KING EDWARD VI
HANDSWORTH GRAMMAR
SCHOOL FOR BOYS

KING EDWARD VI
ACADEMY TRUST
BIRMINGHAM

Year 10

2024 Mathematics 2025

Unit 19 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

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Contents Page

1 Advanced Indices

Indices Recap

Multiplication Law:

$a^m \times a^n = a^{m+n}$

Division Law:

$a^m \div a^n = a^{m-n} \quad (a \neq 0)$

Power Law:

$(a^m)^n = a^{mn}$

Power of Zero Law:

$a^0 = 1 \quad (a \neq 0)$

Other Laws:

$(ab)^n = a^n b^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n} \quad (b \neq 0)$

Negative Indices

Worked Example	Your Turn
Evaluate: a) 3^{-2} b) -3^{-2} c) $(-3)^{-2}$	Evaluate: a) 5^{-3} b) -5^{-3} c) $(-5)^{-3}$

Worked Example	Your Turn
Write $\frac{1}{4^2}$ in index form	Write $\frac{1}{5^3}$ in index form

Fill in the Gaps

Negative Indices

	2^{-1}	2^{-2}	3^{-1}	3^{-2}		4^{0}	2^{-3}			2^{-4}		4^{-3}	1^{-4}
As a Fraction	$\frac{1}{2^1}$								$\frac{1}{5^3}$				
Simplified Fraction	$\frac{1}{2}$	$\frac{1}{4}$			$\frac{1}{25}$			$\frac{1}{7}$			$\frac{1}{27}$		

Worked Example	Your Turn
Simplify: a) $\left(\frac{3}{10}\right)^{-2}$ b) $\left(-\frac{3}{10}\right)^{-2}$	Simplify: a) $\left(\frac{2}{5}\right)^{-3}$ b) $\left(-\frac{2}{5}\right)^{-3}$

Worked Example	Your Turn
Calculate 0.08^{-1} Leave your answer as a decimal.	Calculate 0.016^{-1} Leave your answer as a decimal.

Worked Example	Your Turn
Simplify: a) $(5x^8)^{-3}$ b) $\left(\frac{6p^2q^9}{18p^5q^7}\right)^{-3}$	Simplify: a) $(4d^6)^{-3}$ b) $\left(\frac{2f^9g^5}{6f^8g^7}\right)^{-3}$

Dr Frost 298d and 298f

Worked Example	Your Turn
Write the following in the form ax^b in its simplest form. a) $\frac{1}{x^5}$ b) $\frac{3}{x^5}$ c) $\frac{3}{7x^5}$ d) $\frac{3}{7x^{-5}}$	Write the following in the form ax^b in its simplest form. a) $\frac{1}{d^{10}}$ b) $\frac{9}{d^{10}}$ c) $\frac{9}{13d^{10}}$ d) $\frac{9}{13d^{-10}}$

Expanding Brackets with Indices

Worked Example	Your Turn
Simplify: a) $2a^3(3a^2 + 5a^{-4})$ b) $p^{\frac{1}{2}}\left(2p^{\frac{1}{2}} - p^{-\frac{3}{2}}\right)$ c) $x^2\left(x^{\frac{1}{3}} - x^{\frac{1}{4}}\right)$	Simplify: a) $3a^{-2}(4a^5 + 2a)$ b) $2p^{\frac{1}{3}}\left(3p^{\frac{2}{3}} - p^{-\frac{1}{3}}\right)$ c) $n^{\frac{3}{5}}\left(n^{\frac{1}{2}} + \frac{1}{n^{\frac{1}{2}}}\right)$

Worked Example	Your Turn
Simplify: $(2m^9 - m^{-2})(6m^{-3} + m^5)$	Simplify: $(7x^3 - x^{-4})(4x^{-2} + x^9)$

Fractional Indices

Worked Example

Evaluate:

a) $64^{\frac{1}{2}}$

b) $64^{-\frac{1}{2}}$

c) $\left(\frac{81}{16}\right)^{\frac{1}{4}}$

d) $\left(\frac{81}{16}\right)^{-\frac{1}{4}}$

Your Turn

Evaluate:

a) $64^{\frac{1}{3}}$

b) $64^{-\frac{1}{3}}$

c) $\left(\frac{81}{16}\right)^{\frac{1}{2}}$

d) $\left(\frac{81}{16}\right)^{-\frac{1}{2}}$

Dr Frost 394a, 394b and 394e

Worked Example	Your Turn
Given that $5^{-n} = 0.4$, find the value of $(5^3)^n$	Given that $3^{-n} = 0.2$, find the value of $(3^4)^n$

Fractional Indices

Worked Example	Your Turn
Evaluate: a) $25^{\frac{3}{2}}$ b) $25^{-\frac{3}{2}}$ c) $\left(\frac{36}{25}\right)^{\frac{3}{2}}$ d) $\left(\frac{36}{25}\right)^{-\frac{3}{2}}$	Evaluate: a) $81^{\frac{3}{4}}$ b) $81^{-\frac{3}{4}}$ c) $\left(\frac{81}{256}\right)^{\frac{3}{4}}$ d) $\left(\frac{81}{256}\right)^{-\frac{3}{4}}$

Dr Frost 394f, 394j, 394g and 394k

Fill in the Gaps

Fractional Indices ①

	$4^{\frac{1}{2}}$	$9^{\frac{1}{2}}$			$27^{\frac{1}{3}}$	$1^{\frac{1}{2}}$		$(-8)^{\frac{1}{3}}$	$64^{0.5}$	$125^{\frac{1}{3}}$	$(-27)^{\frac{1}{3}}$	$100{,}000^{\frac{1}{5}}$	$81^{0.25}$	$32^{0.2}$
As a Root	$\sqrt[2]{4}$		$\sqrt[3]{8}$	$\sqrt{36}$			$\sqrt[2]{\ }$							
Integer							7							

