



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

# Year 10

## 2024 Mathematics 2025

### Unit 17 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_



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# Year 10

## 2024 Mathematics 2025

### Unit 17 Booklet – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

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- 4 [Constructions and Loci](#)

# 1 Bounds and Error Intervals

### Worked Example

A number  $z$ , when rounded to the nearest 100, is equal to 6700. Find the upper and lower bound of  $z$ .

### Your Turn

A number  $z$ , when rounded to the nearest 10, is equal to 740. Find the upper and lower bound of  $z$ .

### Worked Example

A number  $x$ , when rounded to 3 decimal places, is equal to 0.007. Find the upper and lower bound of  $x$ .

### Your Turn

A number  $x$ , when rounded to 2 decimal places, is equal to 0.03. Find the upper and lower bound of  $x$ .

### Worked Example

A number  $x$ , when rounded to 3 significant figures, is equal to 612000. Find the upper and lower bound of  $x$ .

### Your Turn

A number  $x$ , when rounded to 2 significant figures, is equal to 35000. Find the upper and lower bound of  $x$ .

### Worked Example

A number  $y$ , when rounded to 1 decimal place, is equal to 8.2.  
Find the error interval for  $y$ .

### Your Turn

A number  $y$ , when rounded to the nearest 10, is equal to 680.  
Find the error interval for  $y$ .



## Fill in the Gaps

$x$	Level of Accuracy	Lower Bound	Upper Bound	Error Interval
30	to the nearest 10	25		$25 \leq x < 35$
700	to the nearest 100		750	
25	to the nearest integer		25.5	
25	to the nearest 5	22.5		
24000	to the nearest 1000			
7.8	to the nearest 0.1			
360	to the nearest 20			
360	to the nearest integer			
6000	to the nearest 100			
200	to the nearest 5			
200	to the nearest 10			
13	to the nearest tenth			
	to the nearest integer		8.5	
		49500	50500	
8				$7 \leq x <$

## Fill in the Gaps

<b><math>x</math></b>	<b>Level of Accuracy</b>	<b>Lower Bound</b>	<b>Upper Bound</b>	<b>Error Interval</b>
6.4	<i>to 1 decimal place</i>	6.35		$6.35 \leq x < 6.45$
7	<i>to the nearest integer</i>		7.5	
7.3	<i>to 1 decimal place</i>	7.25		
5.19	<i>to 2 decimal places</i>			
12.3	<i>to the nearest 0.1</i>			
40	<i>to 1 significant figure</i>		45	
1.5	<i>to 2 significant figures</i>			
0.76	<i>to 2 decimal places</i>			
10	<i>to 1 significant figure</i>			
27	<i>to the nearest integer</i>			
27.9	<i>to 1 decimal place</i>			
654	<i>to 3 significant figures</i>			
	<i>to 1 significant figure</i>	75		
		5.35	5.45	
				$95 \leq x < 150$

## Fill in the Gaps

Value	Rounded to	Lower Bound	Upper Bound	Error Interval	Inequality on a number line
4.2	1 <i>dp</i>	4.15	4.25	$4.15 \leq x < 4.25$	
3.2	1 <i>dp</i>			$\leq x <$	
3.6	1 <i>dp</i>			$\leq x <$	
3.68	2 <i>dp</i>	3.675	3.685	$\leq x <$	
8.63	2 <i>dp</i>			$\leq x <$	
8.43	2 <i>dp</i>				
	2 <i>dp</i>	8.815	8.825		
	2 <i>dp</i>	9.615	9.625		

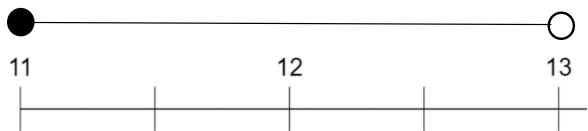
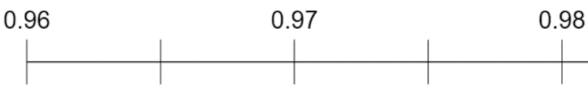

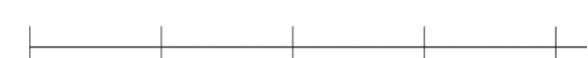

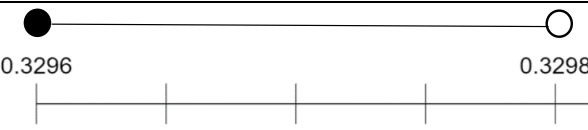


## Fill in the Gaps

				$9.685 \leq x < 9.695$	
9.685	3 dp	9.6845	9.6855		
90.685	3 dp				
58.690	3 dp				
	3 dp				
	3 dp			$\leq x < 812.3275$	
	3 dp			$42.3795 \leq x <$	

## Fill in the Gaps

Value	Rounded to	Lower Bound	Upper Bound	Error Interval	Inequality on a number line
4	$1\ sf$	3.5	4.5	$3.5 \leq x < 4.5$	
40	$1\ sf$	35		$\leq x <$	
30	$1\ sf$			$\leq x <$	
200	$1\ sf$			$\leq x <$	
0.7	$1\ sf$		0.75	$\leq x <$	
0.08	$1\ sf$				
	$1\ sf$			$8.5 \leq x < 9.5$	
	$1\ sf$				

## Fill in the Gaps

12	$2\text{ sf}$	11.5	12.5	$11.5 \leq x < 12.5$	
0.97	$2\text{ sf}$				
760	$2\text{ sf}$				
7.68	$3\text{ sf}$				
9.61	$3\text{ sf}$				
					
	$1\text{ sf}$			$\leq x < 7.5$	
	$2\text{ sf}$			$435 \leq x <$	

## Fill in the Gaps

Number	Rounding	Lower bound	Upper bound	Error interval
4	Nearest integer	3.5	4.5	
40	Nearest ten			$35 \leq x < 45$
40	Nearest integer	39.5	40.5	
50	Nearest integer	49.5		
50	Nearest ten		55	
550		545		
5.5	1 decimal place			
55.5	1 decimal place			
89.6	1 decimal place			
50	1 significant figure			

### **Worked Example**

The number of people on a bus is given as 50, correct to the nearest 10. What is the lowest and highest possible number of people on the bus?

### **Your Turn**

There are 9500 red pandas left in the wild. This number is accurate to the nearest 500. What are the smallest and largest number of red panda that can be left?



### Worked Example

$$p = 5qr$$

$q = 0.709$  correct to 3 significant figures.

$r = 0.071$  correct to 3 decimal places.

Work out the lower bound for the value of  $p$

Give your answer correct to 3 decimal places when appropriate.

### Your Turn

$$a = 5bc$$

$b = 0.124$  correct to 3 decimal places.

$c = 98000$  correct to 2 significant figures.

Work out the lower bound for the value of  $a$

Give your answer correct to 3 decimal places when appropriate.

### Worked Example

$$p = 4q + 2r$$

$q = 907000$  correct to 3 significant figures.

$r = 8.88$  correct to 2 decimal places.

Work out the lower bound for the value of  $p$

Give your answer correct to 3 decimal places when appropriate.

### Your Turn

$$a = 4b + 3c$$

$b = 55.4$  correct to 1 decimal place.

$c = 3.1$  correct to 2 significant figures.

Work out the lower bound for the value of  $a$

Give your answer correct to 3 decimal places when appropriate.

### Worked Example

$$p = 5q - 5r$$

$q = 0.003$  correct to 1 significant figure.

$r = 1.93$  correct to 2 decimal places.

Work out the lower bound for the value of  $p$

Give your answer correct to 3 decimal places when appropriate.

### Your Turn

$$a = 3b - 2c$$

$b = 98.9$  correct to 1 decimal place.

$c = 26.5$  correct to 3 significant figures.

Work out the upper bound for the value of  $a$

Give your answer correct to 5 decimal places when appropriate.

### Worked Example

$$p = \frac{2q}{r}$$

$q = 0.9$  correct to 1 significant figure.

$r = 0.075$  correct to 3 decimal places.

Work out the lower bound for the value of  $p$

Give your answer correct to 3 decimal places when appropriate.

### Your Turn

$$a = \frac{4b}{c}$$

$b = 78.4$  correct to 1 decimal place.

$c = 4150$  correct to 3 significant figures.

Work out the lower bound for the value of  $a$

Give your answer correct to 3 decimal places when appropriate.

### Worked Example

$$p = \frac{q}{r-s}$$

$q = 5$  correct to 1 significant figure.

$r = 0.002$  correct to 1 significant figure.

$s = 0.645$  correct to 3 decimal places.

Work out the lower bound for the value of  $p$

Give your answer correct to 3 decimal places when appropriate.

### Your Turn

$$x = \frac{y}{z-w}$$

$y = 0.786$  correct to 3 decimal places.

$z = 702$  correct to 3 significant figures.

$w = 0.5$  correct to 1 significant figure.

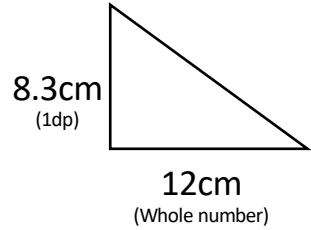
Work out the lower bound for the value of  $x$

Give your answer correct to 5 decimal places when appropriate.

## Worked Example

The height and width of the triangle below have been rounded as shown in brackets.

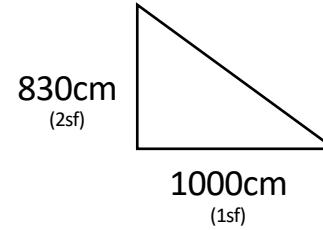
Work out the LB and UB for the **area** of the triangle.



## Your Turn

The height and width of the triangle below have been rounded as shown in brackets.

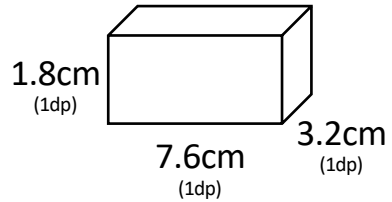
Work out the LB and UB for the **area** of the triangle.



## Worked Example

The dimensions of the cuboid below have been rounded as shown in brackets.

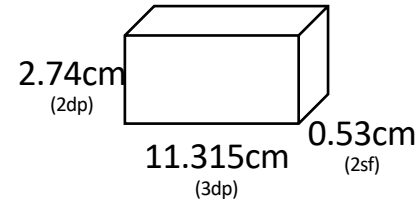
Work out the LB and UB for the **volume** of the cuboid.



## Your Turn

The dimensions of the cuboid below have been rounded as shown in brackets.

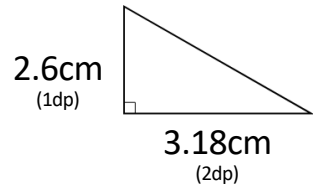
Work out the LB and UB for the **volume** of the cuboid.



## Worked Example

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

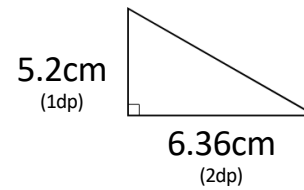
All lengths have been rounded as shown in brackets.



## Your Turn

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

All lengths have been rounded as shown in brackets.

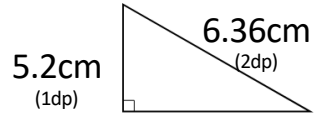




## Worked Example

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

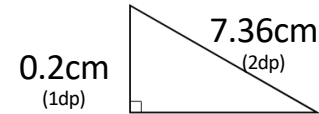
All lengths have been rounded as shown in brackets.



## Your Turn

Use Pythagoras' Theorem to find the LB and UB of the missing sides below.

All lengths have been rounded as shown in brackets.



## Fill in the Gaps

$a$ ,  $b$  and  $c$  are all rounded to the degree of accuracy stated. Find the maximum and minimum values for  $x$ .  
 Values given for  $x_{\max}$  are exact.

$a$	$b$	$c$	Equation	$x_{\max}$	$x_{\min}$
10 (1 sig fig)	12.1 (3 sig fig)	3.4 (2 sig fig)	$\sqrt{ax} = b - c$		
0.5 (1 sig fig)	4.5 (2 sig fig)	-2.0 (2 sig fig)	$\frac{a}{x} = b^2 + 3c$		
5.2 (2 sig fig)	3.4 (2 sig fig)	5 (1 sig fig)	$\frac{ax^2}{b} = c$		
3 (1 sig fig)	4 (1 sig fig)	8 (1 sig fig)	$ax + c = b$		
5 (1 sig fig)	-3 (1 sig fig)	<input type="text"/> (2 sig fig)	$ax = bc$	-3.25	
<input type="text"/> (2 sig fig)	4.3 (2 sig fig)	0.3 (1 sig fig)	$a + x = \frac{b}{c}$	9.35	

## Considering Bounds

### Worked Example

$$a = \frac{\sqrt{b}}{c}$$

$b = 0.24$  correct to 2 decimal places.

$c = 57.2$  correct to 3 significant figures.

By considering bounds, work out the value of  $a$ , giving your answer to a suitable degree of accuracy.

### Your Turn

$$a = \frac{b}{\sqrt{c}}$$

$b = 0.359$  correct to 3 significant figures.

$c = 0.64$  correct 2 decimal places.

By considering bounds, work out the value of  $a$ , giving your answer to a suitable degree of accuracy.

## Truncation

## Worked Example

Truncate 41.53681 to:

- a) 1 decimal place
- b) 2 decimal places
- c) 3 decimal places

## Your Turn

Truncate 11.95291 to:

- a) 1 decimal place
- b) 2 decimal places
- c) 3 decimal places

### Worked Example

A number  $z$ , when truncated to 2 decimal places, is equal to 4.97. Find the upper and lower bound of  $z$ .

