KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Catch Up Tasks – Part 1

DO NOT WRITE INSIDE

KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2026 Mathematics 2027

Catch Up Tasks – Part 2

DO NOT WRITE INSIDE

Contents

1 Graphical Simultaneous Equations

Fluency Practice

Solving Simultaneous Equations Graphically

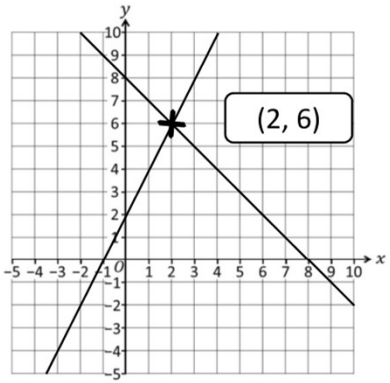
Plot each equation on the same grid.

You may want to rearrange the equations first.

The intersection shows the values (x, y) that satisfy both equations.

Check the solution by substituting values back into the equations.

$y = 2x + 2$ $\quad$ $y + x = 8$



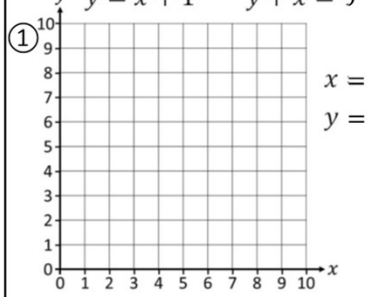
Solution: $x = 2$ $\quad$ $y = 6$

Substitute to check:

$y = 2x + 2$ $\quad$ 6 = 2(2) + 2 ✓

$y + x = 8$ $\quad$ 6 + 2 = 8 ✓

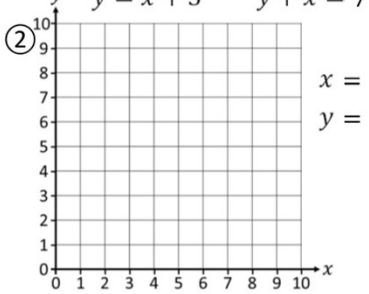
① $y = x + 1$ $\quad$ $y + x = 9$



$x =$

$y =$

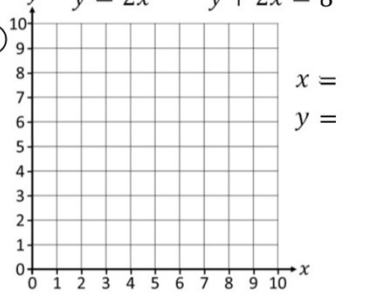
② $y = x + 3$ $\quad$ $y + x = 7$



$x =$

$y =$

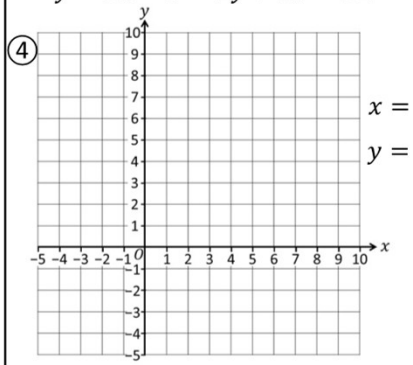
③ $y = 2x$ $\quad$ $y + 2x = 8$



$x =$

$y =$

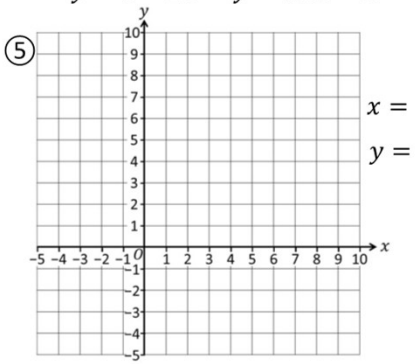
④ $y = 3x - 3$ $\quad$ $2y + 3x = 12$



$x =$

$y =$

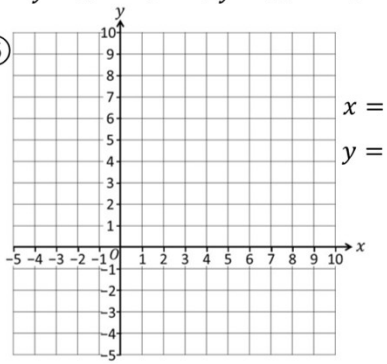
⑤ $y = 5 - 2x$ $\quad$ $y = 0.5x - 5$



$x =$

$y =$

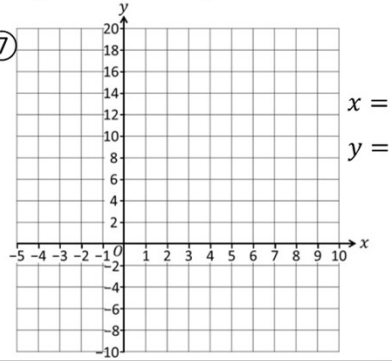
⑥ $y - x = -3$ $\quad$ $2y - 3x = -9$



$x =$

$y =$

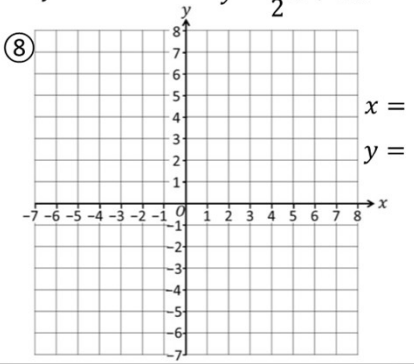
⑦ $2y + 5x = 30$ $\quad$ $y = 6x - 2$



$x =$

$y =$

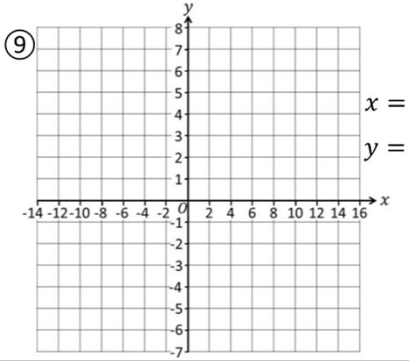
⑧ $4y + 5x = -20$ $\quad$ $y = \frac{1}{2}x + 5.5$



$x =$

$y =$

⑨ $2x = -24 - 6y$ $\quad$ $y - 10 = 2x$



$x =$

$y =$

Plot both lines on the same graph. The point of intersection represents the solution.

1. $y = 3x - 1$

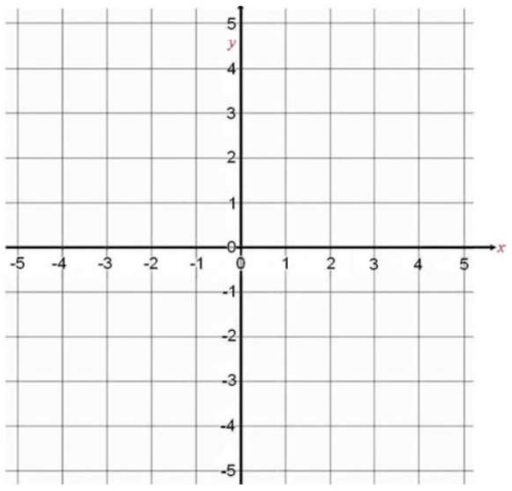
$y = 2x$

$y = 3x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

2. $y = 2x - 1$

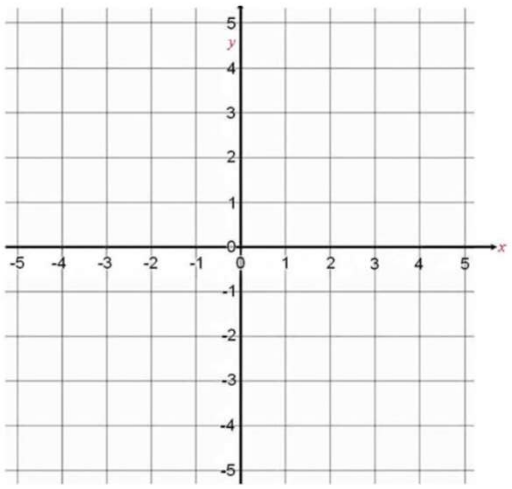
$y = x$

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							

$y = x$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

3. $y = 3x - 2$

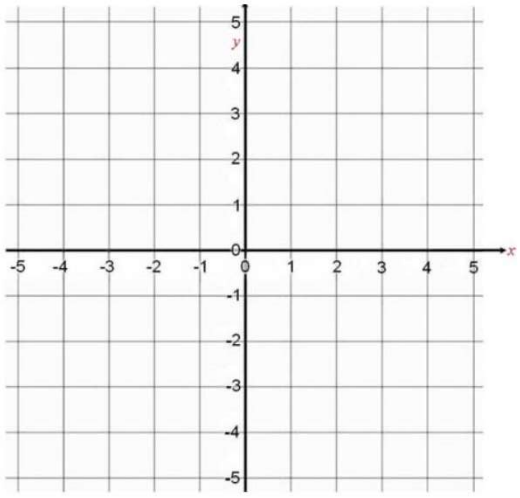
$y = x - 2$

$y = 3x - 2$

x	-3	-2	-1	0	1	2	3
y							

$y = x - 2$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

4. $y = 5 - x$

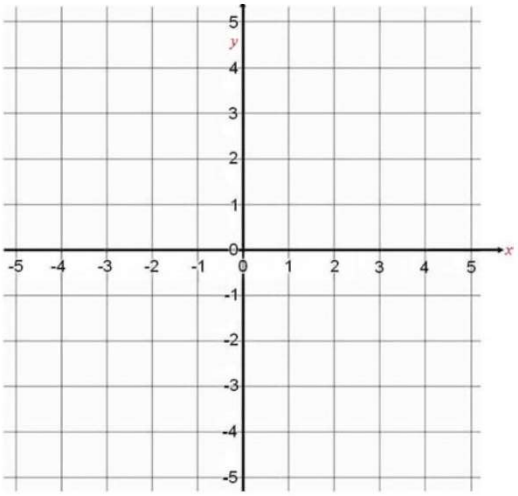
$y = 2x - 1$

$y = 5 - x$

x	-3	-2	-1	0	1	2	3
y							

$y = 2x - 1$

x	-3	-2	-1	0	1	2	3
y							



Point of intersection: (____, ____)

Solution: x=____ y=____

learn by heart

A function is a type of equation, because it contains an equals sign.

When we plot the graph of a function we can see all the pairs of values that make it true. These are also called the solutions of the equation.

Every function has an infinite number of solutions, but if we take any two functions it is possible that there is just one pair of values that solves both of them. To find this solution we can plot graphs of both functions and see where they cross. This is called solving the equations simultaneously - at the same time.

example

By plotting graphs, find estimates for the solution to the simultaneous equations:
$y = 2x + 4$ and $y = 5 - 3x$

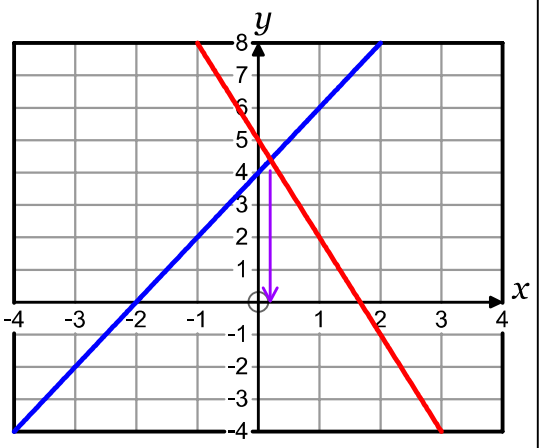
First create tables of values and plot each function:

$y = 2x + 4$

x	y
-1	2
0	4
1	6
2	8

$y = 5 - 3x$

x	y
-1	8
0	5
1	2
2	-1



They cross when x is approximately 0.2 and y is approximately 4.2, so $x \approx 0.2$, $y \approx 4.2$ is an estimate of the solution.

exercise 7o

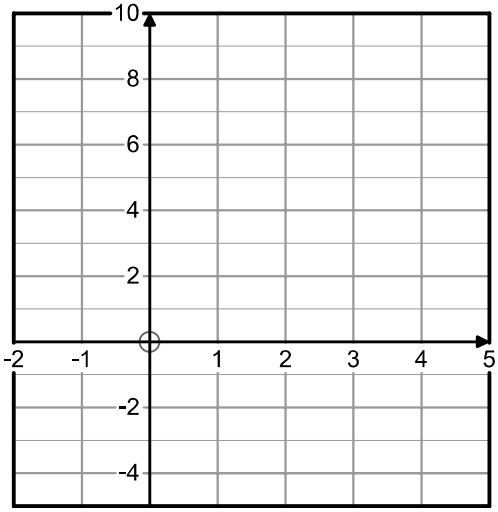
1. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 4x - 5$ and $y = 6 - 2x$

$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 6 - 2x$

x	y
-1	
0	
1	
2	



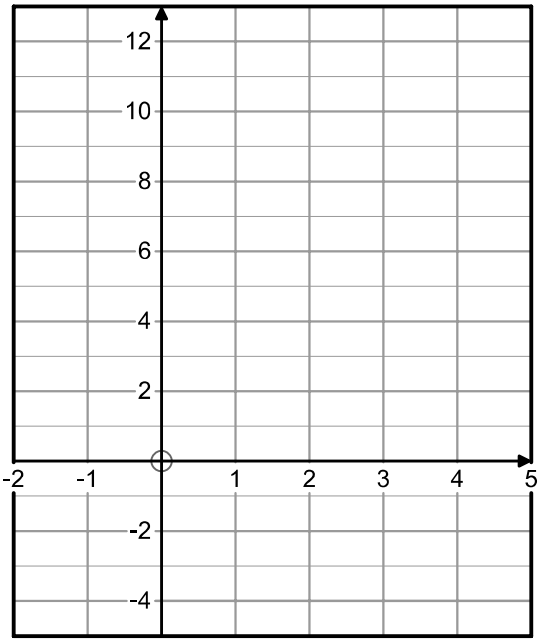
2. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 2x - 3$ and $y = 10 - x$

$y = 2x - 3$

x	y
-1	
0	
1	
2	

$y = 10 - x$

x	y
-1	
0	
1	
2	



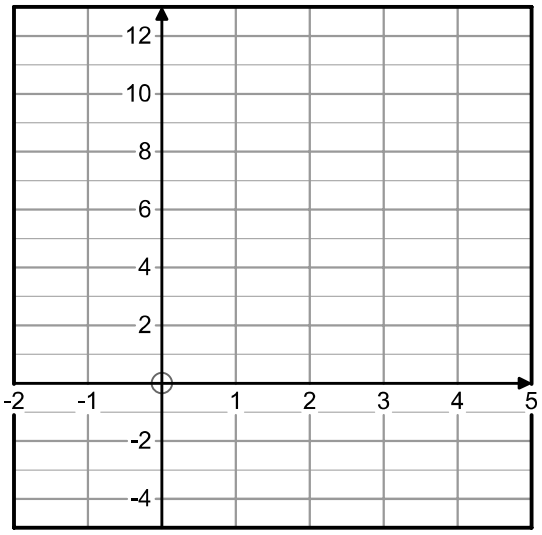
3. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 4x - 5$ and $y = 5 - 2x$

$y = 4x - 5$

x	y
0	
1	
2	
3	

$y = 5 - 2x$

x	y
-1	
0	
1	
2	



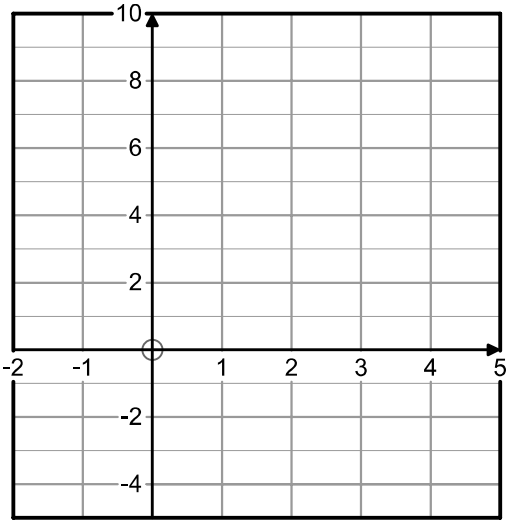
4. By plotting graphs, estimate the solution to the simultaneous equations:
$y = 8 - x$ and $y = 3x - 1$

$y = 8 - x$

x	y
0	
1	
2	
3	

$y = 3x - 1$

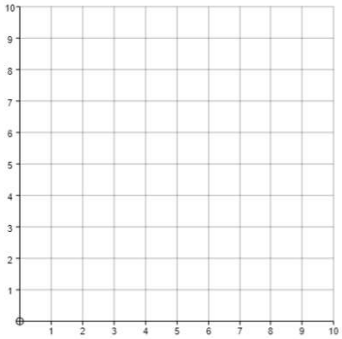
x	y
-1	
0	
1	
2	



Solving Simultaneous Equations Graphically

(a)

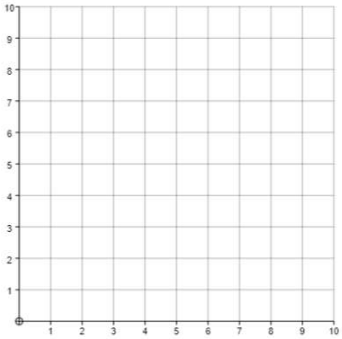
Plot the graphs of
$x + y = 5$ and $x + 3y = 9$



Find the solution to the simultaneous equations $x + y = 5$ and $x + 3y = 9$

(b)

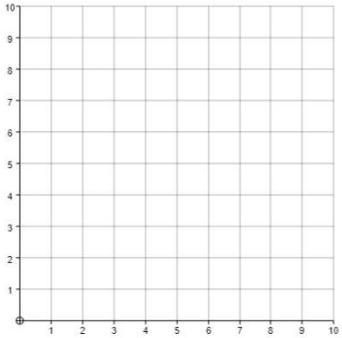
Plot the graphs of
$x + y = 7$ and $5x + 2y = 20$



Find the solution to the simultaneous equations $x + y = 7$ and $5x + 2y = 20$

(c)

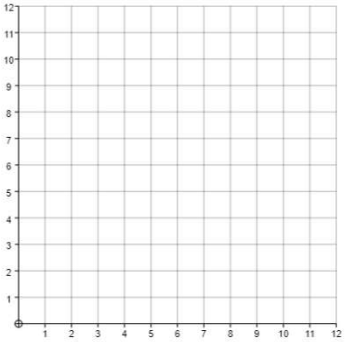
Plot the graphs of
$y = x + 1$ and $x + y = 9$



Find the solution to the simultaneous equations $y = x + 1$ and $x + y = 9$

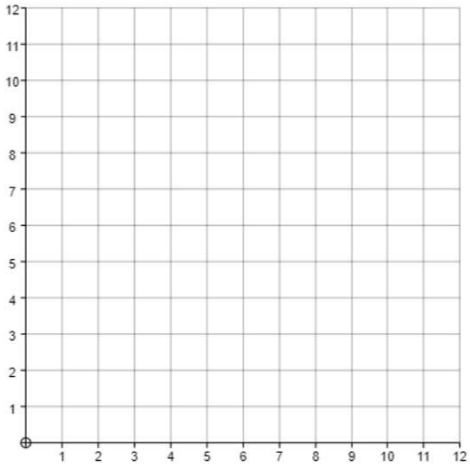
(d)

Plot the graphs of
$y = 2x$ and $2x + y = 12$



Find the solution to the simultaneous equations $y = 2x$ and $2x + y = 12$

(e)



Plot the graphs of $x + y = 8$, $y = x$, $y = \frac{1}{2}x + 5$ and $x + y = 11$

Use the graphs to find the solutions of the simultaneous equations:

(i) $x + y = 8$ and $y = x$

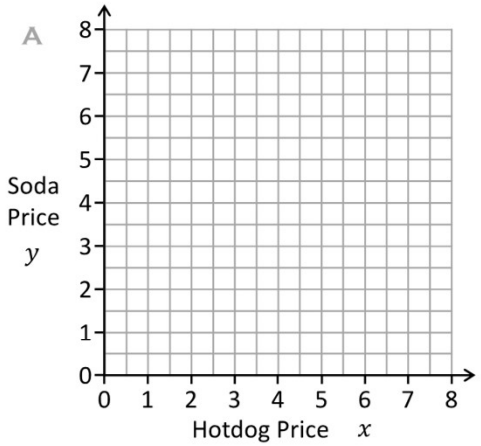
(ii) $x + y = 8$ and $y = \frac{1}{2}x + 5$

(iii) $y = x$ and $x + y = 11$

Purposeful Practice

Simultaneous Equations

A



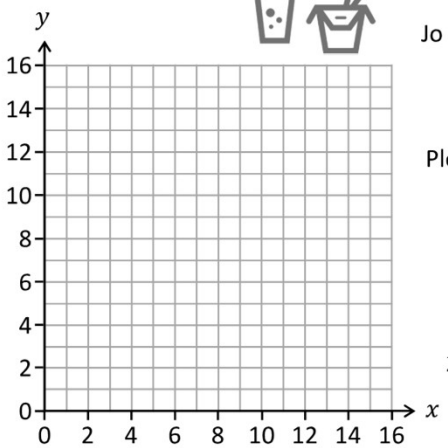
1 hotdog & 1 soda costs £6

1 hotdogs & 2 sodas costs £8

Write 2 equations in x & y to represent these purchases.

Plot both of the equations on the grid.
What do the coordinates of the intersection tell us?