Fractional Indices ②

	$8^{\frac{2}{3}}$	$4^{\frac{3}{2}}$	$8^{\frac{4}{3}}$	$16^{\frac{3}{4}}$		$32^{\frac{2}{5}}$	$8^{\frac{5}{3}}$	$4^{\frac{5}{2}}$	$81^{0.75}$	$243^{0.4}$	$32^{1.2}$
Expanded	$8^{\left(\frac{1}{3}\times 2\right)}$	$4^{\left(\frac{1}{2}\times 3\right)}$									
2 Indices	$\left(8^{\frac{1}{3}}\right)^2$				$\left(27^{\frac{1}{3}}\right)^5$						
1 Index	$(2)^2$										
Integer	4				243						

Worked Example	Your Turn
Simplify: a) $(81a^{16}b^{8})^{\frac{1}{2}}$ b) $(8m^{21}n^{24})^{\frac{2}{3}}$	Simplify: a) $(100a^{8}b^{16})^{\frac{1}{2}}$ b) $(25p^{2}q^{6})^{\frac{3}{2}}$

Worked Example	Your Turn
Simplify: a) $(27p^{18}q^{6})^{-\frac{4}{3}}$ b) $\left(\frac{64a^{13}b^{6}}{27a^{4}}\right)^{-\frac{2}{3}}$	Simplify: a) $(32m^{40}n^{10})^{-\frac{2}{5}}$ b) $\left(\frac{8p^{14}q^{15}}{125p^{5}}\right)^{-\frac{2}{3}}$

Dr Frost 394n and 394o

Worked Example	Your Turn
Given that $x^n = \sqrt{x} \times \frac{1}{x\sqrt[4]{x^3}}$ Find the value of n.	Given that $a^n = a^2 \times \frac{1}{\sqrt[3]{a^2} \times \sqrt{a}}$ Find the value of n.

Dr Frost 394r

Worked Example	Your Turn
Expand and simplify $\left(\frac{3\sqrt{x}}{2}+\frac{2}{\sqrt[3]{x}}\right)^2$ expressing all terms in the form of ax^b	Expand and simplify $\left(\frac{2\sqrt[3]{x}}{3}+\frac{3}{4\sqrt[3]{x}}\right)^2$ expressing all terms in the form of ax^b

Worked Example	Your Turn
Write $10\sqrt[3]{x} + \frac{7}{\sqrt{x}} + \frac{2}{7(\sqrt[3]{x})^2} + \frac{1}{x^7}$ as an expression where all terms are in the form ax^b	Write $\frac{4}{3\sqrt[3]{x}} + \frac{9}{\sqrt{x}} + \sqrt[3]{x} - \frac{2}{x^3}$ as an expression where all terms are in the form ax^b

Worked Example	Your Turn
Full simplify $\frac{\sqrt[3]{27p^2q^7}}{5p^4}$, writing your answer in the form ap^bq^c.	Full simplify $\frac{\sqrt[3]{125x^3y^7}}{7x^6}$, writing your answer in the form ax^by^c.

Worked Example	Your Turn
Write $\dfrac{5m^{\frac{3}{2}}n^3 + 2m^{\frac{1}{2}}}{10m^{\frac{3}{2}}}$ in the form $an^c + bm^d$, where a, b, c and d are constants to be found.	Write $\dfrac{6x^{\frac{8}{5}} - 2x^{\frac{13}{5}}y^2}{3x^{\frac{3}{5}}}$ in the form $ax + bx^c y^d$, where a, b, c and d are constants to be found.

Worked Example	Your Turn
Express $\frac{10x^5 + 6x - 8x^{\frac{2}{3}} + 2x^4}{4\sqrt{x}}$ in the form of $ax^b + cx^d + ex^f + gx^h$	Express $\frac{6x^3 - 3x^{\frac{2}{3}} + 3x + 3x^2}{2\sqrt{x}}$ in the form of $ax^b + cx^d + ex^f + gx^h$

Change of Base

Worked Example	Your Turn
a) Write 27 as a power of 3	a) Write 8 as a power of 2
b) Write 27^4 as a power of 3	b) Write 8^3 as a power of 2
c) Write 27^x as a power of 3	c) Write 8^x as a power of 2
d) Write 8^{2x} as a power of 2	d) Write 8^{3x} as a power of 2

Worked Example	Your Turn
Express 25^{2x-7} in the form 5^y, stating y in terms of x.	Express 8^{8x-6} in the form 2^y, stating y in terms of x.

Dr Frost 358b

Worked Example	Your Turn
Write $64^7 \times 4^5 \times 16^{11}$ in the form 4^k where k is an integer.	Write $2^3 \times 4^{19} \times 8^2$ in the form 2^k where k is an integer.