### Your Turn

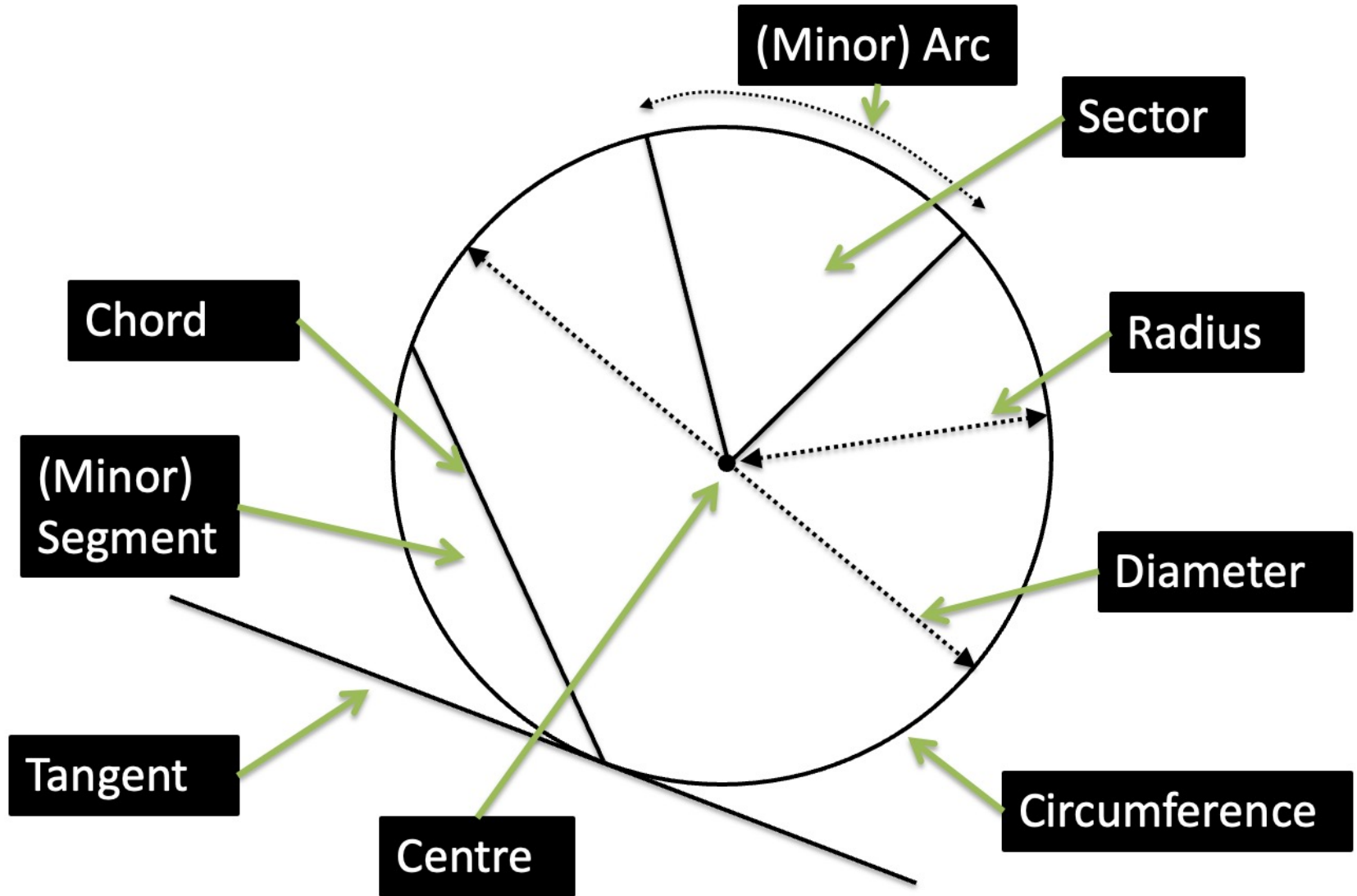
A number  $x$ , when truncated to 3 decimal places, is equal to 0.545. Find the upper and lower bound of  $x$ .

## Extra Notes



## 2 Basic Circle Theorems

## Parts of a Circle



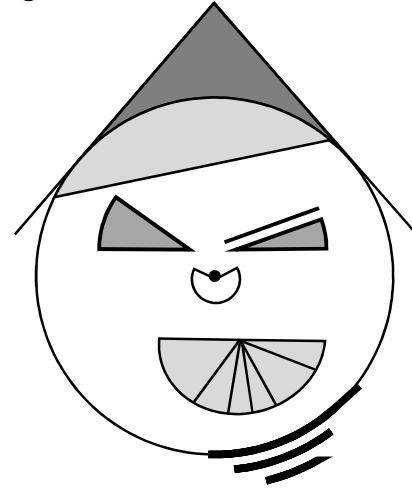
# Fluency Practice

**Circle Vocabulary:** Match each word with its definition.

Arc	Line joining two points on a circumference.
Segment	Perimeter of a circle.
Chord	Part of a circle between a chord and an arc.
Radius	Line touching the circumference of a circle once.
Diameter	Distance from the centre of a circle to the edge.
Circumference	Part of the circumference of a circle.
Tangent	Part of a circle between two radii and an arc.
Sector	Width of a circle.

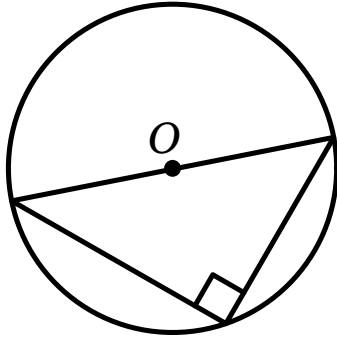
**Circle Vocabulary:** Label the diagram using parts of a circle.

Circumference      Diameter      Chord      Radius      Sector  
Centre      Tangent      Arc      Segment

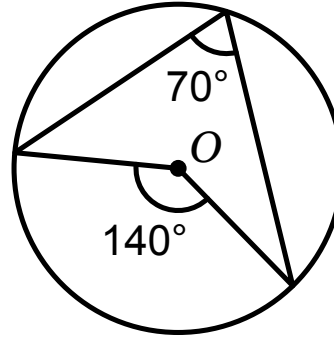


## Circle Theorems 1

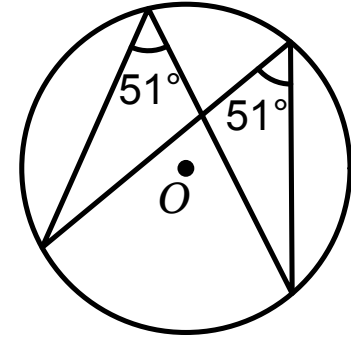
The angle in a semicircle is a right angle.



The angle at the centre is twice the angle at the circumference.



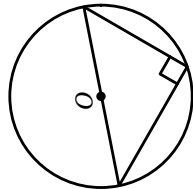
Angles in the same segment are equal.



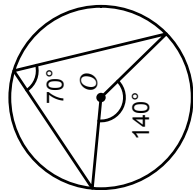
# Fluency Practice

## rules

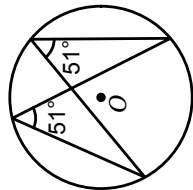
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The angle at the centre is twice the angle at the circumference.

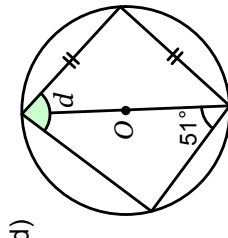
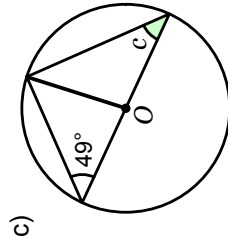
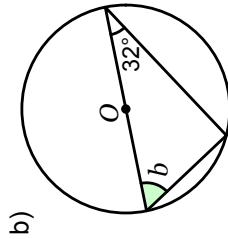
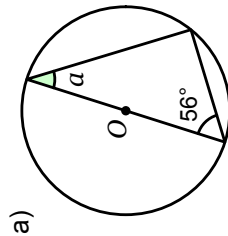


Angles in the same segment are equal.

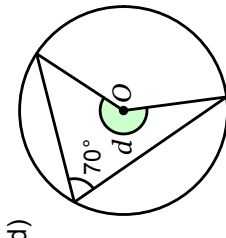
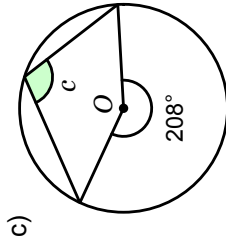
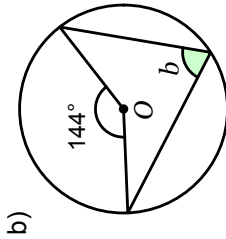
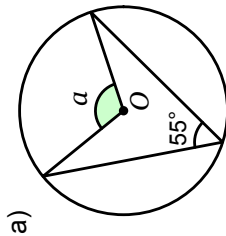


## exercise

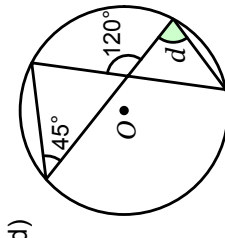
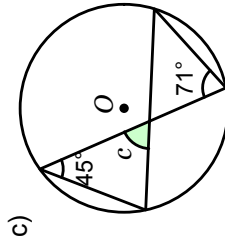
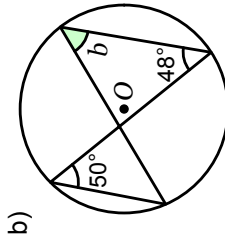
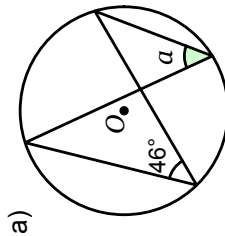
1. Work out the angles marked with letters.



2. Work out the angles marked with letters.

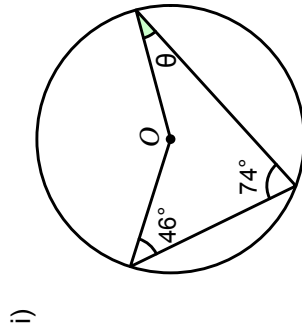
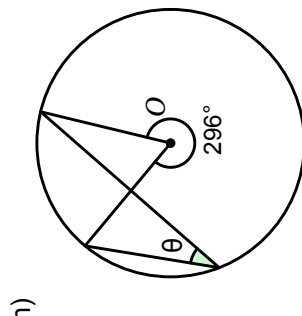
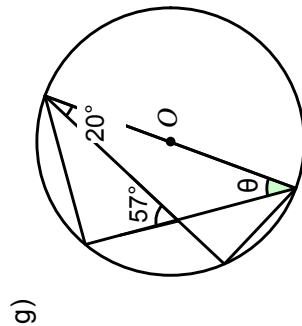
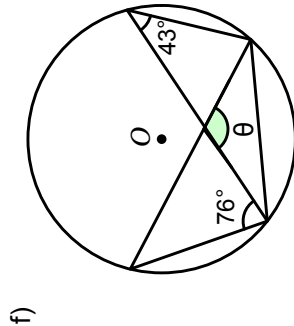
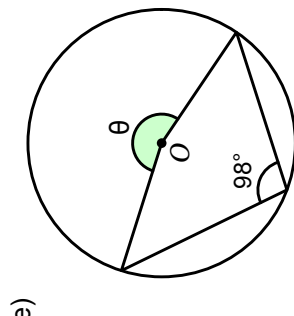
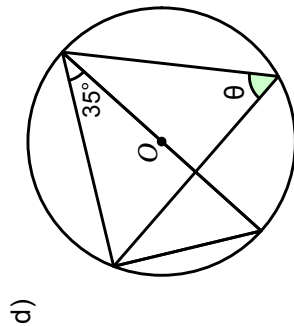
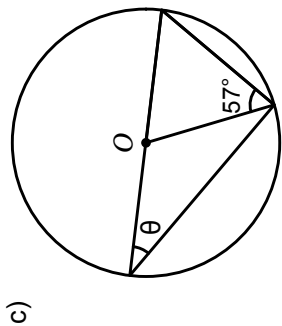
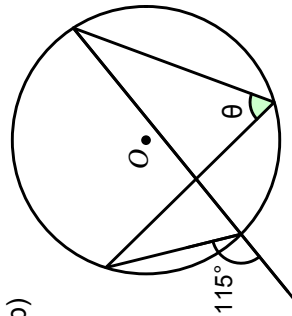
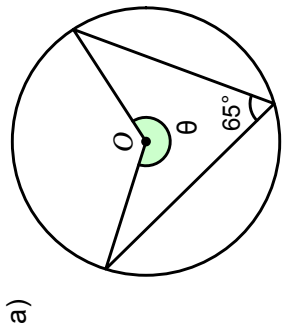


3. Work out the angles marked with letters.

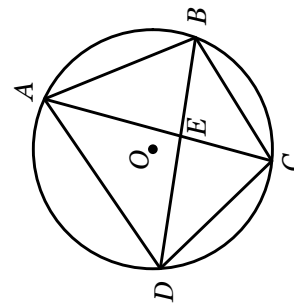


# Fluency Practice

4. Work out the angles marked  $\theta$ .



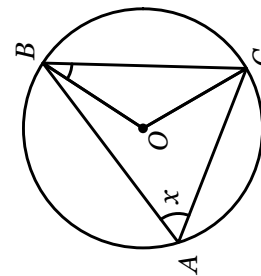
5. Points  $A, B, C$  and  $D$  lie on the circumference of a circle.  $AE$  and  $DE$  are straight lines.



Prove that triangles  $ABE$  and  $DCE$  are similar.

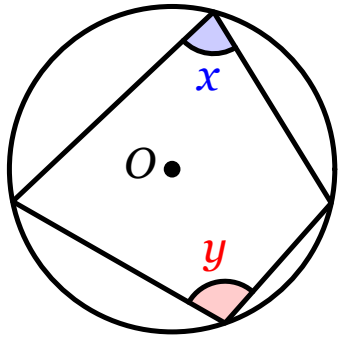
6.  $A, B$  and  $C$  are points on a circle with centre  $O$ .  $\angle BAC = x$

Show that:  $\angle OBC = 90 - x$



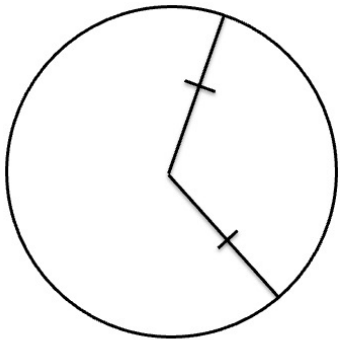
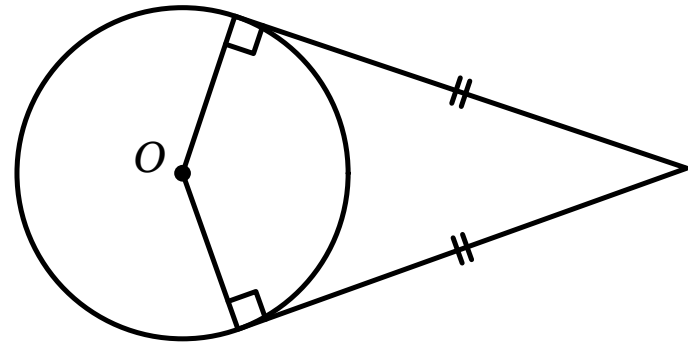
## Circle Theorems 2

Opposite angles of a cyclic quadrilateral sum to  $180^\circ$ .



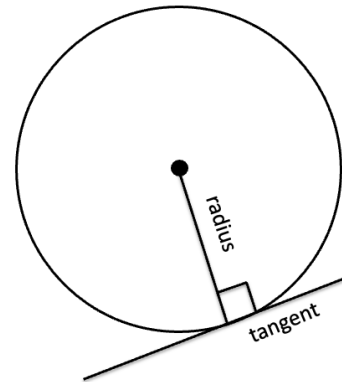
$$x + y = 180^\circ$$

Tangents to a point are equal in length.



**Radius is of constant length**

**Tip:** When you have multiple radii, put a mark on each of them to remind yourself they are the same length.

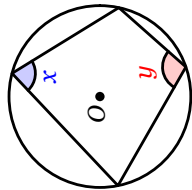


**"Angle between radius and tangent is  $90^\circ$ ".**

# Fluency Practice

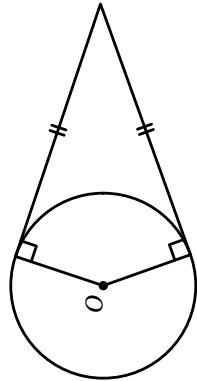
## rules

Opposite angles of a cyclic quadrilateral sum to  $180^\circ$ .



