Year 10

2024   Mathematics   2025

Unit 18 Booklet – Part 1

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Year 10

2024   Mathematics   2025

Unit 18 Booklet – Part 2

HGS Maths

Tasks

Dr Frost Course

Name: _____

Class: _____

Contents Page

1 Advanced Data Handling

Quartiles

Worked Example	Your Turn
Find the lower quartile, upper quartile and interquartile range for:	Find the lower quartile, upper quartile and interquartile range for
1 2 2 3 3 4 5 5	1 2 8 8 9 9 9 10
6 6 8 8 12 13 16 16	10 10 10 12 13 14 14 14
16 17 18 18 18 19 20	15 16 17 17 18 18 18 18
	18 18 20

Mode of Ungrouped Data

	Worked Example		Your Turn

Worked Example

Determine the modal score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the modal score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 237b

Range of Ungrouped Data

	Worked Example		Your Turn

Worked Example

Determine the range of the scores:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn

Determine the range of the scores:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Median of Ungrouped Data

Fluency Practice

|

| | Number of pieces of data: | Position of the median: |

(a)   4, 10, 11, 12, 12, 15, 20

(b)   4, 10, 11, 12, 12, 15

(c)   10, 11, 12, 12, 15

(d)   10, 11, 12, 12

(e)   1, 3, 6, 8, 9, 12

Number of pieces of data:	Position of the median:
7	
11	
10	
41	
24	
	8
	3.5
	40
	21.5

Page 23

Fluency Practice

Number of pets	Frequency	Which pieces of data are in this category?
0	3	1st 2nd 3rd
1	2	4th 5th
2	4	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st 2nd
1		3rd
2		4th 5th 6th 7th 8th
3		9th 10th
4		11th 12th 13th

Number of pets	Frequency	Which pieces of data are in this category?
0	8	
1	9	
2	13	
3	12	
4	9	

Number of pets	Frequency	Which pieces of data are in this category?
0	5	
1	1	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0		1st
1		2nd 3rd 4th
2		5th 6th 7th 8th
3		9th 10th
4		11th 12th

Number of pets	Frequency	Which pieces of data are in this category?
0		1st to 13th
1		14th to 29th
2		30th to 59th
3		60th to 80th
4		81st to 92nd

Number of pets	Frequency	Which pieces of data are in this category?
0	2	
1	1	
2	5	

Number of pets	Frequency	Which pieces of data are in this category?
0	21	1st to 21st
1	15	22nd to…
2	18	
3	25	
4	32	

Number of pets	Frequency	Which pieces of data are in this category?
0	1	
1	3	
2	3	

Number of pets	Frequency	Which pieces of data are in this category?
0	10	
1	12	
2	15	
3	20	
4	5	

	Worked Example			Your Turn	

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	7

Your Turn

Calculate the median score:

Score	Frequency
0	9
1	6
2	2
3	4
4	4
5	8

	Worked Example	Your Turn

Worked Example:

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Your Turn:

Calculate the median score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

	Worked Example	Your Turn

Worked Example

Calculate the median score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	2

Your Turn

Calculate the median score:

Score	Frequency
0	8
1	6
2	2
3	4
4	4
5	8

Quartiles of Ungrouped Data

	Worked Example		Your Turn

Worked Example

Calculate the lower quartile and upper quartile score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	9

Your Turn

Calculate the lower quartile and upper quartile score:

Score	Frequency
0	7
1	6
2	2
3	4
4	4
5	8

Dr Frost 396g

Mean of Ungrouped Data

Worked Example	Your Turn
Calculate the mean score:	Calculate the mean score:

Calculate the mean score:

Score	Frequency
0	2
1	3
2	1
3	2
4	2
5	4

Calculate the mean score:

