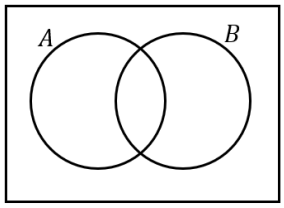
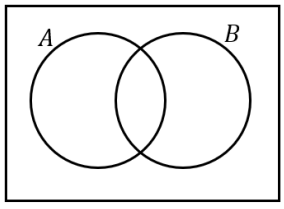
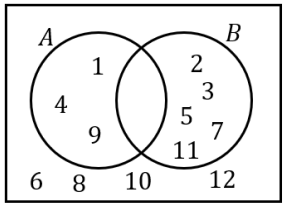
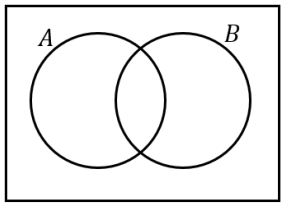
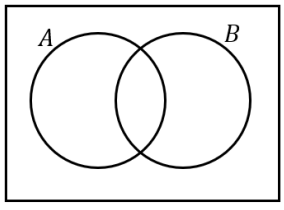
f) $A' \cap B =$

g) $A' \cap B' =$

Worked Example	Your Turn
$\xi = \{1, 2, 3, \ldots, 10\}$ $A = \{2, 4, 6, 8, 10\}$ $B = \{3, 6, 9\}$	$\xi =$ { all whole numbers } $A =$ { factors of 60 } $B =$ { multiples of 3 }
a) $A \cap B =$	a) $A \cap B =$
b) $A \cup B =$	b) $A \cup B =$
c) $A' =$	c) $A' =$
d) $B' =$	d) $B' =$
e) $A \cap B' =$	e) $A \cap B' =$
f) $A' \cap B =$	f) $A' \cap B =$
g) $A' \cap B' =$	g) $A' \cap B' =$

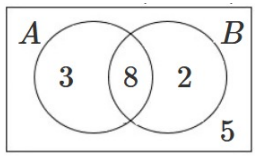
A	B	$A \cap B$	$A \cup B$
$\{1, 2, 3, 4, 5\}$	$\{4, 5, 6, 7, 8\}$	$\{4, 5\}$	$\{1, 2, 3, 4, 5, 6, 7, 8\}$
$\{1, 3, 5, 7\}$	$\{5, 6, 7, 8, 9\}$		
$\{a, b, c, d, e\}$	$\{b, c, d, e, f\}$		
$\{0, 1, 2, 3\}$	$\{4, 5, 6, 7, 8\}$		
Odd numbers from 1 *to* 9 *inclusive*	*Prime numbers less than* 10		
Square numbers less than 20	*Multiples of* 4 *from* 4 *to* 20 *inclusive*		
Even numbers from 2 *to* 12 *inclusive*	*Multiples of* 3 *less than* 15		
$\{1, 4, 7, 10, 13\}$	*Square numbers less than* 20		
Odd numbers from 1 *to* 9 *inclusive*	*Even numbers from* 2 *to* 10 *inclusive*		
$\{5, 6, 7, 8, 9\}$		$\{5, 6\}$	$\{3, 4, 5, 6, 7, 8, 9\}$
	$\{2, 4, 6, 8\}$	$\{2, 4, 6\}$	$\{1, 2, 3, 4, 5, 6, 8\}$
$\{11, 12, 13, 14\}$		$\{13\}$	$\{11, 12, 13, 14\ 17, 19, 23\}$
	$\{4, 5, 6, 7\}$	$\{\}$	$\{0, 1, 2, 3, 4, 5, 6, 7\}$
Square numbers less than 20		$\{1, 4, 16\}$	$\{1, 2, 4, 8, 9, 16\}$
	Factors of 10	$\{5, 10\}$	$\{1, 2, 5, 10, 15, 20\}$

In all questions $\xi = \{Integers\ from\ 1\ to\ 12\ inclusive\}$

Sets	Venn Diagram	$A \cup B$	$A \cap B$	$A' \cap B$	$(A \cup B)'$
$A = \{multiples\ of\ 3\}$ $B = \{factors\ of\ 12\}$			$\{3, 6, 12\}$		$\{5, 7, 8, 10, 11\}$
$A = \{prime\ numbers\}$ $B = \{odd\ numbers\}$					
$A = \{integers\ less\ than\ 7\}$ $B = \{square\ numbers\}$					
					