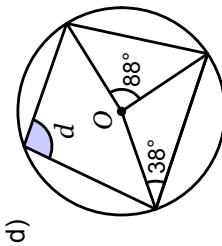
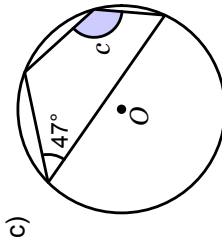
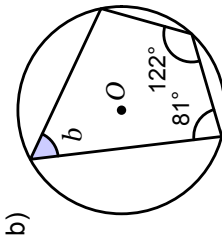
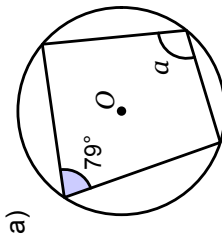
$$x + y = 180^\circ$$

Tangents to a point are equal in length.

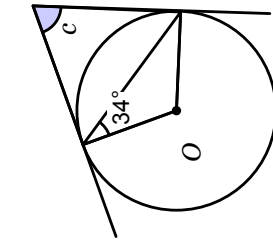
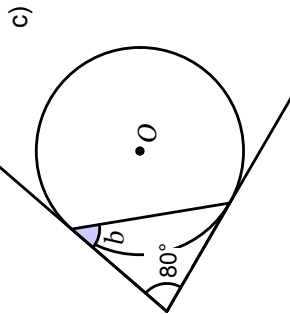
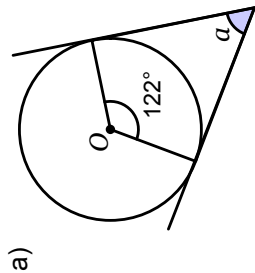


## exercise

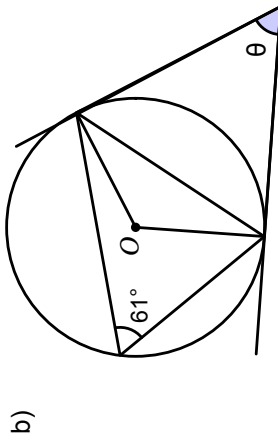
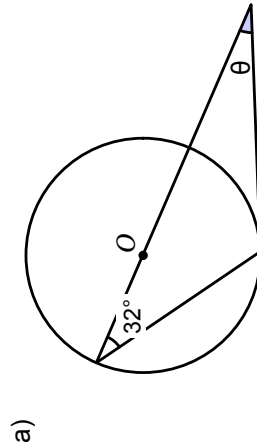
1. Work out the angles marked with letters.



2. Work out the angles marked with letters.



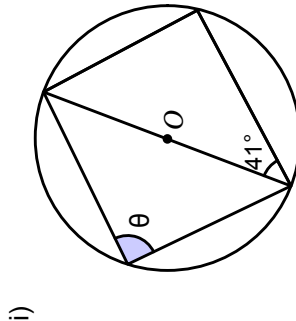
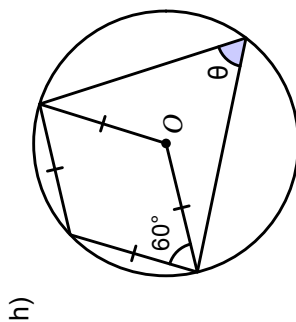
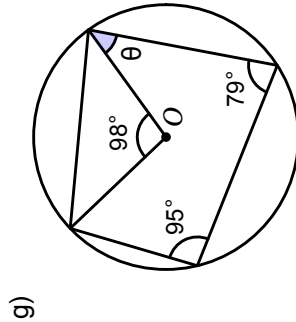
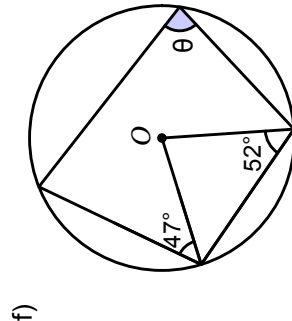
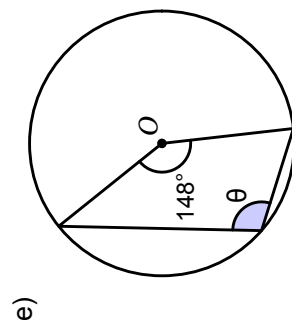
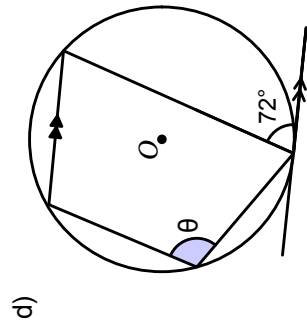
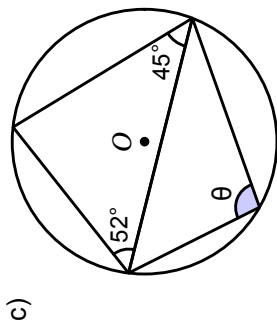
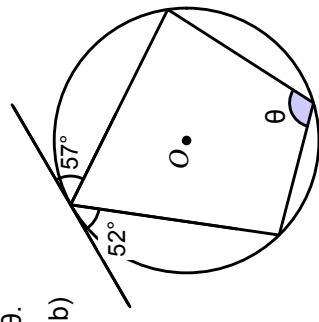
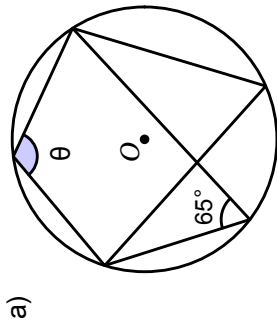
3. Work out the angles marked  $\theta$ .





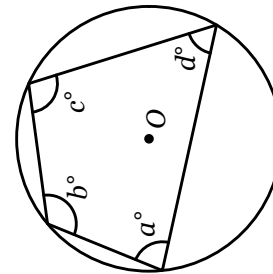
# Fluency Practice

4. Work out the angles marked  $\theta$ .



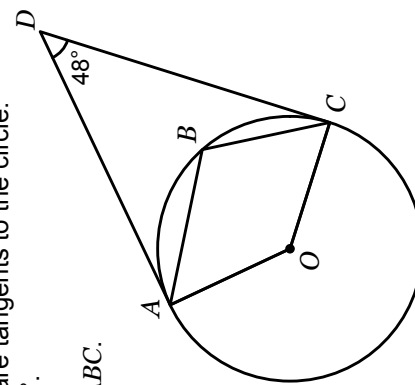
5.  $a : b : c = 4 : 6 : 5$ .

Work out the values of  $a$ ,  $b$ ,  $c$  and  $d$ .



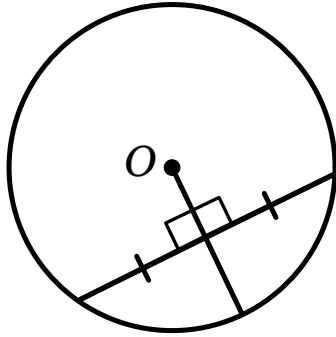
6.  $A$ ,  $B$  and  $C$  are points on the circumference of a circle with centre  $O$ .  
 $AD$  and  $CD$  are tangents to the circle.  
 $\angle ADC = 48^\circ$ .

Work out  $\angle ABC$ .

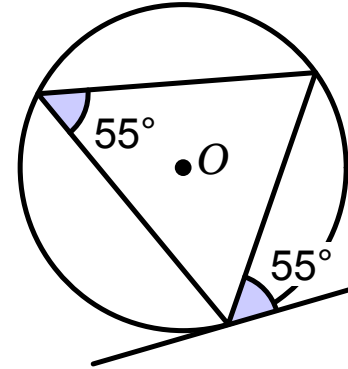


### Circle Theorems 3

The perpendicular from the centre to a chord bisects the chord.



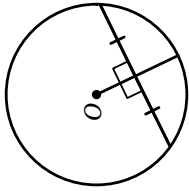
The angle between a chord and a tangent equals the angle in the **alternate segment**.



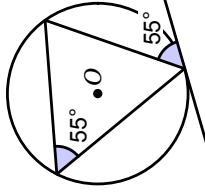
# Fluency Practice

## rules

The perpendicular from the centre to a chord bisects the chord.

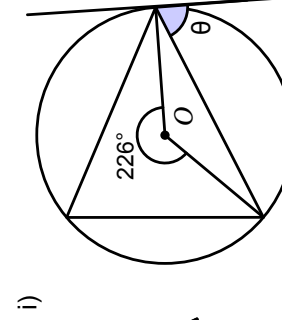
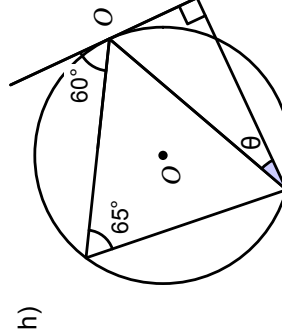
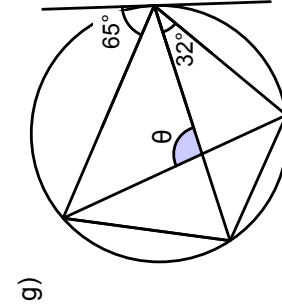
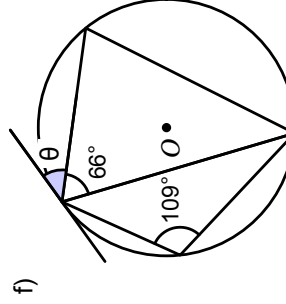
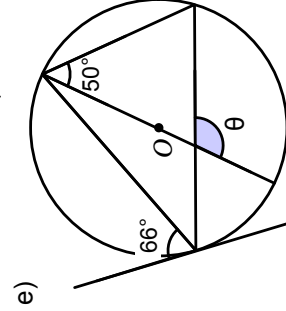
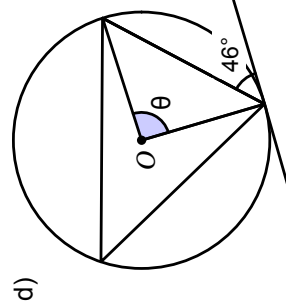
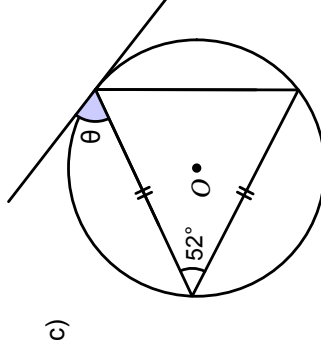
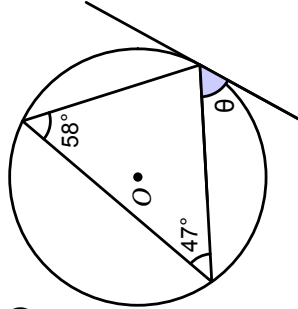
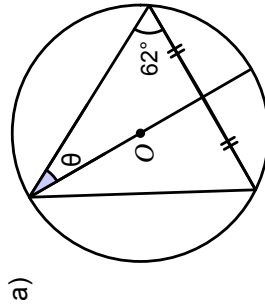


The angle between a chord and a tangent equals the angle in the alternate segment.



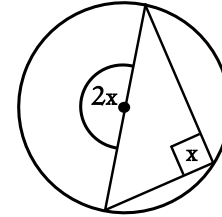
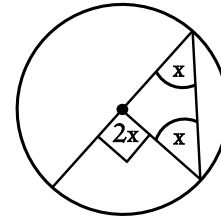
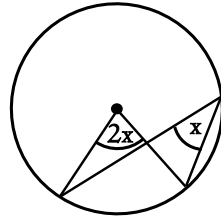
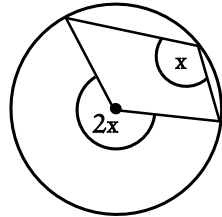
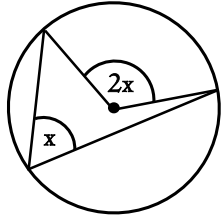
## exercise

1. Work out the angles marked  $\theta$ .



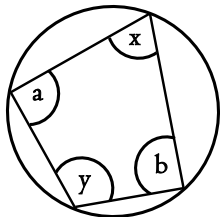
# Circle Theorems Summary

## What you need to remember when answering Circle Theorems questions

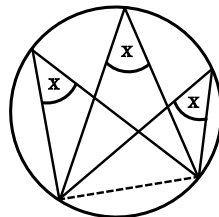


The angle at the centre of a circle is twice the angle at the circumference, so...

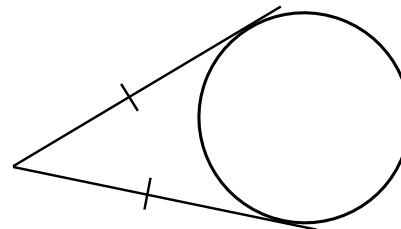
...the angle in a semi-circle is  $90^\circ$



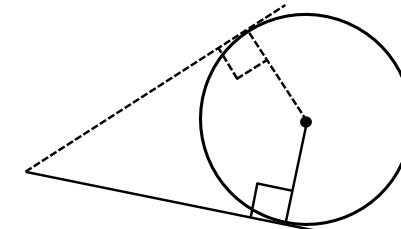
Opposite angles in a cyclic quadrilateral add up to  $180^\circ$



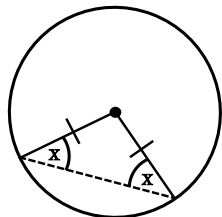
Angles subtended by the same arc [or chord] are equal in size



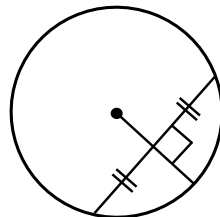
Tangents from the same point are equal in length



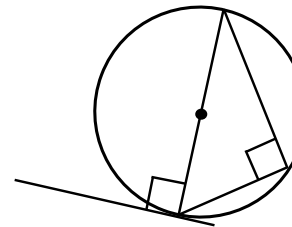
A radius and a tangent form a right angle



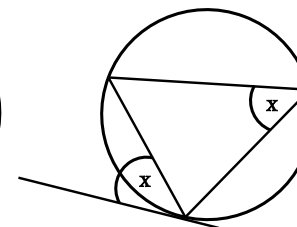
Two radii make an isosceles triangle



The perpendicular bisector of a chord passes through the centre of the circle

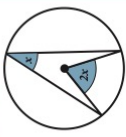


The angle between a tangent and a chord [or diameter] is equal to the angle in the alternate segment

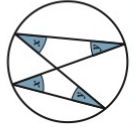


# Geometric Reasoning

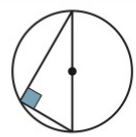
## Circle Theorems



The angle at the centre of a circle is twice the angle at the circumference



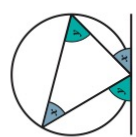
Angles in the same segment are equal



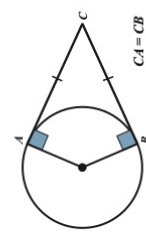
The angle in a semicircle is a right angle (or 90°)



Opposite angles in a cyclic quadrilateral add to 180  
 $a + c = 180$   
 $b + d = 180$

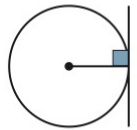


Alternate segment theorem



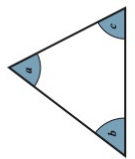
$CA = CB$

Tangents to a circle from an external point are equal in length



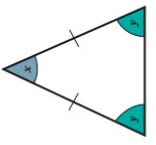
A tangent to a circle is perpendicular (or 90°) to the radius.