Score	Frequency
0	4
1	6
2	2
3	4
4	4
5	8

Dr Frost 236a

Mean from a Frequency Table

Data Set			
7	7	7	7
7			8
8	8	8	8
8			
9	9	9	9

Value	Frequency	Value × Frequency
7	6	42
8	9	72
9	5	45
Totals	20	159

Mean   159   ÷   20   =   7.95

Data Set			
4	4	4	4

Value	Frequency	Value × Frequency
4	13	
5	2	
6	5	
Totals		

Mean   ÷   =

Data Set			
2	2	2	2
2	3	3	3
3	3	3	3
4	4	4	4
4	4	4	4

Value	Frequency	Value × Frequency
2		
3		
4		
Totals	20	

Mean   ÷   20   =

Data Set			
12	12	12	12
12	12	12	12
12	12	12	12
12	14	14	14
14	14	14	14

Value	Frequency	Value × Frequency
12		
13		
14		
Totals		

Mean   ÷   =

Fill in the Gaps

Top Left

Data Set			
4	5	5	5
5	6	6	6
6	6	6	7
7	7	7	7
7	7	7	7

Frequency Table

Value	Frequency	Value × Frequency
4		
5		
6		
7		
Totals		

Mean ÷ =

Top Right

Data Set			

Frequency Table

Value	Frequency	Value × Frequency
2.5	8	
2.6	3	
2.7		
2.8	5	
Totals	20	

Mean ÷ =

Bottom Left

Data Set			

Frequency Table

Value	Frequency	Value × Frequency
8	5	
9	8	
10		
11	6	
Totals	24	

Mean ÷ =

Bottom Right

Data Set			

Frequency Table

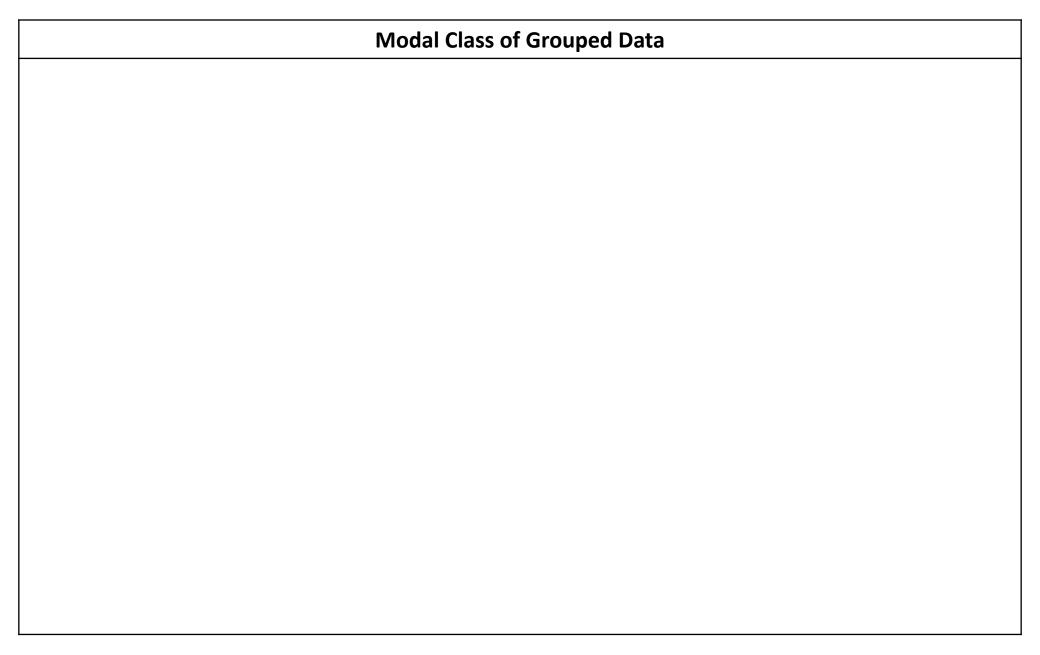| Value	Frequency	Value × Frequency
3	2	
4		
5		
6	12	
Totals	24	

Mean ÷ = 5.25

Worked Example

The table gives information about the numbers of badges gained by the girls in a Guide group.
a) Write down the mode.
b) Find the range.
c) Work out the median
d) Calculate the mean.

Number of badges	Frequency
0	2
1	8
2	4
3	3
4	5
5	3

Fill in the Gaps

Class			Total Frequency (class size)	Most common age (modal age)	Oldest student	Youngest student	Range of ages	Total of all their ages	Mean age	Median age
A	**Age** / 5 / 6	**Frequency** / 3 / 7								
B	**Age** / 7 / 8 / 9	**Frequency** / 3 / / 9	20							
C	**Age** / 10 / 11 / 12 / 13	**Frequency** / / 14 / /	30	13		10	3			
D	**Age** / / / /	**Frequency** / / / /		10 and 11	12		2	108	10.8	

Modal Class of Grouped Data

	Worked Example		Your Turn

Worked Example

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the modal class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Range of Grouped Data

	Worked Example		Your Turn

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Determine the upper and lower bounds for the range:

Mass, x (kg)	Frequency
$10 < x \leq 20$	5
$20 < x \leq 30$	3
$30 < x \leq 50$	2
$50 < x \leq 56$	6
$56 < x \leq 60$	7

Median Class of Grouped Data

	Worked Example	Your Turn

Worked Example

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	5
$10 < x \leq 20$	3
$20 < x \leq 40$	2
$40 < x \leq 46$	6
$46 < x \leq 50$	7

Your Turn

Determine the median class interval:

Mass, x (kg)	Frequency
$0 < x \leq 10$	15
$10 < x \leq 20$	6
$20 < x \leq 40$	4
$40 < x \leq 46$	12
$46 < x \leq 50$	8

Median of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \leq 40$	4
$40 < x \leq 45$	9
$45 < x \leq 50$	26
$50 < x \leq 55$	13
$55 < x \leq 60$	8
$60 < x \leq 65$	20

Use interpolation to estimate the median.
Give your answer correct to 1 decimal place.

Quartiles of Grouped Data

Worked Example

Jack collects the heights of 100 flowers and records the data in the table below.

Height (y cm)	Frequency
$40 < y \leq 50$	7
$50 < y \leq 60$	14
$60 < y \leq 70$	59
$70 < y \leq 80$	11
$80 < y \leq 90$	9

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

Your Turn

James collects the heights of 80 flowers and records the data in the table below.