B

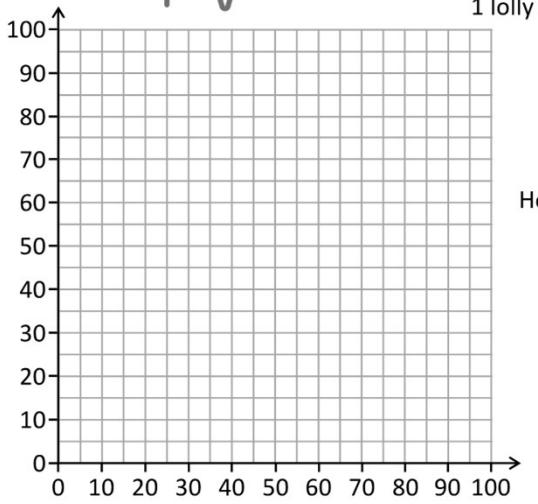


Jo buys 4 sodas and some noodles for £16 and Tye buys 2 sodas and some noodles for £12.

Plot and solve these purchases as simultaneous equations.

Hugh wants to buy
2 sodas and 3 lots of noodles.
How much will it cost?

C



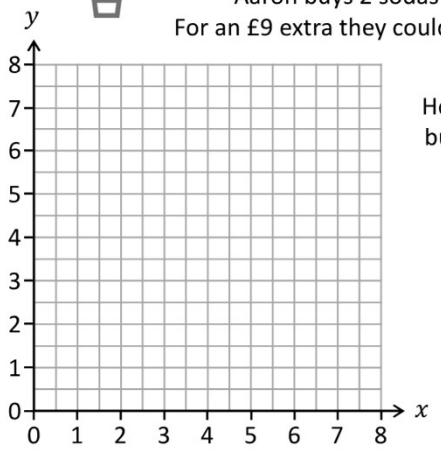
3 lollies and 1 cone costs 90p

1 lolly and 2 cones costs 80p

How much will 5 lollies & 4 cones cost?

D

Aaron buys 2 sodas and 2 tacos for £15.
For an £9 extra they could buy 4 sodas and 3 tacos.

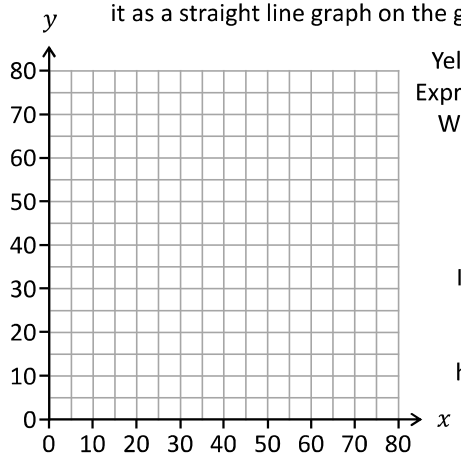


How much does it cost to buy 3 sodas and 4 tacos?

Purposeful Practice

Creating Simultaneous Equations

1) Xavier (x) and Yeliah (y) will share £60.
Create an equation to express this and plot it as a straight line graph on the grid.

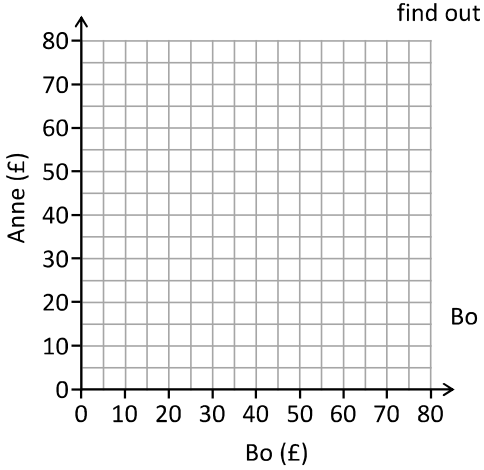


Yeliah wants 5 times Xavier's share.
Express this as an equation and plot it.
What does the intersection show?

Instead, Yeliah's share is half the size of Xavier's share.
Plot the graph and find how much they should both get.

2) Anne and Bo will share £80.
Show this relationship on the grid.

Express and plot these equations & find out how much each person would get.



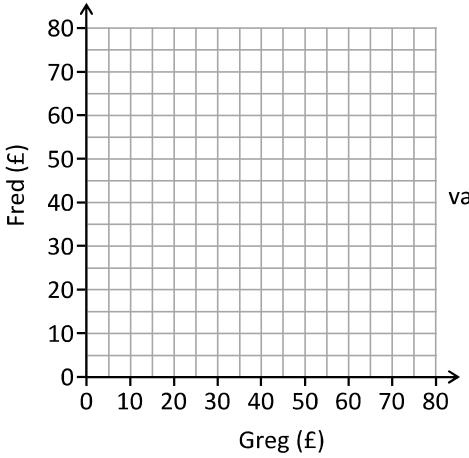
Anne gets £20 more than Bo.

Bo gets £25.

Bo gets 3 times the money Anne gets.

3) Fred and Greg will share £70.
Fred will get £10 more than double what Greg gets.

Express this problem as a pair of simultaneous equations.



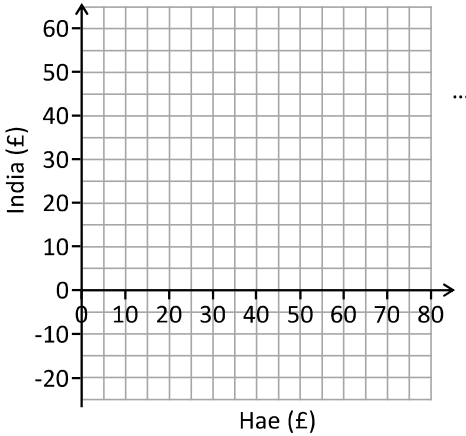
Plot both equations to find the values that satisfy both equations.

If they shared £55 how much would each person get?

4) Hae and India will share £60.

How much do they get each if...

...India gets £15 more than Hae.



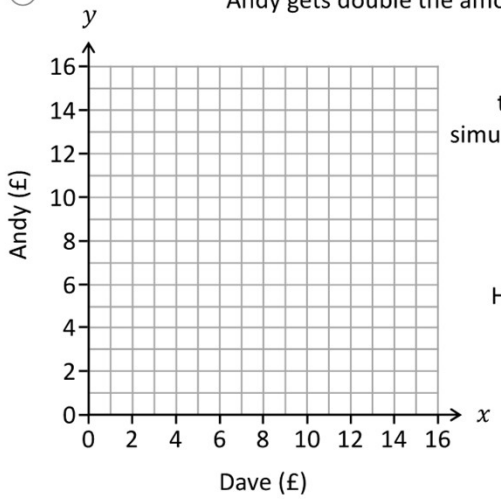
...Hae gets a third of what India gets.

...India gets £15 less than quadruple what Hae gets.

Purposeful Practice

How much!?

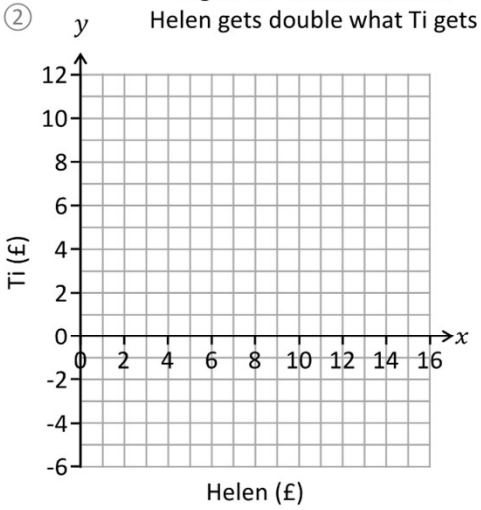
① Andy gets £4 more than Dave.
Andy gets double the amount Dave gets



Represent these two statements as a pair of simultaneous equations on the grid.

How much do they both get?
How much was shared?

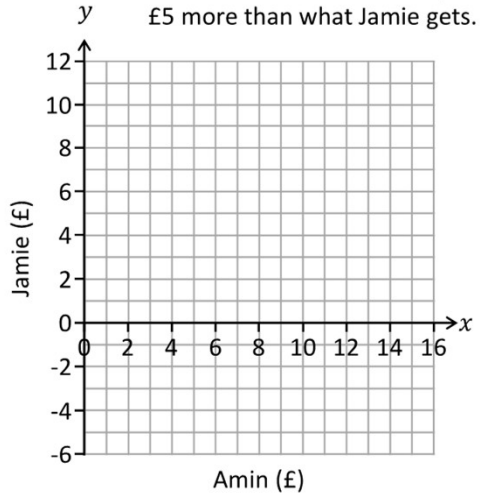
② Ti gets £5 less than Helen.
Helen gets double what Ti gets



Represent these two statements as a pair of simultaneous equations on the grid.

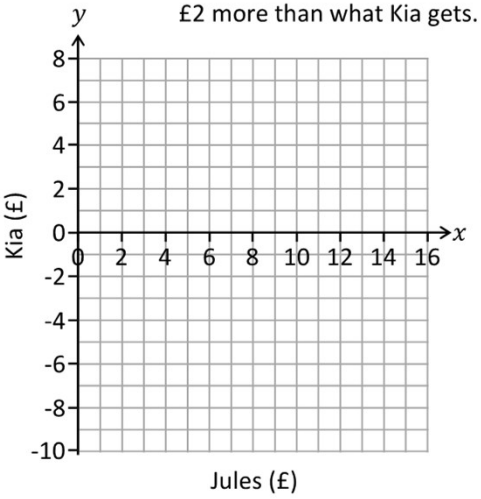
How much do they both get?
How much was shared?

③ Jamie gets £2 more than Amin.
If you double the amount Amin gets, it is £5 more than what Jamie gets.



How much do they both get?
How much was shared?

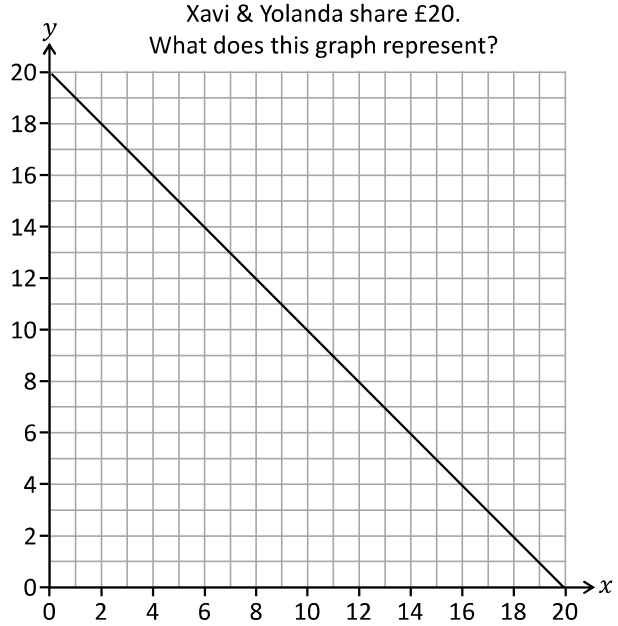
④ Kia gets £8 less than Jules.
If you halve the amount Jules gets, it is still £2 more than what Kia gets.



How much do they both get?
How much was shared?

Purposeful Practice

Xavi & Yolanda share £20.
What does this graph represent?



The money can be split in different ways.
Express and plot each split as an equation.
How much does each person get?

Xavi gets £6.

Xavi =
Yolanda =

Yolanda gets 4 times what Xavi gets.

Xavi =
Yolanda =

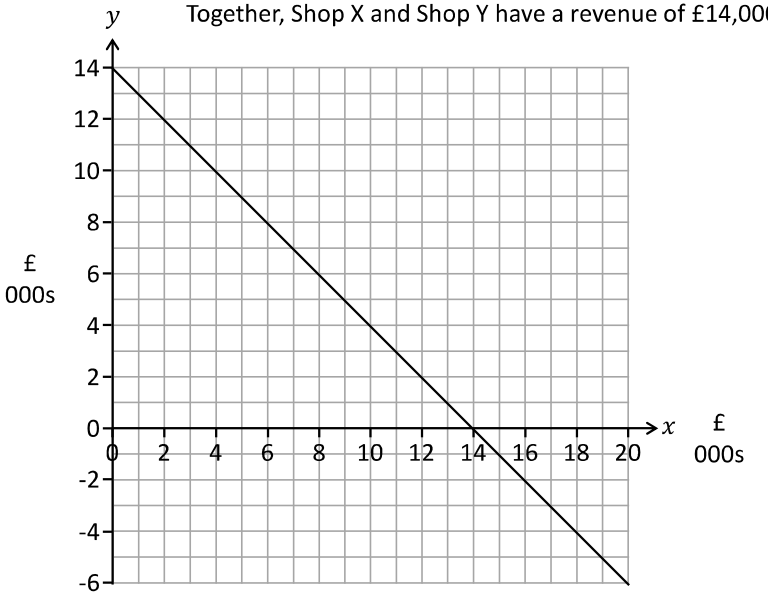
Yolanda gets £2 more than Xavi.

Xavi =
Yolanda =

Yolanda gets a quarter of what Xavi gets.

Xavi =
Yolanda =

Together, Shop X and Shop Y have a revenue of £14,000.



Express each different split of the revenue as an equation on the grid.
Use intersections to solve the simultaneous equations.

Shop Y earnt £6000 less than Shop X.

X =
Y =

Shop X earnt a third of what Shop Y earnt.

X =
Y =

If you halve the amount Shop X earnt, it is £5000 less than Shop Y.

X =
Y =

Shop X's revenue is £6000 greater than triple Shop Y's revenue.

X =
Y =

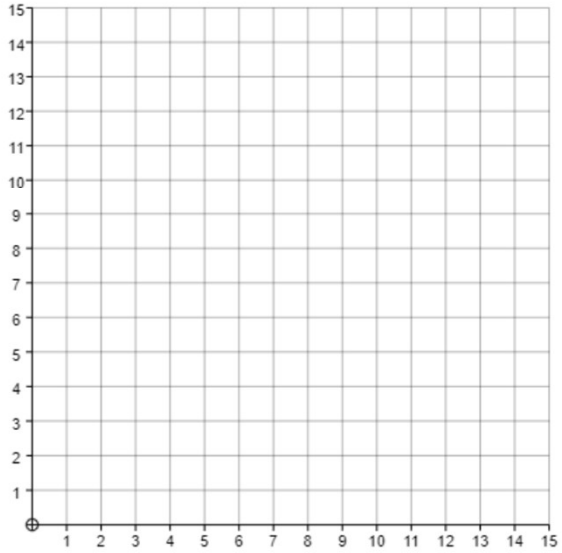
Fluency Practice

Investigating Simultaneous Equations

(a)

Solve the simultaneous equations $x + 2y = 10$, $2x + y = 14$

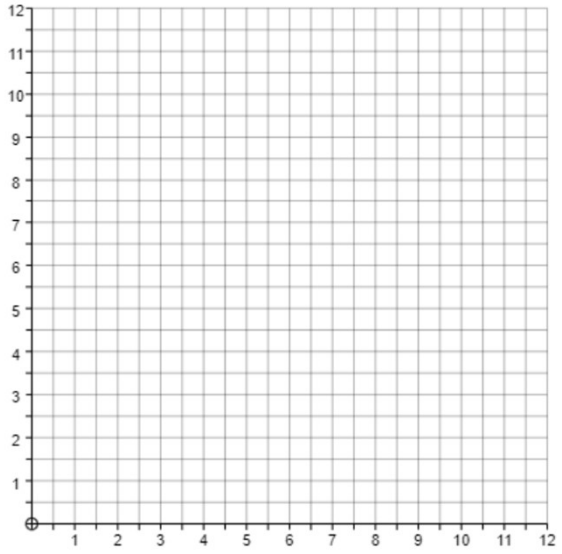
Plot the equations of the lines $x + 2y = 10$ and $2x + y = 14$ on the same axes below. What do you notice?



(b)

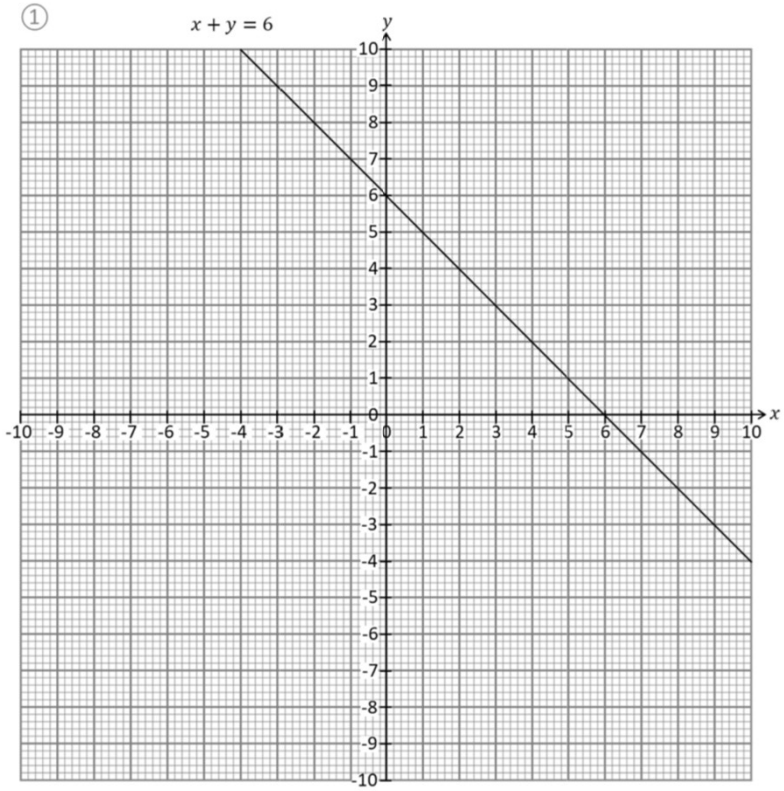
Solve the simultaneous equations $4x + 3y = 24$, $x + 2y = 11$

Plot the equations of the lines $4x + 3y = 24$ and $x + 2y = 11$ on the same axes below. What do you notice?



Plotting Simultaneous Equations

①

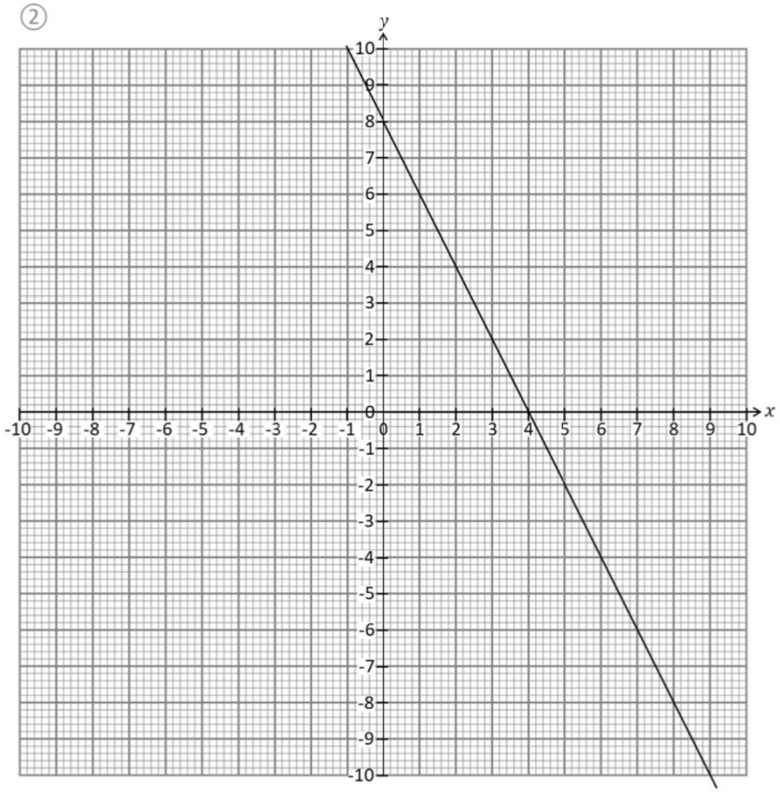


Plot & Solve...

$x + y = 6$ $y = 1$	$x =$ $y =$
$x + y = 6$ $y = 2x$	$x =$ $y =$
$x + y = 6$ $y = 2x - 9$	$x =$ $y =$
$y = 2x - 9$ $x + y = -3$	$x =$ $y =$
$x + y = -3$ $y = 0.5x + 3$	$x =$ $y =$
$x + y = -3$ $y + 2x = 2$	$x =$ $y =$

Plotting Simultaneous Equations

②



Plot & Solve...

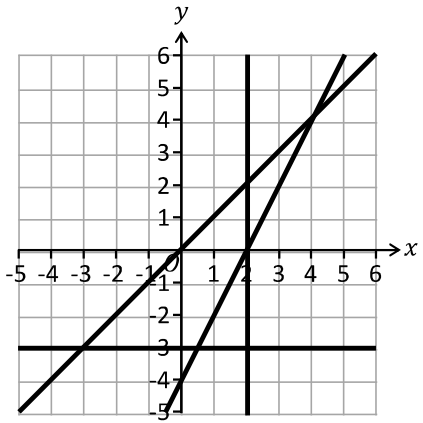
$2x + y = 8$ $y = 2x - 4$	$x =$ $y =$
$y = 2x - 4$ $y + 2x = 4$	$x =$ $y =$
$y + 2x = 4$ $y = 0.5x + 9$	$x =$ $y =$
$2y = x + 18$ $3x + y = -5$	$x =$ $y =$
$y - 2x + 4 = 0$ $2x + 3y = -12$	$x =$ $y =$
$2x + 3y = -12$ $3x - 2y = -4$	$x =$ $y =$

Purposeful Practice

Identifying Simultaneous Equations

① The grid shows 4 equations:

$y = x$ $\quad x = 2$ $\quad y = -3$ $\quad y = 2x - 4$



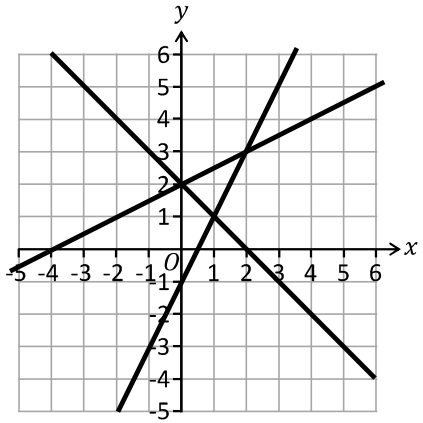
(2, 2) is the solution to which pair of simultaneous equations?