## Triangles

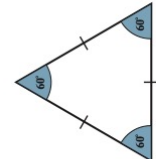


$a + b + c = 180$

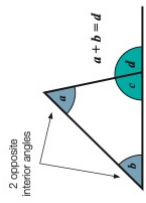
Angles in a triangle add up to 180



Base angles of an isosceles triangle are equal



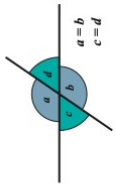
Angles in an equilateral triangle are equal



$a + b = d$

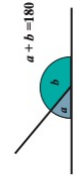
An exterior angle (of a triangle) is equal to the sum of the interior opposite angles

## Lines



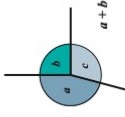
$a = b$   
 $c = d$

Vertically opposite angles are equal



$a + b = 180$

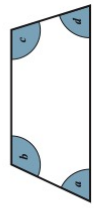
Angles on a straight line add up to 180



$a + b + c = 360$

Angles at a point add up to 360

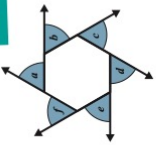
## Quadrilaterals



$a + b + c + d = 360$

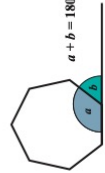
Angles in a quadrilateral add up to 360

## Polygons



$a + b + c + d + e + f = 360$

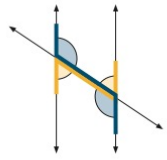
Exterior angles of a polygon add up to 360



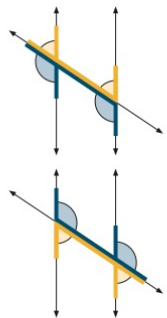
$a + b = 180$

The interior and exterior angle of any polygon add up to 180

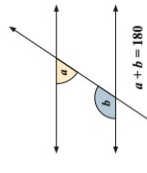
## Parallel lines



Alternate angles are equal



Corresponding angles are equal

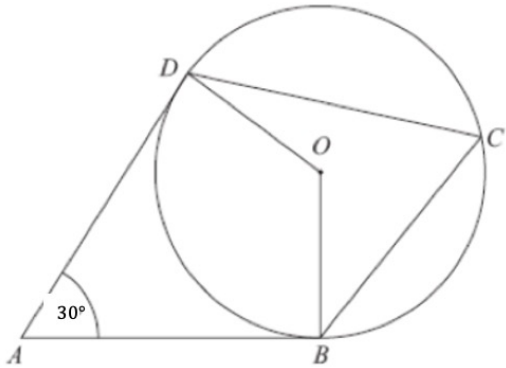


$a + b = 180$

Allied (or co-interior) angles add up to 180

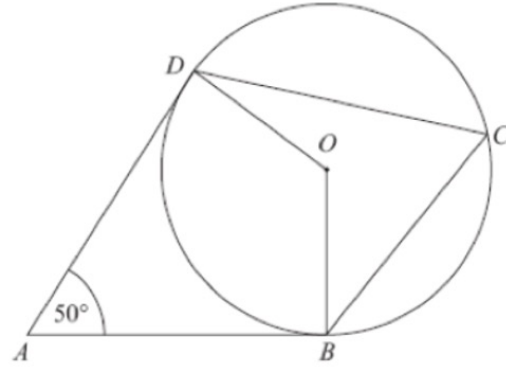
### Worked Example

Calculate the size of angle BCD



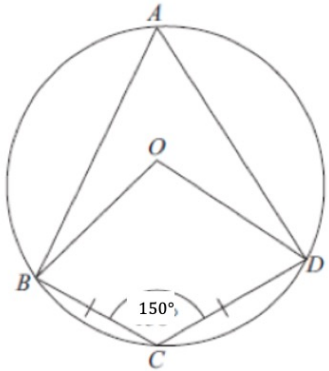
### Your Turn

Calculate the size of angle BCD



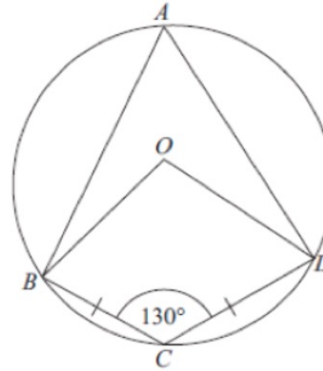
### Worked Example

Calculate the size of angle CDO



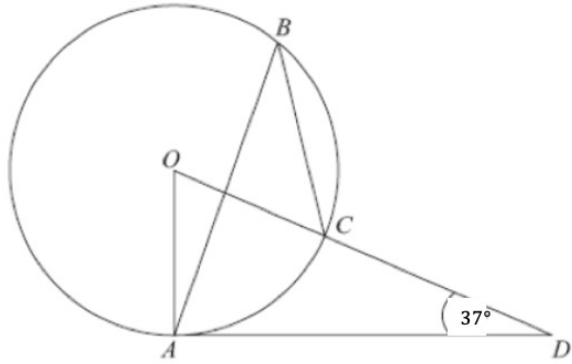
### Your Turn

Calculate the size of angle CDO



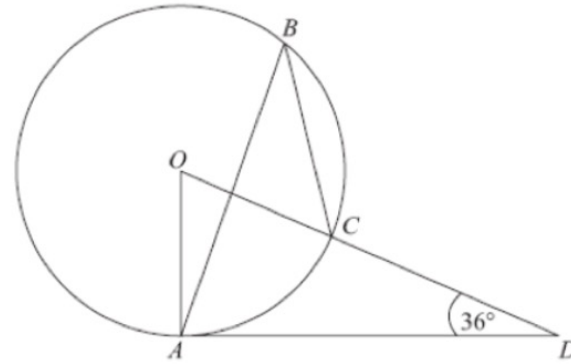
### Worked Example

Calculate the size of angle ABC



### Your Turn

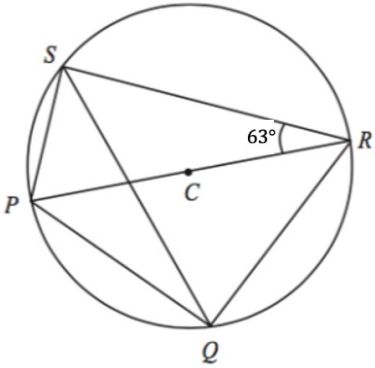
Calculate the size of angle ABC





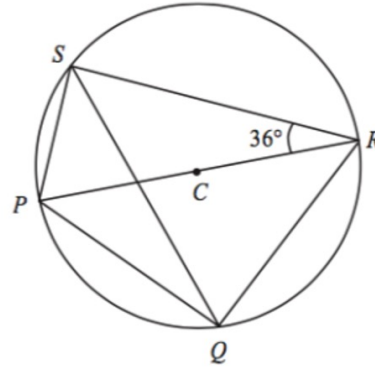
### Worked Example

Calculate the size of angle RQS



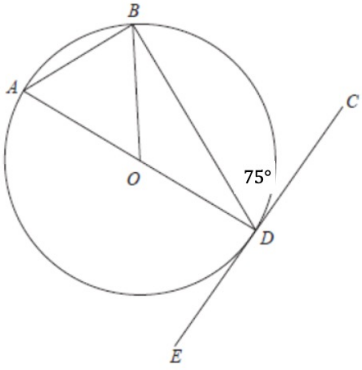
### Your Turn

Calculate the size of angle RQS



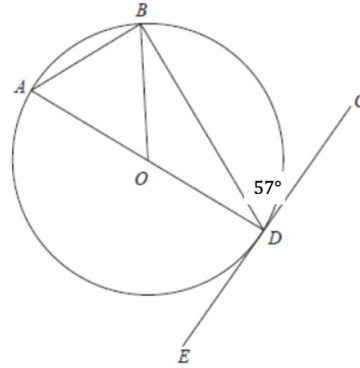
## Worked Example

Calculate the size of angle AOB



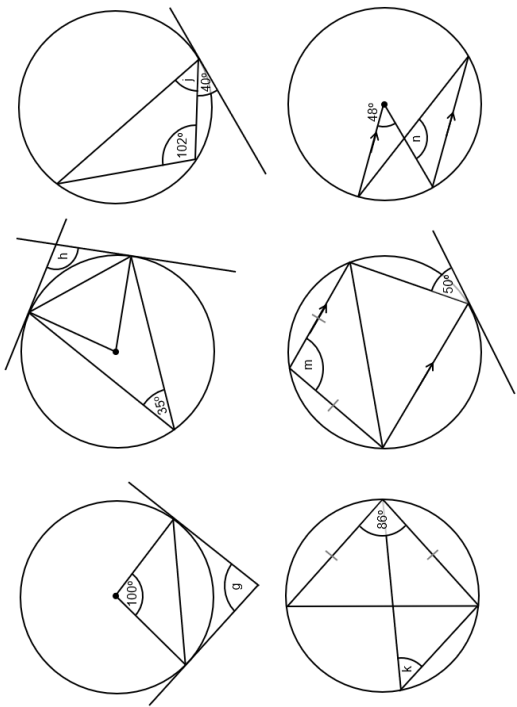
## Your Turn

Calculate the size of angle AOB





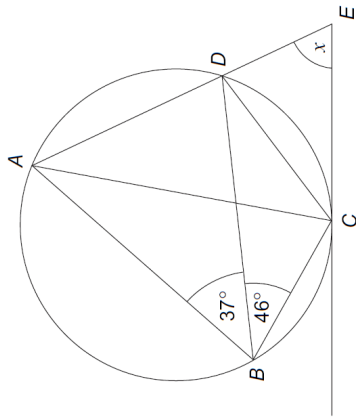
# Fluency Practice



**Incomplete Example:**

For this question, the steps are given below, but the reasons are missing. Complete the table with the correct reasons, and label the calculated angles on the diagram.

The line joining C and E is a tangent to the circle

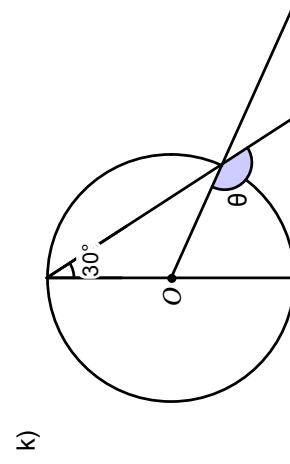
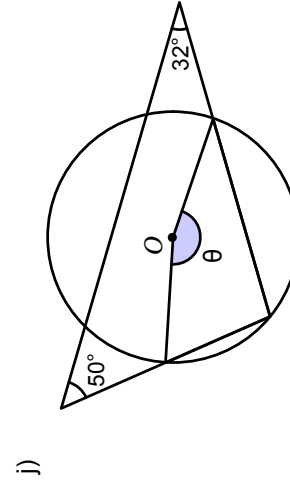
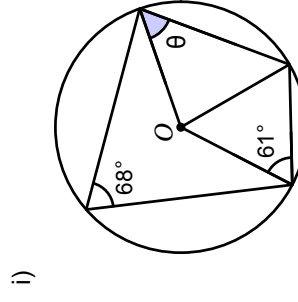
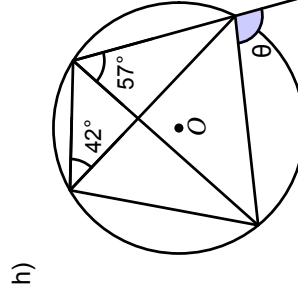
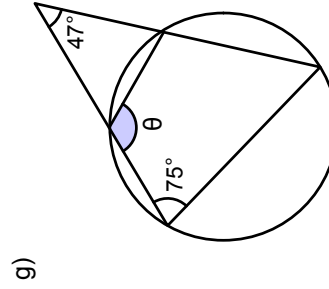
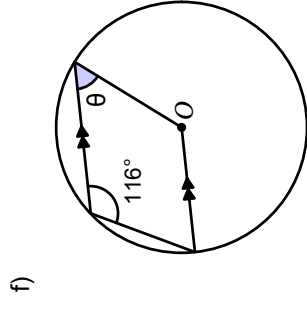
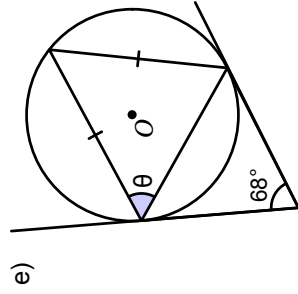
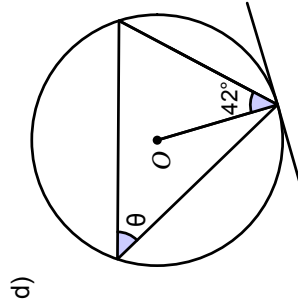
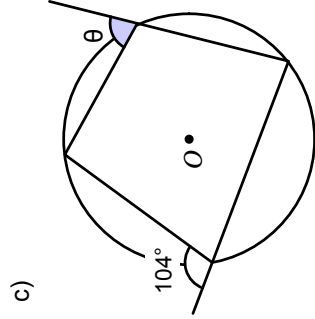
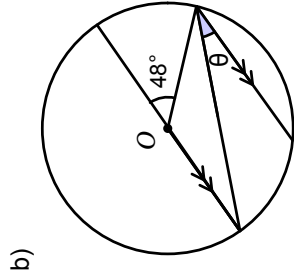
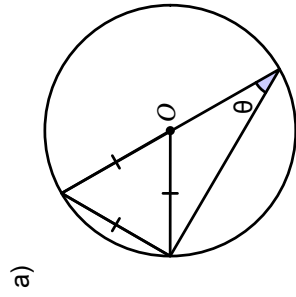


Work out the size of angle  $x$ .