Height (x cm)	Frequency
$35 < x \le 40$	4
$40 < x \le 45$	9
$45 < x \le 50$	26
$50 < x \le 55$	13
$55 < x \le 60$	8
$60 < x \le 65$	20

Use interpolation to estimate the lower quartile and upper quartile.
Give your answer correct to 1 decimal place.

	Worked Example		Your Turn

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	6
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Calculate an estimate for the mean:

Mass, x (kg)	Frequency
$0 < x \leq 8$	3
$8 < x \leq 16$	0
$16 < x \leq 24$	7
$24 < x \leq 32$	4

Dr Frost 312c

Value	Frequency	Midpoint	Midpoint × Freq
$10 \leq x < 12$	6	11	66
$12 \leq x < 14$	9	13	117
$14 \leq x < 16$	5	15	75
Totals	20		258
Estimate of Mean	258 ÷ 20 =		

Value	Frequency	Midpoint	Midpoint × Freq
$20 \leq x < 30$	9	25	225
$30 \leq x < 40$	7	35	
$40 \leq x < 50$	4	45	
Totals	20		
Estimate of Mean	÷ 20 =		

Value	Frequency	Midpoint	Midpoint × Freq
$20 \leq x < 24$	6	22	
$24 \leq x < 28$	10		
$28 \leq x < 32$	5		
$32 \leq x < 36$	4		
Totals	25		
Estimate of Mean	÷ =		

Value	Frequency	Midpoint	Midpoint × Freq
$5 \leq x < 10$	7		
$10 \leq x < 15$	7		
$15 \leq x < 20$	8		
$20 \leq x < 25$	3		
Totals	25		
Estimate of Mean	÷ =		

Fill in the Gaps

Value	Frequency	Midpoint	Midpoint × Freq
100 ≤ x < 120	7		
120 ≤ x < 140	12		
140 ≤ x < 160			
160 ≤ x < 180	2		
Totals	30		
Estimate of Mean	÷		=

Value	Frequency	Midpoint	Midpoint × Freq
0 ≤ x < 2			3
2 ≤ x < 4			9
4 ≤ x < 6			40
6 ≤ x < 8			42
Totals	20		
Estimate of Mean	÷ 20		=

Value	Frequency	Midpoint	Midpoint × Freq
0 ≤ x <			40
≤ x <			240
≤ x <			275
≤ x < 40			
Totals			
Estimate of Mean	÷ 40 = 18.25		

Value	Frequency	Midpoint	Midpoint × Freq
≤ x <			
≤ x <			
≤ x <		55	1430
≤ x <		65	780
Totals			
Estimate of Mean	÷ 50 = 54.2		

In each table, values are grouped into classes of equal width.

Worked Example

Bob asked each of 40 friends how many minutes they took to get to work. The table shows some information about his results.
a) Write down the modal class.
b) Work out the upper and lower bounds for the range.
c) Work out the class in which the median lies.
d) Calculate an estimate for the median.
e) Calculate an estimate for the mean.

Time taken (m minutes)	Frequency
$0 < m \leq 10$	3
$10 < m \leq 20$	8
$20 < m \leq 30$	11
$30 < m \leq 40$	9
$40 < m \leq 50$	9

Extra Notes

Worked Example	Your Turn
Expand and simplify: $(x + 2)(x - 3)(x - 4)$	Expand and simplify: $(x + 4)(x - 3)(x - 2)$

Dr Frost 300c

Worked Example	**Your Turn**
Expand and simplify: $2x(7x - 3)(x - 4)$	Expand and simplify: $4x(7x - 3)(x - 2)$

Worked Example	Your Turn
Expand and simplify: $(5x + 2)(7x - 3)(x - 4)$	Expand and simplify: $(5x + 4)(7x - 3)(x - 2)$

Worked Example	Your Turn
Expand and simplify: $(3x - 2)^3$	Expand and simplify: $(4x - 3)^3$

Fill in the Gaps

Expanded Expression	Factorised Expression
$2x + 8$	$2(x + 4)$
	$3(x - 2)$
	$x(x + 7)$
$5x + 35$	
$8x - 12$	
	$2x(x - 5)$
$x^2 - x$	
	$5x(3 - x)$
$10x^2 + 2x$	
$6x + 9y$	
	$4xy(x + 2)$
$6xy - 4y^2$	
	$(x + 2)(x + 3)$
	$(x + 5)(x - 3)$
$x^2 + 8x + 15$	
$x^2 + 3x + 2$	

Expanded Expression	Factorised Expression
$x^2 - 7x + 10$	
	$(x - 6)(x + 4)$
	$(x + 7)(x - 7)$
$x^2 + 2x - 15$	
$x^2 - 25$	
	$(2x + 1)(x + 5)$
$x^2 - x - 6$	
$x^2 + 3x$	
	$(3x - 1)(x - 2)$
$4x^2 - 25$	
	$(x + 5)^2$
$7x^2 + 10x + 3$	
	$(3x - 1)^2$
$4x^2 + 4x + 1$	
$5x^2 - 14x - 3$	
	$(x - 2)^3$

Worked Example	Your Turn
Expand and simplify: $(3x - 2)^3 - (6x - 5)^2$	Expand and simplify: $(4x - 3)^3 - (5x - 6)^2$

Extra Notes

3 Binomial Expansion (L2FM Only)

Worked Example	Your Turn
Use Pascal's triangle to find the binomial expansion of $(5x - 2)^4$ Give each term in its simplest form.	Use Pascal's triangle to find the binomial expansion of $(2x - 5)^4$ Give each term in its simplest form.