Worked Example	Your Turn
Find the value of each of the following: a) $\sqrt{3^6 \times 16}$ b) $\sqrt[3]{3^6 \times 8}$ c) $\sqrt[4]{3^8 \times 16}$	Find the value of each of the following: a) $\sqrt{2^4 \times 9}$ b) $\sqrt[3]{64 \times 3^3}$ c) $\sqrt[4]{81 \times 256}$

Worked Example	Your Turn
Given that $\frac{12\times\left(\sqrt{32}\right)^{3y-1}}{3\times4^{2y+3}}$ can be written in the form 2^p, find an expression for p in terms of y	Given that $\frac{12\times\left(\sqrt{8}\right)^{2y+2}}{6\times4^{2y+1}}$ can be written in the form 2^p, find an expression for p in terms of y

Equations with Indices

Worked Example	Your Turn
a) Given that $125^q = \frac{1}{25}$ find the value of q b) Given that $z^{-4} = \frac{1}{16}$ find the possible values of z	a) Given that $27^z = \frac{1}{9}$ find the value of z b) Given that $q^{-2} = \frac{1}{25}$ find the possible values of q

Dr Frost 298g and 298h

Worked Example	Your Turn
a) Given that $(5^5)^z = 0.008$ find the value of z b) Given that $(y^{-2})^3 = 0.000001$ find the possible values of y	a) Given that $(5^{-4})^p = 0.04$ find the value of p b) Given that $(z^{-2})^2 = 0.0625$ find the possible values of z

Worked Example	Your Turn
Find the value of x that satisfies: a) $2^x \times 2^{x-3} = 32$ b) $2^{2x} \div 2^{x-3} = 32$	Find the value of x that satisfies: a) $3^x \times 3^{x-2} = 81$ b) $3^{3x} \div 3^{x-2} = 81$

Worked Example	Your Turn
Find the value of x that satisfies: $125^{\frac{1}{4}} \times 5^{2x+3} = 25^{\frac{2}{3}}$	Find the value of x that satisfies: $64^{\frac{1}{4}} \times 4^{3x+1} = 16^{\frac{2}{3}}$

Dr Frost 358d

Worked Example	Your Turn
Solve a) $k^{\frac{2}{3}} = 9$ b) $m^{\frac{2}{3}} = 45$	Solve a) $m^{\frac{4}{3}} = 16$ b) $a^{\frac{2}{5}} = 8$

Dr Frost 395d and 395e

Worked Example	Your Turn
Solve $10x = 810\sqrt[5]{x}$	Solve $7x = 63\sqrt[3]{x}$

Dr Frost 395g

Worked Example	Your Turn
Solve the equation $2x^{\frac{5}{3}} + 5x^{\frac{2}{3}} = 3x^{-\frac{1}{3}}, \qquad x > 0$ Give your answer to 3 significant figures	Solve the equation $2x^{\frac{3}{2}} - 3x^{\frac{1}{2}} = 7x^{-\frac{1}{2}}, \qquad x > 0$ Give your answer to 3 significant figures

Worked Example	Your Turn
Solve the equation $2x^4 - 19x^2 = -35$	Solve the equation $2y^4 - 17y^2 = -36$

Dr Frost 485a

Worked Example	Your Turn
Solve the equation $2z - 13\sqrt{z} = -20$	Solve the equation $4x = 20\sqrt{x} - 21$

Worked Example	Your Turn
Solve the equation $2x^2 + \frac{63}{x^2} = 23$	Solve the equation $2y^2 + \frac{42}{y^2} = 19$

Worked Example	Your Turn
Find the exact solutions of $2 \times 2^{2x+1} = 9 \times 2^{x} - 2$	Find the exact solutions of $27 \times 3^{2x-1} - 26 \times 3^{x} = 3$

Dr Frost 485d

Fill in the Gaps

Original Equation in x	Substitution	Quadratic in y	Solutions for y	Solutions for x
$x^4 - 10x^2 + 21 = 0$	$y = x^2$	$y^2 - 10y + 21 = 0$	$y = 7, y = 3$	$x = \pm\sqrt{7}, \pm\sqrt{3}$
$x^6 = 7x^3 + 8$	$y = x^3$		$y = 8, y = -1$	
$x - 3\sqrt{x} - 10 = 0$				$x = 25, x = 4$
$2^{2x} - 6 \times 2^x + 8 = 0$	$y = 2^x$			
$\sqrt{x} + \frac{1}{\sqrt{x}} = 2$				
$9^x - 28 \times 3^x + 27 = 0$				
$x\sqrt[3]{x} - 13x^{\frac{2}{3}} + 36 = 0$	$y = x^{\frac{2}{3}}$			
$x^3 + 9x + \frac{20}{x} = 0$				
$\left(x - \frac{6}{x}\right)^2 - 6\left(x - \frac{6}{x}\right) + 5 = 0$	$y = \left(x - \frac{6}{x}\right)$			

Extra Notes

2 Calculating with Surds

Multiplying Surds

Worked Example	Your Turn
Simplify: a) $5 \times \sqrt{6}$ b) $\sqrt{5} \times \sqrt{6}$ c) $2\sqrt{5} \times 3\sqrt{6}$	Simplify: a) $\sqrt{5} \times \sqrt{7}$ b) $\sqrt{7} \times 5$ c) $3\sqrt{5} \times 2\sqrt{7}$

Dr Frost 335a and 335c

Worked Example	Your Turn
Simplify: a) $\sqrt{3} \times \sqrt{6}$ b) $4\sqrt{3} \times 5\sqrt{6}$	Simplify: a) $\sqrt{3} \times \sqrt{8}$ b) $7\sqrt{3} \times 2\sqrt{8}$

Worked Example	Your Turn
Simplify a) $\sqrt{6} \times \sqrt{6}$ b) $\left(\sqrt{6}\right)^2$ c) $\left(2\sqrt{6}\right)^2$ d) $2\left(\sqrt{6}\right)^2$ e) $2\left(\sqrt{6}\right)^3$	Simplify a) $\sqrt{7} \times \sqrt{7}$ b) $\left(\sqrt{7}\right)^2$ c) $\left(2\sqrt{7}\right)^2$ d) $2\left(\sqrt{7}\right)^2$ e) $2\left(\sqrt{7}\right)^3$

Dividing Surds

Worked Example	Your Turn
Simplify: a) $\sqrt{60} \div \sqrt{2}$ b) $6\sqrt{60} \div 3\sqrt{2}$	Simplify: a) $\sqrt{90} \div \sqrt{6}$ b) $8\sqrt{90} \div 2\sqrt{6}$