(0.5, −3) is the solution to which pair of simultaneous equations?

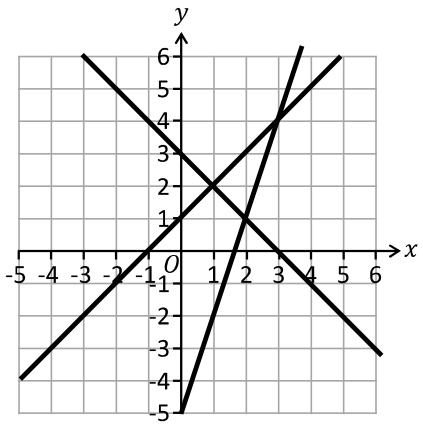
(4, 4) is the solution to which pair of simultaneous equations?

② The grid shows 3 equations. $y = 2x - 1$ $\quad y = \frac{x}{2} + 2$ $\quad y + x = 2$

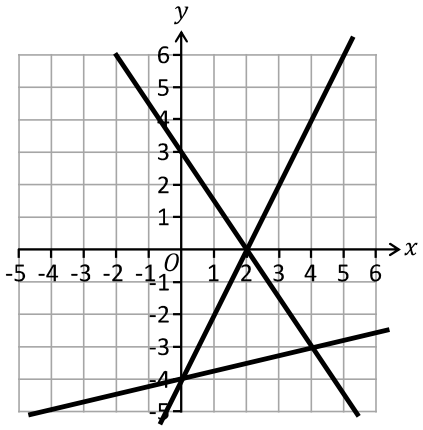
Write down the 3 pairs of simultaneous equations, and their solutions, shown on the grid.



③ What pairs of simultaneous equations, and their solutions, are on the grid?



④ What pairs of simultaneous equations, and their solutions, are on the grid?

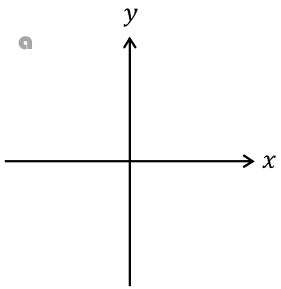


Fluency Practice

Solution Sketching

Sketch each pair of simultaneous equations on the grid. What method can you use to find the solutions?

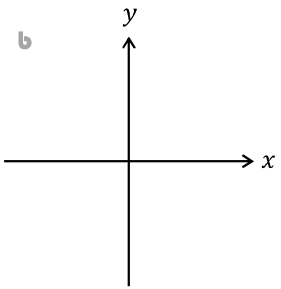
a



$y = x$

$x = 4$

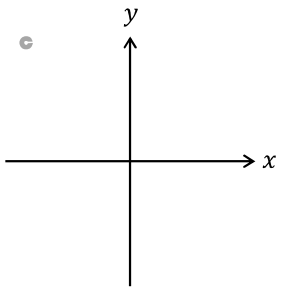
b



$y = 2x$

$y = -4$

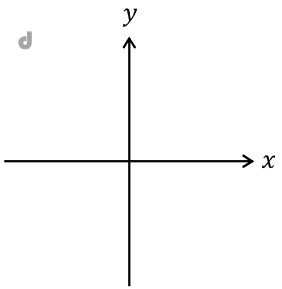
c



$y = 6$

$y = x + 4$

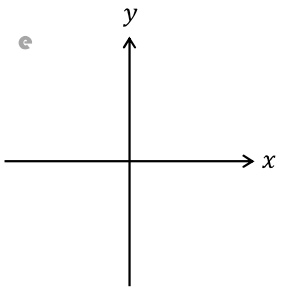
d



$y = x$

$y = x - 2$

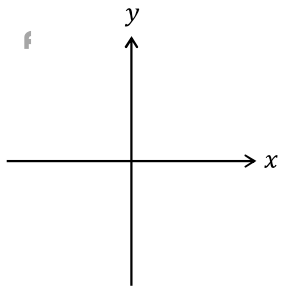
e



$y = -x$

$y = x - 6$

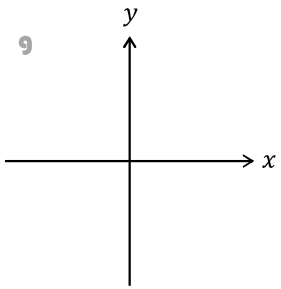
f



$x + y = 8$

$y = 3x$

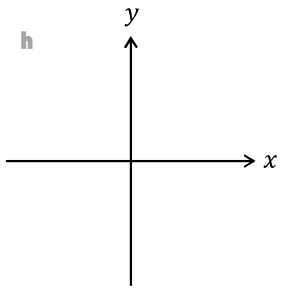
g



$y = -2x$

$x + y = 2$

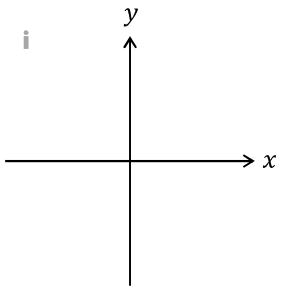
h



$2y - 2x = 10$

$y = x + 5$

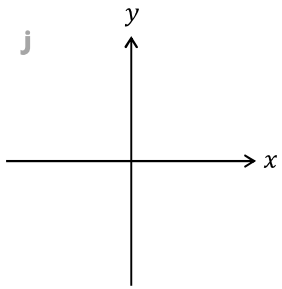
i



$x + y = -5$

$y = 2x - 2$

j

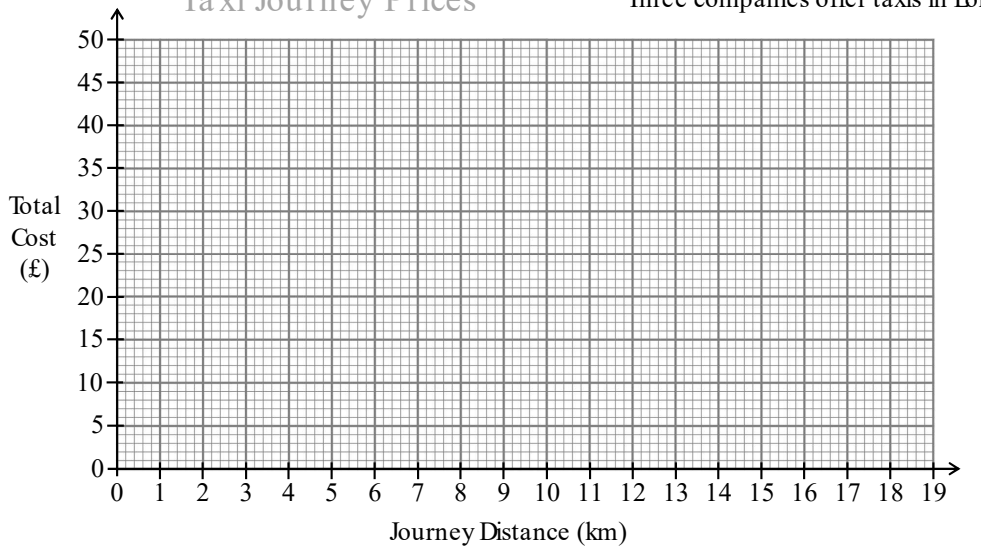


$2x + y = 10$

$y = 3x - 15$

Purposeful Practice

Taxi Journey Prices



Three companies offer taxis in London:

Quick Dash: £3 per mile
Zooom: £2 per mile with a £5 pick-up fee
Kar: 50p per mile with a £20 pick-up fee

Sketch these cost-relationships on the grid.

At what distance is…

…Zooom and Quick Dash the same price?

…Zooom and Kar the same price?

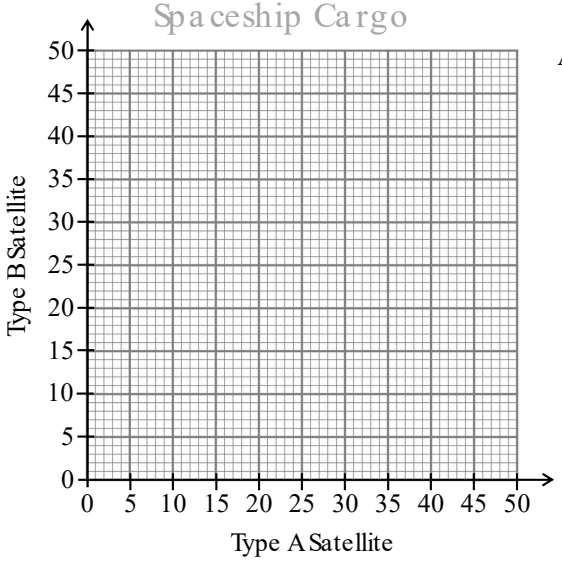
…Quick Dash and Kar the same price?

When is Zooom the cheapest option?

Uri has £22 to spend. Draw this graph and explain any intersections.

Lee wants to travel 13.5 km. *Estimate* the prices they could pay.

Spaceship Cargo



A spaceship will be loaded with 2 different types of mini-satellites (A & B).
In total the spaceship can carry 45 satellites.

Show this limit as a graph on the grid.

In discussion…

…Scientist G argues for an equal amount of A & B satellites.

…Scientist I argues that there should be 15 more B satellites than As onboard.

…Scientist H says there should be double the amount of A satellites compared to B satellites.

Draw each scientist's proposition as a graph and find solutions.

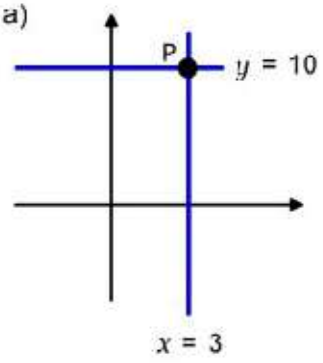
Before launch it is discovered the spaceship can only carry 39 satellites.
Show & find solutions for the new cargo possibilities.

Using the lines above, complete the table below, give your solutions to one decimal place where necessary.

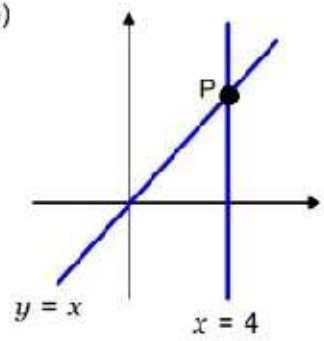
Line:	A	B	C	D
Equation:			$x + 2y = 4$	$x + 4y = 4$
Gradient				
Solution with A:				
Solution with B:				
Solution with C:				
Solution with D:				

10. For each graph, work out the co-ordinates of the point marked P.

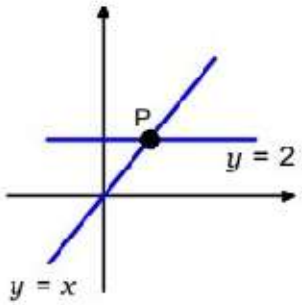
a)



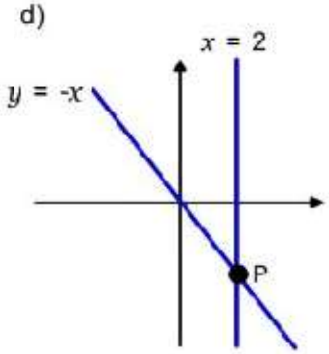
b)



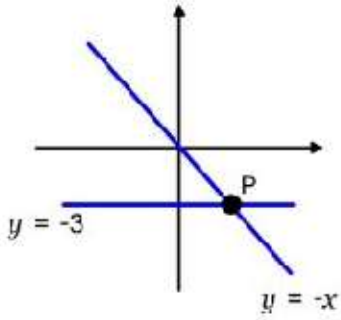
c)



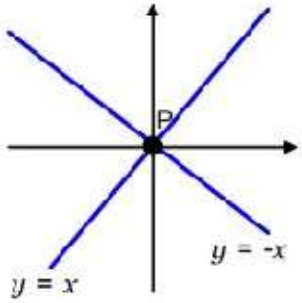
d)



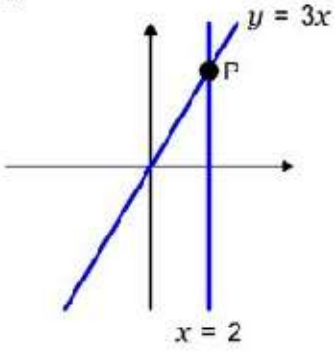
e)



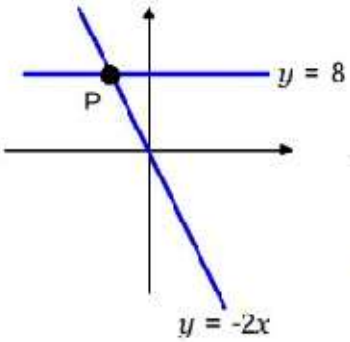
f)



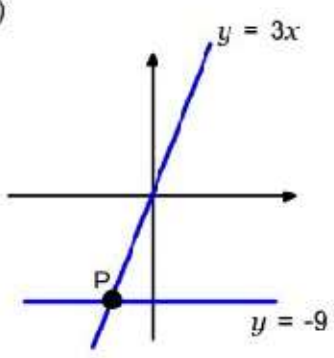
g)



h)



i)



2 Linear Simultaneous Equations

For each statement, circle the correct response.

1	$x = 2$ and $y = 3$ are the solutions to the simultaneous equations $x + y = 5$ and $x - y = 1$	True	False
2	$x = 5$ and $y = 2$ are the solutions to the simultaneous equations $2x + y = 12$ and $x + 5y = 15$	True	False
3	$x = 4$ and $y = 1$ are the solutions to the simultaneous equations $3x + 4y = 16$ and $x - y = 3$	True	False
4	$x = 6$ and $y = 3$ are the solutions to the simultaneous equations $2x - 3y = 6$ and $4x - y = 21$	True	False
5	$x = 3$ and $y = -2$ are the solutions to the simultaneous equations $x + y = 1$ and $x - y = 5$	True	False
6	$x = 4$ and $y = -5$ are the solutions to the simultaneous equations $2x + y = 13$ and $x - 2y = 14$	True	False
7	$x = 1.5$ and $y = -1$ are the solutions to the simultaneous equations $6x - 2y = 11$ and $4x + 3y = 9$	True	False
8	$x = \frac{1}{2}$ and $y = \frac{3}{2}$ are the solutions to the simultaneous equations $x + y = 2$ and $7x - y = 2$	True	False
9	$x = -0.5$ and $y = -4$ are the solutions to the simultaneous equations $2x - y = 3$ and $2x + 3y = -11$	True	False
10	$x = \frac{2}{3}$ and $y = 5$ are the solutions to the simultaneous equations $3x - y = -3$ and $\frac{3}{2}x + 2y = 11$	True	False

1. Solve:

a) $4a + 3b = 44$
$3a + 3b = 39$

b) $2a + b = 23$
$a + b = 16$

c) $5a + 2b = 27$
$5a + 4b = 49$

d) $3a + 2b = 16$
$4a + 6b = 28$

e) $2a + b = 14$
$4a + 6b = 36$

f) $6a + 5b = 39$
$2a + 3b = 17$

2. Explain why it is not possible to solve the pair of simultaneous equations:

$x + y = 9$
$2x + 2y = 18$

3. Select the correct pair of solutions to the simultaneous equations shown in the box:

$p + q = 7$
$5p + 6q = 36$

a) $p = 3$
$q = 4$

b) $p = 6$
$q = 1$

c) $p = 1$
$q = 6$

4. Solve:

a) $2a + 5b = 25$
$3a + 4b = 27$

b) $3a + 5b = 17$
$5a + 2b = 22$

c) $4a + 3b = 33$
$6a + 5b = 51$

d) $2a + 3b = 5$
$2a + 5b = 11$

e) $3a + 2b = -11$
$5a + 4b = -21$

f) $3a + 7b = -6$
$2a + 3b = 1$

5. Match pairs of equations with their solutions:

A $3x + y = 8$ $3x + 4y = 14$	B $2x + y = -2$ $3x + 2y = -1$	P $x = 3$ $y = 7$	Q $x = 1$ $y = 4$
C $3x + 2y = 11$ $2x + 3y = 14$	D $4x + 3y = -7$ $2x + 2y = -6$	R $x = 4$ $y = 2$	S $x = 2$ $y = 2$
E $6x + 5y = 34$ $4x + 3y = 22$	F $2x + 4y = -12$ $x + 3y = -9$	T $x = -3$ $y = 4$	U $x = -2$ $y = -1$
G $x + y = 10$ $2x + 5y = 41$	H $x + 2y = -4$ $2x + 3y = -7$	V $x = 2$ $y = -5$	W $x = 0$ $y = -3$

1. Solve:

a) $4p - 3q = 10$
$3p + 6q = 24$

b) $8t - 3u = 61$
$t - u = 7$

c) $2k + 3p = 30$
$4k - 5p = 16$

d) $-x + 3y = -11$
$4x + 2y = 16$

e) $-5c + 2d = 21$
$2c - 3d = -15$

f) $-6x + 4y = -42$
$-4x - 5y = -5$

2. Select the correct pair of solutions to the simultaneous equations shown in the box:

$-2g + 3h = 0$
$3g - h = 7$

a) $g = 2$
$h = -1$

b) $g = -6$
$h = 4$

c) $g = 3$
$h = 2$

3. Jake and Lucy are trying to solve the simultaneous equations shown.

$4x + 3y = 20$
$3y - 3x = 6$

Jake ***adds*** the two equations.
Lucy ***subtracts*** the two equations.

Who has correctly eliminated one unknown?

4. Solve:

a) $3a + 2b = -13$
$2a - 3b = -39$

b) $2c + 3d = 7$
$-c + 6d = -26$

c) $g + 3h = -3$
$h - 2g = -8$

d) $-3x = y + 8$
$2x - y = -7$

e) $4u + 3w - 3 = 0$
$3u - 2w = 32$

f) $-2x + 5y = -14$
$-2y + 5x = -7$

5. Spot the mistake!

a) $4a + 3b = 18$
$5a - 3b = 9$

$4a + 3b = 18$
$5a - 3b = 9$ −
$a = 9$

$36 + 3b = 18$

$a = 9, b = -6$

b) $2a + 3b = 4$
$4a + 5b = 10$

$4a + 6b = 4$
$4a + 5b = 10$ −
$b = -6$

$2a - 18 = 4$

$a = 11, b = -6$

c) $-a + 3b = 3$
$a - b = 1$

$-a + 3b = 3$
$a - b = 1$ +
$4b = 4$
$b = 1$

$a - 1 = 1$

$a = 2, b = 1$

Spot The Error*

*one error is the use of an inefficient method.

Solve **A**

$3x - 2y = 8$

$x + y = 2$

Solution

$3x - 2y = 8$

$2x + 2y = 2$ +

$5x = 10$

$x = 2$

$y = 0$

Solve **B**

$2x + y = 17$

$x - 2y = -1$

Solution

$2x + y = 17$

$2x - 4y = -2$ -

$-3y = 15$

$y = -5$

$x = 11$

Solve **C**

$3x - 5y = 27$

$2x + 5y = 8$

Solution

$2x = 8 - 5y$

$3x = 1.5(8 - 5y)$

$1.5(8-5y) - 5y = 27$

$12 - 7.5y - 5y = 27$

$12 - 27 = 12.5y$

$-1.2 = y$

$2x - 6 = 8; x = 7$

Sc

3x

5x

So

y =

5x

5x

2x

13

13

y =

When you have finished, work out the correct answer for each pair of equation

Fluency Practice

$2x = y \qquad y = x + 8$

$2x = y = x + 8$

$2x = x + 8$

$x = 8$

$2x = y \qquad 2(8) = 16$

$x \rightarrow (8, 16) \leftarrow y$

Simultaneous Equations

a) $2x = y \qquad y = x + 4$

$2x = x + 4$

b) $3x = y \qquad y = x + 12$

c) $3x = y \qquad y = x - 4$

d) $x + 3 = y$

$y = 2x - 2$

---

e) $x + 7 = y$

$y = 3x - 9$

f) $x + 10 = y$

$y = 3x + 18$

g) $2x + 9 = y$

$y = x + 12$

h) $2x + 6 = y$

$y = 3x + 4$

i) $y = 2x + 8$

$y = 4x - 6$

---

j) $y = 2x + 11$

$y = 5 - x$

k) $y = \frac{x}{2}$

$y = x - 6$

l) $y = \frac{x}{2} + 3$

$y = x - 1$

m) $y = \frac{x}{3} + 2$

$y = 2x - 3$

n) $2y = 3x + 3$

$y = x + 4$

---

o) $2y = 2x + 10$

$y = x + 5$

p) $y = x - 6$

$3y = 2x - 16$

q) $2y = 2x - 22$

$y = \frac{x}{2} - 7$

r) $x + 3 = y$

$y = \frac{x + 11}{2}$

s) $2y + 10 = 2x$

$y = 2x - 11$

t) $2y = 3x - 2$

$3y = 2x + 7$

Fluency Practice

Manipulating Equations to Solve

Use $x + y$ to solve the other equations (the variable values will be different each time).