			$\{1, 8\}$	$\{\}$	$\{3, 5, 6, 7, 9, 10, 11, 12\}$
		$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$	$\{6, 9, 12\}$	$\{5, 7, 8, 10, 11\}$	

Worked Example

Calculate:



$P(A) =$

$P(B) =$

$P(A') =$

$P(B') =$

$P(A \cap B) =$

$P(A \cup B) =$

$P(A' \cap B) =$

$P(A \cap B') =$

$P(A' \cap B') =$

$P((A \cap B)') =$

$P(A' \cup B) =$

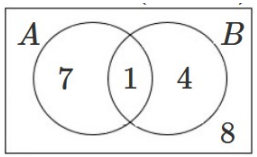
$P(A \cup B') =$

$P(A' \cup B') =$

$P((A \cup B)') =$

Your Turn

Calculate:



$P(A) =$

$P(B) =$

$P(A') =$

$P(B') =$

$P(A \cap B) =$

$P(A \cup B) =$

$P(A' \cap B) =$

$P(A \cap B') =$

$P(A' \cap B') =$

$P((A \cap B)') =$

$P(A' \cup B) =$

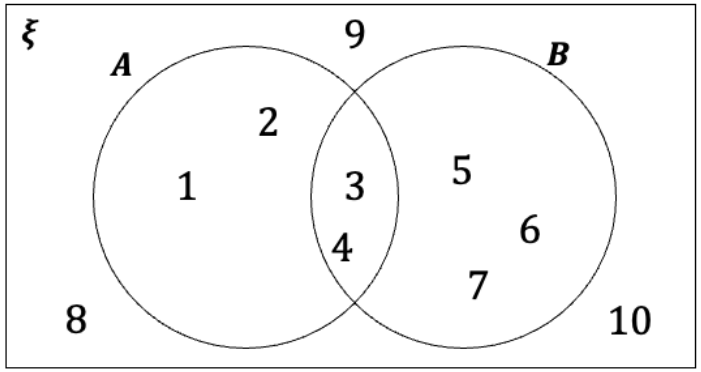
$P(A \cup B') =$

$P(A' \cup B') =$

$P((A \cup B)') =$

Cardinality of Sets

Worked Example



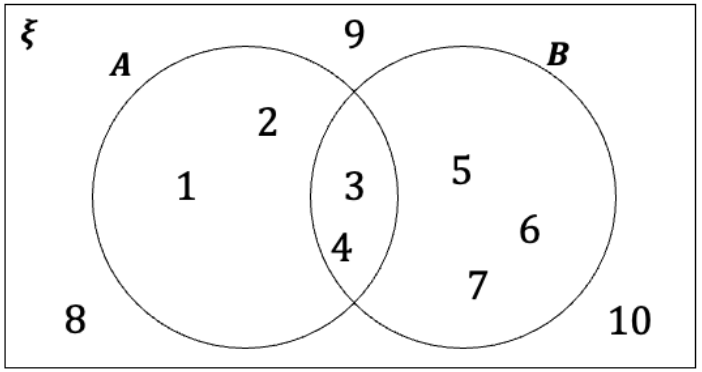
a) $n(A) =$

b) $n(A \cap B) =$

c) $n(A' \cap B) =$

d) $n(A' \cup B) =$

Your Turn



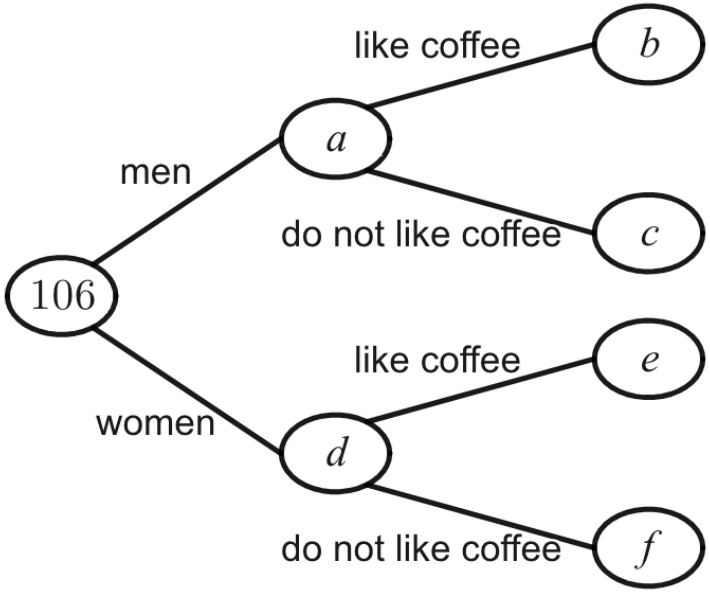
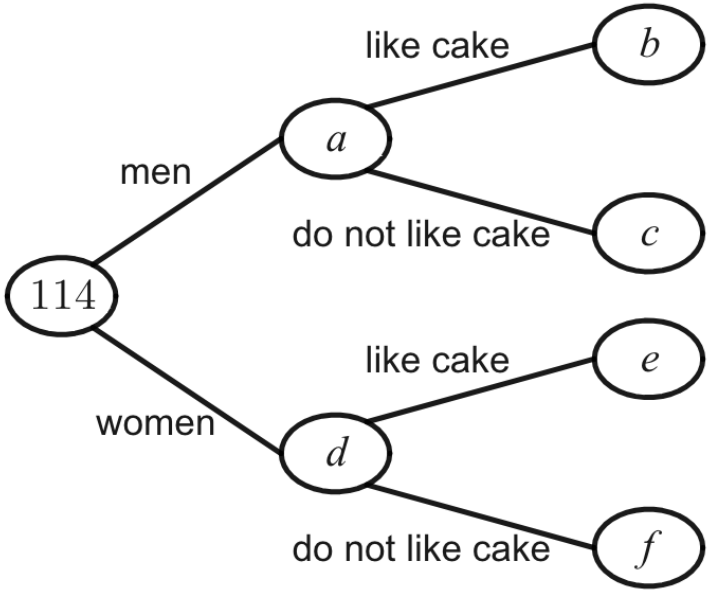