Worked Example	Your Turn
Simplify: a) $\sqrt{60} \div \sqrt{3}$ b) $12\sqrt{60} \div 2\sqrt{3}$	Simplify: a) $\sqrt{90} \div \sqrt{2}$ b) $12\sqrt{90} \div 3\sqrt{2}$

Worked Example	Your Turn
Simplify fully $(\sqrt{10})x^2 \div \sqrt{\frac{160}{x^6}}$	Simplify fully $(\sqrt{11})x^3 \div \sqrt{\frac{99}{x^4}}$

Adding and Subtracting Surds

Worked Example	Your Turn
Simplify: a) $2\sqrt{5} + 5\sqrt{5}$ b) $2\sqrt{20} + 5\sqrt{5}$ c) $2\sqrt{20} + 5\sqrt{10}$	Simplify: a) $2\sqrt{6} + 5\sqrt{6}$ b) $2\sqrt{54} + 5\sqrt{6}$ c) $2\sqrt{20} + 5\sqrt{15}$

Dr Frost 336a and 336b

Worked Example	Your Turn
Simplify: $\frac{2\sqrt{20}+5\sqrt{5}}{\sqrt{5}}$	Simplify: $\frac{2\sqrt{54}-5\sqrt{6}}{\sqrt{6}}$

Expanding Brackets with Surds

Worked Example	Your Turn
Expand and simplify: a) $2\left(5+\sqrt{3}\right)$ b) $-\sqrt{3}\left(5+\sqrt{3}\right)$ c) $\sqrt{12}(5+\sqrt{3})$ d) $\sqrt{12}(\sqrt{5}+\sqrt{3})$	Expand and simplify: a) $-2\left(\sqrt{3}-5\right)$ b) $\sqrt{3}\left(\sqrt{3}-5\right)$ c) $\sqrt{27}(\sqrt{3}-5)$ d) $-\sqrt{27}(\sqrt{3}-\sqrt{5})$

Dr Frost 337a and 337b

Worked Example	Your Turn
Expand and simplify: a) $(2-\sqrt{3})(4+\sqrt{3})$ b) $(2-5\sqrt{3})^2$	Expand and simplify: a) $(\sqrt{3}-2)(\sqrt{3}+4)$ b) $(5\sqrt{3}-2)^2$

Dr Frost 337c and 337f

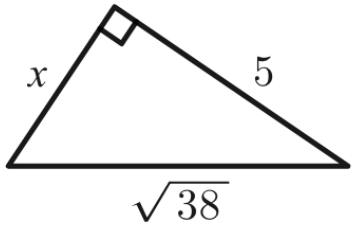
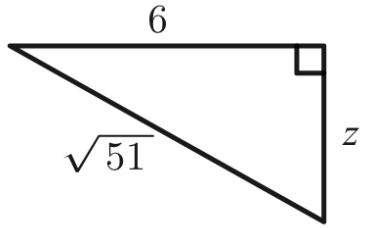
Worked Example	Your Turn
Expand and simplify: a) $\left(2-\sqrt{20}\right)\left(4+\sqrt{5}\right)$ b) $(2-2\sqrt{20})(4+5\sqrt{5})$	Expand and simplify: a) $\left(\sqrt{54}-2\right)\left(\sqrt{6}+4\right)$ b) $(2\sqrt{54}-2)(5\sqrt{6}+4)$

Dr Frost 337d and 337e

Worked Example	Your Turn
Expand and simplify: $(3+\sqrt{2})^2-(3-\sqrt{2})^2$	Expand and simplify: $(2+\sqrt{3})^2-(2-\sqrt{3})^2$

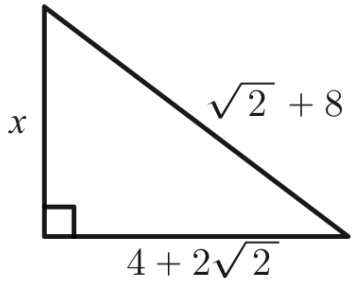
Worked Example	Your Turn
Expand and simplify: $(\sqrt{6}-1)(4+\sqrt{6})(5-\sqrt{6})$	Expand and simplify: $(\sqrt{11}-2)(3-\sqrt{11})(1-\sqrt{11})$

Dr Frost 337g

Worked Example	Your Turn
Given that all lengths are in centimetres, find the exact value of x. 	Given that all lengths are in centimetres, find the exact value of z. 

Worked Example

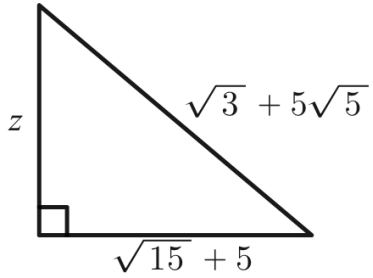
Given that all lengths are in centimetres, find the exact value of x.



Give your answer in simplified surd form.

Your Turn

Given that all lengths are in centimetres, find the exact value of z.



Give your answer in simplified surd form.

Worked Example	Your Turn
Express b and c in terms of a: $\left(a+\sqrt{12}\right)^2 = b + c\sqrt{3}$	Express b and c in terms of a: $\left(a+\sqrt{8}\right)^2 = b + c\sqrt{2}$

Worked Example	Your Turn
Find the value of a and b: $\left(a - 3\sqrt{5}\right)^2 = b - 42\sqrt{5}$	Find the value of a and b: $\left(a - 2\sqrt{3}\right)^2 = b - 20\sqrt{3}$

Rationalising Surds

Worked Example	Your Turn
Rationalise: a) $\frac{3}{\sqrt{7}}$ b) $\frac{3}{\sqrt{6}}$ c) $\frac{3}{2\sqrt{6}}$	Rationalise: a) $\frac{10}{\sqrt{11}}$ b) $\frac{10}{\sqrt{5}}$ c) $\frac{10}{4\sqrt{5}}$

Dr Frost 392a and 392c

Worked Example	Your Turn
Express $\frac{30}{\sqrt{6}} - \sqrt{24}$ in the form $a\sqrt{6}$ where a is an integer to be found.	Express $\frac{21}{\sqrt{7}} + \sqrt{28}$ in the form $a\sqrt{7}$ where a is an integer to be found.