$x + y = 8$

$2x + 2y = 16$	$3x + 3y = 24$	$x + 2y = 13$	$x + 4y = 20$	$2x + y = 18$
$2x + y = 11$	$3x + 2y = 18$			
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

$x + y = 5$

$5x + 5y = 25$	$x + 6y = 15$	$2x + 4y = 10$	$3x + 6y = 6$	$x + 4y = 35$
$5x + y = 21$				
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

$2x + y = 12$

$4x + 2y = 24$	$6x + y = 28$	$x + 3y = 26$	$3x + 5y = 39$	$8x + 7y = 42$
$4x + y = 22$				
$x =$	$x =$	$x =$	$x =$	$x =$
$y =$	$y =$	$y =$	$y =$	$y =$

Fluency Practice

Substitution Solving

a

If $3x + 2 = 14$

and $y + x = 9$

$y = ?$

b

If $2x - 5 = 7$

and $y + 2x = 15$

$y = ?$

c

If $5x - 8 = 7$

and $2y + 2x = 18$

$y = ?$

d

If $8 - x = 3$

and $3y + 3x = 27$

$y = ?$

e

If $19 - 2x = 7$

and $2x + 2y = 16$

$y = ?$

f

If $3(x + 5) = 21$

and $6y + 5x = 10$

$y = ?$

g

If $3(2x + 5) = 33$

and $3(y + x) = 36$

$y = ?$

h

If $4x + 13 = 5$

and $3y + 2x = 26$

$y = ?$

i

If $\frac{3x + 1}{2} = 11$

and $4x - 2y = 22$

$y = ?$

j

If $2(5 - 2x) = 26$

and $8 - 3x = 4y$

$y = ?$

Fluency Practice

Mixed Written Methods to Solve Simultaneous Equations

Do you prefer a method?

Eliminate (subtract)

$3x + y = 19$
$x + y = 9$

$x =$ $y =$

Eliminate (add)

$3x - y = 7$
$x + y = 13$

$x =$ $y =$

Balance & Eliminate

$3x + y = 9$
$x + 2y = 13$

$x =$ $y =$

Substitute

$x = 2y + 3$
$x + 2y = 19$

$x =$ $y =$

Rearrange & Substitute

$x - y = 6$
$x + 3y = 14$

$x =$ $y =$

$4x + 2y = 18$
$4x - y = 3$

$5x + 3y = 32$
$2x - 3y = -4$

$x - 2y = 6$
$3x + y = 32$

$3x + 2y = 14$
$y = 3x - 2$

$x = 9 - y$
$2x + 3y = 21$

$3x + 2y = 15$
$3x + y = 12$

$x + y = 8$
$3x + 2y = 18$

$3x + 5y = 28$
$2x + 5y = 22$

$x = 3y - 2$
$3x + 2y = 16$

$x + 3y = 2$
$2x + y = 9$

$2x + 3y = 23$
$6x + y = 29$

$x - y = 8$
$4x + 2y = 44$

$x + 4y = 22$
$3x - 4y = 18$

Fluency Practice

Solving Simultaneous Equations using Substitution

Ex)

$$x + y = 12$$
$$y = x + 2$$

$$x + (\boldsymbol{x} + \mathbf{2}) = 12$$
$$2x + 2 = 12$$
$$2x = 10$$
$$x = 5$$

$$y = (\mathbf{5}) + 2$$
$$y = 7$$

a) $x + y = 10$, $y = x + 4$

b) $x + y = 10$, $y = x - 6$

c) $2x + y = 9$, $y = x - 3$

d) $x + y = 13$, $y = 2x + 4$

---

e) $x + 2y = 14$, $y = x + 1$

f) $x + 3y = 14$, $y = x + 2$

g) $2x - y = 7$, $y = x - 2$

h) $x + 2y = 17$, $y = 2x + 1$

i) $x + 3y = 33$, $y = 2x - 3$

---

j) $2x + 3y = 30$, $y = 3x - 1$

k) $2x + 3y = 28$, $x = 4y + 3$

l) $3x + 2y = 15$, $y = 9 - 2x$

m) $4x + 2y = 32$, $y - 4x = 4$

n) $5x - y = 12$, $2y - 6x = -8$

Fluency Practice

Substitute & Solve

1) $a = b + 1$
$a + b = 5$ $a =$ $b =$

2) $a = b + 3$
$a + b = 9$ $a =$ $b =$

3) $a = 3b$
$a + b = 8$ $a =$ $b =$

4) $a = b - 4$
$a + b = 10$ $a =$ $b =$

5) $x = 2y$
$2x + y = 20$ $x =$ $y =$

6) $a = 2b + 1$
$a + b = 13$ $a =$ $b =$

7) $a = 3b + 2$
$a + b = 6$ $a =$ $b =$

8) $g = h + 3$
$2g + h = 21$ $g =$ $h =$

9) $b = 2a + 3$
$a + 3b = 23$ $a =$ $b =$

10) $a - b = 4$
$a + 3b = 12$ $a =$ $b =$

11) $a - b = 4$
$2a + b = 17$ $a =$ $b =$

12) $m - n = 2$
$2m - n = 10$ $m =$ $n =$

13) $a + b = 6$
$a + 2b = 11$ $a =$ $b =$

14) $k + p = 6$
$2k + p = 9$ $k =$ $p =$

15) $a + b = 10$
$3a + 2b = 22$ $a =$ $b =$

16) $a + 2b = 13$
$2a + b = 11$ $a =$ $b =$

17) $x - 2y = -2$
$3x - y = 9$ $x =$ $y =$

18) $2a + 3b = 23$
$3a + 2b = 17$ $a =$ $b =$

Purposeful Practice

1) In each question say which variable we want to eliminate

a) $5x + 3y = 22$
$2x + 3y = 10$

b) $5x + 3y = 22$
$5x + 2y = 10$

c) $5x + 3y = 22$
$-5x + 2y = 10$

d) $4x - y = 25$
$x + y = 10$

e) $-3x - y = 25$
$-3x + 2y = 10$

2) For each part in question 1, would we add or subtract?

a) b) c) d) e)

3) Tom says that we need to add these to eliminate the x.
Mark says you need to subtract them to eliminate the y.
Who do you agree with? Why?

$$3x - y = 9$$
$$-3x + y = -9$$

4) Tarquin says, *"you add the equations because the equations have different signs in the middle."*

$$11x + 4y = 19$$
$$11x - 3y = 13$$

5) Now solve each pair of equations in question one.

(1) Complete this table, to solve each pair of simultaneous equations.

Equations	Solved for 1st Variable	Solved for 2nd Variable
$3x - y = 1$ $x + y = 3$	$3x - y = 1$ $+ \quad x + y = 3$ $4x \quad = 4$ $x \quad = 1$	Sub $x = 1$ into either equation: $1 + y = 3$ $y = 2$
$2x - y = 2$ $x + y = 7$		
$4x + y = 9$ $2x - y = 3$		
$-x + 2y = 13$ $x + y = 8$		
$2x + y = 7$ $x + y = 4$		
$3x + y = 9$ $2x + y = 7$		
$5x - 2y = 13$ $3x + 2y = 3$		

(2) Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$5x + 2y = 11$ $3x - 4y = 4$	$\times 2$ → $10x + 4y = 22$ $3x - 4y = 4$	$10x + 4y = 22$ $+ \quad 3x - 4y = 4$ $13x = 26$ $x = 2$	Sub $x = 2$ into any equation. $5(2) + 2y = 11$ $10 + 2y = 11$ $2y = 1$ $y = \frac{1}{2}$
$3x + 2y = 8$ $2x - y = 3$			
$3x + 2y = 11$ $2x - y = -3$			
$2x + 3y = 11$ $3x + y = 13$			
$2x - 3y = 8$ $x - 5y = 11$			
$x + 6y = 0$ $3x - 2y = -10$			
$5x - 4y = 24$ $2x = y + 9$			

(3) Complete this table, to solve each pair of simultaneous equations. One of the equations will need to be 'scaled'.

Equations	Scaled	Solved for 1st Variable	Solved for 2nd Variable
$2x + 3y = 5$ $5x - 2y = -16$	$\times 2$ → $4x + 6y = 10$ $\times 3$ → $15x - 6y = -48$	$4x + 6y = 10$ $+ \quad 15x - 6y = -48$ $19x = -38$ $x = -2$	Sub $x = -2$ into any equation. $2(-2) + 3y = 5$ $-4 + 3y = 5$ $3y = 9$ $y = 3$
$2x + 5y = 24$ $4x + 3y = 20$			
$2x + 3y = 11$ $3x + 4y = 15$			
$3x + 8y = 27$ $-4x - 3y = -13$			
$2x + 3y = 14$ $8x - 5y = 5$			
$9x - 4y = -20$ $5x - 6y = -13$			
$2y = 3x - 5$ $1 = 5x - 4y$			

Fluency Practice

A1 Solve $x + y = 1$ $x - y = 5$	**A2** Solve $5x + y = 17$ $x + y = 3$	**A3** Solve $6x - 5y = 9$ $6x + 3y = 33$	**A4** Solve $x + 5y = -13$ $4x - 5y = 48$
B1 Solve $8x - y = 7$ $12x - 8y = 6$	**B2** Solve $3x - 2y = 13$ $x - y = 5$	**B3** Solve $2x - 3y = 3$ $3x + 6y = 1$	**B4** Solve $3x + 5y = 7$ $9x + 11y = 13$
C1 Solve $5x + 3y = 14$ $2x + 2y = 4$	**C2** Solve $4x + 5y = 13$ $3x - 2y = 27$	**C3** Solve $2x + 7y = 12$ $3x + 8y = 13$	**C4** Solve $3x + 2y = 7$ $4x - 3y = 15$
D1 Solve $y = 4x - 8$ $y = 3x - 5$	**D2** Solve $y = 4x - 2$ $y = 9x - 12$	**D3** Solve $3x + 2y = 61$ $y = 2x - 22$	**D4** Solve $3x - 2y = -24$ $y = 5 - 2x$

Fluency Practice

Q	Solve the simultaneous equations:	Answers
1	$7x + 2y = 29$ and $3x + 2y = 17$	
2	$7x + 2y = 29$ and $y = 2x - 2$	
3	$7x + 2y = 13$ and $3x + 2y = 1$	
4	$7x + 2y = 13$ and $y = 2x - 10$	
5	$7x + 2y = -20$ and $3x + 2y = -12$	
6	$7x + 2y = -20$ and $x = y + 1$	
7	$7x - 2y = 29$ and $3x - 2y = 17$	
8	$7x - 2y = 29$ and $y = x - 7$	
9	$7x - 2y = -20$ and $3x - 2y = -12$	
10	$7x - 2y = -20$ and $y = x + 5$	
11	$-7x - 2y = 29$ and $-3x - 2y = 17$	
12	$-7x - 2y = 29$ and $x = 1 - y$	

Q	Solve the simultaneous equations:	Answers
13	$3x + 4y = 11$ and $-3x + y = 14$	
14	$3x + 4y = 11$ and $y = 3x + 14$	
15	$5x - 0.7y = 46$ and $6x + 0.7y = 9$	
16	$5x - 0.7y = 46$ and $y = -6x$	
17	$2x + 3y = 9$ and $5x + 7y = 23$	
18	$2x + 3y = 9$ and $x = 5 - y$	
19	$3x - 2y = -5$ and $2x - 4y = 2$	
20	$3x - 2y = -5$ and $x = 2y + 1$	
21	$3x + 2y = 7$ and $4x - 3y = 15$	
22	$3x + 2y = 7$ and $y = 2x - 7$	
23	$ax + by = c$ and $ax + dy = e$	
24	$ax + by = c$ and $y = dx + e$	

(a) Solve
$$4x - y = 17$$
$$x = y + 2$$

(b) Solve
$$2x + y = 6$$
$$y = 4x + 3$$

(c) Solve
$$3x + 7y = 13$$
$$y = x - 11$$

(a) Solve
$$4x - 3y = 7$$
$$3y = x + 5$$

(b) Solve
$$y + 1 = 3x$$
$$2x - 3y = 24$$

(c) Solve
$$3x + 5y = 29$$
$$y + 11 = 5x$$

(a) Solve
$$4x + 6y = 74$$
$$11 - y = 2x$$

(b) Solve
$$y - 8 = 6x$$
$$4x + 5y + 28 = 0$$

(c) Solve
$$8 - x = 3y$$
$$10 - 3x = 5y$$

(a) Given that $7x = 2y + 34$ and $3x + 5y + 3 = 0$, find the value of $x^2 + y^2$

(b) Solve $\frac{3x+1}{2} = y$
$$5y - 4x = 13$$

(c) Find the coordinates of intersection of the straight lines with equations
$$y = 3x - 2$$
$$x + 3y = 19$$

Question 1: Solve the following simultaneous equations by using elimination.

(a) $6x + y = 18$, $4x + y = 14$

(b) $4x + 2y = 10$, $x + 2y = 7$

(c) $9x - 4y = 19$, $4x + 4y = 20$

(d) $2x + y = 36$, $x - y = 9$

(e) $6x - 3y = 12$, $4x - 3y = 2$

(f) $3x - 6y = 6$, $2x - 6y = 3$

(g) $8x + 7y = 39$, $8x + 2y = 34$

(h) $x + 3y = 38$, $x + 6y = 53$

(i) $6x + 3y = 48$, $6x + y = 26$

(j) $2x - 4y = 10$, $2x + 3y = 24$

(k) $5x - 2y = 120$, $5x + y = 165$

(l) $x - 2y = 8$, $x - 3y = 3$

(m) $3x + 2y = 54$, $2x - 2y = 16$

(n) $7x - 4y = 80$, $3x - 4y = -80$

(o) $5x - 2y = -23$, $5x - 6y = -39$

(p) $6x + 2y = -26$, $2x + 2y = -10$

(q) $x - 5y = 65$, $2x - 5y = 85$

(r) $10x - 10y = -40$, $10x + 4y = 16$

Question 2: Solve the following simultaneous equations by using elimination.

(a) $3x + 2y = 23$, $2x - y = 6$

(b) $3x - 3y = 9$, $2x + y = 12$

(c) $4x + 2y = 34$, $3x + y = 21$

(d) $9x - 4y = 59$, $2x - y = 12$

(e) $2x + 8y = 43$, $x + 3y = 18$

(f) $6x + 3y = 45$, $2x - 2y = 12$

(g) $5x + 4y = 130$, $x + 6y = 130$

(h) $10x - 15y = 25$, $x - 2y = 1$

(i) $3x + 8y = 97$, $2x + 4y = 58$

(j) $3x - y = 4$, $5x + 4y = 52$

(k) $4x + 9y = 10$, $2x + 3y = 2$

(l) $5x - 3y = 33$, $3x - 9y = 63$

(m) $2x + 4y = -2$, $4x + 2y = -10$

(n) $8x + 4y = -28$, $3x - 12y = 30$

(o) $15x - 4y = 82$, $5x - 9y = 12$

(p) $12x + 3y = 9$, $2x + 11y = -9$

(q) $9x - 7y = 111$, $x - 2y = 16$

(r) $8x - y = 4$, $3x + 8y = -166$

Question 3: Solve the following simultaneous equations by using elimination.

(a) $2x + 2y = 14$, $5x - 3y = 19$

(b) $2x + 3y = 1$, $7x + 2y = -22$

(c) $5x + 3y = 22$, $2x + 4y = 20$

(d) $5x - 6y = 28$, $4x - 4y = 24$

(e) $3x + 2y = 7$, $2x + 9y = 43$

(f) $3x + 3y = -6$, $4x - 4y = -24$

(g) $3x + 8y = 31$, $5x + 3y = 31$

(h) $7x - 15y = 2.5$, $3x - 2y = 5.5$

(i) $3x + 2y = 53$, $2x + 5y = 72$

(j) $5x - 3y = 18$, $2x + 4y = 54$

(k) $2x + 9y = 11$, $9x + 3y = -63$

(l) $2x - 4y = 4$, $5x - 3y = 24$

(m) $3x + 3y = 42$, $2x + 4y = 38$

(n) $6x + 2y = -2$, $4x - 3y = 29$

(o) $4x - 4y = 8$, $5x - 3y = 18$

(p) $4x + 3y = 9$, $5x + 2y = 13$

(q) $4x - 2y = 18$, $2x - 3y = 15$

(r) $5x + 2y = 38$, $2x - 3y = 19$

Question 4: Solve the following simultaneous equations by rearranging and then using elimination.

(a) $x = 10 - y$, $2x + y = 17$

(b) $x - 4 = y$, $x + 3y = 12$

(c) $2x + 6y = 4$, $x = 12 + 2y$

(d) $3x = 10 + 5y$, $3y = 52 - 4x$

(e) $2x + y - 18 = 0$, $3y = 7x + 80$

(f) $6x + 2y + 6 = 0$, $7x - 5y - 93 = 10$

Apply

Question 1: The cost of buying a coffee and a tea in a cafe is £4.
The cost of buying a coffee and three teas in a cafe is £7.
Work out the cost of buying a coffee and the cost of buying a tea.

Question 2: The sum of Rosemary's age and Hannah's age is 102 years.
The difference between Rosemary's age and Hannah's age is 52 years.
Rosemary is older than Hannah.
Find the age of each woman by using simultaneous equations.

Question 3: Five adult tickets and three child tickets for a movie cost £58.
Two adult tickets and eight child tickets for a movie cost £47.
Find the cost of each type of ticket.

Question 4: Four chairs and two tables cost £218.
Six chairs and seven tables cost £587.
Find the total cost of buying twenty chairs and Yve tables.

Question 5: A plumber charges a price for each hour, £h, and a Yxed charge, £c.
A 5 hour job costs £155 in total.
A 8 hour job costs £230 in total.
How much would a job that lasts 2 hours cost?

Question 6: Barry buys 200 pieces of stationery for £76.
Of the 200 pieces of stationery, x of them are rulers that cost 50p each and y of them are pens that cost 20p each.
Find how many rulers Barry buys and how many pens he buys.

Question 7: In a greengrocers, 4kg of bananas and 3kg of apples costs £7.50
In the same greengrocers, 3kg of bananas and 5kg of apples costs £8.10
How much would 2kg of bananas and 2kg of apples cost?

Question 8: Can you spot any mistakes in the question below?

2. $x = 3y + 9$
$x = 5y + 13$

3. $x = 3y + 9$
$3x - 5y = 10$

4. $6x + 5y - 8 = 0$
$4x - 3y - 18 = 0$

5. $y = 2x$
$3x + 2y = 21$

6. $y = 6x - 3$
$8 - 5x = y$

7. $9x + 12y = -6$
$6x - 5y = -17$

8. $x = 5$
$3x - 2y = 13$

9. $5x + 3y = 8$
$5x - 4y = 7$

10. $y = \frac{3}{5}(x - 3)$
$y = \frac{3}{4}x + 1$

11. $y = 2x + 1$
$3x + \frac{5y}{2} = 8$

12. Make up and solve an example in simultaneous equations which is solved more readily by the method of comparison than by either of the other two methods of elimination.

13. Make up and solve an example in simultaneous equations which is solved more readily by the method of substitution than by either of the other two methods.

14. Make up and solve an example solved more readily by the method of addition and subtraction than by the other two methods.

Solve the following pairs of simultaneous equations:

1. $x + 4y = 1,$
   $x + 7y = 4.$
2. $x + 2y = 1,$
   $3x + 5y = 1.$
3. $5x + 3y = 1,$
   $7x + y = 11.$
4. $x - y = 5,$
   $3x - 4y = 16.$
5. $5x + 4y = 3,$
   $2x + 3y = 4.$
6. $3x + 4y = 1.$
   $5x + 3y = 9.$
7. $\frac{x}{3} + \frac{y}{2} = 4, x - \frac{y}{2} = 4.$
8. $\frac{x}{3} + \frac{y}{6} = 5, \frac{x}{2} - \frac{y}{9} = 1$
9. $2x + y = 9,$
   $3x + 2y = 9.$
10. $7x - 5y + 5 = 0,$
    $3x + 2y - 2 = 0.$
11. $\frac{2}{x} + \frac{3}{y} = 2, \frac{4}{x} + \frac{3}{y} = 3.$
12. $\frac{6}{x} + \frac{1}{y} = 2, \frac{4}{x} + \frac{2}{y} = 2.$
13. $\frac{9}{x} - \frac{4}{y} = 5, \frac{3}{x} + \frac{2}{y} = 0.$
14. $\frac{5}{x} - \frac{2}{y} = 3, \frac{6}{x} + \frac{7}{y} = 13.$
15. $5/x - 1/y = 1,$
    $3/x + 2/y = 11.$
16. $\frac{2}{x} - \frac{1}{y} = 7, \frac{3}{x} + \frac{2}{y} = 28.$
17. $\frac{2}{x} + \frac{1}{y} = 1, \frac{5}{x} - \frac{3}{y} = 19.$
18. $x + \frac{3}{y} = 5, 3x - \frac{2}{y} = 4.$
19. $5x + \frac{2}{y} = 4, 13x + \frac{4}{y} = 11.$
20. $\frac{2}{x} - \frac{4}{y} = 2, \frac{8}{x} + \frac{12}{y} = 1.$
21. $\frac{8}{x} - 2y = 10, \frac{6}{x} - y = 6.$

Solve the following pairs of simultaneous equations by substituting the value of x given by the first equation into the second equation.

22. $x = y - 1,$
    $x + 2y = 8.$
23. $x = 3y - 2,$
    $x + 5y = 14..$
24. $x = y - 2,$
    $5x - y = 6.$
25. $x = 3y - 17,$
    $x + 2y = 8.$
26. $x = 3y + 5,$
    $x + y = 1.$
27. $x = 5y + 13,$
    $2x + y = 4.$

Solve the following pairs of simultaneous equations:

28. $x + y = 5 = 3x - 2y.$
29. $3x - y = 7 = 4x + y.$
30. $x = 5 - x - y = y - 2x.$
31. $3x + 2y = 5x - y = 13.$
32. $4x - 3y = 6 - 2x = y.$

33. If $3x - 2y = 11, x - 5y = 8,$ find the values of $2x + y$ and $x - 2y$

34. If $2x + 5y = 19, 3x + 4y = 11,$ find the values of $x + 3y$ and $3x - y$

35. If $\frac{3}{x} + \frac{4}{y} = 3, \frac{9}{x} - \frac{4}{y} = 1,$ find the values of $\frac{1}{x} + \frac{1}{y}$ and $\frac{1}{x} - \frac{1}{y}$.