Worked Example	Your Turn
Use Pascal's triangle to find the first three terms, in descending powers of x, of the binomial expansion of $(2x - 3)^6$ Give each term in its simplest form.	Use Pascal's triangle to find the first three terms, in descending powers of x, of the binomial expansion of $(3x - 2)^5$ Give each term in its simplest form.

Dr Frost 501c

Fluency Practice

Binomial Expansion

(a) Expand and simplify $(x+2)^3$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^3	2^0	x^3
3	x^2	2^1	$6x^2$
3	x^1	2^2	
1	x^0	2^3	

$$= x^3 + 6x^2 +$$

(b) Expand and simplify $(x-5)^3$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^3	$(-5)^0$	x^3
3	x^2	$(-5)^1$	$-15x^2$

$$= x^3 - 15x^2 +$$

(c) Expand and simplify $(x+y)^4$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	x^4	y^0	
4	x^3	y^1	
6	x^2	y^2	

(d) Expand and simplify $(2x+1)^4$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	$(2x)^4$	1^0	$16x^4$

(e) Expand and simplify $(3x-2)^5$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	$(3x)^5$	$(-2)^0$	
5	$(3x)^4$	$(-2)^1$	

(f) Expand and simplify $(4-y)^5$

Pascal's Triangle	Powers of 1st term	Powers of 2nd term	Simplified
1	4^5	$(-y)^0$	

Worked Example	Your Turn
Find the first three terms, in ascending powers of x, of the binomial expansion of $(3x - 2)^8$. Give each term in its simplest form.	Find the first three terms, in ascending powers of x, of the binomial expansion of $(2x - 3)^7$. Give each term in its simplest form.

Worked Example	Your Turn
Find the first three terms, in ascending powers of x, of the binomial expansion of $\left(3x - \frac{2}{3}\right)^7$ Give each term in its simplest form.	Find the first three terms, in ascending powers of y, of the binomial expansion of $\left(3y + \frac{3}{4}\right)^5$ Give each term in its simplest form.

Worked Example	Your Turn
Find the first three terms, in ascending powers of x, of the binomial expansion of $(2 - gx)^7$. Give each term in its simplest form.	Find the first three terms, in ascending powers of y, of the binomial expansion of $(2 + fy)^9$. Give each term in its simplest form.

Worked Example	Your Turn
Find the expansion of $(2x - 3y^2)^5$	Find the expansion of $(3x^2 - 2y)^4$

Fluency Practice

More Binomial Expansion

(a) Expand and simplify $(x - 2)^6$

$1 \times x^6 \times (-2)^0$
$+6 \times x^5 \times (-2)^1$

(b) Expand and simplify $\left(3x + \frac{1}{2}\right)^4$

$1 \times (3x)^4 \times \left(\frac{1}{2}\right)^0$

(c) Expand and simplify $(x^2 + 5)^3$

(d) Expand and simplify $\left(\frac{x}{3} - 1\right)^5$

(e) Expand and simplify $\left(\frac{3}{2} - 5y\right)^4$

(f) Expand and simplify $\left(2x - \sqrt{3}\right)^3$

Worked Example	Your Turn
Find the coefficient of x^3 in the expansion of $(2 - 3x)^4$	Find the coefficient of x^4 in the expansion of $(3 - 2x)^5$

Worked Example	Your Turn
Find the coefficient of x^{20} in $(1 - 3x^5)^7$	Find the coefficient of x^{20} in $(1 - 2x^4)^7$

Worked Example	Your Turn
The coefficient of x^3 in the expansion of $(3 - cx)^5$ is 720. Find the possible value(s) of the constant c.	The coefficient of x^3 in the expansion of $(2 - cx)^5$ is -1080. Find the possible value(s) of the constant c.

Worked Example	Your Turn
In the expansion of $(p + qx)^5$, where p is a positive constant and q is a negative constant, the numerical term is 3125 and the coefficient of x^2 is 31250. Hence find the values of p and q	In the expansion of $(py + q)^6$, where p and q are positive constants, the coefficient of y^2 is 34560 and the coefficient of y^6 is 729. Hence find the values of p and q.

Extra Notes

4 Solving Quadratics

Multiplication by Zero

$(a - 3) \times 2 = 0$ $a =$ _____

$(a - 7) \times a = 0$ $a =$ _____

$a \times a = 0$ $a =$ _____

$(a + 5)(a - 3) = 0$ $a =$ _____

$a^2 + 6a + 8 = 0$ $a =$ _____

$a^2 + 8a + 16 = 0$ $a =$ _____

Worked Example	Your Turn
What values of x satisfy the equation $x(x - 9) = 0$?	What values of x satisfy the equation $(x + 6)x = 0$?