Worked Example	Your Turn
Express $\frac{4-\sqrt{32}}{\sqrt{2}}$ in the form $a+b\sqrt{2}$ where a and b are constants to be found.	Express $\frac{14-\sqrt{28}}{\sqrt{7}}$ in the form $a+b\sqrt{7}$ where a and b are constants to be found.

Worked Example	Your Turn
Express $\left(\frac{2}{\sqrt{6}}\right)^5$ in the form $\frac{a\sqrt{6}}{27}$ where a is an integer.	Express $\left(\frac{2}{\sqrt{3}}\right)^7$ in the form $\frac{a\sqrt{3}}{81}$ where a is an integer.

Worked Example	Your Turn
A rectangle has area $64\ cm^2$ and a width of $\sqrt{32}\ cm$. Find the length of the rectangle in the form $a\sqrt{b}$	A rectangle has area $60\ cm^2$ and a width of $\sqrt{12}\ cm$. Find the length of the rectangle in the form $a\sqrt{b}$

Conjugates

Is $\sqrt{3} - 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $\sqrt{3} + 1$?

Is $-\sqrt{3} + 1$ the conjugate of $1 + \sqrt{3}$?

Is $1 - \sqrt{3}$ the conjugate of $1 + \sqrt{3}$?

Is $-1 - \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{3}$ the conjugate of $1 - \sqrt{3}$?

Is $1 + \sqrt{5}$ the conjugate of $1 - \sqrt{5}$?

Is $1 - 3\sqrt{5}$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $1 + 3\sqrt{5}$?

Is $3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} + 1$?

Is $-3\sqrt{5} - 1$ the conjugate of $3\sqrt{5} - 1$?

Worked Example	Your Turn
Rationalise the denominator of $\frac{6}{\sqrt{3}+1}$ giving your answer in the form $a + b\sqrt{3}$	Rationalise the denominator of $\frac{5}{\sqrt{6}-1}$ giving your answer in the form $a + b\sqrt{6}$

Worked Example	Your Turn
Rationalise the denominator of $\frac{8}{\sqrt{7}+\sqrt{3}}$ Give your answer in the form $a\sqrt{7}+b\sqrt{3}$	Rationalise the denominator of $\frac{4}{\sqrt{6}-\sqrt{2}}$ Give your answer in the form $a\sqrt{6}+b\sqrt{2}$

Worked Example	Your Turn
Rationalise the denominator of $\frac{2+\sqrt{5}}{\sqrt{5}-2}$ Give your answer in the form $a+b\sqrt{5}$	Rationalise the denominator of $\frac{9+\sqrt{6}}{\sqrt{6}-1}$ Give your answer in the form $a+b\sqrt{6}$

Worked Example	Your Turn
Rationalise the denominator of $\frac{3\sqrt{5}+5}{5-2\sqrt{5}}$ Give your answer in the form $a+b\sqrt{5}$	Rationalise the denominator of $\frac{4\sqrt{2}-3}{3-2\sqrt{2}}$ Give your answer in the form $a+b\sqrt{2}$

Fill in the Gaps

Question	Working		Answer
$\frac{5}{\sqrt{3}}$	$\times\frac{\sqrt{3}}{\sqrt{3}}$	$=\frac{5\sqrt{3}}{\sqrt{9}}$	$=\frac{5\sqrt{3}}{3}$
$\frac{\sqrt{3}}{\sqrt{7}}$	$\times\frac{\sqrt{7}}{\sqrt{7}}$		
$\frac{5\sqrt{5}}{\sqrt{6}}$			
$\frac{2+\sqrt{3}}{\sqrt{5}}$	$\times\frac{\sqrt{5}}{\sqrt{5}}$	$=\frac{\sqrt{5}(2+\sqrt{3})}{\sqrt{25}}$	$=\frac{2\sqrt{5}+\sqrt{15}}{5}$
$\frac{3-\sqrt{5}}{\sqrt{2}}$			
$\frac{1+\sqrt{2}}{2\sqrt{3}}$			
$\frac{\sqrt{2}-3\sqrt{5}}{5\sqrt{2}}$			

Question	Working		Answer
$\frac{3}{2+\sqrt{2}}$	$\times\frac{2-\sqrt{2}}{2-\sqrt{2}}$	$=\frac{3(2-\sqrt{2})}{4-\sqrt{4}}$	$=\frac{6-3\sqrt{2}}{2}$
$\frac{8}{4-\sqrt{3}}$			
$\frac{\sqrt{5}}{6+\sqrt{5}}$			
$\frac{3\sqrt{5}}{3-\sqrt{7}}$			
$\frac{7+\sqrt{2}}{3-\sqrt{2}}$	$\times\frac{3+\sqrt{2}}{3+\sqrt{2}}$	$=\frac{(7+\sqrt{2})(3+\sqrt{2})}{9-\sqrt{4}}$	$=\frac{23+10\sqrt{2}}{7}$
$\frac{1-\sqrt{8}}{5+\sqrt{2}}$			
$\frac{a+\sqrt{b}}{a\sqrt{b}}$			