Solve the following pairs of simultaneous equations:

36. $\frac{x}{y} = \frac{3}{4}, x + y = 14.$

37. $\frac{x+y}{2x+y} = \frac{3}{5}, x - y = 1.$

38. $\frac{x+2}{y} = 2, x + y = 10.$

39. $\frac{x+1}{x+y} = \frac{1}{2}, \frac{x}{y} = \frac{1}{2}.$

40. $\frac{x-3}{y} = 1, x + y = 13.$

Patrons are reminded to try to use the substitution method where possible. Look for an x or y to isolate from one equation. Then substitute this into the *other* equation. For example solve

$$\begin{matrix} 3x+y=7 \\ 2x+5y=2 \end{matrix}.$$

From the first we can see that $y=7-3x$. Substituting this into the second we find $2x+5(7-3x)=2$ which solves to $x=\frac{33}{13}$. We then place this value into $y=7-3x$ to discover $y=-\frac{8}{13}$. So $(x,y)=(\frac{33}{13},-\frac{8}{13})$

Present answers in the form $(x,y)=(-1,\frac{1}{2})$. (Don't forget the brackets!)

1. $\begin{matrix} x+y=3 \\ x-y=2 \end{matrix}$.
2. $\begin{matrix} x-y=4 \\ x+y=7 \end{matrix}$.
3. $\begin{matrix} 2x+y=3 \\ x-2y=2 \end{matrix}$.
4. $\begin{matrix} x+2y=4 \\ 3x-2y=-7 \end{matrix}$.
5. $\begin{matrix} 5x-y=4 \\ 4x-5y=0 \end{matrix}$.
6. $\begin{matrix} 2x-3y=5 \\ 3x+2y=2 \end{matrix}$.
7. $\begin{matrix} 4x-2y=-9 \\ 3x+5y=3 \end{matrix}$.
8. $\begin{matrix} x-2y=2 \\ 3x+2y=1 \end{matrix}$.
9. $\begin{matrix} 3x-y=3 \\ 2x+3y=-1 \end{matrix}$.
10. $\begin{matrix} y-2x=5 \\ 5x-7y=2 \end{matrix}$.
11. $\begin{matrix} x+3y=4 \\ 5x-2y=6 \end{matrix}$.
12. $\begin{matrix} 4x-y=2 \\ 3x+4y=1 \end{matrix}$.
13. $\begin{matrix} 5x+4y=1 \\ x-3y=0 \end{matrix}$.
14. $\begin{matrix} x+y=-7 \\ 2x+3y=4 \end{matrix}$.
15. $\begin{matrix} 2x+y=2 \\ x-6y=1 \end{matrix}$.
16. $\begin{matrix} 3x+2y=1 \\ y-2x=-3 \end{matrix}$.
17. $\begin{matrix} a-b=2 \\ 3a-2b=-4 \end{matrix}$.
18. $\begin{matrix} 4x+7y=10 \\ 3x-y=-2 \end{matrix}$.
19. $\begin{matrix} 2x+3y=1 \\ 3x-4y=2 \end{matrix}$.
20. $\begin{matrix} 5x+y=7 \\ 4x-\frac{1}{2}y=2 \end{matrix}$.
21. $\begin{matrix} x+5y=0 \\ 3x+4y=-1 \end{matrix}$.
22. $\begin{matrix} \frac{x+y}{2}-\frac{x-y}{3}=1 \\ x-2y=2 \end{matrix}$.
23. $\begin{matrix} \frac{x+2y}{3}-\frac{x-3y}{7}=x \\ x+3y=2 \end{matrix}$.
24. $\begin{matrix} x+ay=0 \\ 2x+3y=-1 \end{matrix}$.

Fluency Practice

25. $\begin{matrix} kx + y = 4 \\ 2x - 3y = 2 \end{matrix}$.

26. $\begin{matrix} ax + 4y = 6 \\ bx - y = 5 \end{matrix}$.

27. $\begin{matrix} kx + y = 1 \\ 5x - ky = m \end{matrix}$.

28. $\begin{matrix} x + y = 1 \\ ax + by = 1 \end{matrix}$.

29. $\begin{matrix} x + ay = 3 \\ ax + by = 4 \end{matrix}$.

30. $\begin{matrix} 4x + y = 3 \\ ax + by = c \end{matrix}$.

31. $\begin{matrix} x + by = 2 \\ \frac{x + ay}{2} - \frac{x - y}{3} = 1 \end{matrix}$.

32. $\begin{matrix} \frac{ax + 1}{2} + \frac{by + 2}{3} = 1 \\ \frac{5x + 1}{3} + \frac{ay + 1}{2} = 1 \end{matrix}$.

33. $\begin{matrix} 23x + 21y = 1 \\ 21x + 23y = -1 \end{matrix}$.

Fluency Practice

Solve and check each result:

1. $3x + 4y = 2a$
   $5x + 6y = 4a$

2. $2ax + 3by = 4ab$
   $5ax + 4by = 3ab$

3. $ax + by = 1$
   $a'x + b'y = 1$

4. $x - y = 2n$
   $mx - ny = m^2 + n^2$

5. $2bx + ay = 4b + a$
   $abx - 2aby = 4b + a$

6. $ax - by = a^2 + b^2$
   $bx + ay = 2(a^2 + b^2)$

7. $ax + by = c$
   $mx + ny = d$

8. $bx + ay = a + b$
   $ab(x - y) = a^2 - b^2$

9. $c^2x - d^2y = c - d$
   $cd(2dx - cy) = 2d^2 - c^2$

10. $\frac{x + m}{y - n} = \frac{n}{m}$
    $x + y = 2n$

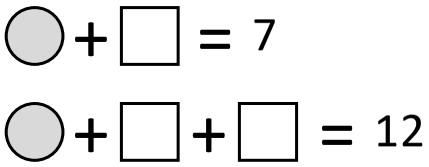
11. $(a + 1)x - by = a + 2$
    $(a - 1)x + 3by = 9a$

12. $(a - b)x - (a + b)y = a^2 + b^2$
    $bx + ay = 0$

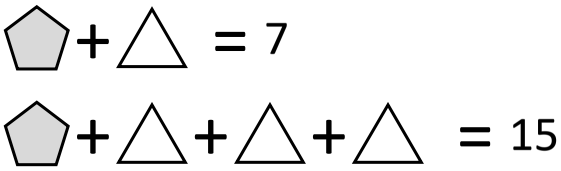
Simultaneous Equations

Find the value of each shape in each pair of equations.

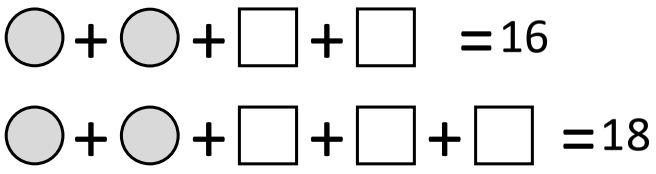
a)



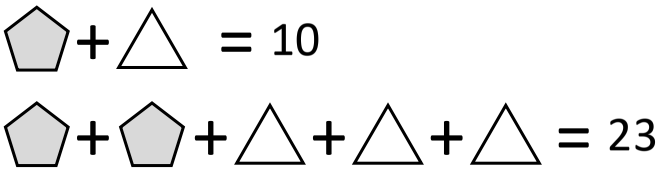
b)



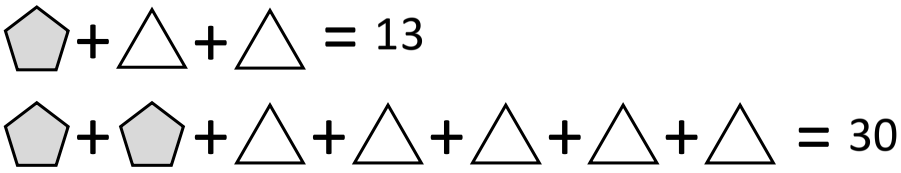
c)



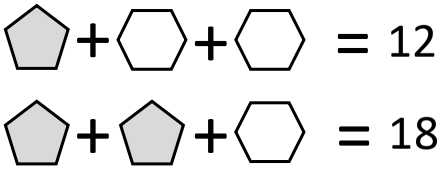
d)



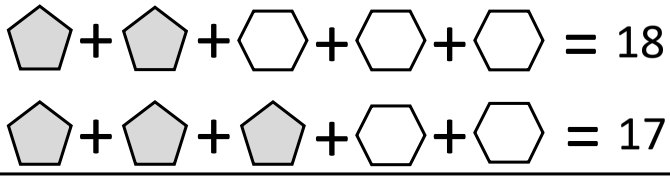
e)



f)

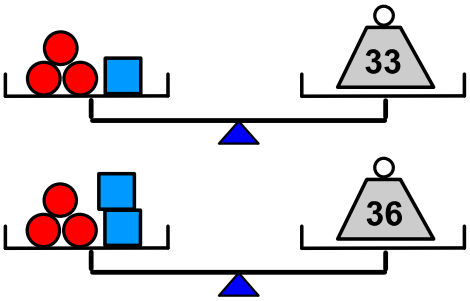


g)

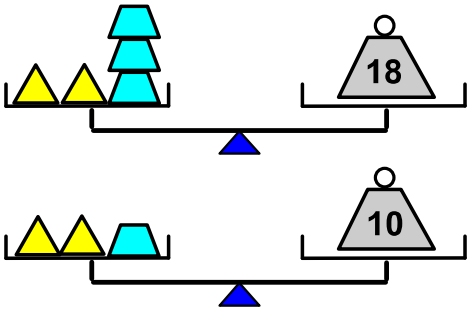


Work out the value of each shape.

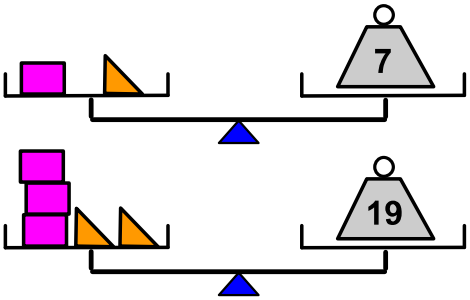
a)



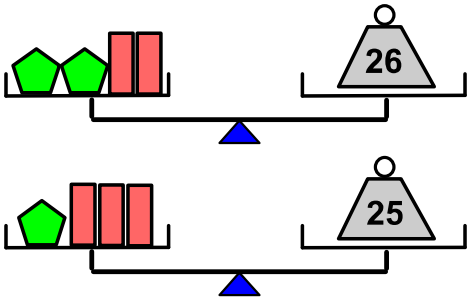
b)



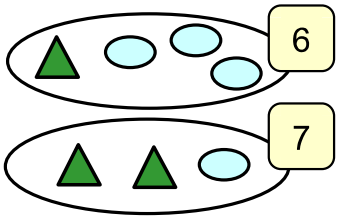
c)



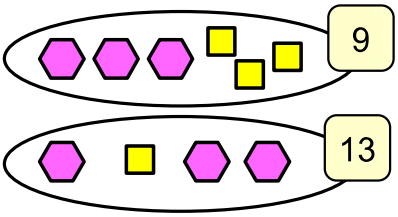
d)



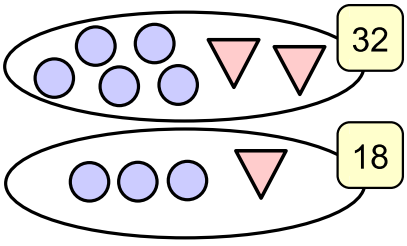
e)



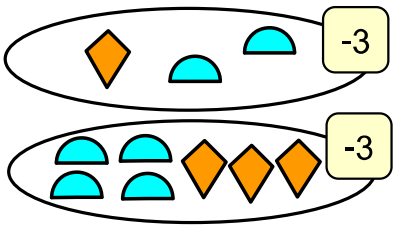
f)



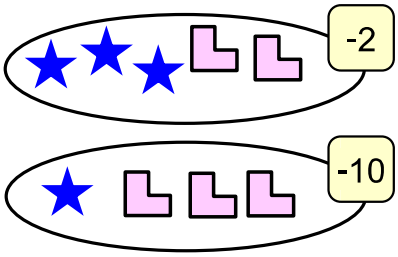
g)



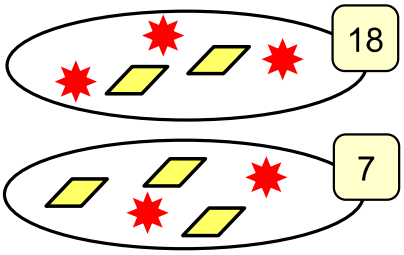
h)



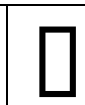
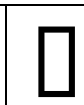
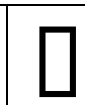
i)



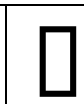
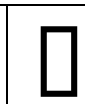
j)



				12
				14
				13
				11

				16
				8
				8
				14

£	⓪	¥	¥	26
⓪	£	£	£	40
$	$	£	£	46
¥	⓪	⓪	⓪	17

				32
				22
				30
				34

				18
				11
				18
				21

a	*b*	*b*	*c*	15
a	*b*	*c*	*c*	24
d	*a*	*a*	*a*	14
d	*d*	*d*	*d*	20

Purposeful Practice

Forming & Solving Simultaneous Equations using Substitution

① 1 t-shirt and 1 jumper cost £25.
The jumper costs £5 more than the t-shirt.

② 1 scarf and 1 hat cost £16.
The hat costs £2 more than the scarf.

③ 1 dress and 1 skirt cost £35.
The skirt is £5 cheaper than the dress.

④ 1 coffee and 2 teas cost £11.
A tea is £1 more expensive than a coffee.

⑤ 2 cupcakes and 1 muffin cost £8.
A muffin is double the price of a cupcake.

⑥ 3 notebooks and 2 books cost £33.
A book costs £4 more than a notebook.

⑦ 3 t-shirts and 2 hoodies cost £81.
For the cost of 1 hoodie you
could buy 3 t-shirts

⑧ 2 pairs of sweatpants and 3 jumpers cost £89.
For the cost of 1 jumper you could
buy 2 pairs of sweatpants and
have £3 left over.

⑨ 5 litres of washing up liquid and
2 litres of professional detergent cost £80.
The cost of 1 litre of detergent is £4 less than
the price of 3 litres of washing up liquid.

(a) Maryam buys 5 pears and 2 kiwi fruits for £3.90. At the same grocers, Dave buys 3 pears and 1 kiwi fruit for £2.20. Find the cost of one pear and the cost of one kiwi fruit.
(b) Peter buys 2 pencils and 3 pens for £1.70. Bola buys 4 pencils and 9 pens for £4.60. Find the cost of one pencil and the cost of one pen.

(c) Yusuf visits a café and orders 2 coffees and 3 cakes for £10.35. Maria orders 5 coffees and 2 cakes for £15.70 at the same café. How much would Tia pay for 3 coffees and 2 cakes at the café?
(d) Two numbers have a sum of 57 and a difference of 36. Find the two numbers.

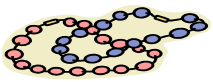
(e) Cards are 65p each and gift wrap sheets are 45p each at a gift shop. Owen buys 21 items from the shop and spends £11.85. How many cards and how many gift wrap sheets did Owen buy?
(f) Terri has a loose change jar that contains only 5p and 10p coins. There are 134 coins in the jar and their total value is £9. Find the number of 5p coins and the number of 10p coins in the jar.

(g) Flour comes in bags of 1.5 kg or 2 kg. A baker has thirty bags of flour and their total weight is 51.5 kg. Find the number of 1.5 kg bags and the number of 2 kg bags the baker has.
(h) A farmer keeps ducks and pigs in the same field. The total number of animals in the field is 54. The difference between the number of pig's feet and the number of duck's feet is 72. Find the number of pigs and the number of ducks in the field.

1. The sum of two numbers is 130, while their difference is 38. Find the two numbers.

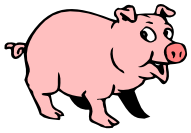
2. Becky and Peter's ages add to 53. If Becky is 3 years younger than Peter, what are their ages?

3. A necklace is made from 164 purple and blue beads. There are 8 more purple beads than blue beads. How many of each colour bead are there?



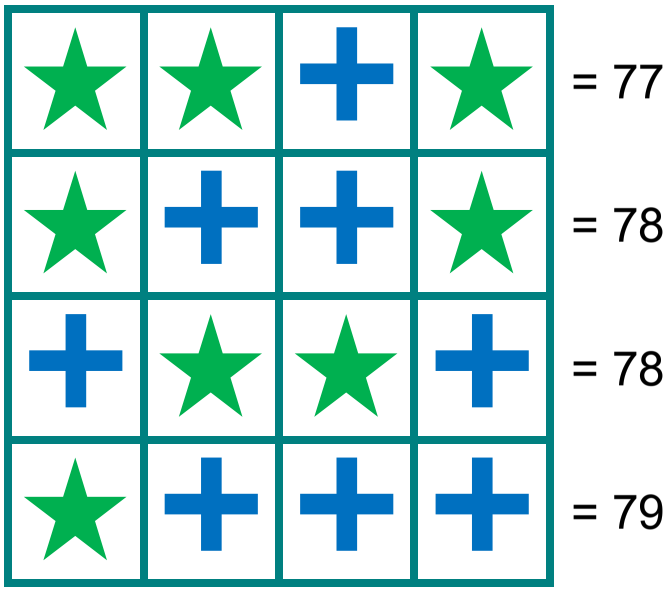
4. Ten Choco bars and six nutty bars cost £5. Three Choco bars and one Nutty bar cost £1.30. Find the cost of each chocolate bar.

5. Some chickens and pigs are in a field. How many of each animal are there if there are 32 heads and 80 legs in total?



6. Peter bought a mixture of large postcards (35p) and small postcards (29p). He bought 20 postcards in total, costing £6.28. How many of each size of postcard did he buy?

7. Find the value of each of the symbols in the grid:



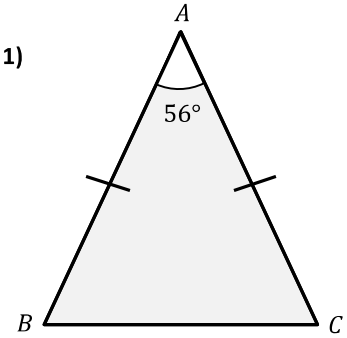
8. Five apples and three pears cost 86 pence. Seven apples and five pears cost £1.30. Find the cost of each piece of fruit.

9. Two adults and three children went to the cinema, and the total cost was £32.40. Three adults and five children cost £51.20. What were the individual prices of adult and child tickets ?

10. A slot machine takes only 20p and 50p coins. The machine contains a total of 140 coins worth £45.10. How many of each type of coin are in the machine?

Purposeful Practice

1)

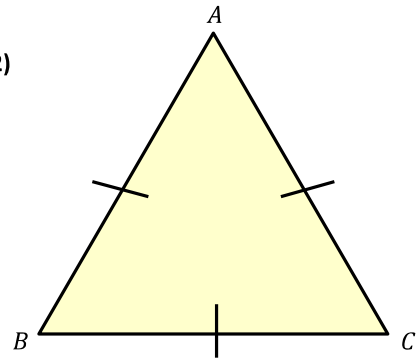


$\angle ABC = 2x + 5y + 4$

$\angle ACB = 5x + 6y - 5$

What are the values of x and y?

2)



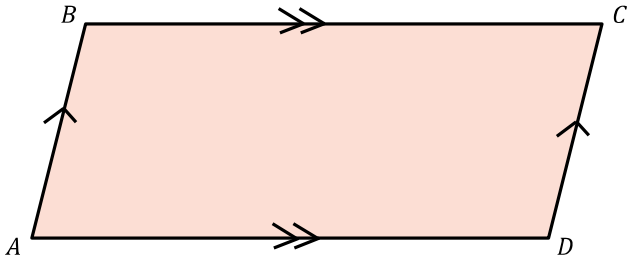
$\angle ABC = 5x + 4y$

$\angle ACB = 8x - y + 1$

$\angle BAC = 4x + 3y + 13$

What are the values of x and y?

3)



$\angle ABC = 9x + 7y + 7$

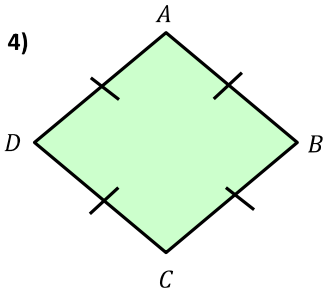
$\angle BCD = 4x + y + 7$

$\angle CDA = 6x + 4y + 58$

$\angle DAB = 3x + 2y + 2$

What is the size of each angle?

4)



$AB = 4x + y + 5$

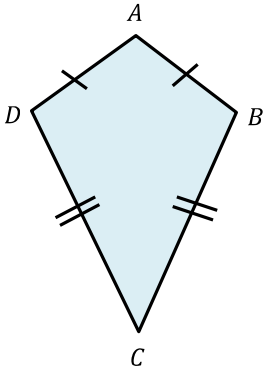
$BC = 3x + 3y$

$CD = x + 4y + 2$

$DA = 2x + 4y - 1$

How long is each side?