Step	Reason
Angle DCE = $46^\circ$	
Angle ACD = $37^\circ$	
Angle CAE = $46^\circ$	
$x = 180 - 46 - 46 - 37$	
$x = 51^\circ$	

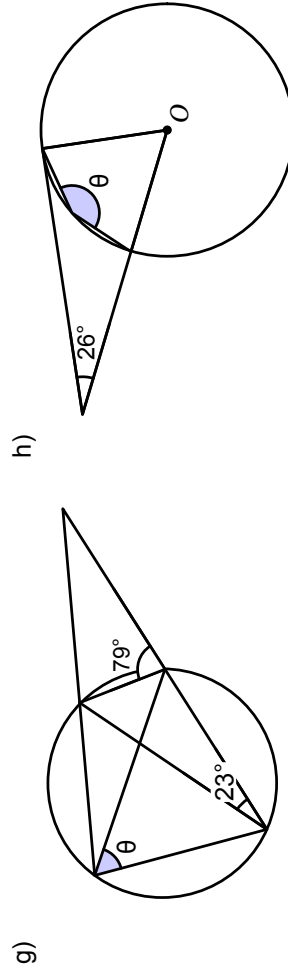
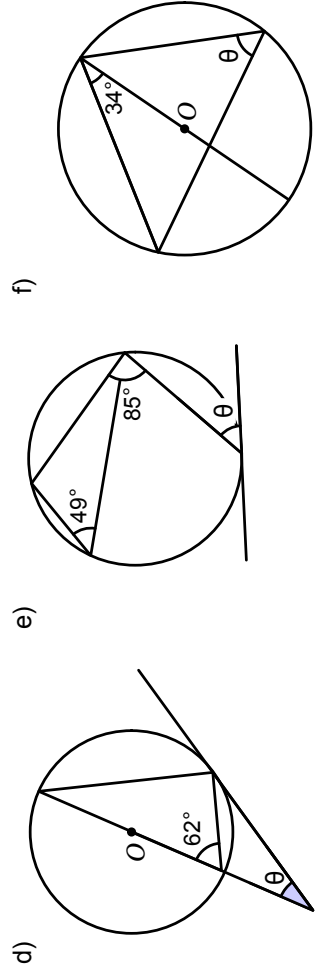
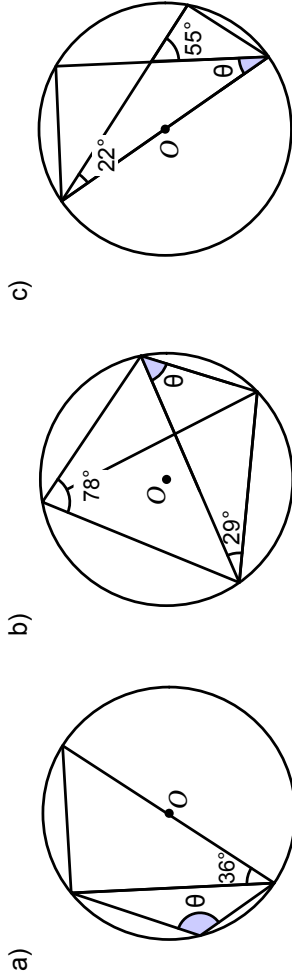
# Fluency Practice

1. Work out the angles marked  $\theta$ .



# Fluency Practice

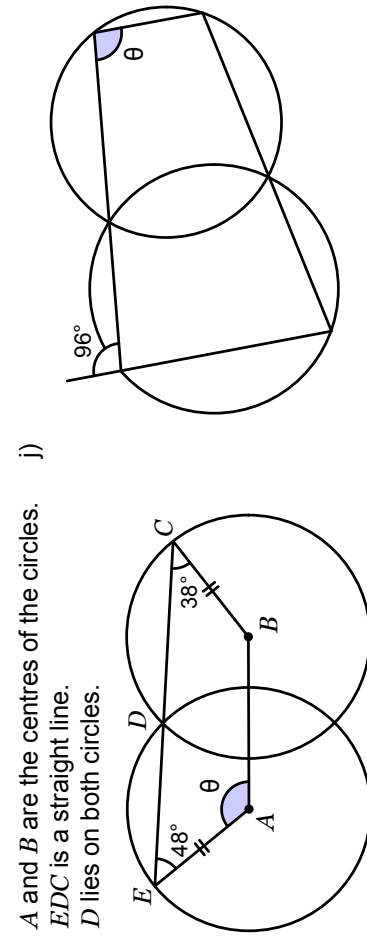
1. Work out the angles marked  $\theta$ .



i)  $A$  and  $B$  are the centres of the circles.

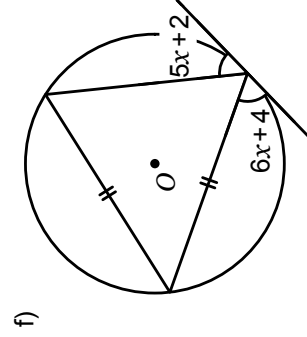
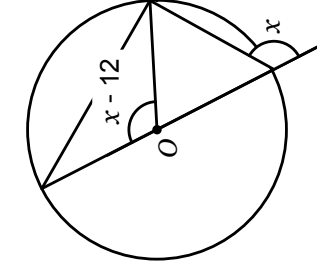
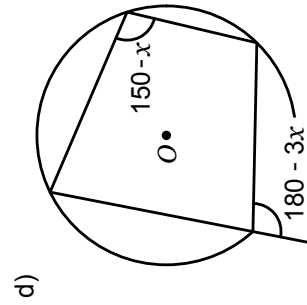
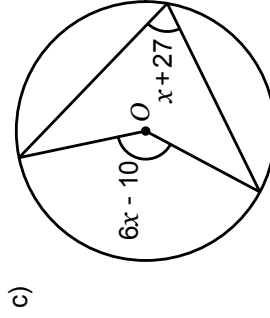
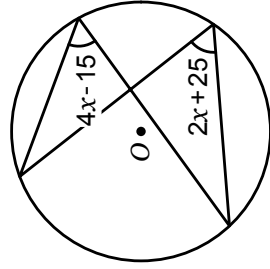
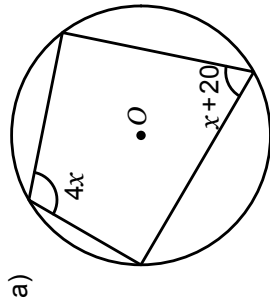
$EDC$  is a straight line.

$D$  lies on both circles.

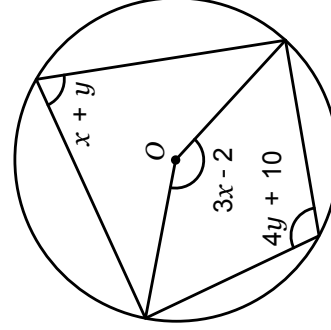
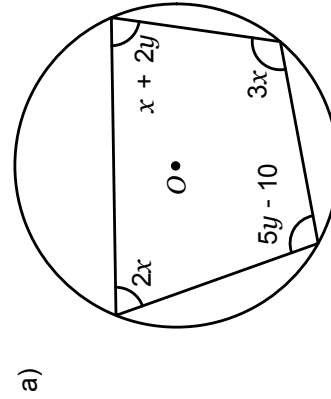


# Fluency Practice

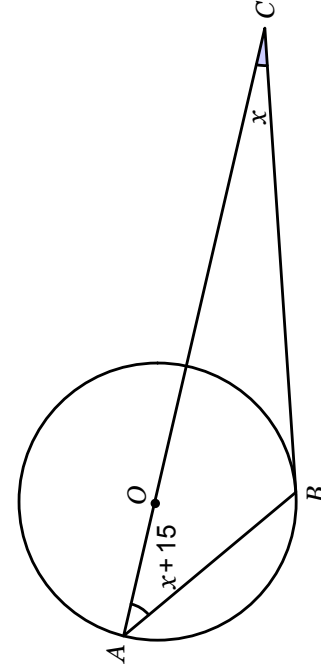
1. Work out value of  $x$  in each diagram:



2. Work out values of  $x$  and  $y$  in each diagram:

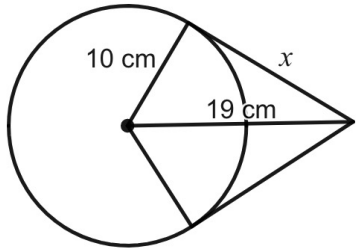


3.  $BC$  is a tangent to the circle.  
Work out the value of  $x$ .



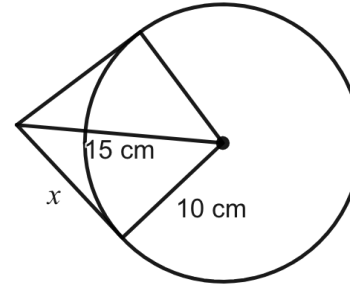
## Worked Example

Find the value of  $x$



## Your Turn

Find the value of  $x$





## Extra Notes

### 3 Direct and Inverse Proportion

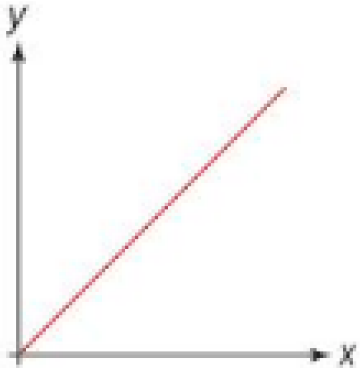
## Direct Proportion

$y$  is directly proportional to  $x$   
 $y$  is proportional to  $x$   
 $y$  varies directly to  $x$

$$y \propto x$$

$$y = kx$$

$k$  is called the constant of proportionality



The graph of  $y = kx$   
is a straight line that  
passes through  
the origin.

### Worked Example

$y$  is directly proportional to  $x$

When  $y = 20, x = 2$

- a) Find  $y$  when  $x = 5$
- b) Find  $x$  when  $y = 200$

### Your Turn

$b$  is directly proportional to  $a$

When  $b = 30, a = 5$

- a) Find  $b$  when  $a = 2$
- b) Find  $a$  when  $b = 3000$

### Worked Example

$y$  is directly proportional to the square of  $x$

When  $y = 36, x = 3$

- a) Find  $y$  when  $x = 5$
- b) Find  $x$  when  $y = 400$

### Your Turn

$b$  is directly proportional to the square of  $a$

When  $b = 12, a = 2$

- a) Find  $b$  when  $a = 3$
- b) Find  $a$  when  $b = 300$

### Worked Example

$y$  is directly proportional to the cube of  $x$

When  $y = 32, x = 2$

- a) Find  $y$  when  $x = 5$
- b) Find  $x$  when  $y = 108$

### Your Turn

$b$  is directly proportional to the cube of  $a$

When  $b = 54, a = 3$

- a) Find  $b$  when  $a = 4$
- b) Find  $a$  when  $b = 16$

### Worked Example

$y$  is directly proportional to the square root of  $x$

When  $y = 36, x = 16$

- a) Find  $y$  when  $x = 25$
- b) Find  $x$  when  $y = 900$

### Your Turn

$b$  is directly proportional to the square root of  $a$

When  $b = 36, a = 144$

- a) Find  $b$  when  $a = 49$
- b) Find  $a$  when  $b = 243$

## Fill in the Gaps

General Statement	General Equation	Table of Values	Value of $k$	Specific Equation	When $x = 5$ , $y = ?$	When $y = 24$ , $x = ?$								
$y \propto x$	$y = kx$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"></td> </tr> </table>	$x$	1	2	10	$y$	3			$k = 3$	$y = 3x$	$y = 3 \times 5$ $y = 15$	$24 = 3 \times x$ $x = 8$
$x$	1	2	10											
$y$	3													
$y \propto x$	$y = kx$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">80</td> </tr> </table>	$x$	1	2	10	$y$	8		80				$x = 3$
$x$	1	2	10											
$y$	8		80											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"></td> </tr> </table>	$x$	1	2	10	$y$					$y = 2.5x$	$24 = 2.5 \times x$ $x = 9.6$	
$x$	1	2	10											
$y$														
$y \propto x$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">10</td> <td style="padding: 2px 5px;"></td> </tr> </table>	$x$	1	2	10	$y$		10					
$x$	1	2	10											
$y$		10												
$y \propto x^2$	$y = kx^2$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">600</td> </tr> </table>	$x$	1	2	10	$y$			600	$k = 6$			$24 = 6 \times x^2$ $x = 2$
$x$	1	2	10											
$y$			600											
$y \propto x^2$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">150</td> </tr> </table>	$x$	1	2	10	$y$			150				
$x$	1	2	10											
$y$			150											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;"></td> </tr> </table>	$x$	1	2	10	$y$		1		$k = 0.5$			
$x$	1	2	10											
$y$		1												



## Fill in the Gaps

General Statement	General Equation	Table of Values	Value of $k$	Specific Equation								
$y \propto x^3$	$y = kx^3$	<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>2</td> <td>4</td> </tr> <tr> <td><math>y</math></td> <td>3</td> <td></td> <td></td> </tr> </table>	$x$	1	2	4	$y$	3			$k = 3$	
$x$	1	2	4									
$y$	3											
$y \propto \sqrt{x}$		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>4</td> <td>25</td> </tr> <tr> <td><math>y</math></td> <td></td> <td>24</td> <td></td> </tr> </table>	$x$	1	4	25	$y$		24			
$x$	1	4	25									
$y$		24										
$y \propto x$		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>4</td> <td>10</td> </tr> <tr> <td><math>y</math></td> <td></td> <td>3</td> <td></td> </tr> </table>	$x$	1	4	10	$y$		3			
$x$	1	4	10									
$y$		3										
	$y = k\sqrt[3]{x}$	<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>8</td> <td>125</td> </tr> <tr> <td><math>y</math></td> <td></td> <td>20</td> <td></td> </tr> </table>	$x$	1	8	125	$y$		20			
$x$	1	8	125									
$y$		20										
		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>4</td> <td>10</td> </tr> <tr> <td><math>y</math></td> <td></td> <td>32</td> <td>200</td> </tr> </table>	$x$	1	4	10	$y$		32	200		
$x$	1	4	10									
$y$		32	200									
		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td><math>y</math></td> <td></td> <td>3</td> <td>7.5</td> </tr> </table>	$x$	1	4		$y$		3	7.5	$k = 1.5$	
$x$	1	4										
$y$		3	7.5									
		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>4</td> <td></td> </tr> <tr> <td><math>y</math></td> <td></td> <td><math>\frac{32}{3}</math></td> <td>24</td> </tr> </table>	$x$	1	4		$y$		$\frac{32}{3}$	24	$k = \frac{2}{3}$	
$x$	1	4										
$y$		$\frac{32}{3}$	24									
$y \propto x^3$		<table border="1"> <tr> <td><math>x</math></td> <td>2</td> <td><math>\sqrt{5}</math></td> <td></td> </tr> <tr> <td><math>y</math></td> <td></td> <td>25</td> <td><math>27\sqrt{5}</math></td> </tr> </table>	$x$	2	$\sqrt{5}$		$y$		25	$27\sqrt{5}$		
$x$	2	$\sqrt{5}$										
$y$		25	$27\sqrt{5}$									
		<table border="1"> <tr> <td><math>x</math></td> <td>1</td> <td>8</td> <td></td> </tr> <tr> <td><math>y</math></td> <td></td> <td><math>2a</math></td> <td><math>4a</math></td> </tr> </table>	$x$	1	8		$y$		$2a$	$4a$	$k = a$	
$x$	1	8										
$y$		$2a$	$4a$									

## Fill in the Gaps

### Direct Proportion – Method Breakdown

Complete the table. Use the equation with the known constant ( $k$ ) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality ( $k$ )	Equation Re-write	Question
$y$ is directly proportional to $x$	$y = kx$	When $x = 9$ , $y = 45$	$45 = k(9)$			When $x = 10$ , $y =$
$y$ is directly proportional to $x$ squared	$y = kx^2$	When $x = 3$ , $y = 36$	$36 = k(3)^2$			When $x = 5$ , $y =$
$y$ is directly proportional to $x$ cubed		When $x = 4$ , $y = 128$				When $x = 3$ , $y =$
$y$ is directly proportional to the square root of $x$	$y = k\sqrt{x}$	When $x = 25$ , $y = 15$	$15 = k\sqrt{25}$			When $x = 100$ , $y =$
	$y = k\sqrt[3]{x}$	When $x = 8$ , $y = 20$				When $x = 64$ , $y =$
	$y = kx$	When $x = 5$ , $y = 40$				When $x = 2.5$ , $y =$
$y$ is directly proportional to $x$ squared		When $x = 4$ , $y = 96$				When $x = 10$ , $y =$
$y$ is directly proportional to the square root of $x$		When $x = 81$ , $y = 81$				When $x = 36$ , $y =$
		When $x = 5$ , $y = 500$	$500 = k(5)^3$			When $x = 3$ , $y =$
$y$ is directly proportional to the cube root of $x$		When $x = 1000$ , $y = 70$				When $x = 8$ , $y =$
$y$ is directly proportional to $x$		When $x = 16$ , $y = 56$				When $y = 49$ , $x =$
$y$ is directly proportional to $x$ squared		When $x = 3$ , $y = 4.5$				When $y = 72$ , $x =$
$y$ is directly proportional to $x$ cubed		When $x = 2$ , $y = 1.6$				When $y = 12.8$ , $x =$

### Worked Example

$y$  is directly proportional to  $x + 2$

When  $y = 20$ ,  $x = 2$

Find  $y$  when  $x = 5$

### Your Turn

$y$  is directly proportional to  $x + 2$

When  $y = 12$ ,  $x = 2$

Find  $y$  when  $x = 8$

### Worked Example

$y$  is directly proportional to  $x^2 + 4$

When  $y = 52$ ,  $x = 3$

Find  $y$  when  $x = 5$

### Your Turn

$y$  is directly proportional to  $2x^2$

When  $y = 36$ ,  $x = 3$

Find  $y$  when  $x = 5$

## Inverse Proportion

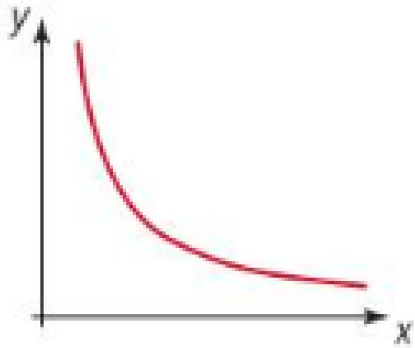
$y$  is inversely proportional to  $x$   
 $y$  varies inversely or indirectly to  $x$

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$k$  is called the constant of proportionality

The graph of  $y = \frac{k}{x}$  is a reciprocal graph.