Worked Example	Your Turn
Show that $\frac{3-\sqrt{54}}{3+\sqrt{6}}$ can be written in the form $a+b\sqrt{6}$ where a and b are integers	Show that $\frac{9-\sqrt{75}}{2-\sqrt{3}}$ can be written in the form $a+b\sqrt{3}$ where a and b are integers

Worked Example	Your Turn
Show that $\frac{4+2\sqrt{54}}{\sqrt{6}-1}$ can be written in the form $a+b\sqrt{6}$ where a and b are integers	Show that $\frac{8+5\sqrt{12}}{\sqrt{3}+1}$ can be written in the form $a+b\sqrt{3}$ where a and b are integers

Worked Example	Your Turn
Show that $\dfrac{3-\sqrt{24}}{\left(1-\sqrt{6}\right)^2}$ can be written in the form $\frac{a+b\sqrt{6}}{c}$ where a, b and c are integers in their simplest form	Show that $\dfrac{\sqrt{150}-3}{\left(1+\sqrt{6}\right)^2}$ can be written in the form $\frac{a+b\sqrt{6}}{c}$ where a, b and c are integers in their simplest form

Worked Example	Your Turn
Solve $(2-\sqrt{2})x=6$ giving your answer in the form $a+b\sqrt{2}$	Solve $(\sqrt{5}+2)x=2$ giving your answer in the form $a+b\sqrt{5}$

Worked Example	Your Turn
Solve $3x - \sqrt{45} = \sqrt{6}x$ giving your answer in the form $a\sqrt{5} + b\sqrt{30}$	Solve $\sqrt{6}x - \sqrt{20} = 2x$ giving your answer in the form $a\sqrt{30} + b\sqrt{5}$

Worked Example	Your Turn
A rectangle has an area of $\left(2+\sqrt{2}\right)\ cm^2$ and a width of $(3\sqrt{2}-4)\ cm$. Find the length of the rectangle in the form $a+b\sqrt{2}$	A rectangle has an area of $\left(15-6\sqrt{3}\right)\ cm^2$ and a width of $(2\sqrt{3}-3)\ cm$. Find the length of the rectangle in the form $a+b\sqrt{3}$

Worked Example	Your Turn
Rationalise: $\frac{4}{\frac{1}{\sqrt{3}}+\sqrt{3}}$	Rationalise: $\frac{3}{\sqrt{2}+\frac{1}{\sqrt{2}}}$

Worked Example	Your Turn
Find in its simplest form $a : b$, given: $a = \sqrt{5} + \sqrt{c}$ $b = \sqrt{80} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 16$	Find in its simplest form $a : b$, given: $a = \sqrt{7} + \sqrt{c}$ $b = \sqrt{63} + \sqrt{d}$ c and d are positive integers $c : d = 1 : 9$

Extra Notes

3 Algebraic Fractions

Simplifying Algebraic Fractions

Worked Example	Your Turn
Simplify $\frac{6xy^2}{10x^2y}$	Simplify $\frac{8x^2y}{12xy^3}$

Fill in the Gaps

Question	Write Each Term Separately	Simplify Each Variable	Answer
$\frac{10a^2}{2a}$	$\frac{10 \times a^2}{2 \times a}$	$5 \times a$	$5a$
$\frac{9b}{3b}$	$\frac{9 \times b}{3 \times b}$		
$\frac{2a}{4b}$	$\frac{2 \times a}{4 \times b}$		
$\frac{12a^3}{3a}$			
$\frac{5ab}{a}$			
$\frac{15a^2}{5ab}$			
$\frac{24ab}{4bc}$	$\frac{24 \times a \times b}{4 \times b \times c}$		
$\frac{6ab^2}{3a^2b}$			

Question	Write Each Term Separately	Simplify Each Variable	Answer
$\frac{4a^7}{2a^3}$	$\frac{4 \times a^7}{2 \times a^3}$	$2 \times a^4$	
$\frac{16b^5}{2b^2}$			
$\frac{2a^9}{10a^2}$			
$\frac{5a^6}{2a}$			
$\frac{25ab^7}{5b^2}$	$\frac{25 \times a \times b^7}{5 \times b^2}$		
$\frac{14a^4b^8}{2a^3b^6}$			
$\frac{9ab^5}{3a^2b^3}$			
$\frac{15a^4b}{25a^2b^2}$			

Worked Example	Your Turn
Simplify a) $\frac{8(2z-1)}{7z(2z-1)}$ b) $\frac{(4z-7)(8z-3)}{(7z-1)(4z-7)}$	Simplify a) $\frac{7x(5x-1)}{9(5x-1)}$ b) $\frac{(6x-4)(3x-7)}{(3x-7)(6x-9)}$

Dr Frost 387a

Worked Example	Your Turn
Simplify a) $\frac{x+3}{2x^2+6x}$ b) $\frac{5x+25}{4x^2+20x}$	Simplify a) $\frac{x+4}{4x^3+16x^2}$ b) $\frac{6x^2+9x}{6x^4+9x^3}$

Dr Frost 387b and 387c

Worked Example	Your Turn
Simplify a) $\frac{x+5}{x^2+x-20}$ b) $\frac{x^2-x-2}{x^2-4}$	Simplify a) $\frac{x+3}{x^2-x-12}$ b) $\frac{x^2-9}{x^2+6x+9}$

Dr Frost 387d and 387e

Worked Example	Your Turn
Simplify $\frac{2x^2 + 14 + 24}{3x^2 - 15x - 108}$	Simplify $\frac{2x^2 + 14x + 24}{3x^2 + 15x + 18}$