Worked Example	Your Turn
What values of x satisfy the equation $(x - 9)(x + 5) = 0$?	What values of x satisfy the equation $(x + 6)(x - 5) = 0$?

Worked Example	**Your Turn**
Solve the equation $(2x - 3)(3x + 1) = 0$	Solve the equation $(3x + 2)(2x - 1) = 0$

Worked Example	**Your Turn**
Solve the equation $x^2 + 2x - 8 = 0$	Solve the equation $x^2 + 2x - 15 = 0$

Dr Frost 367b

Worked Example	**Your Turn**
Solve the equation $x^2 - 49 = 0$	Solve the equation $x^2 - 64 = 0$

Worked Example	Your Turn
Solve the equation $3x^2 + 2x = 0$	Solve the equation $2x^2 - 3x = 0$

Worked Example	Your Turn
Solve the equation $x^2 - 4x + 4 = 0$	Solve the equation $x^2 + 14x + 49 = 0$

Fill in the Gaps

#	Quadratic	Factorised	Equations	Solutions
1	$x^2 + 6x + 5 = 0$	$(x+5)(x+1) = 0$	$x+5=0$ $x+1=0$	
2	$x^2 + 7x + 10 = 0$	$(x+5)(x+2) = 0$	$x+5=0$ $x+\ =0$	$x=-5$ $x=$
3	$x^2 + 8x + 15 = 0$	$(x+5)(x+3) = 0$		
4	$x^2 + 7x + 12 = 0$	$(x+4)(x+\ ) = 0$		
5	$x^2 + 8x + 16 = 0$	$(x+4)(x+\ ) = 0$		
6	$x^2 + 10x + 24 = 0$			
7	$x^2 + 14x + 24 = 0$			
8	$x^2 + 14x + 40 = 0$			
9		$(x+\ )(x+\ ) = 0$	$x+8=0$ $x+5=0$	
10				$x=-40$ $x=-1$
11	$x^2 + 4x - 5 = 0$	$(x+5)(x-1) = 0$	$x+5=0$ $x-1=0$	
12	$x^2 - 4x - 5 = 0$	$(x-5)(x+1) = 0$	$x-5=0$ $x+1=0$	
13	$x^2 + 4x - 12 = 0$	$(x+6)(x-2) = 0$	$x+6=0$ $x-2=0$	
14	$x^2 + x - 12 = 0$	$(x+4)(x-3) = 0$		
15	$x^2 - 11x - 12 = 0$	$(x-12)(x\ \ ) = 0$		
16	$x^2 + 2x - 8 = 0$	$(x+4)(x\ \ ) = 0$		
17	$x^2 - 2x - 8 = 0$			
18	$x^2 + 7x - 8 = 0$			
19	$x^2 - 7x - 8 = 0$			
20		$(x-5)(x-1) = 0$		
21				$x=5$ $x=2$
22				$x=5$ $x=5$
23				$x=-5$ $x=5$

Fill in the Blanks

Solving Quadratics by Factorising

Filling the Gaps

Quadratic Equation	Factorised Equation	1st Equation	2nd Equation	1st Solution	2nd Solution
$x^2 + 8x + 15 = 0$	$(x + 5)(x + 3) = 0$	$x + 5 = 0$	$x + 3 = 0$	$x = -5$	$x = -3$
$x^2 - 8x - 20 = 0$					
$x^2 - x - 20 = 0$					
$x^2 - x - 6 = 0$					
$x^2 - 5x + 6 = 0$					
$x^2 - 5x + 4 = 0$					
$x^2 - 4x + 4 = 0$					
	$(x - 7)(x + 3) = 0$				
		$x - 8 = 0$	$x + 5 = 0$		
		$x + 6 = 0$			$x = -7$
				$x = 3$	$x = -5$

Page 135

Worked Example	Your Turn
Solve the equation $5x^2 + 13x - 6 = 0$	Solve the equation $5x^2 + 7x - 6 = 0$

Worked Example	Your Turn
Solve the equation $4x^2 - 9 = 0$	Solve the equation $16x^2 - 81 = 0$

Worked Example	Your Turn
Solve the equation $x^2 - x = 12$	Solve the equation $x^2 = 2x + 3$

Worked Example	Your Turn
Solve the equation $12x^2 + 10x - 12 = 0$	Solve the equation $18x^2 - 15x - 18 = 0$

Worked Example	Your Turn
Solve the equation $x(x - 2) = 15$	Solve the equation $(x - 3)(x + 2) = 6$

Worked Example	**Your Turn**
Solve $5x - \dfrac{3}{x} = 14$	Solve $3x + \dfrac{14}{x} = 13$

Worked Example	Your Turn
Solve the following equation: $$x + 2 = -\frac{4}{3x - 7}$$	Solve the following equation: $$-\frac{7}{3x + 4} = x - 6$$

Worked Example	Your Turn
The width of a rectangular field is 3 m shorter than its length. The area of the field is 126 m². Show that $x^2 + bx + c = 0$ where x is the length of the field and b and c are constants to be found.	The length of a rectangular swimming pool is 6 m longer than its width. The area of the swimming pool is 144 m². Show that $y^2 + by + c = 0$ where y is the width of the swimming pool and b and c are constants to be found.