a) $n(B) =$

b) $n(A \cup B) =$

c) $n(A \cup B') =$

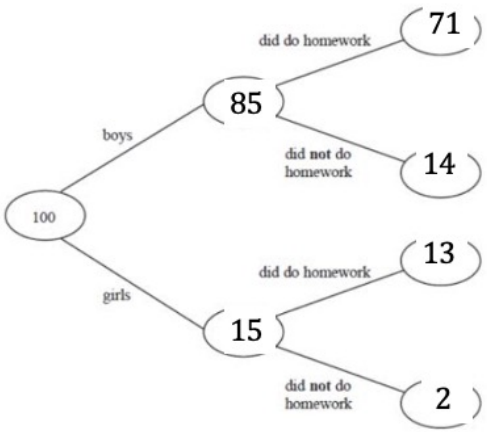
d) $n(A \cap B') =$

Frequency Trees

Worked Example	Your Turn
106 people are asked if they like coffee. 63 of these people are men. 60 of the 106 people like coffee. 21 of the women do not like coffee. Use this information to complete the frequency tree.  	114 people are asked if they like cake. 58 of these people are men. 57 of the 114 people like cake. 24 of the women do not like cake. Use this information to complete the frequency tree. 

Worked Example

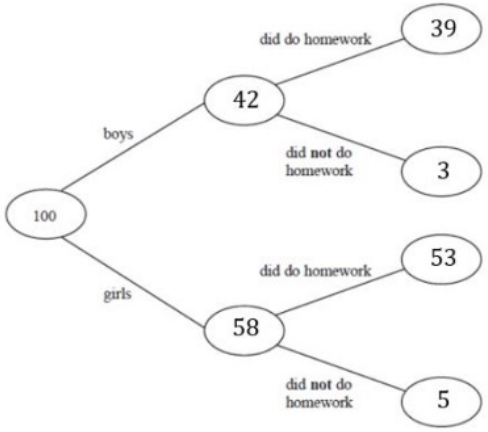
100 students had some homework.
The frequency tree shows the number of students who did the homework.



A boy is chosen at random.
Write down the probability that they did not do their homework.

Your Turn

100 students had some homework.
The frequency tree shows the number of students who did the homework.



A girl is chosen at random.
Write down the probability that they did do their homework.