5)



$AB = 4x + y - 7$

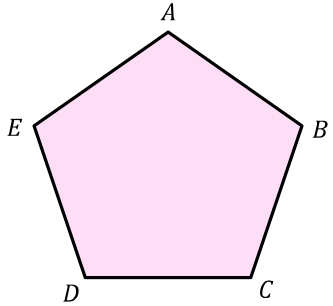
$BC = 5x + 3y - 1$

$CD = 8x + 2y - 5$

$DA = 2x + y + 1$

How long is each side?

6)

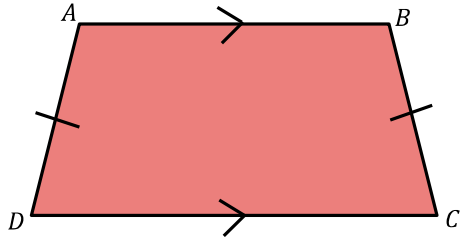


$\angle EAB = 5x + 7y + 14$

$AB = x + 2y$

This is a regular pentagon. Its perimeter is 115cm. What are the values of x and y?

7)



$\angle BCD = 15x + 6y + 16$

$\angle CDA = 10x + 7y + 22$

$AD = 3x + 3y$

$BC = 13x - 2y$

What are the values of the given angles and sides?

Purposeful Practice

subsections

a total of **60** people were observed

of these,

25% of the men had blue eyes

40% of the women had blue eyes

altogether there were **14** people with blue eyes

how many males and how many females were observed?

98 people were interviewed

of these,

30% of the women were born in London

33⅓% of the men were born in London

altogether **31** people were born in London

how many females and how many males gave information in the interviews?

41 people bought a newspaper

of these,

25% of the men were over 60

40% of the women were over 60

altogether **14** people were over 60

how many females and how many males bought newspapers?

a total of **76** people were asked about driving

of these,

20% of the women did not own a car

37½% of the men did not own a car

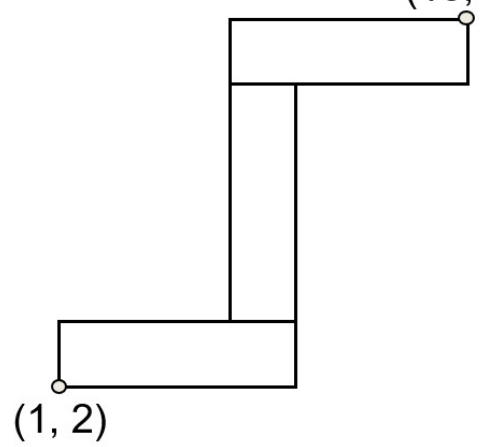
altogether, **25** people did not own a car

how many males and how many females were asked?

Fluency Practice

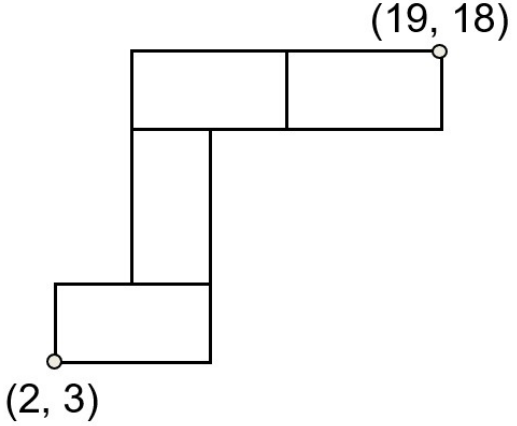
rectangles on a grid how long and wide are each of the rectangles?

(1)

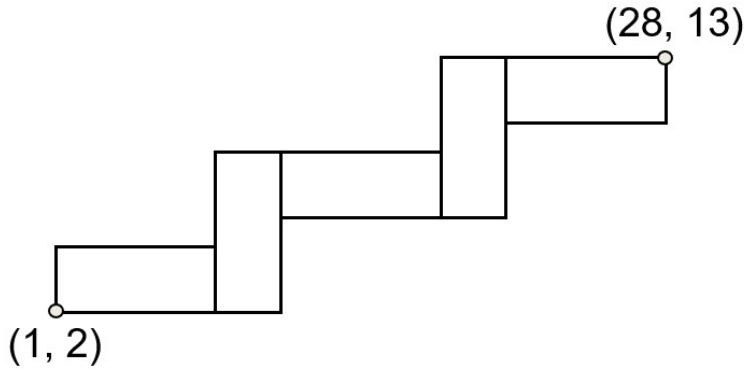


the rectangles in each question are all congruent

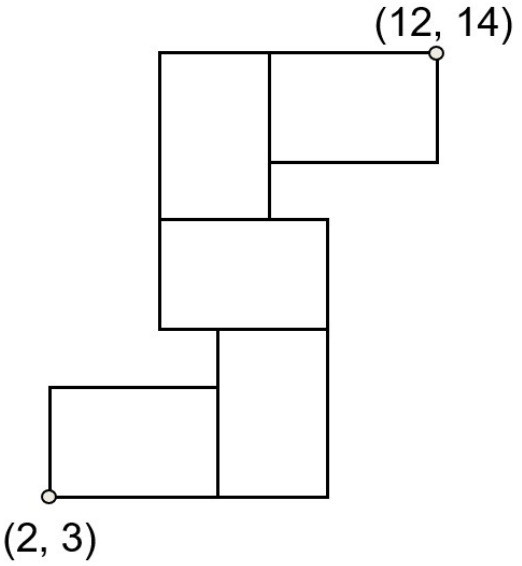
(2)



(3)



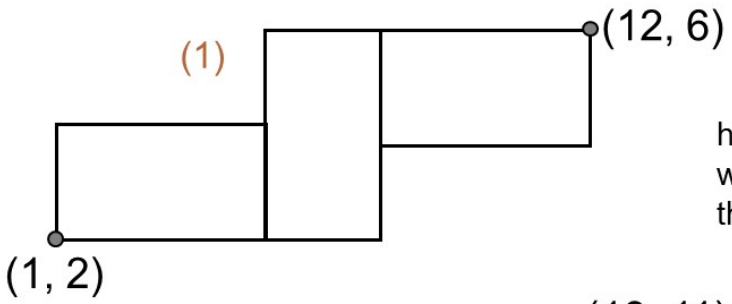
(4)



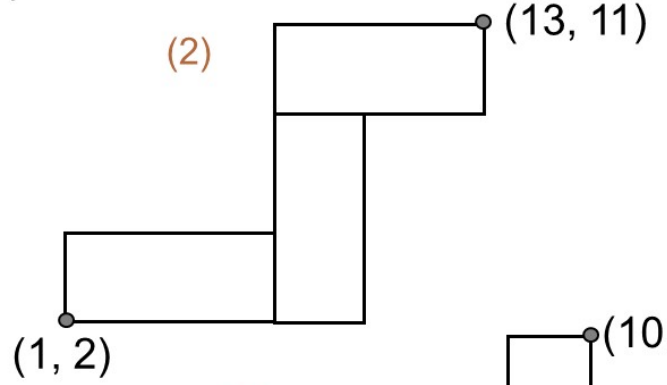
Fluency Practice

rectangles on a grid

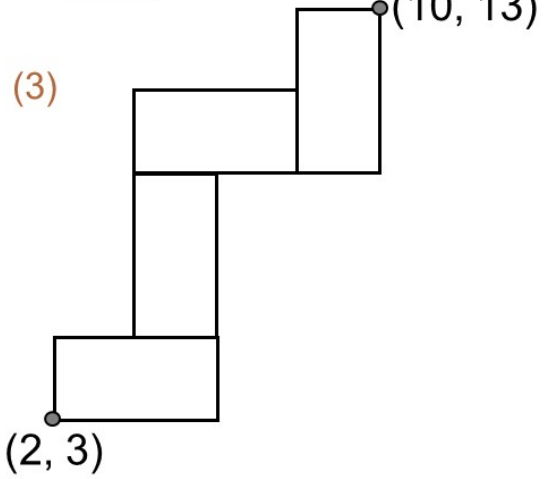
(1)



(2)

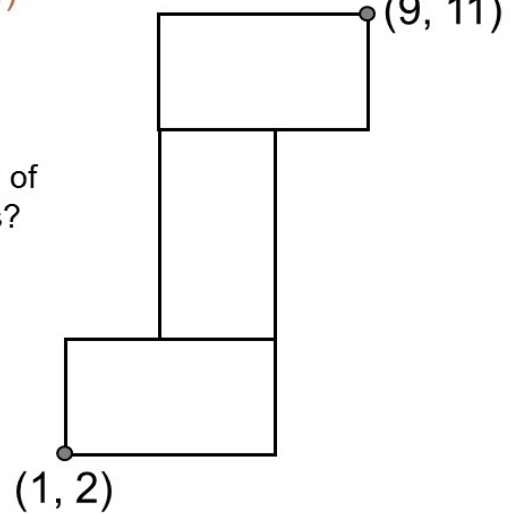


(3)



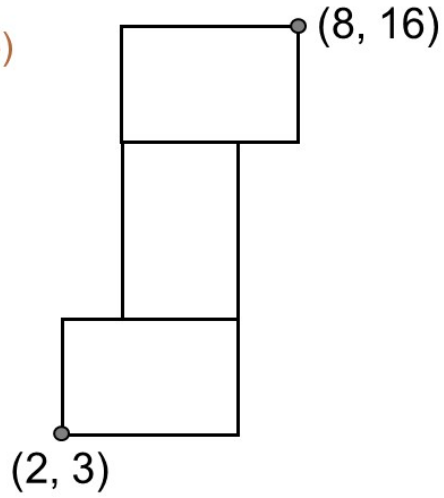
the rectangles in each question are all congruent

(4)



how long and wide are each of the rectangles?

(5)



Solve:

(a) $2x + y = 10$
$x + y = 7$

(b) $4x + y = 9$
$x + y = 3$

(c) $3x + 2y = 22$
$x + 2y = 10$

(d) $5x + 2y = 17$
$3x + 2y = 11$

Solve:

(a) $2x + y = 14$
$x - y = 1$

(b) $3x - y = 1$
$x + y = 7$

(c) $3x + 2y = 25$
$x - 2y = 3$

(d) $x - 3y = 2$
$2x + 3y = 22$

Solve:

(a) $5x + y = 5$
$3x + y = 7$

(b) $6x - y = 9$
$5x - y = 7$

(c) $4x + 2y = 22$
$3x - 2y = 6$

(d) $x - 3y = 4$
$4x + 3y = 1$

(e) $x + y = 0$
$x - y = 6$

(f) $5x + 2y = 13$
$x + 2y = 9$

(g) $3x + 2y = 16$
$x - 2y = 4$

(h) $3x - y = 9$
$5x + y = 11$

(i) $4x + y = 8$
$2x + y = 7$

(j) $5x - 2y = 7$
$4x + 2y = 11$

David buys 5 biscuits and 3 cakes for £3.95. Samira buys 8 biscuits and 3 cakes for £5.15. Find the cost of one biscuit and the cost of one cake.

Jim is thinking of two numbers. When he adds them together he gets 20. When he subtracts one from the other he gets 30. What are Jim's two numbers?

Solve:

(a) $x + 2y = 7$
$3x + y = 6$

(b) $x + 3y = 11$
$4x + y = 22$

(c) $3x - y = 7$
$5x - 2y = 10$

(d) $x - y = 5$
$2x - 5y = 4$

Solve:

(a) $x + y = 5$
$3x - 2y = 5$

(b) $7x - y = 1$
$x + 3y = 19$

(c) $2x + 5y = 24$
$3x - y = 2$

(d) $4x - 2y = 14$
$3x + y = 23$

Solve:

(a) $x + 2y = 13$
$2x + 3y = 20$

(b) $7x - 4y = 5$
$x + 2y = 11$

(c) $2x + 5y = 5$
$3x - 2y = 17$

(d) $4x - 2y = 14$
$x - 3y = -4$

Solve:

(a) $5x + 2y = 31$
$x - 4y = 4$

(b) $4x + y = 5$
$2x + 3y = 10$

(c) $2x - 3y = 16$
$7x - 2y = 39$

(d) $x - 5y = 6$
$3x + 2y = 1$

(e) $3x + 4y = 14$
$x - y = -7$

(f) $x + 7y = 15.5$
$2x - 5y = -7$

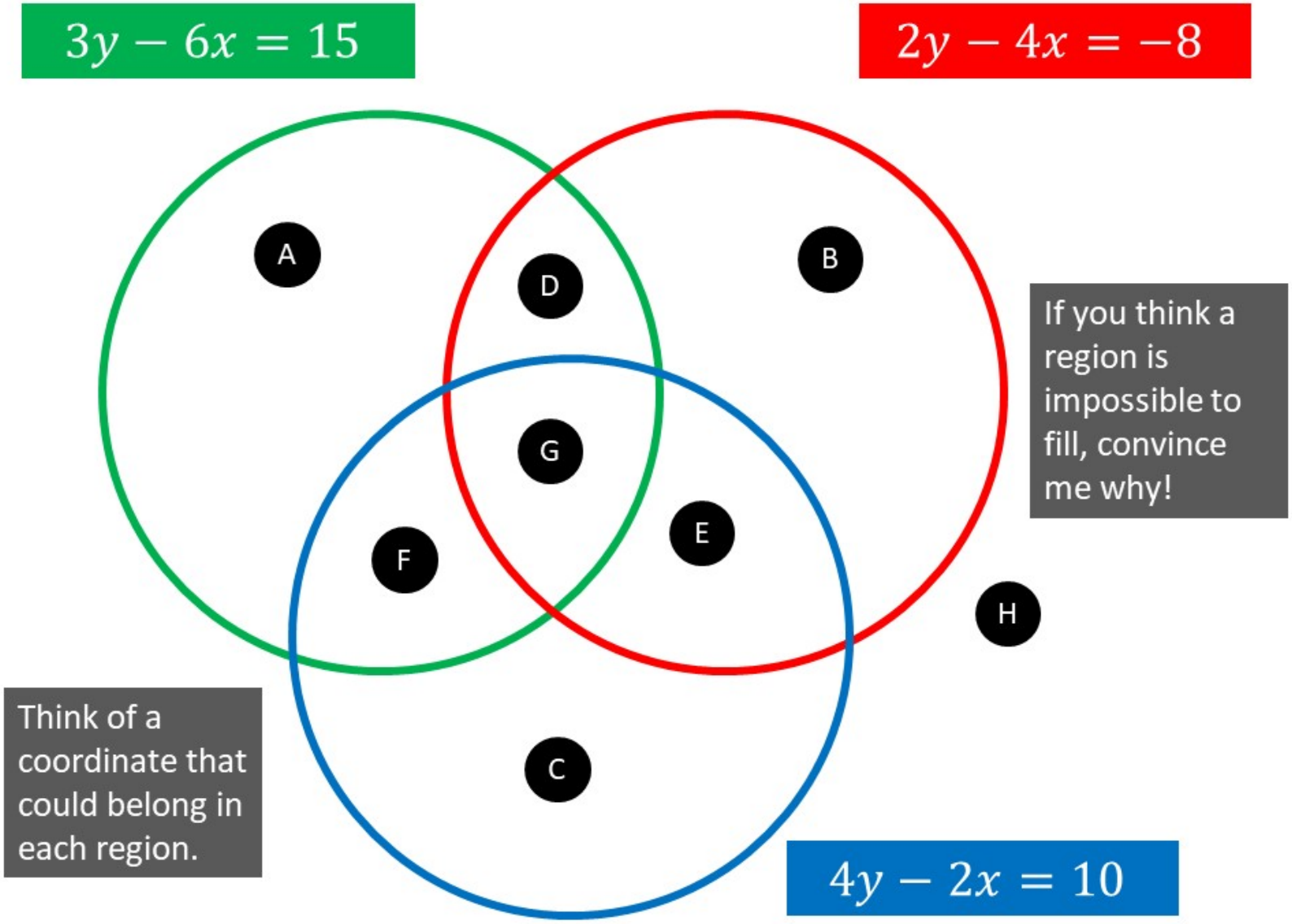
Three apples and two oranges costs £2.53. Five apples and three oranges costs £4.12. Find the cost of one apple and the cost of one orange.

Fluency Practice

(a)	(b)	(c)	(d)
Solve $2x + 3y = 14$ $x + 3y = 10$	Solve $5x + 2y = 37$ $3x - 2y = 3$	Solve $x + 4y = 17$ $3x + 4y = 19$	Solve $2x + y = 4$ $3x + 2y = 5$
(e)	**(f)**	**(g)**	**(h)**
Solve $5x - y = 36$ $x + 3y = 4$	Solve $7x + 4y = 6$ $3x + 2y = 4$	Solve $6x - y = 4$ $2x - 4y = 5$	Solve $4x + 2y = 19$ $x + 3y = 16$
(i)	**(j)**	**(k)**	**(l)**
Solve $2x - 3y = 20$ $3x + 5y = 11$	4 burgers and 2 sausages costs £4.70. 3 burgers and 5 sausages costs £5.80. Find the cost of one burger and one sausage.	The sum of two numbers is 10.3. The difference between the same two numbers is 2.84. Find the two numbers.	Find the coordinates of the point where the lines $2x + 3y = 21$ and $3x - y = 4$ meet.

1. Write some **sets** of coordinates that fit the equations.
   Place sets that fit both in the middle. The first is done for you.

Problem Solving



3 Advanced Ratio

Fluency Practice

Ratio & Formulae

$a : b = 3 : 4$

$\frac{a}{b} = \frac{3}{4}$ → $a = \frac{3}{4}b$

$\frac{b}{a} = \frac{4}{3}$ → $b = \frac{4}{3}a$

① $a : b = 2 : 5$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} =$ → $b =$

② $a : b = 1 : 3$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} = \frac{1}{\quad}$ → $b =$

③ $a : b = 6 : 5$

$\frac{a}{b} =$ → $a =$

$\frac{b}{a} =$ → $b =$

④ $a : 4 = 3 : b$

→ $a =$

→ $= b$

⑥ $a : 2 = b : 9$

→ $\frac{a}{\quad} =$

→ $a =$

→ $= b$

⑦ $3 : a = 4 : b$

→

→ $a =$

→ $= b$

⑧ $a : 2 = 3 : 7$

→

→ $a =$

⑨ $5 : a = 2 : 3$

→

→ $a =$

⑩ $4 : 7 = 3 : 5a$

→ $= \frac{\quad}{3}$

→ $= a$

⑪ If, $a = \frac{4}{5}b$

$a : b =$

⑫ If, $b = \frac{8}{7}a$

$a : b =$

⑬ If, $a = 1.25b$

$a : b =$

⑭ If, $b = 1.8a$

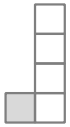
$a : b =$

Fluency Practice

Comparing Ratios For each ratio, express a in terms of b… and b in terms of a. Complete each sentence without using fractions.

① $a : b = 1 : 4$

$a : b$



$\frac{a}{b} = \frac{1}{\_}$ $\frac{b}{a} = \frac{4}{\_}$

$a = \frac{1}{\_} b$ $b = \_\, a$

b is _____ times the size of a.

② $a : b =$

$a : b$

$\frac{a}{b} = \frac{1}{\_}$ $\frac{b}{a} = \frac{\_}{1}$

$a = \_\, b$ $b =$

a is _____ times the size of b.

③ $a : b =$

$a : b$

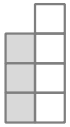
$\frac{a}{b} =$ $\frac{b}{a} = \frac{\_}{2}$

$a =$ $b =$

b is _____ times the size of a.

④ $a : b =$

$a : b$



$\frac{a}{b} =$ $\frac{b}{a} =$

$a =$ $b =$

a is _____ times the size of b.

⑤ $a : b =$

$a : b$

$\frac{a}{b} =$ $\frac{b}{a} = \frac{\_}{4}$

$a =$ $b =$

b is _____ times the size of a.

a is _____ times the size of b.

⑥ $a : b =$

$a : b$

$\frac{a}{b} =$

$a = \frac{4}{\_} b$ $b =$

b is _____ % larger than a.

⑦ $a : b = 5 : 2$

$a : b$

$a =$ $b =$

a is _____ times the size of b.

b is _____ % smaller than a.

⑧ $a : b =$

$a : b$

$a = \frac{5}{7} b$ $b =$

a is _____ times the size of b.

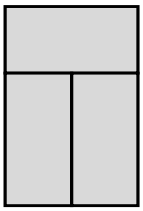
⑨ $a : b$ $a : b =$

$a =$ $b =$

b is $2\frac{2}{3}$ times the size of a.

a is _____ % smaller than b.

Ratios & Formulas

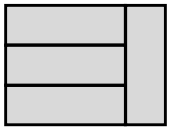


This compound shape is made from 3 congruent rectangles.

For one small rectangle…

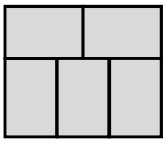
a) What is the formula for the length in terms of width?

b) What is the ratio **length : width** ?

1) 

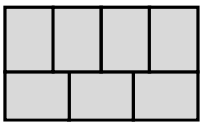
Formula _____

Ratio _____

2) 

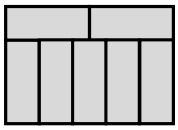
Formula for length ⇨

length : width = ⇨

3) 

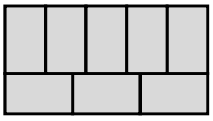
Formula _____

Ratio _____

4) 

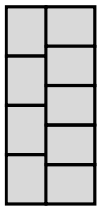
Formula _____

Ratio _____

5) 

Formula _____

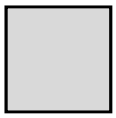
Ratio _____

6) 

Formula _____

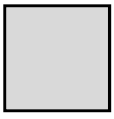
Ratio _____

For these **squares**, write a formula for **a** in terms of **b**.
Convert the formula into the ratio **a : b**

7) a  2b

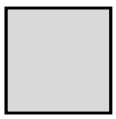
Formula _____

a : b _____

8) 2a  3b

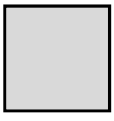
Formula _____

a : b _____

9) 4a  3b

Formula _____

a : b _____

10) 8a  10b

Formula _____

a : b _____

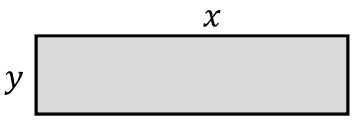
Ratios & Formulas

These rectangles are **similar**.