Multiplying and Dividing Algebraic Fractions

Worked Example	Your Turn
Simplify: a) $\frac{6x}{2y^2} \times \frac{4y}{5x}$ b) $\frac{6x}{2y^2} \div \frac{4y}{5x}$	Simplify: a) $\frac{5a^2}{2b} \times \frac{5b}{30a^3}$ b) $\frac{5a^2}{2b} \div \frac{5b}{30a^3}$

Dr Frost 388a and 388b

Question	Write as a Single Fraction	Simplify Numerator and Denominator	Simplified Answer (where possible)
$\frac{x}{4} \times \frac{2x}{3}$	$\frac{x \times 2x}{4 \times 3}$	$\frac{2x^2}{12}$	$\frac{x^2}{6}$
$\frac{x}{6} \times \frac{4y}{5}$	$\frac{x \times 4y}{6 \times 5}$	$\frac{4xy}{30}$	
$\frac{2}{x} \times \frac{3xy}{5}$	$\frac{2 \times 3xy}{x \times 5}$		
$\frac{4x}{3y} \times \frac{2y}{x}$			
$\frac{2}{7x} \times \frac{3xy}{4}$			
$\frac{x^2}{8} \times \frac{4y}{x}$			
$\frac{2y}{x} \times \frac{9x^2y}{4}$			
$\frac{10y}{x^2} \times \frac{3xy^2}{5}$			
$\frac{4yz}{3} \times \frac{x^2}{6y^3}$			
$\frac{2x^3}{15yz} \times \frac{5x^2y^2}{z^3}$			
$\frac{\square}{5y^2} \times \frac{4x^2y}{\square}$	$\frac{\square \times 4x^2y}{5y^2 \times \square}$	$\frac{24x^3y}{15y^2}$	
$\frac{12x}{5yz} \times \frac{\square}{\square}$			$\frac{6x^3y}{25z}$

Question	Write as a Multiplication	Simplify Numerator and Denominator	Simplified Answer (where possible)
$\frac{x}{4} \div \frac{x}{2y}$	$\frac{x}{4} \times \frac{2y}{x}$	$\frac{2xy}{4x}$	$\frac{y}{2}$
$\frac{3x}{y} \div \frac{1}{xy}$	$\frac{3x}{y} \times \frac{xy}{1}$	$\frac{3x^2y}{y}$	
$\frac{2y}{3} \div \frac{4}{y}$	$\frac{2y}{3} \times \frac{y}{4}$		
$\frac{xy}{5} \div \frac{3y}{10}$			
$\frac{4y}{3x} \div \frac{xy}{6}$			
$\frac{x^2}{8} \div \frac{3x}{4y}$			
$\frac{6xy}{5} \div \frac{x}{y}$			
$\frac{5x}{3} \div \frac{10}{xy}$			
$\frac{2x}{y} \div \frac{4xy}{9}$			
$\frac{4}{5xy} \div \frac{2y^2}{x^2}$			
$\frac{\square}{y} \div \frac{\square}{3y^2}$	$\frac{\square}{y} \times \frac{3y^2}{\square}$	$\frac{6xy^2}{8y}$	
$\frac{5x}{2y} \div \frac{\square}{\square}$			$10y$

Worked Example	Your Turn
a) Express $\frac{2x-4}{6x+12} \times \frac{3x+12}{10x-20}$ as a single fraction in its simplest form.	a) Express $\frac{5x-5}{4x+16} \times \frac{8x+32}{15x+45}$ as a single fraction in its simplest form.
b) Express $\frac{20x-80}{9x+36} \div \frac{10x-30}{3x+12}$ as a single fraction in its simplest form.	b) Express $\frac{20x+60}{9x+36} \div \frac{10x+30}{3x+6}$ as a single fraction in its simplest form.

Dr Frost 388e and 388f

Worked Example	Your Turn
Simplify fully: $\frac{2x^2 + 7x - 15}{x^2 - 36} \times \frac{2x + 12}{2x^3 - 3x^2}$	Simplify fully: $\frac{2x^2 - 17x + 21}{x^2 - 49} \times \frac{5x^2 + 15x}{2x^2 - 3x}$

Worked Example	Your Turn
Simplify fully: $\frac{3x^2+8x+5}{x^2-25} \div \frac{3x^2+5x}{5x^2-25x}$	Simplify fully: $\frac{3x^2-x-14}{9x^2-4} \div \frac{x+2}{3x^2+2x}$

Adding and Subtracting Algebraic Fractions

Fill in the Gaps

Question	With a Common Denominator	Unsimplified Answer	Simplified Answer (where possible)
$\frac{x}{4}+\frac{7x}{20}$	$\frac{5x}{20}+\frac{7x}{20}$	$\frac{12x}{20}$	
$\frac{7x}{18}-\frac{2x}{9}$	$\frac{7x}{18}-\frac{4x}{18}$		
$\frac{2x}{3}+\frac{x}{4}$	$\frac{\square}{12}+\frac{\square}{12}$		
$\frac{17x}{30}+\frac{x}{10}$	$\frac{17x}{\square}+\frac{3x}{\square}$		
$\frac{x}{6}+\frac{11x}{24}$			
$\frac{3x}{4}-\frac{7x}{36}$			
$\frac{7}{2x}+\frac{3}{x}$	$\frac{\square}{2x}+\frac{\square}{2x}$		
$\frac{6}{5x}-\frac{9}{20x}$			
$\frac{5}{x}+\frac{2}{x^2}$	$\frac{\square}{x^2}+\frac{2}{x^2}$		
	$\frac{\square}{xy}-\frac{\square}{xy}$	$\frac{3-y}{xy}$	
	$\frac{\square}{4x^2}+\frac{\square}{4x^2}$	$\frac{7x+6}{4x^2}$	
$\frac{3}{10xy}-\frac{2}{x^2}$			