Tree Diagrams

Mutually Exclusive and Independent Events

If A and B are **mutually exclusive events**, they can't happen at the same time. Then:

P(A **or** B) = P(A) + P(B)

If A and B are **independent events**, then the outcome of one doesn't affect the other. Then:

P(A **and** B) = P(A) × P(B)

Worked Example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag. It is replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are red
b) Neither counter is red
c) The counters are different colours

Your Turn

There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag. It is replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are purple
b) Neither counter is purple
c) The counters are different colours

Worked Example	Your Turn
In bag A there are 2 white and 5 red counters. In bag B there are 7 white counters and 3 red counters. A person takes at random one counter from A and one counter from B. a) Draw a probability tree diagram to represent the situation. b) Find the probability that the counters are the same colour. c) Find the probability that the counters are different colours.	In bag A there are 4 white and 7 red counters. In bag B there are 9 white counters and 5 red counters. A person takes at random one counter from A and one counter from B. a) Draw a probability tree diagram to represent the situation. b) Find the probability that the counters are the same colour. c) Find the probability that the counters are different colours.

Worked Example

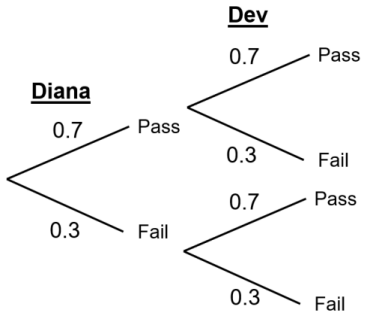
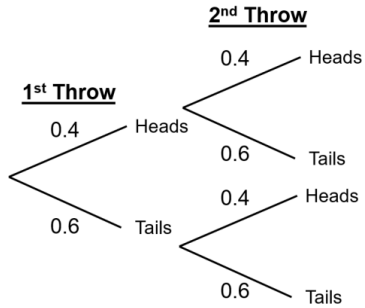
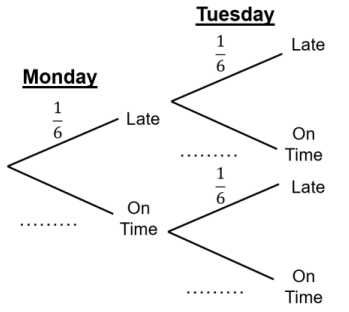
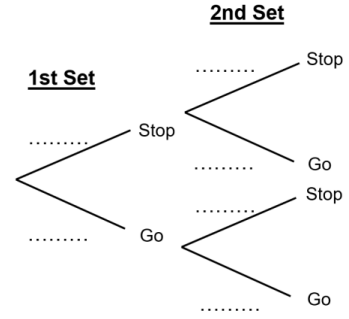
A person plays a game of tennis and then a game of golf. They can only win or lose each game. The probability of winning tennis is 0.3. The probability of winning golf is 0.7. The results of each game are independent of each other.

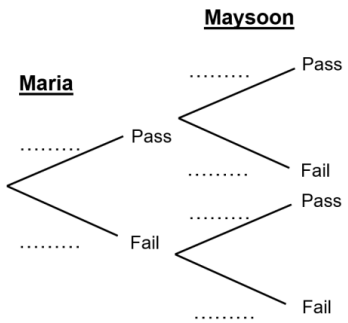
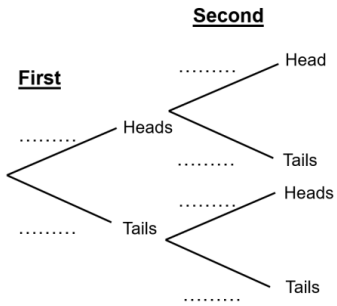
a) Draw a probability tree to represent this information.
b) Calculate the probability that the person win both games.
c) Calculate the probability that the person wins one and loses one.
d) Calculate the probability that the person wins at least one game.

Your Turn

A person plays a game of tennis and then a game of golf. They can only win or lose each game. The probability of winning tennis is 0.6. The probability of winning golf is 0.35. The results of each game are independent of each other.

a) Draw a probability tree to represent this information.
b) Calculate the probability that the person loses both games.
c) Calculate the probability that the person wins one and loses one.
d) Calculate the probability that the person loses at least one game.