### Worked Example

$y$  is inversely proportional to  $x$

When  $y = 5, x = 2$

- a) Find  $y$  when  $x = 5$
- b) Find  $x$  when  $y = 0.5$

### Your Turn

$b$  is inversely proportional to  $a$

When  $b = 10, a = 3$

- a) Find  $b$  when  $a = 5$
- b) Find  $a$  when  $b = 0.25$

### Worked Example

$y$  is inversely proportional to the square of  $x$

When  $y = 6, x = 10$

- a) Find  $y$  when  $x = 5$
- b) Find  $x$  when  $y = 1.5$

### Your Turn

$b$  is inversely proportional to the square of  $a$

When  $b = 6, a = 5$

- a) Find  $b$  when  $a = 10$
- b) Find  $a$  when  $b = 6$

### Worked Example

$y$  is inversely proportional to the cube of  $x$

When  $y = 8, x = 10$

- a) Find  $y$  when  $x = 2$
- b) Find  $x$  when  $y = 15.625$

### Your Turn

$b$  is inversely proportional to the cube of  $a$

When  $b = 5, a = 2$

- a) Find  $b$  when  $a = 10$
- b) Find  $a$  when  $b = 0.625$



### Worked Example

$y$  is inversely proportional to the square root of  $x$

When  $y = 4, x = 25$

- a) Find  $y$  when  $x = 4$
- b) Find  $x$  when  $y = 2.5$

### Your Turn

$b$  is inversely proportional to the square root of  $a$

When  $b = 4, a = 9$

- a) Find  $b$  when  $a = 16$
- b) Find  $a$  when  $b = 6$

## Fill in the Gaps

General Statement	General Equation	Table of Values	Value of $k$	Specific Equation	When $x = 6$ , $y = ?$	When $y = 10$ , $x = ?$								
$y \propto \frac{1}{x}$	$y = \frac{k}{x}$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">4</td><td style="padding: 2px 5px;">8</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td style="padding: 2px 5px;">48</td><td></td><td></td></tr> </table>	$x$	1	4	8	$y$	48			$k = 48$	$y = \frac{48}{x}$	$y = \frac{48}{6} = 8$	$x = \frac{48}{10} = 4.8$
$x$	1	4	8											
$y$	48													
$y \propto \frac{1}{x}$	$y = \frac{k}{x}$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">5</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td style="padding: 2px 5px;">120</td><td></td><td style="padding: 2px 5px;">24</td></tr> </table>	$x$	1	2	5	$y$	120		24				$x = \frac{120}{10} = 12$
$x$	1	2	5											
$y$	120		24											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td></td><td></td><td></td></tr> </table>	$x$	1	5	10	$y$					$y = \frac{30}{x}$		$x = \frac{30}{10} = 3$
$x$	1	5	10											
$y$														
$y \propto \frac{1}{x}$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">20</td><td style="padding: 2px 5px;">100</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td></td><td style="padding: 2px 5px;">30</td><td></td></tr> </table>	$x$	5	20	100	$y$		30					
$x$	5	20	100											
$y$		30												
$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">3</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td></td><td></td><td style="padding: 2px 5px;">40</td></tr> </table>	$x$	1	2	3	$y$			40	$k = 360$			$x = \sqrt{\frac{360}{10}} = 6$
$x$	1	2	3											
$y$			40											
$y \propto \frac{1}{x^2}$		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">2</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td></td><td></td><td style="padding: 2px 5px;">3</td></tr> </table>	$x$	1	2	10	$y$			3				
$x$	1	2	10											
$y$			3											
		<table border="1" style="width: 100%; border-collapse: collapse;"> <tr><td style="padding: 2px 5px;"><math>x</math></td><td style="padding: 2px 5px;">1</td><td style="padding: 2px 5px;">5</td><td style="padding: 2px 5px;">10</td></tr> <tr><td style="padding: 2px 5px;"><math>y</math></td><td></td><td style="padding: 2px 5px;">4</td><td></td></tr> </table>	$x$	1	5	10	$y$		4		$k = 20$			
$x$	1	5	10											
$y$		4												

## Fill in the Gaps

General Statement	General Equation	Table of Values	Value of $k$	Specific Equation												
$y \propto \frac{1}{x^2}$	$y = \frac{k}{x^2}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">100</td> </tr> </table>	$x$	1	2	5	$y$	100			$k = 100$					
$x$	1	2	5													
$y$	100															
$y \propto \frac{1}{\sqrt{x}}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">25</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">5</td> </tr> </table>	$x$	1	4	25	$y$	5								
$x$	1	4	25													
$y$	5															
$y \propto \frac{1}{x^3}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">125</td> </tr> </table>	$x$	1	2	10	$y$	125								
$x$	1	2	10													
$y$	125															
	$y = \frac{k}{\sqrt[3]{x}}$	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">125</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">20</td> </tr> </table>	$x$	1	8	125	$y$	20								
$x$	1	8	125													
$y$	20															
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">10</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">7.5</td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;">3</td> </tr> </table>	$x$	1	4	10	$y$	7.5				3				
$x$	1	4	10													
$y$	7.5															
	3															
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">0.75</td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;">0.12</td> </tr> </table>	$x$	1	2		$y$	0.75				0.12			$k = 3$	
$x$	1	2														
$y$	0.75															
	0.12															
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;"><math>\frac{1}{12}</math></td> </tr> <tr> <td style="padding: 2px 5px;"></td> <td colspan="3" style="padding: 2px 5px;"><math>\frac{1}{30}</math></td> </tr> </table>	$x$	1	4		$y$	$\frac{1}{12}$				$\frac{1}{30}$			$k = \frac{1}{6}$	
$x$	1	4														
$y$	$\frac{1}{12}$															
	$\frac{1}{30}$															
$y \propto \frac{1}{x^3}$		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;"><math>\sqrt{3}</math></td> <td style="padding: 2px 5px;">2</td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td colspan="3" style="padding: 2px 5px;">3</td> </tr> </table>	$x$	1	$\sqrt{3}$	2	$y$	3								
$x$	1	$\sqrt{3}$	2													
$y$	3															
		<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="padding: 2px 5px;"><math>x</math></td> <td style="padding: 2px 5px;">0.5</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;"></td> </tr> <tr> <td style="padding: 2px 5px;"><math>y</math></td> <td style="padding: 2px 5px;"><math>4a</math></td> <td style="padding: 2px 5px;"></td> <td style="padding: 2px 5px;"><math>\frac{a}{16}</math></td> </tr> </table>	$x$	0.5	2		$y$	$4a$		$\frac{a}{16}$	$k = a$					
$x$	0.5	2														
$y$	$4a$		$\frac{a}{16}$													

## Fill in the Gaps

**Inverse Proportion – Method Breakdown** Complete the table. Use the equation with the known constant ( $k$ ) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality ( $k$ )	Equation Re-write	Question
$y$ is inversely proportional to $x$	$y = \frac{k}{x}$	When $x = 8$ , $y = 2$	$2 = \frac{k}{8}$			When $x = 2$ , $y =$
$y$ is inversely proportional to $x$ squared		When $x = 4$ , $y = 0.5$	$0.5 = \frac{k}{(4)^2}$			When $x = 2$ , $y =$
	$y = \frac{k}{x^3}$	When $x = 2$ , $y = 5$				When $x = 1$ , $y =$
	$y = \frac{k}{\sqrt{x}}$	When $x = 25$ , $y = 4$				When $x = 100$ , $y =$
		When $x = 8$ , $y = 4$	$4 = \frac{k}{\sqrt[3]{8}}$			When $x = 64$ , $y =$
$y$ is inversely proportional to $x$		When $x = 2$ , $y = 2.5$				When $x = 20$ , $y =$
$y$ is inversely proportional to $x$ cubed		When $x = 4$ , $y = 0.25$				When $x = 10$ , $y =$
$y$ is inversely proportional to the square root of $x$		When $x = 100$ , $y = 3$				When $x = 9$ , $y =$
$y$ is inversely proportional to the cube root of $x$		When $x = 125$ , $y = 10$				When $x = 8$ , $y =$
$y$ is inversely proportional to $x$ squared		When $x = 10$ , $y = 2$				When $x = 5$ , $y =$

### Worked Example

$y$  is inversely proportional to  $x + 3$

When  $y = 52$ ,  $x = 3$

Find  $y$  when  $x = 5$

### Your Turn

$y$  is inversely proportional to  $2x + 1$

When  $y = 30$ ,  $x = 4$

Find  $y$  when  $x = 7$

## Fill in the Gaps

Type	Statement	k-Formula	k value $x = 2, y = 4$	Final Formula
y is proportional to x	$y \propto x$	$y = kx$		
x is proportional to y				
y is inversely proportional to x	$y \propto \frac{1}{x}$	$y = \frac{k}{x}$		
x is inversely proportional to y				
y is proportional to the square of x				
x is proportional to the square of y				
x is proportional to $\sqrt{y}$				
Y is inversely proportional to $\sqrt{x}$				
Y is proportional to $x^3$				
x is proportional to 3 more than y				

## Fill in the Gaps

**Direct & Inverse Proportion – Method Breakdown** Complete the table. Use the equation with the known constant ( $k$ ) to answer the question.

Relationship in Words	Equation	Known Values	Substitution	Constant of Proportionality ( $k$ )	Equation Re-Write	Question
$y$ is directly proportional to $x$	$y = kx$	When $x = 9$ , $y = 45$	$45 = k(9)$			When $x = 10$ , $y =$
$y$ is inversely proportional to $x$	$y = \frac{k}{x}$	When $x = 8$ , $y = 2$				When $x = 2$ , $y =$
$y$ is directly proportional to $x$ squared		When $x = 3$ , $y = 36$	$36 = k(3)^2$			When $x = 5$ , $y =$
	$y = kx^3$	When $x = 4$ , $y = 128$				When $x = 3$ , $y =$
	$y = \frac{k}{x^2}$	When $x = 4$ , $y = 0.5$				When $x = 2$ , $y =$
$y$ is inversely proportional to $x$ cubed		When $x = 2$ , $y = 5$				When $x = 1$ , $y =$
$y$ is directly proportional to the square root of $x$	$y = k\sqrt{x}$	When $x = 25$ , $y = 15$				When $x = 100$ , $y =$
	$y = k\sqrt[3]{x}$	When $x = 8$ , $y = 20$				When $x = 64$ , $y =$
	$y = \frac{k}{\sqrt{x}}$	When $x = 25$ , $y = 4$				When $x = 100$ , $y =$
$y$ is inversely proportional to the cube root of $x$		When $x = 8$ , $y = 4$				When $x = 64$ , $y =$
$y$ is directly proportional to $x$ squared		When $x = 3$ , $y = 4.5$				When $y = 72$ , $x =$
$y$ is inversely proportional to the square root of $x$		When $x = 100$ , $y = 3$				When $x = 9$ , $y =$

## Worked Example

$m$	16	25
$n$	625	256

Select the correct statement below.

- $n \propto \frac{1}{\sqrt{m}}$
- $n \propto \frac{1}{m}$
- $n \propto \frac{1}{m^2}$

## Your Turn

$c$	1	4
$d$	32	2

Select the correct statement below.

- $d \propto \frac{1}{\sqrt{c}}$
- $d \propto \frac{1}{c}$
- $d \propto \frac{1}{c^2}$



### Worked Example

$x$  is inversely proportional to  $y^2$

$y$  is directly proportional to  $\sqrt[3]{z}$

Given that  $x = 10$  and  $z = 512$  when  $y = 7$  find a formula for  $x$  in terms of  $z$

### Your Turn

$x$  is directly proportional to  $y^3$

$y$  is inversely proportional to  $\sqrt{z}$

Given that  $x = 10$  and  $z = 36$  when  $y = 5$  find a formula for  $x$  in terms of  $z$

### Worked Example

$q$  is proportional to  $t^3$

$t$  is decreased by 30%

Work out the percentage decrease in  $q$

### Your Turn

$y$  is proportional to  $z^2$

$z$  is decreased by 80%

Work out the percentage decrease in  $y$

### Worked Example

$t$  is inversely proportional to  $z^3$   
 $z$  is decreased by 50%  
Find the percentage increase in  $t$

### Your Turn

$y$  is inversely proportional to  $p^2$   
 $p$  is decreased by 50%  
Find the percentage increase in  $y$