The area of the trapezium is 219 mm².
All measurements are in mm.

$3y + 3$

$3y + 5$

$4 - y$

Show that $6y^2 + by + c = 0$, where b and c are integers to be determined.

The area of the trapezium is 216 mm².
All measurements are in mm.

$4 - x$

$2 - x$

$3x + 5$

Show that $2x^2 + bx + c = 0$, where b and c are integers to be determined.

Worked Example	Your Turn
Solve: a) $x^2 - 28 = 53$ b) $5\sqrt{x} = 20$	Solve: a) $\frac{\sqrt{x}}{3} = 4$ b) $24 + x^3 = 88$

Dr Frost 200g

Quadratic Formula

Worked Example	Your Turn
Write down the values of a, b and c in: a) $5x^2 + 2x - 3 = 0$ b) $x^2 + 2x - 3 = 0$ c) $x^2 + 2x = 4x - 3$	Write down the values of a, b and c in: a) $5x^2 - 2x + 3 = 0$ b) $x^2 - 2x + 3 = 0$ c) $x^2 - 2x = -4x + 3$

Intelligent Practice

Questions	a	b	c
$3x^2 + 5x + 1 = 0$			
$0 = 3x^2 + 5x + 1$			
$0 = 3x^2 + 5x + 2$			
$3x^2 + 4x + 2 = 0$			
$0 = 3x^2 + 4x - 2$			
$3x^2 - 4x + 2 = 0$			
$x^2 - 4x + 2 = 0$			
$x^2 + 2 - 4x = 0$			
$1 + 2x - 4x^2 = 0$			
$1 + 2x = 4x^2$			

Intelligent Practice

Questions	a	b	c
$2x = 4x^2 + 1$			
$1 = 4x^2 + 2$			
$4x^2 + 2x = 0$			
$4x^2 + 2 = 0$			
$2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 0$			
$-2(2x^2 + 1) = 2x$			
$-2(2x^2 + 1) = 2x + 2$			
$-2(2x^2 + 1) = x^2 + 2x + 2$			
$-2(2x^2 + x + 1) = x^2 + 2x + 2$			

Discriminant

Worked Example	Your Turn
Calculate the discriminant of $5x^2 + 6x - 7$	Calculate the discriminant of $-6x^2 + 7x + 8$

Dr Frost 492a

Worked Example	Your Turn
By calculating the discriminant, work out the number of real solutions to the equation $4x^2 - 2x - 9 = 0$	By calculating the discriminant, work out the number of real solutions to the equation $-4x^2 - 7x - 4 = 0$

Dr Frost 492b

Worked Example	Your Turn
Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 9x - 2 = 0$	Solve the following quadratic equation, leaving your answer in exact form: $x^2 - 10x + 3 = 0$

Dr Frost 417a

Worked Example	Your Turn
Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 + 7x - 2 = 0$	Solve the following quadratic equation, giving your answer accurate to 2 decimal places: $3x^2 - 9x + 2 = 0$

Worked Example	Your Turn
Solve the equation $4x^2 = 7x + 1$ giving your answers correct to two decimal places.	Solve the equation $7x^2 = 4x + 1$ giving your answers correct to two decimal places.

Quadratic Equation	a, b and c	$b^2 - 4ac$	$x = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$	$x = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$	Solutions to 3sf
$x^2 + 5x + 1 = 0$	$a = 1, b = 5, c = 1$	$5^2 - 4 \times 1 \times 1 = 21$	$x = \dfrac{-5 + \sqrt{21}}{2}$	$x = \dfrac{-5 - \sqrt{21}}{2}$	
$2x^2 + 5x + 1 = 0$	$a = 2, b = 5, c = 1$	$5^2 - 4 \times 2 \times 1 = 17$			
$2x^2 - 5x + 1 = 0$	$a = 2, b = -5, c = 1$	$(-5)^2 - 4 \times 2 \times 1 = 17$	$x = \dfrac{5 + \sqrt{17}}{4}$		
$x^2 - 7x + 3 = 0$					
$2x^2 - 7x + 3 = 0$					
$5x^2 + x - 2 = 0$					
	$a = 3, b = 5, c = 2$				
			$x = \dfrac{-9 + \sqrt{89}}{4}$	$x = \dfrac{-9 - \sqrt{89}}{4}$	

Fill in the Gaps

Using the Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1) Identify a, b & c and substitute into the quadratic formula.
2) **Remember!** The square root of the discriminant can be positive or negative –
    You must complete the formula for both!
3) Give your answers to 2 decimal places