Question	Tree Diagram	Probability	
The probability of passing a music exam is 0.7. Diana and Dev both sit the music exam. Complete the tree diagram and calculate the probability of each outcome.		$P(PP) = 0.7 \times 0.7 =$	0.49
		$P(PF) = 0.7 \times 0.3 =$	
		$P(FP) = 0.3 \times 0.7 =$	
		$P(FF) = 0.3 \times 0.3 =$	
The probability of a biased coin landing on tails is 0.4. The coin is tossed twice. Complete the tree diagram and calculate the probability of each outcome.		$P(HH) = 0.4 \times 0.4 =$	
		$P(HT) =$ × =	
		$P(TH) =$ × =	
		$P(TT) =$ × =	
The probability of Abby being late for work is $\frac{1}{6}$. Abby works Monday and Tuesday. Complete the tree diagram and calculate the probability of each outcome.		$P(LL) =$ × =	
		$P(LO) =$ × =	
		$P(OL) =$ × =	
		$P(OO) =$ × =	
The probability of stopping at traffic lights is $\frac{3}{8}$. Jameela drives through two sets of traffic lights. Complete the tree diagram and calculate the probability of each outcome.			

Question	Tree Diagram	Probability	
Two students, Maria and Maysoon each sit their driving theory exam. Complete the tree diagram and calculate the probability of each outcome.		$P(PP) =$ $\times$ $=$	
		$P(PF) =$ $\times$ $=$	
		$P(FP) =$ $\times$ $=$	
		$P(FF) = 0.6 \times 0.6 =$	0.36
A biased coin is tossed once and then tossed again for a second time. Complete the tree diagram and calculate the probability of each outcome.		$P(HH) = 0.2 \times$ $=$	0.04
		$P(HT) =$ $\times$ $=$	
		$P(TH) =$ $\times$ $=$	
		$P(TT) =$ $\times$ $=$	
A car travels through two sets of traffic lights. The probability of stopping at each set is the same. Complete the tree diagram and calculate the probability of each outcome.		$P(SS) =$ $\times$ $=$	
		$P(SG) = \frac{3}{7} \times$ $=$	
		$P(GS) =$ $\times$ $=$	
		$P(GG) =$ $\times$ $=$	
There are 12 red or blue balls in a box. There are more blue balls than red balls. A ball is removed at random, the colour recorded, then replaced. A second ball is then removed. Complete the tree diagram and probabilities.		$P(RR) =$ $\times$ $=$	
		$P(RB) =$ $\times$ $=$	$\frac{35}{144}$
		$P(BR) =$ $\times$ $=$	
		$P(BB) =$ $\times$ $=$	

Worked Example

There are counters in a bag.

Colour	Red	Blue	Yellow
Number	5	10	15

One counter is taken out the bag. It is not replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are red
b) Neither counter is red
c) The counters are different colours

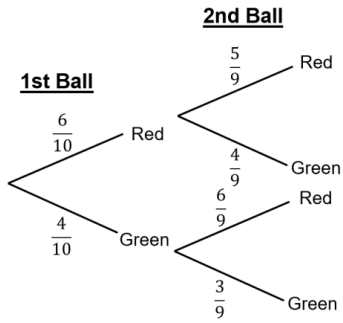
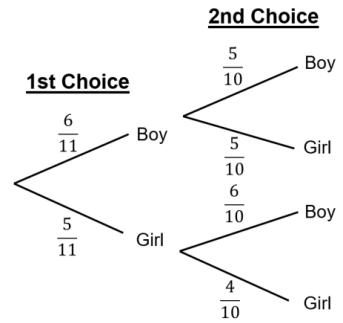
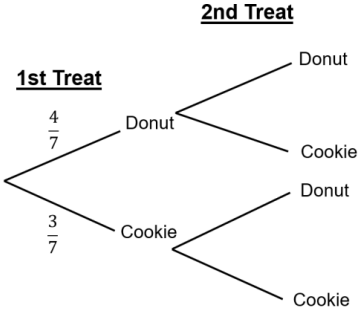
Your Turn

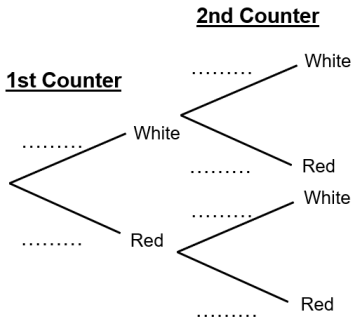
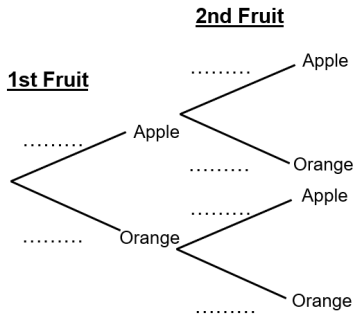
There are counters in a bag.