### Worked Example

$r$  is inversely proportional to  $s^2$

$r = 34$  when  $s = 3d$

Find  $r$  when  $s = d$

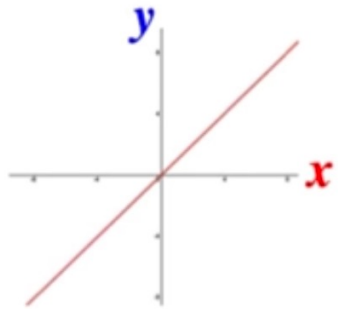
### Your Turn

$y$  is inversely proportional to  $x^3$

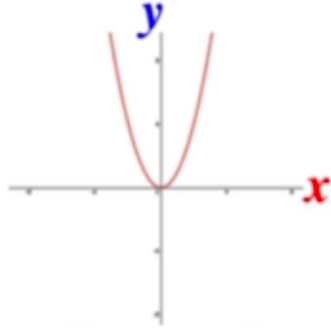
$y = 99$  when  $x = 2c$

Find  $y$  when  $x = 3c$

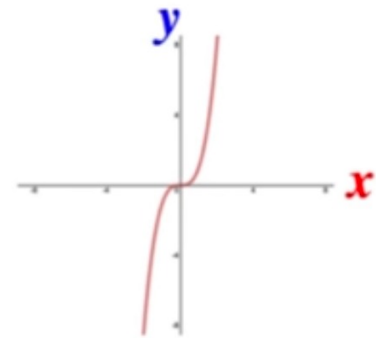
# Graphs



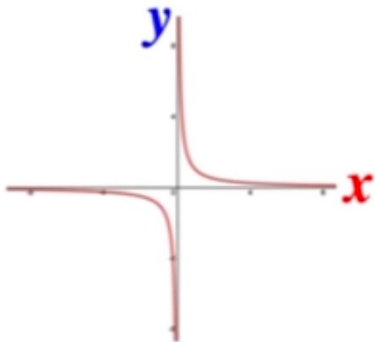
$$y = kx$$



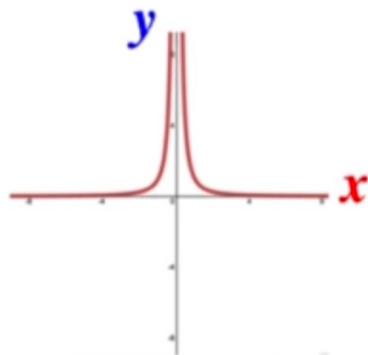
$$y = kx^2$$



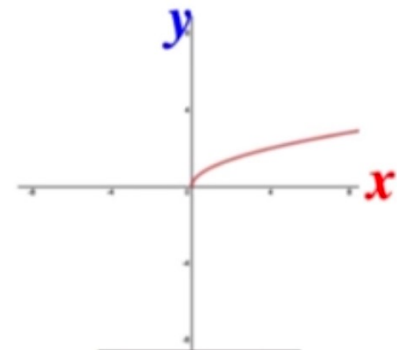
$$y = kx^3$$



$$y = \frac{k}{x}$$



$$y = \frac{k}{x^2}$$

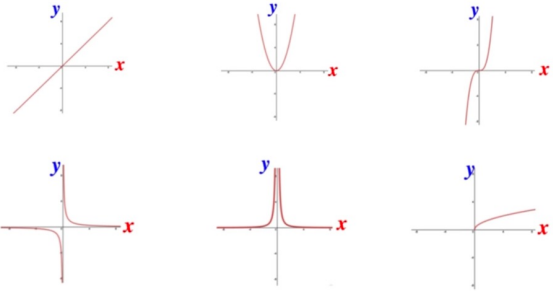


$$y = k\sqrt{x}$$

# Fluency Practice

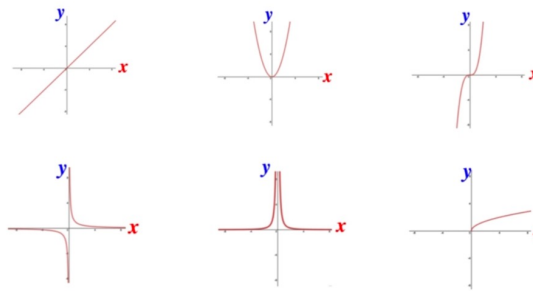
$y$  is proportional to the square of  $x$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



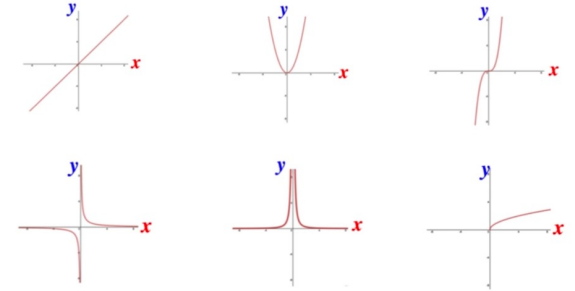
$y \propto x$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



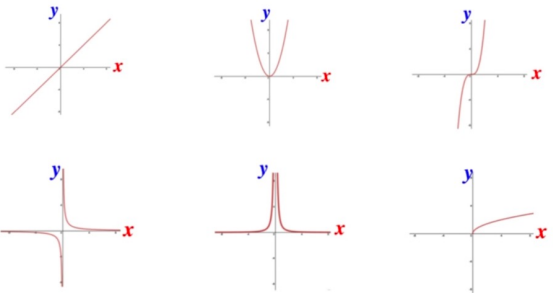
$y$  is inversely proportional to  $x$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



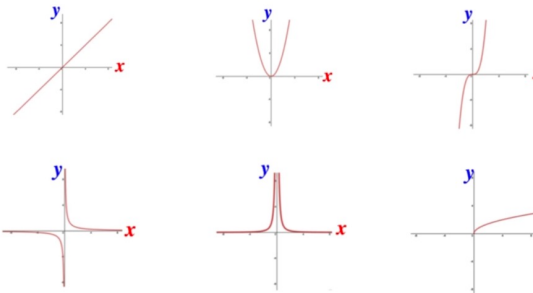
$y \propto x^3$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



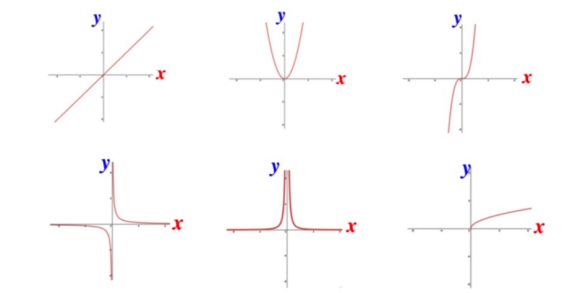
$y$  is inversely proportional to the square of  $x$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



$y \propto \sqrt{x}$

Which of the following could be the graph demonstrating between  $y$  and  $x$ ?



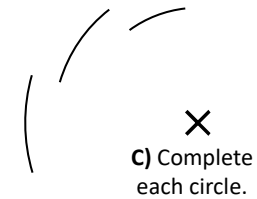
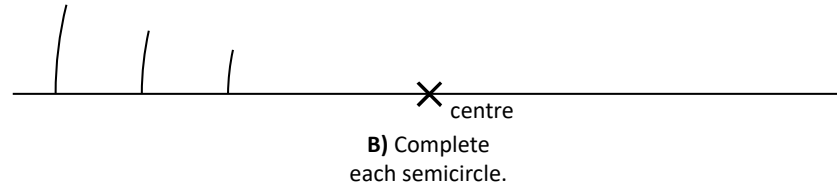
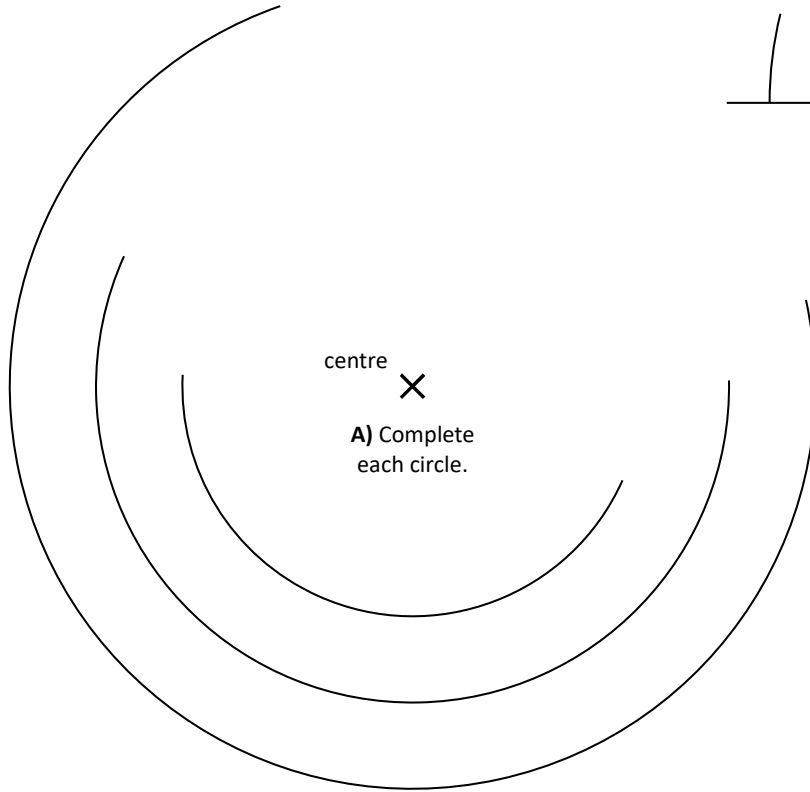
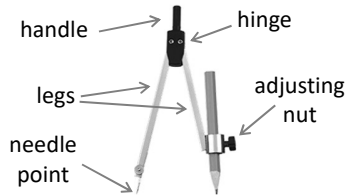
## Extra Notes

## 4 Constructions and Loci



# Fluency Practice

## Compass Skills



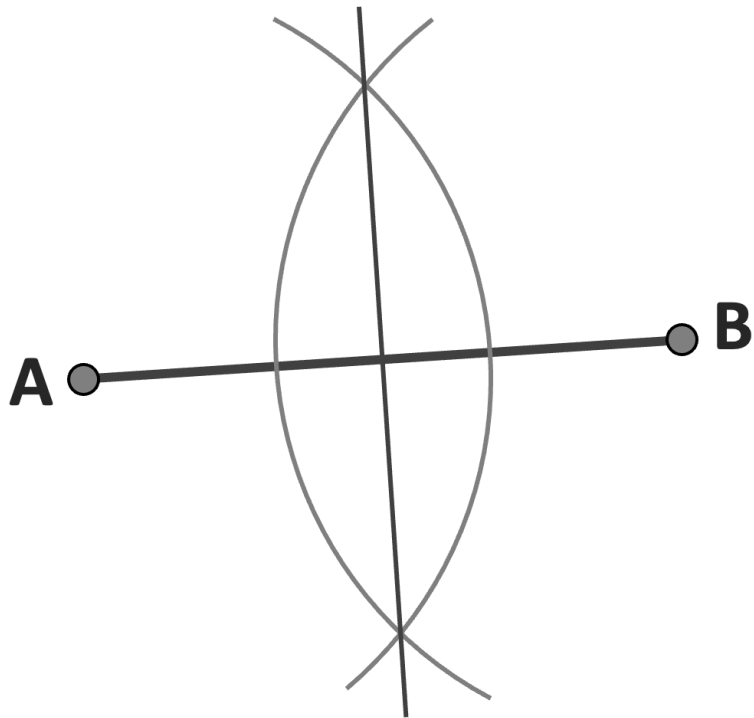
### Quick Tips:

- When the compass is closed, the point and the pencil point should meet.
- Make sure the legs are not loose (this needs tight hinge screws).
- Place paper underneath the worksheet to make the point hold.
- When drawing, try to only hold the handle with a finger and thumb.

## Perpendicular Bisector

Draw two points on your page and label them A and B.  
Join them with a straight line.  
Construct its perpendicular bisector.

- 1) Draw two equal arcs.
- 2) Connect the intersections with a straight line.
- 3) This line is the perpendicular bisector and contains all the points equidistant from A and B.



## Worked Example

Construct the perpendicular bisector of the line:



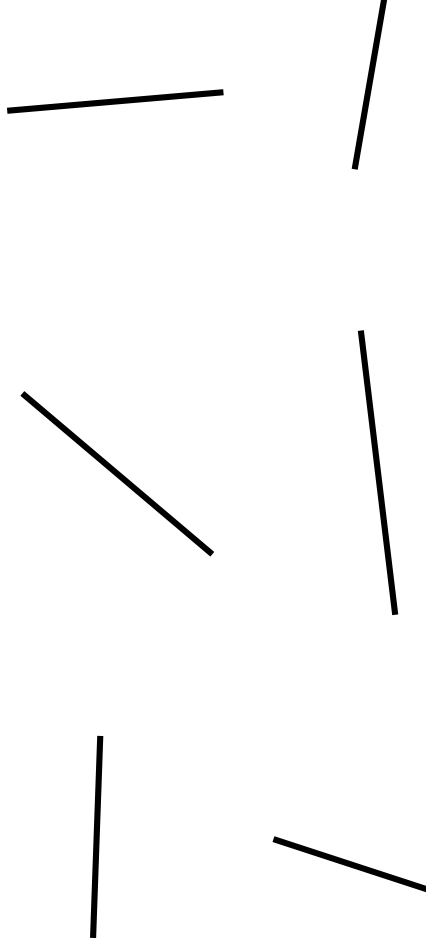
## Your Turn

Construct the perpendicular bisector of the line:

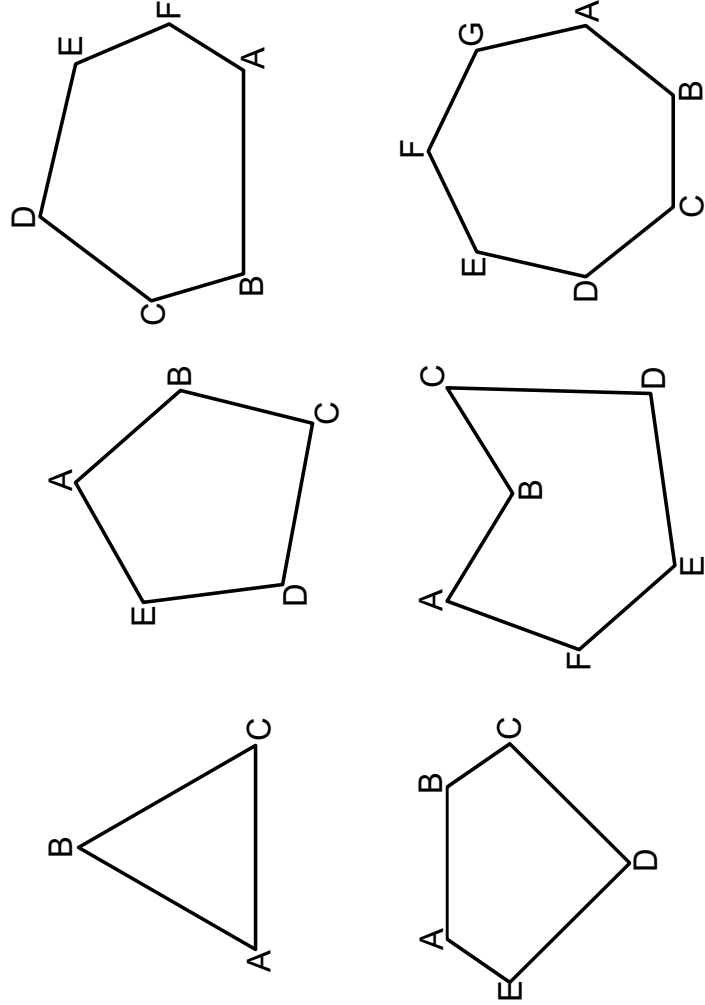


# Fluency Practice

**Q1** Construct the perpendicular bisector of the following lines.



**Q2** Construct the perpendicular bisector of the line **AB** in each of the following.

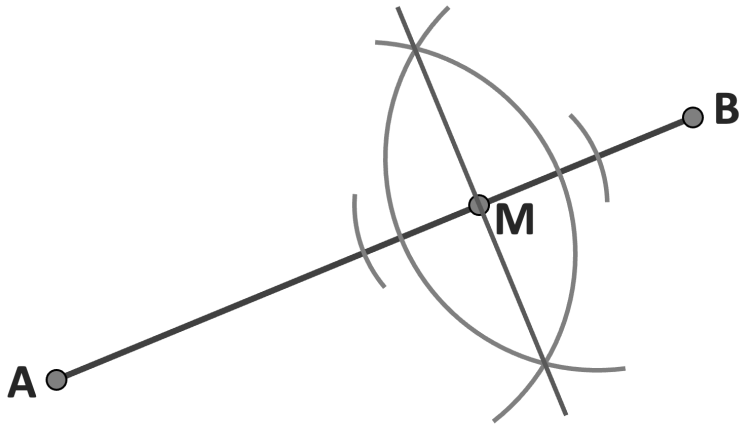


## Perpendicular Line at a Point 1

M is a point on the line AB.