Worked Example	Your Turn
Simplify a) $\frac{5x+2}{3} + \frac{4x-3}{2}$ b) $\frac{5x+2}{3} - \frac{4x-3}{2}$	Simplify a) $\frac{4x-5}{2} + \frac{6x-1}{3}$ b) $\frac{4x-5}{2} - \frac{6x-1}{3}$

Dr Frost 389a

Worked Example	Your Turn
Simplify $\frac{7}{4x} + \frac{8}{5x}$	Simplify $\frac{4}{3x} - \frac{5}{8x}$

Worked Example	Your Turn
Simplify a) $\frac{3}{5x-3}+\frac{8}{4x+1}$ b) $\frac{3}{5x-3}-\frac{8}{4x+1}$	Simplify a) $\frac{5}{3x+4}+\frac{2}{7x-4}$ b) $\frac{5}{3x+4}-\frac{2}{7x-4}$

Dr Frost 389f and 389g

Worked Example	Your Turn
Express $\frac{5}{8x+12} - \frac{1}{6x+9}$ as a single fraction in the form $\frac{A}{B(Cx+D)}$ where A, B, C, and D are integers to be found.	Express $\frac{x}{30x+5} - \frac{3}{12x+2}$ as a single fraction in the form $\frac{Ax+B}{C(Dx+E)}$ where A, B, C, D, and E are integers to be found.

Worked Example	Your Turn
Simplify $6 - \frac{2x^2 - 13x + 20}{(2x - 5)(3x + 2)}$ giving your answer in the form $\frac{ax+b}{cx+d}$	Simplify $1 - \frac{x - 4}{(2x + 5)(3x + 5)}$ giving your answer in the form $\frac{ax^2+bx+c}{(dx+e)(fx+g)}$

Worked Example	Your Turn
Simplify $\frac{x}{x+1} - \frac{4x+3}{(2x+3)(x+1)}$ giving your answer in the form $\frac{ax+b}{cx+d}$	Simplify $\frac{3x}{3x+2} - \frac{5x-4}{(3x+2)(2x+5)}$ giving your answer in the form $\frac{a(x+b)}{cx+d}$

Worked Example	Your Turn
Simplify $\frac{10}{x^2 - 14x + 24} - \frac{1}{x - 12}$ Leave the denominator in factorised form when applicable.	Simplify $\frac{7}{y^2 + 6y - 40} - \frac{6}{y - 4}$ Leave the denominator in factorised form when applicable.

Worked Example	Your Turn
Simplify $\frac{2}{a^2-36}-\frac{3}{a^2+7a+6}$ Leave the denominator in factorised form when applicable.	Simplify $\frac{2}{y^2-9}-\frac{5}{y^2+7y+12}$ Leave the denominator in factorised form when applicable.

Worked Example	Your Turn
Write as a single simplified fraction: $3-(x-4)\div\frac{x^2-16}{x-5}$	Write as a single simplified fraction: $5-(x-2)\div\frac{x^2-4}{x+3}$

Worked Example

Express

$$x - 1 - \frac{5x^2 - 16x + 3}{2x + 5} \div \frac{5x^2 - 21x + 4}{x^2 - 9x + 20}$$

as a single fraction in the form $\frac{ax^2+bx+c}{dx+e}$

Your Turn

Express

$$x - 3 - \frac{3x^2 + 7x}{3x^2 + x - 14} \div \frac{2x + 5}{3x^2 - 8x + 4}$$

as a single fraction in the form $\frac{ax^2+bx+c}{dx+e}$

Solving Equations with Algebraic Fractions

Worked Example	Your Turn
Solve $\frac{x+4}{2}+\frac{x+1}{5}=5$	Solve $\frac{x-4}{2}+\frac{x-1}{5}=2$

Worked Example	Your Turn
Solve $\frac{x+1}{3} - \frac{x-3}{5} = 1$	Solve $\frac{x+2}{3} - \frac{x-6}{5} = 2$

Worked Example	Your Turn
Solve $\frac{3m+6}{2} - \frac{m-4}{4} = 3m-6$	Solve $\frac{4x-2}{2} - \frac{4x-1}{3} = x+4$

Worked Example	Your Turn
Solve $\frac{3m-5}{5} = \frac{3m-2}{2} - 2m + 3$	Solve $\frac{3x+4}{2} = \frac{4x-1}{2} - 3x + 1$

Worked Example	Your Turn
Solve $\frac{4}{x+6}+\frac{5}{x+8}=1$	Solve $\frac{4}{x+3}+\frac{5}{x+4}=2$

Worked Example	Your Turn
Solve $\frac{3}{x-6}+\frac{4}{x-9}=1$	Solve $\frac{3}{x-2}+\frac{4}{x-3}=3$

Worked Example	Your Turn
Solve the following equation: $\frac{4}{3x^2 + x - 14} - \frac{3}{x - 2} = 2$ Give your answer in exact form.	Solve the following equation: $\frac{4}{x - 2} - \frac{3}{2x^2 - x - 6} = 2$ Give your answer in exact form.

Rearranging Formulae with Algebraic Fractions

Worked Example	Your Turn
Make x the subject: $\frac{y}{a} + \frac{3y}{x-2} = 5$	Make x the subject: $\frac{5p}{x+3} + \frac{p}{b} = 2$

Worked Example	Your Turn
Make x the subject: $\frac{1}{x} - \frac{1}{y} = \frac{1}{z}$	Make p the subject: $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$

Worked Example	Your Turn
Make x the subject: $\frac{5x}{A} - \frac{b}{c} = \frac{4x - d}{a}$	Make x the subject: $\frac{3x}{E} - \frac{f}{g} = \frac{5x - h}{F}$

Extra Notes