Colour	Purple	Orange	Green
Number	10	45	5

One counter is taken out the bag. It is not replaced. Then another counter is taken out the bag. Find the probability that:

a) Both counters are purple
b) Neither counter is purple
c) The counters are different colours

Question	Tree Diagram	Probability	
There are 6 red balls and 4 green balls in a bag. Two balls are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(RR) = \frac{6}{10} \times \frac{5}{9} =$	$\frac{30}{90}$
		$P(RG) = \frac{6}{10} \times \frac{4}{9} =$	$\frac{24}{90}$
		$P(GR) =$ $\times$ $=$	
		$P(GG) =$ $\times$ $=$	
There are 6 boys and 5 girls in a football team. Two team members are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(BB) =$ $\times$ $=$	
		$P(BG) =$ $\times$ $=$	
		$P(GB) =$ $\times$ $=$	
		$P(GG) =$ $\times$ $=$	
There are 4 donuts and 3 cookies in a tin. Riaz chooses two treats at random. Complete the tree diagram and calculate the probability of each outcome.		$P(DD) =$ $\times$ $=$	
		$P(DC) =$ $\times$ $=$	
		$P(CD) =$ $\times$ $=$	
		$P(CC) =$ $\times$ $=$	
There are 7 blue pens and 5 red pens in a pencil case. Two pens are chosen at random. Complete the tree diagram and calculate the probability of each outcome.		$P(BB) =$	
		$P(BR) =$	
		$P(RB) =$	
		$P(RR) =$	

Question	Tree Diagram	Probability	
There are some white counters and some red counters in a bag. Two counters are taken from the bag at random. Complete the tree diagram and calculate the missing probabilities.		$P(WW) =$ $\times$ $=$	
		$P(WR) =$ $\times$ $=$	
		$P(RW) =$ $\times$ $=$	
		$P(RR) = \frac{5}{8} \times \frac{4}{7} =$	$\frac{20}{56}$
There are some apples and some oranges in a fruit bowl. Two pieces of fruit are chosen at random. Complete the tree diagram and calculate the missing probabilities.		$P(AA) =$ $\times \frac{2}{9} =$	
		$P(AO) =$ $\times$ $=$	
		$P(OA) =$ $\times$ $=$	
		$P(OO) =$ $\times$ $=$	
Milo has some black socks and some grey socks in a drawer. He chooses two socks at random. Draw a tree diagram and calculate the missing probabilities.		$P(BB) =$ $\times$ $=$	
		$P(BG) =$ $\times \frac{5}{11} =$	
		$P(GB) =$ $\times$ $=$	
		$P(GG) =$ $\times$ $=$	
Adrianna buys some sausage rolls and some cheese pasties from the bakery. She chooses two items at random to eat for lunch. Draw a tree diagram and calculate the missing probabilities.		$P(SS) =$ $\times$ $=$	
		$P(SC) =$ $\times$ $=$	
		$P(CS) =$ $\times$ $=$	
		$P(CC) =$ $\times$ $=$	$\frac{12}{110}$

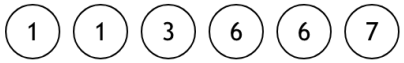
Worked Example

There are 8 counters in a bag.
There is a number on each counter.

Martha takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **odd** number.

Your Turn

There are 6 counters in a bag.
There is a number on each counter.



Emily takes at random three of the counters.
She works out the **product** of the numbers on the three counters.
Work out the probability that the product is an **even** number.

Worked Example

There are 9 counters in a bag.
There is a number on each counter.

Zoe takes at random three of the counters.
She works out the **sum** of the numbers on the three counters.
Work out the probability that the sum is an **odd** number.

Your Turn

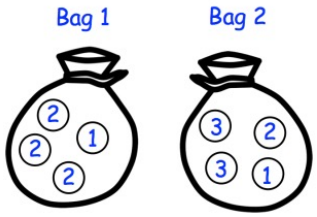
There are 7 tiles in a sack.
There is a number on each tile.

Jules takes at random three of the tiles.
She **adds** together the numbers on the three counters to get a total.
Find the probability that her total is an **even** number.

Dr Frost 354f

Worked Example

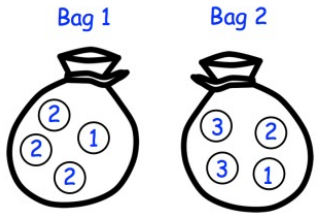
There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.
If it is labelled 2, he puts the disc in bag 2.
If it is labelled 1, he does not put the disc in bag 2.
He then chooses a disc at random from bag 2.
He then adds the numbers of the two discs he selected to give his score.
Find the probability that his score is 5.