$x^2 + 5x + 3 = 0$	$a = 1$ $b = 5$ $c = 3$	$x = \dfrac{-(\ )\pm\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ )\pm\sqrt{(\quad)}}{(\quad)}$	$x = \dfrac{-(\ )-(\ )}{(\ )}$ $x = \dfrac{-(\ )+(\ )}{(\ )}$	$x = -0.70$ or $x =$
$x^2 + 6x - 2 = 0$	$a = 1$ $b =$ $c =$	$x = \dfrac{-(\ )\pm\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ )\pm\sqrt{(\quad)}}{(\quad)}$	$x = \dfrac{-(\ )-(\ )}{(\ )}$ $x = \dfrac{-(\ )+(\ )}{(\ )}$	$x = 0.32$ or $x =$
$2x^2 + 8x + 3 = 0$	$a =$ $b =$ $c =$	$x = \dfrac{-(\ )\pm\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$	$x = \dfrac{-(\ )\pm\sqrt{(\quad)}}{(\quad)}$		
$4x^2 + 3x - 6 = 0$		$x = \dfrac{-(\ )\pm\sqrt{(\ )^2 - 4(\ )(\ )}}{2(\ )}$			
$5x^2 - 4x - 8 = 0$					

2) Use the quadratic formula to solve these equations. Give your answers to 2 decimal places.

a) $2x^2 + 9x = -3$

b) $4x^2 - 3 = 7x$

c) $5x = -8 + 5x^2$

1 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$x^2 + 5x + 1 = 0$	1	5	1	$x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(1)}}{2(1)}$	$x = \dfrac{-5 \pm \sqrt{25-4}}{2}$	$x = \dfrac{-5 \pm \sqrt{21}}{2}$	x = -4.79 & x = -0.209
1	$x^2 + 6x + 1 = 0$							
2	$x^2 + 6x + 2 = 0$							
3	$x^2 + 7x + 2 = 0$							
4	$x^2 + 7x - 2 = 0$							
5	$x^2 - 7x - 2 = 0$							
6	$-x^2 - 7x - 2 = 0$							
7	$-x^2 - 7x + 2 = 0$							
8	$2 - 7x - x^2 = 0$							

Fill in the Gaps

#	Equation						
9		1	7	3			
10		1	7	4			
11		1	-7	4			
12					$x=\dfrac{-(-7)\pm\sqrt{(-7)^2-4(-1)(4)}}{2(-1)}$		
13						$x=\dfrac{-(-8)\pm\sqrt{64+16}}{2(-1)}$	
14		-1					$x=\dfrac{-8\pm\sqrt{84}}{-2}$
15	$\dfrac{1}{2}x^2 + 4x + 5 = 0$	$\dfrac{1}{2}$					
16	$\dfrac{1}{2}(x^2 + 4x + 5) = 0$						
17	$\dfrac{1}{3}(x^2 + 4x + 5) = 0$						
18	$\dfrac{2x^2 + 4x + 5}{3} = 0$						

2 Complete the table below.

Q	Equation	a	b	c	Substitution	Simplification	Exact Answer	Answers to 3sf
Ex	$2x^2 + 5x + 1 = 0$	2	5	1	$x = \dfrac{-5 \pm \sqrt{5^2 - 4(2)(1)}}{2(2)}$	$x = \dfrac{-5 \pm \sqrt{25-8}}{4}$	$x = \dfrac{-5 \pm \sqrt{17}}{4}$	**x = -2.28** **&** **x = -0.219**
1	$2x^2 + 6x + 1 = 0$							
2	$2x^2 + 6x + 2 = 0$							
3	$2x^2 + 6x - 2 = 0$							
4	$2x^2 - 6x + 2 = 0$							
5	$2x^2 - 6x - 2 = 0$							
6	$3x^2 - 6x - 2 = 0$							
7	$4x^2 - 6x - 2 = 0$							
8	$4x^2 - 6x + 2 = 0$							

Fill in the Gaps

9	$2 - 6x + 4x^2 = 0$							
10	$2 - 6x - 4x^2 = 0$							
11	$1 - 3x - 2x^2 = 0$							
12	$\dfrac{1}{2} - \dfrac{3}{2}x - x^2 = 0$							
13	$\dfrac{1}{6} - \dfrac{1}{2}x - \dfrac{1}{3}x^2 = 0$							
14	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = 0$							
15	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = 1$							
16	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = x$							
17	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = x^2$							
18	$\dfrac{1}{3}x^2 + \dfrac{1}{2}x - \dfrac{1}{6} = -x^2$							

Worked Example	Your Turn
Solve the following equation: $$\frac{x+5}{x+6} = -\frac{6}{5x-2}$$ Give your answer in exact form.	Solve the following equation: $$-\frac{3x}{3x+5} = \frac{x-3}{x+2}$$ Give your answer in exact form.

Worked Example	Your Turn
A right-angled triangle has sides $(x + 2)$ cm, $(2x - 1)$ cm and $(2x + 3)$ cm.	A right-angled triangle has sides $(2y - 4)$ cm, $(2y - 1)$ cm and $(y + 6)$ cm.

Worked Example:

A right-angled triangle has sides $(x + 2)$ cm, $(2x - 1)$ cm and $(2x + 3)$ cm.

$(x + 2)$ cm

$(2x + 3)$ cm

$(2x - 1)$ cm

Find the value of x.
Give your answer correct to 2 decimal places.