What can we **deduce** about the **relationship** between x and y?

Length : Width

4 : 1



Length : Width

$x : y$

The length and width are in the same ratio.

$4 : 1 = x : y$

Therefore, $\frac{4}{1} = \frac{x}{y}$ and $\frac{1}{4} = \frac{y}{x}$

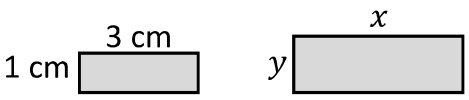
We can rearrange these equations to express x in terms of y and vice-versa.

$x = 4y$ $\quad y = \frac{x}{4}$

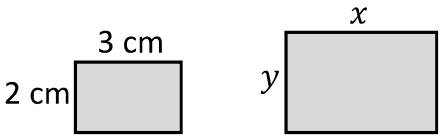
For each similar pair, express their equal ratios and convert this into a formula.

Rearrange the formula to express x in terms of y.

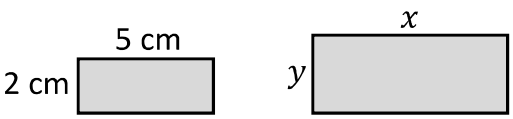
A)



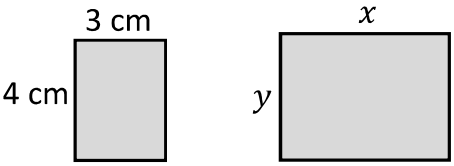
B)



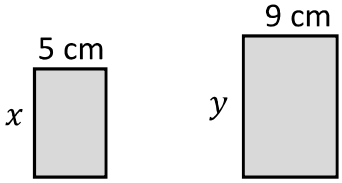
C)



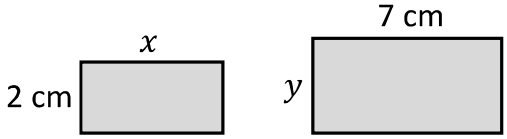
D)



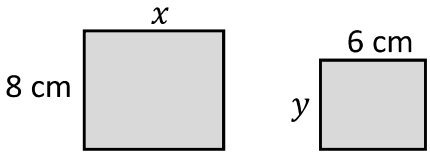
E)



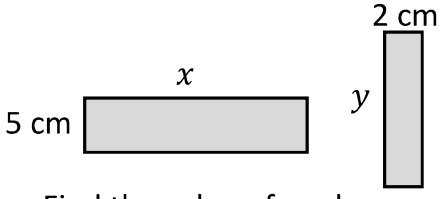
F)



G)



H)



Find the value of x when y = 12

Find the value of y when x = 8

Show each of the formulas using the equivalent ratios below.

(y+ x) : (y-x) is equivalent to k : 1

Show that $y = \frac{x(k+1)}{k-1}$

(b+ m) : (b-m) is equivalent to k : 1

Show that $b = \frac{m(k+1)}{k-1}$

(p+ y) : (p-y) is equivalent to k : 3

Show that $p = \frac{y(k+3)}{k-3}$

(p+ q) : (p-q) is equivalent to k : 5

Show that $p = \frac{q(k+5)}{k-5}$

(3x+ y) : (x-3y) is equivalent to k : 2

Show that $x = \frac{y(3k+2)}{k-6}$

(x+ 2y) : (3x-4y) is equivalent to k : 2

Show that $x = \frac{4y(k+1)}{(3k-2)}$

Fluency Practice

Changing Ratios scaling

Tia and Ben share some pens in the ratio 1:2
Ben spends ≤3. The remaining pens in the ratio 2:3

	then	now	
	1:2	2:3	Ben's pens
multiplicative part	· 2 ↓	↓	change
(Tia's pens)	2:4	2:3	−1
	↓	↓ · 3	↓
	6:12	6:9	−3

−≤3

① Rosa and Roe share some pens in the ratio 1:3
Roe spends ≤5. The remaining pens in the ratio 2:5
How many pens does Roe have now?

	then	now	
	1:3	2:5	
multiplicative part	· 2 ↓	↓	change
	↓	↓	↓

② Gen and Gil share some pens in the ratio 3:5
Gil spends ≤16. The remaining pens in the ratio 9:7
How many pens did they have altogether?

then	now
3:5	9:7
↓	↓

③ Mia and Maisie share some pens in the ratio 3:4
Maisie spends ≤20. The remaining pens in the ratio 2:1
How many pens does Maisie have now?

3:4 2:1

④ Ken and Kelsie share some pens in the ratio 4:3
Kelsie spends ≤3. The remaining pens in the ratio 3:2
How many pens did they start with?

⑤ Ann and Ais share some pens in the ratio 3:2
Ann spends ≤18. The remaining pens in the ratio 3:5
How many pens did Ann start with?

⑥ Abag has contained red and blue discs in the ratio 4:5
After 4 blue discs are removed the ratio is now 2:3
How many discs are there now?

⑦ In a store there are 50% more bananas than apples
15 bananas are added so the apple:banana ratio = 3:7
How many apples are there?

⑧ Abag has 25% more triangles than circles
After 15 triangles are added the circle:triangle ratio is 5:7
How many triangles are there now?

1. The ratio of boys to girls in a group is 7 : 5.
   15 more boys join the group.
   The ratio is now 12 : 5.
   How many students are there in total?

2. The ratio of green pens to red pens in a drawer is 5 : 1.
   15 red pens are added to the drawer.
   The ratio becomes 5 : 4.
   How many pens of each colour are now in the drawer?

3. In an animal sanctuary the ratio of dogs : cats : donkeys is 5: 4 : 2
   8 more cats arrive at the sanctuary.
   The ratio is now 5 : 8 : 2.
   How many donkeys are there?

4. Jim likes to buy shirts of particular colours.
   The ratio of blue, black and white shirts is 1 : 3 : 4.
   There are 6 more black shirts than blue shirts.
   How many shirts are there in total?

5. Julia keeps pigs, sheep and goats in the ratio of 2 : 3 : 5.
   She has 9 more goats than pigs.
   How many sheep does she have?

6. Bill buys sweets for his friends, Tom, John and Kevin.
   He gives out the sweets in the ratio 4 : 5 : 2.
   John has 12 more sweets than Kevin.
   How many sweets does each friend have?

7. Green, white and yellow counters are placed in a bag in the ratio of 4 : 3 : 2.
   20 green counters are removed and the ratio becomes 2 : 3 : 2.
   How many of each counter are left in the bag?

8. Burgers, hot-dogs, and sausage rolls are sold in the ratio of 5 : 6 : 7.
   10 more hot-dogs than burgers were sold.
   How many items were sold in total?

Fluency Practice

Person A and Person B share money in a ratio. A gives B some money. Now they have the same amount …

A	B	Ratio	A gives B	Question
James	Jessie	5 : 3	$4	How much did Jessie have to start?
Jessie	Brock	9 : 7	$2	How much did Jessie have to start?
Brock	Ash	3 : 1	$8	How much did they start with altogether?
Ash	Misty	5 : 1	$6	How much did Ash start with?
Misty	Brock	3 : 1	$1	How much do they both have now?
Ash	James	9 : 5	$1	How much did Ash start with?
Brock	Jessie	3 : 1	$0.50	How much do they both have now?

Purposeful Practice

A bag contains black and white cubes in the ratio 1:2
3 black cubes are added and the ratio becomes 3:5

then	$x : 2x$	$= 1 : 2$	$15 : 30$
now	$x + 3 : 2x$	$= 3 : 5$	$18 : 30$

$$\frac{x+3}{2x} = \frac{3}{5}$$
$$5(x+3) = 2x(3)$$
$$5x + 15 = 6x$$
$$15 = x$$

Changing Ratios using algebra

① A bag contains black and white cubes in the ratio 1:2
4 black cubes are added and the ratio becomes 3:5
How many white cubes are there?

then	$x : 2x$	$= 1 : 2$
now		$= 3 : 5$

② In a bag, black cubes : grey cubes = 1:3
2 black cubes are added and
the ratio becomes 2:5
How many black cubes are there now?

$x : 3x$

③ In a bag, red cubes : grey cubes = 2:3
10 red cubes are added and
the ratio becomes 3:2
How many red cubes are there now?

④ In a bag, red cubes : blue cubes = 3:4
6 red cubes are added and
the ratio becomes 4:5
How many red cubes are there now?

⑤ In a bag, green : blue cubes = 4:5
2 green cubes are **removed** and
the ratio becomes 3:4
How many cubes are there now?

$4x - 2 :$

⑥ In a bag, grey : blue cubes = 3:7
4 grey cubes are removed and
the ratio becomes 2:5
How many blue cubes are there?

⑦ In a sports club, adults : teens = 5:8
2 adults join **and** 8 teens join and
the ratio becomes 1:2
How many adults are there now?

$5x + 2 :$

⑧ In a sports club, adults : teens = 2:3
2 adults join and 4 teens **leave** and
the ratio becomes 5:4
How many people are in the club now?

⑨ There are 80% more adults than teens in a sports club.
9 adults leave and 2 teens join and
the adult:teen ratio becomes 3:2
Now 3 teens join the club. As a percentage,
how many more adults than teens are there?

(a) The ratio of white socks to black socks is 1 : 4 in a drawer. Four white socks are added to the drawer and the ratio of white socks to black socks becomes 5 : 12. Find the number of white socks and black socks there were initially.

(b) Lily and Mary have marbles in the ratio 5 : 6. Lily gets two more marbles and now the ratio is 7 : 8. How many marbles did each girl have initially?

(c) The ratio of the number of boys to girls at a party is 3 : 4 Six boys leave the party. The ratio of the number of boys to girls at the party is now 5 : 8 Work out the number of girls at the party.

(d) The ratio of pigeons to ducks in a park is 3 : 2. When 5 pigeons fly away, the ratio of pigeons to ducks becomes 5 : 4. How many ducks and pigeons were there originally?

(e) Bill and Chuck share some sweets in the ratio 7 : 3. Bill gives 3 sweets to Chuck and now the ratio is 5 : 3. How many sweets did each have initially?

(f) There are two bags containing counters, bag A and bag B. The ratio of counters in bag A to bag B is 3 : 4. Twelve counters are taken from bag B and added to bag A and the number of counters in each bag is now the same. How many counters were there originally in each bag?

A A bag contains red and blue counters in the ratio 1 : 3.

When four red counters are added to the bag, the ratio changes to 1 : 2.

Work out the number of blue counters in the bag.

B A fruit bowl contains apples and bananas in the ratio 4 : 5.

Two apples are removed, changing the ratio to 2 : 3.

Work out the total number of pieces of fruit that remain in the bowl.

C In a group of people, the ratio of right-handed to left-handed people is 8 : 1.

Two right-handed people join the group. This changes the ratio to 25 : 3.

Work out the total number of people that are now in the group.

D At a company, 75% of the employees were men.

After six women joined the company, the ratio of men to women was 2 : 1.

Work out how many men work at the company.

E After a certain number of matches, the ratio of wins, draws and losses for a football team is 3 : 1 : 2.

If the team win both of their next two matches, they will have won 55% of their matches.

Work out how many matches the team has played so far.

F The colours of cars in a car park are shown in the table.

	Red	Blue	Grey	White	Black
Freq.	3	5	2	1	4

Work out the smallest number of cars that would need to enter the car park (without any cars leaving) for the ratio of blue to white cars to change to 2 : 5.

G There are 108 people in a hall. The ratio of adults to children is 5 : 4.

How many adults must leave to hall for the ratio to change to 1 : 1 ?

H The ratio of red pencils to green pencils in a box is 4 : 1.

What is the smallest possible number of pencils that would have to be added to change the ratio to 5 : 2 ?

I A box contains gold, silver and bronze tokens in the ratio 5 : 7 : 6.

Six gold tokens are added.

A token is then selected at random from the box.

The probability that it is gold is $\frac{7}{20}$.

Work out the probabilty that it is silver.

J Helen has 1.5 litres of pink paint, made from a mixture of red and white paint in the ratio 1 : 4.

She adds enough red paint to change the ratio to 1 : 3.

How much white paint must she add to change the ratio back to 1 : 4 ?

A In a company, the ratio of the number of men to the number of women is 2 : 3.

$\frac{1}{4}$ of the men are part-time.

$\frac{1}{3}$ of the women are part-time.

There are 12 part-time workers at the company. Work out how many workers there are in total.

B On a bookshelf, the ratio of fiction to non-fiction books is 2 : 5.

The ratio of paperback books to hardback books on the shelf is 3 : 7.

Work out the smallest possible number of books on the shelf.

C At a gallery, there is a photography exhibition. The ratio of portrait to landscape photographs on show is 5 : 3.

60% of the portrait photographs are in colour. 24 of the portrait photographs are not in colour.

Work out the total number of photographs at the exhibition.

D The ratio of men to women in an orchestra is 3 : 2.

35% of the men play a stringed instrument.
45% of the women play a stringed instrument.

What percentage of all of the members of the orchestra play a stringed instrument?

E The table shows the prices for hiring boats at a lake.

	Rowing boat	Pedal boat
30 mins	£10	£12
1 hour	£16	£18

One day, the ratio of rowing boats to pedal boats hired was 2 : 3.

$\frac{5}{8}$ of the rowing boats hired were for 1 hour.

$\frac{5}{9}$ of the pedal boats hired were for 1 hour.

Given that there were 16 pedal boats hired for 30 minutes, work out the total sales income for the day.

F A box contains a number of tokens, of 4 different types. Each token is either blue or white, and either round or square.

The ratio of blue to white tokens in the box is 3 : 4.

The ratio of round blue tokens to square blue tokens is 1 : 2.

The ratio of round white tokens to square white tokens is 3 : 1.

What fraction of all the tokens are round?

G Pupils at a school all study exactly one foreign language.
The ratio of pupils who study French, German and Spanish is 4 : 3 : 2.
One of the French classes has 24 pupils, which is $\frac{2}{9}$ of all the pupils who study French.

Work out the number of pupils who study Spanish at the school.

Purposeful Practice

ratio problems (i)

(1) the ratio of shares owned by Asterix and Cleopatra is in the ratio 3 : 5

if Asterix gives Cleopatra 2 of his shares then the ratio will be 1 : 3

how many shares do they each have initially?

(2) Jan and Kim own numbers of marbles that are in the ratio 5 : 6

Jan gains 2 more marbles and the ratio is now 7 : 8

how many marbles do each own initially?

(3) the ratio of Ann's age to Bob's age is 3 : 4

in 7 years time this ratio will be 4 : 5

(i) what are their ages now?
(ii) after how many years (from now) will the ratio be 5 : 6?

(4) the ratio of cockles to winkles in a bag of sea shells is 1 : 2

when 6 winkles are accidentally spilt from the bag the ratio is 3 : 5

how many of each shell were there initially?

(5) the ratio of geese to ducks on a lake is 4 : 9

when 10 extra geese land on the lake (and there are no duck changes) the ratio is 2 : 3

how many of each were there initially?

(6) the ratio of the width to depth (when they are measured in metres) of a dam is 5 : 7

when 10 extra metres is added to the width of the dam (and the depth remains the same) the ratio is 5 : 6

how wide and deep was the dam initially?

Purposeful Practice

ratio problems (ii)

(1)

bag A contains 7n counters
bag B contains 3n counters

3 counters are taken from bag A and put in bag B

the ratio of counters in bag A to bag B is now 5 : 3

how many counters were in the bags initially?

(2)

box C contains 5k pens
box D contains 2k pens

8 pens are taken from box C and placed in box D

the ratio of pens in box C to box D is now 3 : 2

how many pens were there in each box initially?

(3)

the ratio of pairs of white socks to pairs of red socks is 9 : 5

9 pairs of the white socks are dyed red

the ratio of pairs of white socks to pairs of red socks is now 3 : 2

how many pairs of each colour were there initially?

(4)

Sapna and Rob have many paperweights, in the ratio of 8 : 3

Sapna, being a generous person, gives Rob 14 of her paperweights

the ratio is now 6 : 5

how many paperweights did each have initially?

explore what other ratios could have been created

Purposeful Practice

ratio problems (iii) (simplest form involves integers)

(1) the ratio **a : b** = 3 : 4

what is the ratio

(i) b : 3a – b in its simplest form?

(ii) 2a + b : 4a – b in its simplest form?

(2) the ratio **a : b** = 5 : 6

the ratio **b : c** = 9 : 10

what is the ratio
(i) a : c in its simplest form?

(ii) a + b : 9b – 2a in its simplest form?

(3) the ratio **a : b** = 4 : 5

what is the ratio

(i) a + 2b : 3a – 2b in its simplest form?

(ii) 3b – 2a : a + 9b in its simplest form?

(4) the ratio **a : b** = 4 : 3

the ratio **a : c** = 2 : 5

what is the ratio
(i) b : c in its simplest form?

(ii) 2a – b : 2b + a in its simplest form?

(5) the ratio **a : b** = 2 : 5

the ratio **d : e** = 10a : 3b

what is the ratio
(i) d : e in its simplest form?

(ii) a + 2b : 2a + b in its simplest form?

(6) the ratio **a : b** = 3 : 2

the ratio **b : c** = 5 : 6

what is the ratio
(i) a : c in its simplest form?

(ii) 2a + b : 2b + c in its simplest form?

Purposeful Practice

ratio problems (iv)

(1)

the ratio

(a number add 2 : the number add 4)

is the same as the ratio

(the same number add 6 : this number add 9)

what is the number?

(2)

the ratio

(a number take 1 : the number add 3)

is the same as the ratio

(the same number add 1 : this same number add 6)

what is the number?

(3)

the ratio

(twice a number add 1 : this number take 1)

is the same as the ratio

(twice the same number add 8 : this same number add 2)

what is the number?

(4)

the ratio

(a number add 1 : the number add 4)

is the same as the ratio

(twice the same number take 5 : this same number add 20)

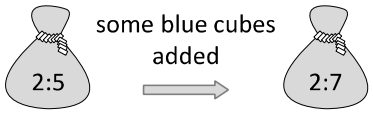
what is the number?

[find two solutions if you can]

Purposeful Practice

Cube Ratios

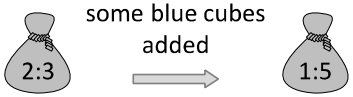
① red : blue



What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

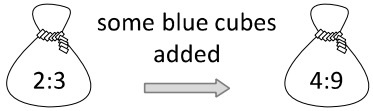
② red : blue



What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were added?

③ red : blue

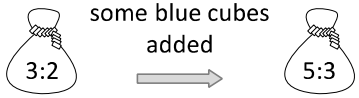


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

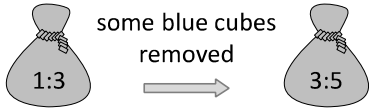
If 6 cubes were added, how many are there now?

④ red : blue



Why is this impossible?

⑤ red : blue

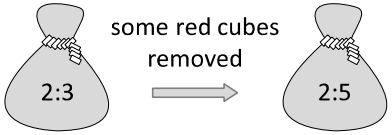


What is the minimum number of cubes that were in the bag?

What is the minimum number of cubes that were removed?

If 12 cubes were removed, how many are there now?

⑥ red : blue

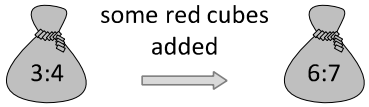


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were removed?

If 20 cubes were removed, how many are there now?

⑦ red : blue

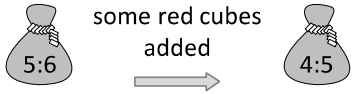


What is the minimum number of cubes that are now in the bag?

What is the minimum number of cubes that were added?

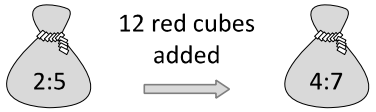
If 30 cubes were added, how many were there?

⑧ red : blue



Why is this impossible?

⑨ red : blue



How many cubes are there now?

ratio problems involving algebra

(1) if $a : 7 = b : 4$ then $(a + b) : 11 = (a – b) : 3$

(2) if $a : b = c : d$ then $a : c = (a + b) : (c + d)$

(3) if $a : b = c : d$ then $(a + b) : (a – b) = (c + d) : (c – d)$

(4) if $3a : 5b : 6c = p : q : r$ then $a : b : c = 10p : 6q : 5r$

(5) if $a : b = b : c$ then $(b + c) : c = (a – c) : (b – c)$

(6) if $a : b = b : c$ then $(a + b) : a = (a – c) : (a – b)$

Purposeful Practice

Ratio Worded Problems			
(a)	**(b)**	**(c)**	**(d)**
A pencil case contains 5 blue pencils, 3 red pencils and 7 black pencils. What fraction of the pencils are red? Give your answer in its simplest form.	In a florist shop, the ratio of roses to tulips to gerberas is 7 : 2 : 3. What fraction of the flowers in the shop are roses or tulips? Give your answer in its simplest form.	The ratio of the number of boys to the number of girls in a school is 4 : 7. There are 91 girls in the school. Work out the number of boys in the school.	The books in a library are fiction or non-fiction in the ratio 3 : 2. If there are 573 fiction books, how many books are there in the library in total?
(e)	**(f)**	**(g)**	**(h)**
A map has a scale of 1 : 500. Martha measures a distance of 5 cm on the map. What actual distance in m does this correspond to?	Madeira cake mixes flour, sugar and butter in the ratio 5 : 3 : 2. How much flour and butter is required to mix with 75 g sugar?	In a car park there are twice as many white cars as black cars, and three times as many red cars as white cars. Write the ratio of cars red : white : black in its simplest form.	A recipe to make 8 pancakes needs 200 ml milk and 120 g flour. How much milk and flour is need to make 20 pancakes?
(i)	**(j)**	**(k)**	**(l)**
The angles in a quadrilateral are in the ratio 5 : 7 : 4 : 2. Find the size of each of the angles.	120 ml of orange drink is made with cordial and water in the ratio 1 : 4. To make the drink stronger 10 ml more of cordial is added. What is the ratio of cordial to water for the new drink?	Ishaq builds a model rocket. The height of the model rocket is 16 cm, and the actual height of the model is 48 m. Find the ratio of the model rocket to the real rocket.	Jim mixes 1 litre of red and white paint in the ratio 1 : 3. How many ml of red paint should be added so that the ratio becomes 8 : 15 ?