Your Turn

There are two bags with numbered discs as shown.



A person chooses a disc at random from bag 1.
If it is labelled 1, he puts the disc in bag 2.
If it is labelled 2, he does not put the disc in bag 2.
He then chooses a disc at random from bag 2.
He then adds the numbers of the two discs he selected to give his score.
Find the probability that his score is 4.

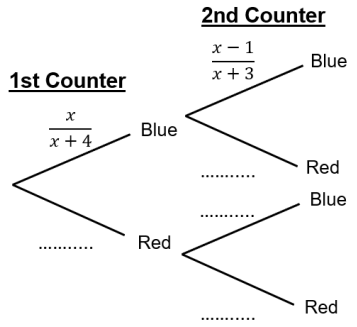
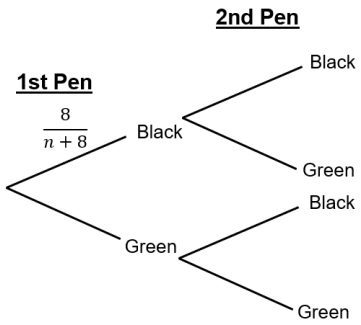
Review

...with replacement:
The item is returned before another is chosen. The probability of each event on each trial is fixed.

...without replacement:
The item is not returned.
- Total balls decreases by 1 each time.
- Number of items of this type decreases by 1.

Note that if the question doesn't specify which, e.g. "You pick two balls from a bag", then PRESUME WITHOUT REPLACEMENT.

Worked Example	Your Turn
A person throws a biased coin two times. The probability of flipping a heads on any throw is p a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails	A person throws a biased coin two times. The probability of flipping a heads on any throw is q a) Draw a probability tree diagram to represent the situation. b) Find an expression for the probability of flipping: i) Two heads ii) Two tails iii) One heads and one tails

Question	Tree Diagram	Probability
There are x blue counters and 4 red counters in a bag. Two counters are chosen at random without replacement. Complete the tree diagram and find expressions for each of the probabilities.		$P(BB) = \frac{x}{x+4} \times \frac{x-1}{x+3}$ $P(BR) =$ × $P(RB) =$ × $P(RR) =$ ×
There are 8 black pens and n green pens in a pencil case. Gloria chooses two pens at random from the pencil case. Complete the tree diagram and find expressions for each of the probabilities.		$P(BB) = \frac{8}{n+8} \times$ $P(BG) =$ × $P(GB) =$ × $P(GG) =$ ×
There are n biscuits in a tin. There are some digestives and five shortbreads. Ayyan takes two biscuits from the tin at random and eats them. Draw a tree diagram and find expressions for each of the probabilities.		$P(DD) =$ × $P(DS) =$ × $P(SD) =$ × $P(SS) =$ ×
A jar contains x lime sweets and some pear sweets. The number of pear sweets is one more than the number of lime sweets. Two sweets are chosen at random. Draw a tree diagram and find expressions for each of the probabilities.		$P(LL) =$ × $P(LP) =$ × $P(PL) =$ × $P(PP) =$ ×

Worked Example	Your Turn
There are n sweets in a bag. 4 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{2}{5}$. Find n	There are n sweets in a bag. 6 of the sweets are orange. The rest are yellow. Hannah takes at random a sweet from the bag. She eats the sweet. Hannah then takes at random another sweet from the bag and eats it. The probability that Hannah eats two orange sweets is $\frac{1}{3}$. Find n

Worked Example	Your Turn
There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{90}$ Work out the number of blue counters in the bag at the start.	There are counters in a bag. At the start, 7 are red and the rest are blue. A person takes at random a counter from the bag. They do not put it back in the bag. Then they take another counter at random. The probability that the first counter is blue, and the second counter is red is $\frac{21}{80}$ Work out the number of blue counters in the bag at the start.

Extra Notes

5 Capture-Recapture

Worked Example	Your Turn
I catch 12 fish in a pond and mark them with a red dot. The next day I catch 8 fish, of which, 3 have red dots on them. Estimate the population of fish in the pond.	I catch 18 fish in a pond and mark them with a red dot. The next day I catch 7 fish, of which, 2 have red dots on them. Estimate the population of fish in the pond.

Extra Notes