Your Turn:

A right-angled triangle has sides $(2y - 4)$ cm, $(2y - 1)$ cm and $(y + 6)$ cm.

$(2y - 1)$ cm

$(2y - 4)$ cm

$(y + 6)$ cm

Find the value of y.
Give your answer correct to 2 decimal places.

Worked Example	**Your Turn**
The surface area of the cylinder is 1158 cm².	The surface area of the cylinder is 1457 cm².

Worked Example side:

13.9 cm

x

Find the value of x.

Give your answer correct to 1 decimal place.

Your Turn side:

19.2 cm

x

Find the value of x.

Give your answer correct to 1 decimal place.

Worked Example	Your Turn
A semi-cylinder has surface area 663 and a length of 17 cm, as shown on the diagram below.	A semi-cylinder has surface area 2088 and a length of 29.8 cm, as shown on the diagram below.

17 cm

x cm

29.8 cm

x cm

Work out the diameter of the semi-cylinder.
Give your answer correct to 1 decimal place.

Work out the diameter of the semi-cylinder.
Give your answer correct to 1 decimal place.

Extra Notes

5 Factor Theorem (L2FM Only)

Algebraic Division

Worked Example	Your Turn
Given that $2x^3 + 5x^2 - 11x + 4 = (2x - 1)(Ax^2 + Bx + C)$ Use algebraic long division to work out the values of the constants A, B, and C.	Given that $2x^3 + 3x^2 - 29x + 30 = (2x - 3)(Ax^2 + Bx + C)$ Use algebraic long division to work out the values of the constants A, B, and C.

Dr Frost 497a

Worked Example	**Your Turn**
Given that $3x^4 + 11x^3 - 14x^2 - 32x + 32 = (3x - 4)(Ax^3 + Bx^2 + Cx + D)$ Work out the values of the constants A, B, C, and D.	Given that $2x^4 + x^3 - 16x^2 + 3x + 18 = (2x - 3)(Ax^3 + Bx^2 + Cx + D)$ Work out the values of the constants A, B, C, and D.

Dr Frost 497c

Worked Example	Your Turn
Given that	Given that
$$f(x) = 4x^3 - 28x^2 + 35x + 25$$	$$f(x) = 4x^3 - 20x^2 + 13x + 12$$
Select which of the following are factors of $f(x)$	Select which of the following are factors of $f(x)$
$\circ$ $(2x + 1)$	$\circ$ $(2x + 1)$
$\circ$ $(2x - 5)$	$\circ$ $(2x - 3)$
$\circ$ both	$\circ$ both
$\circ$ neither	$\circ$ neither

Dr Frost 498a

Worked Example	Your Turn
Given that $(2x + 1)$ is a factor, factorise $f(x) = 18x^3 - 51x^2 + 20x + 25$	Given that $(2x + 3)$ is a factor, factorise $f(x) = 6x^3 + 23x^2 + 9x - 18$

Dr Frost 498c

Worked Example	Your Turn
Factorise $f(x) = 2x^3 + 7x^2 - 10x - 24$	Factorise $f(x) = 4x^3 + 20x^2 + 13x - 12$

Dr Frost 498d

Cubic Expression	Is $(x-1)$ a factor?	Is $(x-2)$ a factor?	Is $(x+1)$ a factor?	Is $(x+3)$ a factor?	Factorised Expression
$x^3 + 2x^2 - 13x + 10$	Yes	Yes	No	No	$(x-1)(x-2)(x+5)$
$x^3 - 5x^2 + 2x + 8$	No	Yes	Yes	No	$(x-2)(x+1)$
$x^3 - 2x^2 - 21x - 18$	No	No	Yes	Yes	$(x+3)(x+1)$
$x^3 + x^2 - 14x + 24$		Yes			$(x-2)$
$x^3 - 10x^2 + 23x - 14$	Yes				$(x-1)$
$x^3 + 8x^2 - x - 8$					
$x^3 - 4x^2 - 11x + 30$					
$x^3 - x^2 - 16x + 16$					
$x^3 + 3x^2 - 18x - 40$					
$x^3 - 8x^2 + 13x - 6$					
$2x^3 + 5x^2 - 23x + 10$					

Worked Example	Your Turn
$f(x) = 4x^3 + bx^2 + 33x + 18$ where b is a constant Given that $(2x + 3)$ is a factor of $f(x)$, find the value of b.	$f(x) = 4x^3 + bx^2 - 15x - 4$ where b is a constant Given that $(2x + 1)$ is a factor of $f(x)$, find the value of b.

Dr Frost 498f

Worked Example	Your Turn
Solve $8x^3 - 14x^2 + 3x = 0$	Solve $3x^3 + 7x^2 - 6x = 0$

Dr Frost 500c

Worked Example	Your Turn
Given that $x = -\dfrac{1}{2}$ is a solution to the equation $2x^3 - 9x^2 + 3x + 4 = 0$ Find all the solutions to the equation.	Given that $x = -\dfrac{3}{4}$ is a solution to the equation $8x^3 - 38x^2 + 27x + 45 = 0$ Find all the solutions to the equation.

Extra Notes