Purposeful Practice

Ratio 1

① Some money is shared in the ratio 2 : 3 : 5

The largest share is £45 larger than the smallest.

How much money is there in total?

② Trees in a forest,
evergreen : deciduous = 5 : 2

For ever-green trees,
healthy : unhealthy = 4 : 7

There were 35 unhealthy evergreen trees.
How many deciduous trees were there?

③ Write the ratio 9 : 4 in the form n : 1 where n Is a decimal

$a : b = 5 : 3$

How many times larger is a than b?

④ The value of y is 50% more than the value of x.
Express the ratio $x : y$ using integers.

a : b = 3 : 2
a : c = 4 : 7

Express the ratio a : b : c

⑤ There are triangles, hexagons & circles in a bag.

triangles to hexagons = 2:3
hexagons to circles = 6:7

If there are less than 100 shapes in the bag, what is the greatest number of hexagons in the bag?

⑥ Jan has some money.
Caleb has £6 more than four times the money Jan has.

In total they have £51.

Express this as a ratio Jan : Caleb

⑦ $\frac{1}{7} : \frac{3}{5} = x : 1$

Calculate the value of x.

⑧ The points A, B, C and D lie in order on a straight line.
AB : BD = 1:3
AC : CD = 5:7
Work out AB : BC : CD

⑨ Greg scored 20 more than Kim.
Tan scored triple Kim's score.
In total they scored 80 points at archery.

Find the ratio
Kim's score : Greg's score

⑩ a : b = 2 : 5 and 4b = 5c
Work out a : c in its simplest form.

⑪ b is one-quarter of c
$2a = 3c$
Express the ratio $a : b : c$ using integers.

⑫ Ann has £28 in savings.
Bill has no savings.

Each week from now,
Ann saves £6 & Bill saves £4.

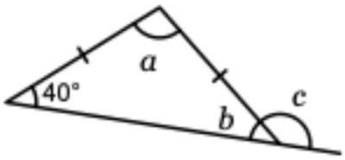
How many weeks will it take for Ann & Bill to have savings in the ratio 5 : 2?

Purposeful Practice

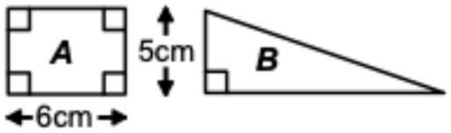
Ratio 1

① A distance is split in the ratio 5 : 7 : 2 The smallest distance is 3.5 km smaller than the largest. What is the total distance?	② Electricity generation, renewable : non-renewable = 9 : 5 For renewable energy, solar : wind = 11 : 8 18 MW of power was generated from wind power. How much energy was from non-renewable sources?	③ Write the ratio 4 : 2.5 in the form n : 1 where n is a decimal $a : b = 3 : \frac{2}{3}$ How many times larger is a than b?	④ The value of y is 20% more than the value of x. Express the ratio $x : y$ using integers. a : b = 2 : 3 b : c = 7 : 8 Express the ratio a : c
⑤ There are octagons, pentagons & squares in a bag. octagons to pentagons = 4:9 octagons to squares = 5:8 If there are less than 1000 shapes in the bag, what is the greatest number of squares in the bag?	⑥ On a test, Will scored double the lowest score. Dan scored 140 marks more than the lowest score. Will's score : Dans score = 1:4 What was the lowest score?	⑦ $\frac{2}{5} : \frac{6}{7} = x : 1$ Calculate the value of x.	⑧ The points A, B, C and D lie in order on a straight line. AB : BD = 1:2 AC : CD = 3:1 Work out AB : BC : CD
⑨ Wai has £8 less than Toby. Cindy has 4 times the money Wai has. In total they have £260. Find the ratio of money Toby : Cindy	⑩ a : b = 4 : 5 and 3b = 10c Work out a : c in its simplest form.	⑪ b is two-fifths of c $3a = 7c$ Express the ratio $a : b : c$ using integers.	⑫ Cane has £27 in savings. Dale has £28 savings. Each week from now, Cane saves £6 & Dale saves £3. How many weeks will it take for Cane & Dale to have savings in the ratio 3 : 2?

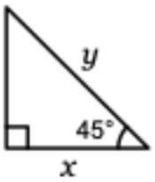
A. Work out the ratio $a : b : c$ in the simplest form.



B. Shapes A and B have the same height and the same area. Work out the ratio *perimeter of A : perimeter of B* in the simplest form.



C. Work out the ratio $x : y$ in the form $1 : n$.



D. Given that:

$$2x - 9 : x - 4 = x : 4$$

Work out the value of x.

E. A shop sells a type of cereal in two packet sizes. The larger packets cost 40% more than the smaller packets.

Work out the ratio of the cost of a smaller packet to the cost of a larger packet in the form $1 : n$.

F. Jason sells red, blue and green raffle tickets. He starts with 120 of each colour.

Jason managed to sell red, blue and green tickets in the ratio 3 : 2 : 1.
168 of the tickets remained **unsold**.

Work out how many of each colour ticket **were sold**.

G. Penny has a small block of iron and a small block of silver.

The iron is in the shape of a cuboid, with dimensions 31mm, 25mm and 43mm.

The silver has a mass of 324g and has a density of 10.5g/cm^3.

Work out the ratio of the volume of iron to the volume of silver in the form *1 : n*.

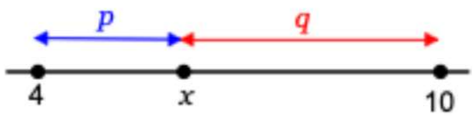
H. The prices of two games are in the ratio $a : b$.

If both prices were decreased by £10, the ratio would become 3 : 5.

If both prices were increased by £5, the ratio would become 3 : 4.

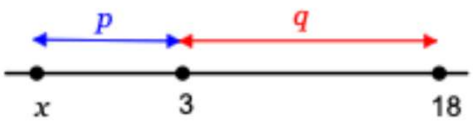
Work out the ratio $a : b$ in the simplest form.

I. In the diagram, the dots represent points on a number line.



Given that $p : q = 1 : 3$, work out x.

J. In the diagram, the dots represent points on a number line.



Given that $p : q = 2 : 5$, work out x.

Question 1: The table below shows information about the number of medals won by 6 countries in the 2016 Olympic Games.

Country	Gold	Silver	Bronze	Total
Spain	7	4	6	17
France	10	18	14	42
Germany	17	10	15	42
Italy	8	12	8	28
Japan	12	8	21	41
Australia	8	11	10	29

(a) Write down the ratio of the total number of medals won by Italy to the total number of medals won by France.
Give your answer in its simplest form.

(b) Write down the ratio of the total number of gold medals won to the total number of bronze medals won.
Give your answer in its simplest form.

Question 2: There are between 30 and 40 sweets in a bag.
Ben and Daisy share the sweets in the ratio 4:5.
There are no sweets remaining in the bag.

How many sweets were in the bag to begin with?

Question 3: George buys some first class and second class stamps to put on some letters.
Each first class stamp costs 67p
Each second class stamp costs 58p
The ratio of first class stamps to second class stamps that he buys is 1:4
George bought 20 stamps.

How much did the stamps cost George?

Question 4: 1,935 people visit a library during one week.
The ratio children : adults is 1 : 4
How many more adults than children visited the library?

Question 5: The sizes of the interior angles of a triangle are in the ratio 1:3:8
Calculate the difference in size between the largest and smallest angles.

Question 6: Mr Rodgers is organising a trip to a football match for 84 children.
The ratio of teachers on the trip to children on the trip is 1:6
Each bus holds 30 passengers.
Show four buses are needed.

Question 7: Isaac and Victoria share money in the ratio 2:5
Victoria receives £6.60
Work out the difference between how much money Isaac and Victoria receive.

Question 8: Charlotte and Melissa booked theatre tickets costing £180.
They have a voucher that entitles them to 20% off the total price.
Charlotte and Melissa share the total cost of the tickets in the ratio 5:7.

Work out how much more Melissa pays than Charlotte.

Question 9: Barnaby has 288 counters in a bag.
The counters are red, yellow and white.

$\frac{3}{8}$ of the counters are red.

The other counters are yellow and white in the ratio 1:4

Work out how many counters of each colour there are.

Question 10: The sizes of the interior angles of a pentagon are in the ratio 1:2:5:5:7
Calculate the size of the largest

Question 11: Jack is 10 years old.
Kylie is 17 years old.
Vanessa is 23 years old.
Kylie and Vanessa share £16 in the ratio of their ages.
Kylie gives 20% of her share to Jack.
Vanessa gives a quarter of her share to Jack.
How much money does Jack receive?

Question 12: Two numbers are in the ratio 3:7
One of the numbers is 42
There are two possible values for the other number.
What are the two possible values?

Question 13: Alex is 12 years older than George.
Carl is three times older than Alex
The sum of their ages is 68
Find the ratio of George's age to Carl's age to Alex's age.

Question 14: Felicity has two boxes of counters, each with an equal ratio of black and white beads.
In box A, 54 of the beads of black and 36 are white.
There are 162 white beads in box B.

How many beads are there in the two boxes in total?

Question 15: Sophie is making 400 scones.
She uses butter, sugar and flour in the ratio 2:1:9
Here are the costs of those ingredients.

Butter	£2.20 per 500g
Sugar	£1.60 per kilogram
Flour	60p per 1.5kg

The total mass of the butter, sugar and flour in each scone is 30g

Work out the total cost of these ingredients for the 400 scones.

Question 16: Copper costs £5 per kilogram.
Zinc costs £3.20 per kilogram.
Copper and zinc are mixed in the ratio 4:1 to make brass.

Work out the cost of 7 kilograms of brass.

Question 17: Geraint has 2p and 50p coins in the ratio 20 : 3

Write the ratio of the value of the 2p coins to the value of 50p coins in its simplest form.

Question 18: Mrs Chambers is organising a school trip to a museum for year 7 and year 8.
She needs to work out the total cost of the museum tickets and bus hire.
The table below shows the museum ticket prices.

Visitor Age	Price
0 - 3	free
4 - 12	£4.50
13 - 17	£6.50
18+	£11.50

Each bus has 51 seats and costs £125

Altogether 300 students want to go on the trip.
The ratio of the number of students to the number of teachers is 25:1
The ratio of the number of students in year 7 to the number of students in year 8 is 8:7

At the time of the trip, all of the students in year 7 are 11 or 12 years old.
Of year 8 students, the ratio of number of 12 year olds to 13 year olds is 2:3.
Work out the total price of the school trip.

Question 19: Alannah, Colin and Hannah visit the park and collect conkers and acorns.

In total they collect the same number of conkers and acorns.

Alannah, Colin and Hannah collect conkers in the ratio 4:7:9
Alannah, Colin and Hannah collect acorns in the ratio 3:1:5

(a) Did Hannah collect more conkers or acorns?

(b) What is the lowest possible number of conkers they could have collected?

Question 20: A bag contains red, yellow and blue beads.

The ratio of red beads to yellow beads is 2:3
The ratio of yellow beads to blue beads is 5:4

Work out what fraction of the beads are red.

Question 21: In a box

the number of blue counters and the number green counters are in the ratio 7:4
the number of green counters and the number of red counters are in the ratio 3:1

The total number of counters in the bag is 444.

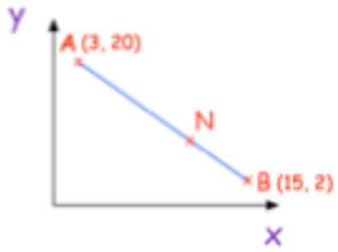
How many green counters are in the bag?

Question 22: The ratio of the red cards to black cards in a deck is 3:10
2 more red cards are added to the deck.
The ratio of red cards to black cards is now 1:3

Work out the number of black cards in the deck.

Question 23: A is the point with coordinates (3, 20)
B is the point with coordinates (15, 2)
N is a point of the line AB such that AN : NB = 2 : 1

Find the coordinates of the point N.



Question 24: Bag A contains 5x coins.
Bag B contains 3x coins.
8 coins are taken from Bag B and put into Bag A
The ratio of coins in Bag A to Bag B is now 11:5

Work out the total number of coins.

Question 25: A cone has radius 4cm and perpendicular height of 9cm.
A sphere has a radius of 6cm

Volume of a sphere	$\frac{4}{3}\pi r^3$
Volume of a cone	$\frac{1}{3}\pi r^2 h$

Calculate the ratio of the volume of the cone to the volume of the sphere.

Question 26: On 1st March 2001, the ratio of Freddie's age to his mother's age was 1:11
On 1st March 2018, the ratio of Freddie's age to his mother's age was 2:5
Write the ratio of Freddie's age to his mother's age on 1st March 2030

Question 27: The ratio 25 000 000 000 : 500 can be written in the form $n : 1$
Work out the value of n

Give your answer in standard form

Question 28: The distance of Mercury from the Sun is 5.7×10^7 km
The distance of Neptune from the Sun is 4.3×10^9 km

Work out the ratio of the distance of Mercury from the Sun to the distance of Neptune from the Sun.

Question 29: Class 10D make some cakes using milk chocolate, dark chocolate or white chocolate.
Some of the cakes contain nuts and the rest do not.

The ratio of the number of milk chocolate cakes to dark chocolate cakes is 10:3
The ratio of the number of white chocolate cakes to milk chocolate cakes is 1:6

Of the milk chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 1:8

Of the dark chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 3:2

Of the white chocolate cakes, the ratio of the number of cakes containing nuts to not containing nuts is 2:5

What percentage of the cakes contain nuts?

Purposeful Practice

gcse ratio problems (based on Edexcel)

(1) Riley and Dan share some jelly beans
Dan gets 40% of the jelly beans
write down the ratio of the number of jelly beans
Riley gets to the number that Dan gets

(2) there are 20 boxes of eggs with 6 eggs in each box
write, as a ratio, the number of eggs in four of the boxes to the number of eggs that there are altogether
what does this ratio simplify to?

(3) Harry, Ryan and Kara share £405 in the ratio 3 : 2 : 4
how much money does Kara get?

(4) Arnold and Lara share some money in the ratio 7 : 13
what percentage of the money does Lara get?

(5) a path is made of white tiles and grey and
¼ of the tiles are white
(i) what is the ratio of white tiles to grey tiles ?

there are 72 tiles altogether
(*ii)* work out the number of grey tiles

(6) there are only red buttons, yellow buttons and orange buttons in a jar
these are in the ratio 2 : 4 : 9 respectively

what percentage of buttons in the jar that are orange?

(7) 2.5 kg of plums cost £5.10
work out the cost of 3.5 kg of plums at the same price

(8) there are only red, orange and green tennis balls in a bag
there are three times as many orange as red
and four times as many green as orange

Hannah takes a tennis ball at random from the bag
work out the probability that Hannah picks a green one

(9) the ratio of the cost of one metre of cotton to the cost of one metre of silk (same widths) is 2 : 7

if 2m of cotton cost £8,
how much does 9m of silk cost?

(10) this recipe for flapjacks makes 12 pieces

120 g butter
140 g light brown sugar
250 g porridge oats
2 tablespoons syrup

how much of each ingredient is needed to make 18 pieces?

(11) Kim is 6 years older than Jim
Tim is twice as old as Kim
the sum of their three ages is 90 years

find the ratio of Jim's age : Kim's age : Tim's age
in the simplest possible form

Purposeful Practice

gcse ratio problems (based on Edexcel)

(12) one Saturday, some adults and some children were in a the Grand theatre, Wolverhampton

the ratio of the number of adults to the number of children was 7 : 4

each person sat in the circle or the stalls

¾ of the children had seats in the stalls
68 children had seats in the circle

there are exactly 1200 seats in the theatre

on this particular Saturday, were there people on more than 60% of the seats?

(13) Jay bakes 280 cakes,

four different kinds of cake are baked:
vanilla, banana, lemon and chocolate

- 2 out of every 7 of the cakes are vanilla cakes
- 35% of the cakes are banana cakes
- the ratio of the number of lemon cakes to the number of chocolate cakes is 1 : 5

are there more chocolate cakes than vanilla cakes?

you must show your working to support your answer

(14) it normally takes 5 cleaners 4½ hours to clean all of the rooms in a hotel

today there are only 3 cleaners to clean all of the rooms

each cleaner is paid at a rate of £8.10 per hour (or part of an hour) that he/she works

how much will **each** cleaner be paid today?

(15) Theo drove 54 km from Liverpool to Manchester
he then drove 62 km from Manchester to Sheffield

Theo's average speed from Liverpool to Manchester was 72 km/h

Theo then took 75 minutes to drive from Manchester to Sheffield

work out Theo's average speed for his total drive from Liverpool to Sheffield

(16) a gold bullion bar has a mass of 12.4 kg

the density of gold is 19.32 g/cm^3

work out the volume of the gold bar

give your answer correct to 1 decimal place

gcse ratio problems (based on Edexcel)

(17) the density of apple juice is 1.04 grams per cm^3
the density of fruit syrup is 1.4 grams per cm^3
the density of carbonated water is 0.99 grams per cm^3

30 cm^3 of apple juice are mixed with 10 cm^3 of fruit syrup and 280 cm^3 of carbonated water to make a drink with a volume of 320 cm^3

work out the density of the drink

give your answer correct to 2 decimal places

(18) there are only blue, black and red pens in a box

the ratio of the number of blue to the number of black is 2 : 5

the ratio of the number of black to the number of red is 4 : 1

there are less than 100 pens altogether in the box

what is the greatest possible number of red pens in the box?

(19) Jack and Jill cycled along the same 40 km route
Jack took 2½ hours to cycle the 40 km

Jill started to cycle 10 minutes after Jack started to cycle
Jill caught up with Jack when they had both cycled 24 km
Jack and Jill both cycled at constant speeds

work out Jill's speed

(20) the points *A*, *B*, *C* and *D* lie in order along a straight line

$AB : BD = 2 : 5$
$AC : CD = 11 : 10$

work out the ratio $AB : BC : CD$

(21) white shapes and black shapes are used in a game

some shapes are circles and the other shapes are squares

the ratio of white shapes to black shapes is 3 : 7

the ratio of white circles to white squares is 4 : 5

the ratio of black circles to black squares is 2 : 5

work out what fraction of all the shapes are circles

1

(a) A box contains only red pens and blue pens.

The ratio red : blue = 3 : 2

There are 24 **red** pens.

Work out the number of pens in the box.

[2 marks]

(b) Another box contains only green pens and black pens.

$\frac{3}{4}$ of the pens are **green**.

What is the ratio of green pens to black pens?

[1 mark]

2

Amy has some red beads and some blue beads.
She uses the beads to make two necklaces, A and B.

Necklace A
24 beads
3 times as many red beads as blue beads

Necklace B
35 beads
number of red beads : number of blue beads = 3 : 2

How many **more** red beads than blue beads does she use?

[4 marks]

3

There are 60 balls in a bag.
The balls are either blue, red or yellow.

The ratio of blue balls to red balls is 5 : 3
The ratio of red balls to yellow balls is 1 : 4

Work out how many **red** balls are in the bag.

[3 marks]

4 Ahmed, Ben and Carla share some money.

Ahmed gets $\frac{3}{7}$ of the money.

Ben and Carla share the rest of the money in the ratio 3 : 2

Carla gets £64

Work out how much Ahmed gets.

[5 marks]

5 Jack sells packs of envelopes.
There are three types of pack, Economy (E), Standard (S) and Luxury (L).

One week he sells 270 packs in the ratio

E : S : L = 5 : 3 : 2

He makes a profit of

12p a pack on Economy packs

15p a pack on Standard packs

20p a pack on Luxury packs.

Work out the **total** profit he makes selling envelopes.

[3 marks]

6 Three of the notes on a piano keyboard are C, F and G.
Each note has a frequency, measured in Hertz (Hz)

frequency of C : frequency of F = 3 : 4

frequency of F : frequency of G = 8 : 9

(a) Show that frequency of C : frequency of G = 2 : 3

[2 marks]

(b) The frequency of C is 261.6 Hz

Work out the frequency of G.

[2 marks]

7 A blend of tea contains Darjeeling tea and Assam tea in the ratio

Darjeeling : Assam = 4 : 1

Darjeeling tea costs £14 per kilogram.

Assam tea costs £42 per kilogram.

Work out the cost of 250 grams of this blend.

[5 marks]

8 A, B and C are such that

A : B = 1 : 5

B is $\frac{3}{4}$ of C

Work out the ratio A : C

[3 marks]

9 $0.4a = 0.75b$

Work out the ratio $a : b$

Give your answer in its simplest form.

[3 marks]

10 Bob and Kai share £x in the ratio 2 : 3

How much money does Kai receive?
Circle the correct expression.

[1 mark]

$£\frac{2}{3}x$ $£\frac{2}{5}x$ $£\frac{3}{5}x$ $£\frac{5}{8}x$

Bob gives Kai £8 from his share.
The amount of money they each have now is in the ratio 2 : 5

Work out the value of x.
You **must** show your working.
Do **not** use Trial and Improvement.

[4 marks]