Construct a line perpendicular to AB through M.

- 1) Use your compass to find two points on the line equidistance from M.
- 2) Construct a perpendicular bisector of these two points.



## Worked Example

Construct a perpendicular to the line which passes through the marked point:



## Your Turn

Construct a perpendicular to the line which passes through the marked point:

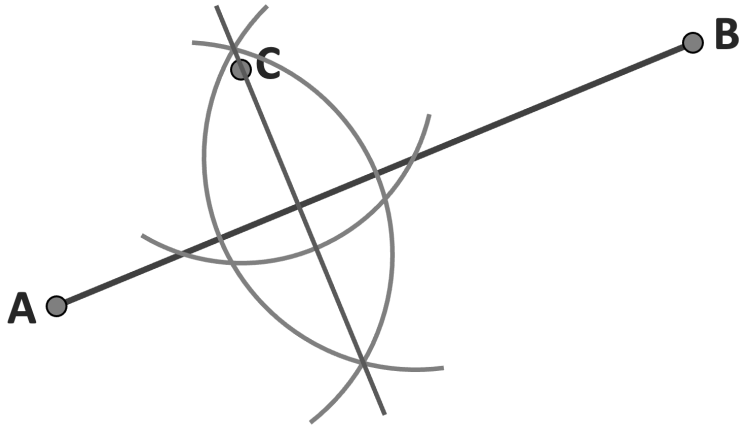




## Perpendicular Line at a Point 2

Construct a line perpendicular to AB through C, which is a point not on AB.

- 1) Use your compass to find two points on the line equidistance from C.
- 2) Construct a perpendicular bisector of these two points.



## Worked Example

Construct a perpendicular to the line which passes through the marked point:



X

## Your Turn

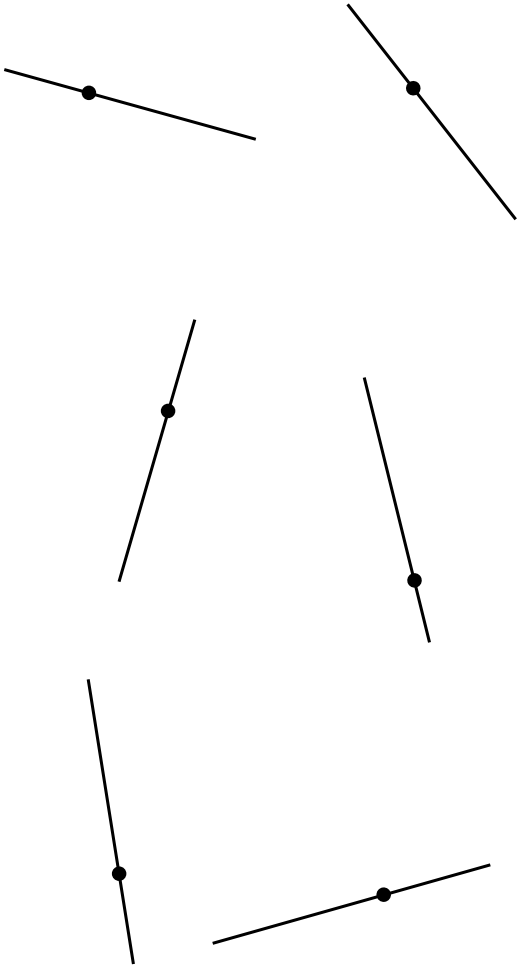
Construct a perpendicular to the line which passes through the marked point:

X

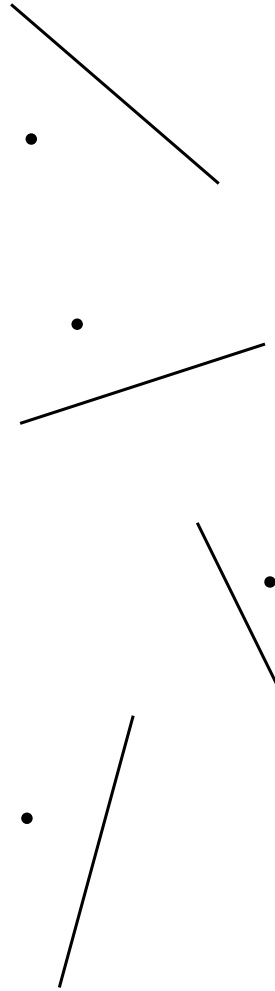


# Fluency Practice

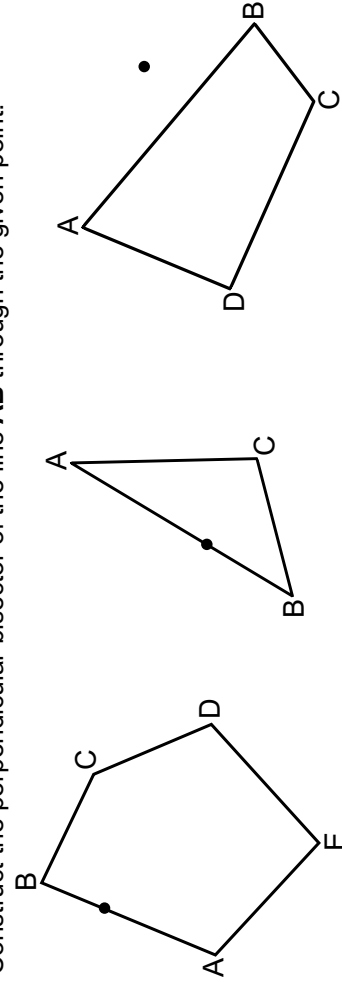
**Q3** Construct the perpendicular bisector of the following lines through the given point.



**Q4** Construct the perpendicular bisector of the following lines through the given point.



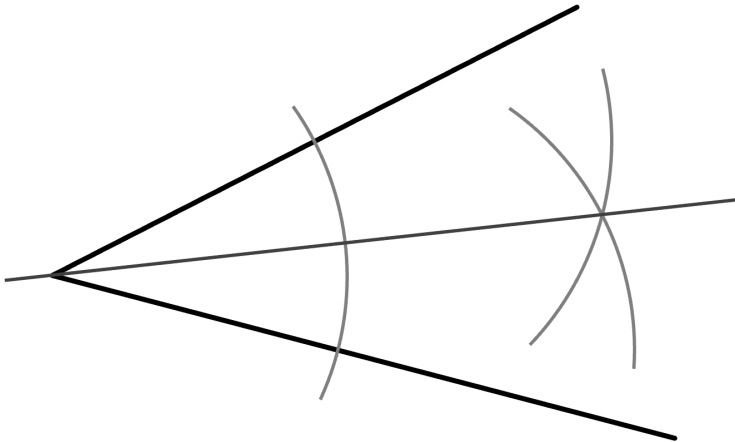
**Q5** Construct the perpendicular bisector of the line **AB** through the given point.



## Angle Bisector

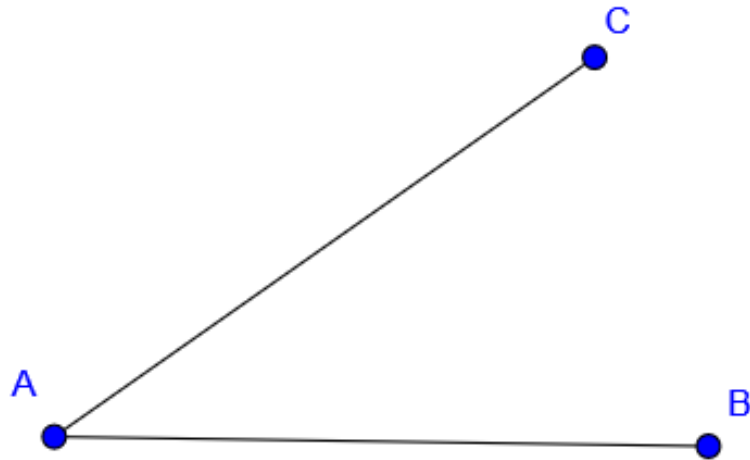
Draw an acute angle on your page.  
Construct its angle bisector.

- 1) Draw an arc from the vertex.
- 2) Draw two more equal arcs from the intersections.
- 3) Join the new intersection up to the vertex.
- 4) This line is the angle bisector and contains all points equidistant from both arms of the angle.



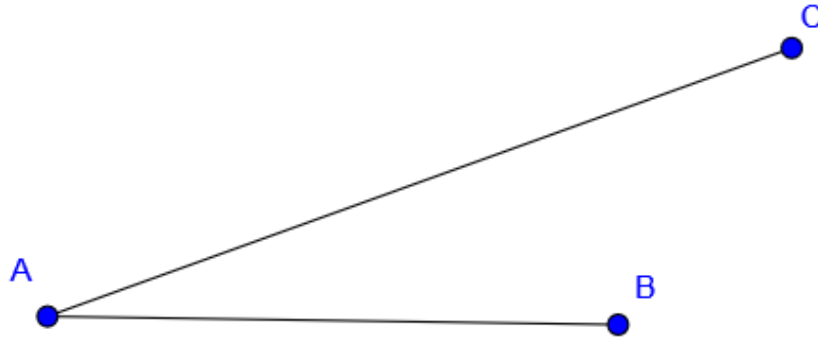
## Worked Example

Bisect angle BAC:



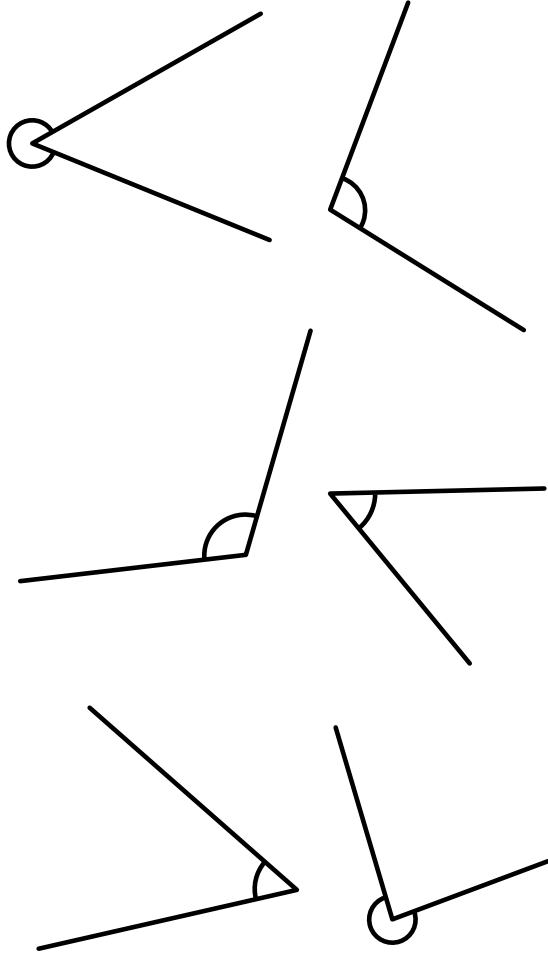
## Your Turn

Bisect angle BAC:

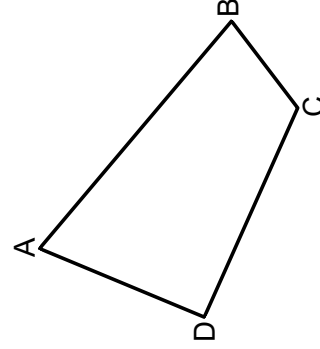
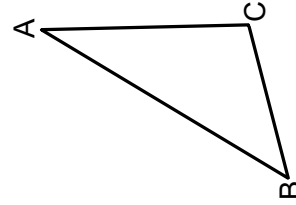
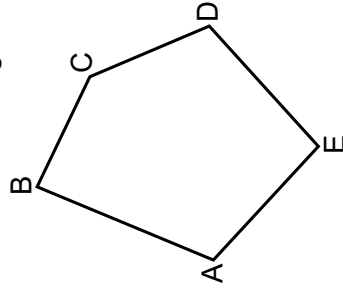


# Fluency Practice

**Q6** Construct the angle bisector of the following angles.



**Q7** Construct the angle bisector of  $\angle ABC$  for each of the following shapes..



**Q8** Construct a  $90^\circ$  angle.

**Q9** Construct a  $45^\circ$  angle.



## Constructing Triangles

You can construct a unique triangle when you know:

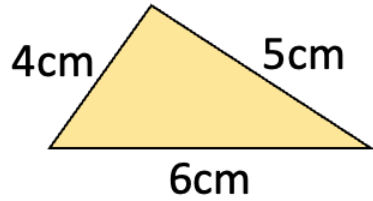
Two sides and the angle between them (**SAS**)

Two angles and a side (**ASA**)

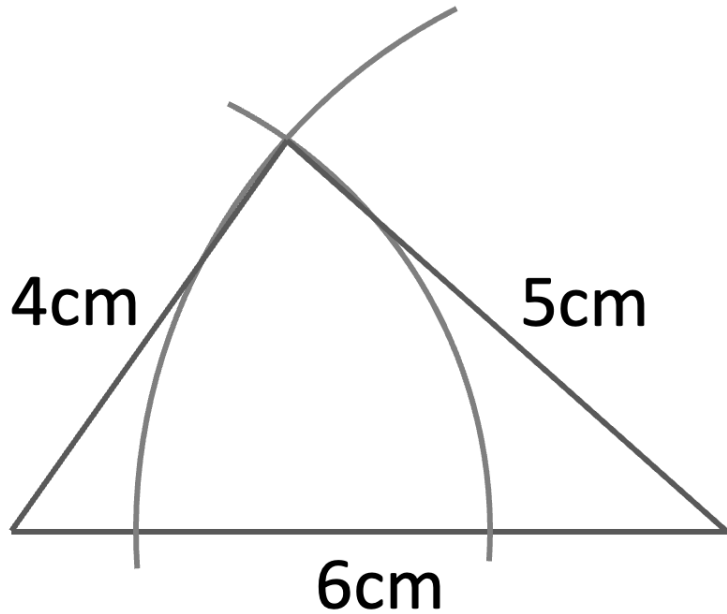
Three sides (**SSS**)

## SSS

Using a ruler and compass only, construct the following SSS triangle accurately.



- 1) Draw a 6cm line with a ruler.
- 2) Draw two arcs with lengths 4cm and 5cm from each end of the line.
- 3) Join the ends of the line to the intersection.



## Worked Example

Construct a triangle with:

- A side length of 10 *cm*
- A side length of 6 *cm*
- A side length of 8 *cm*

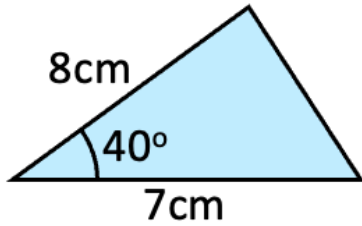
## Your Turn

Construct a triangle with:

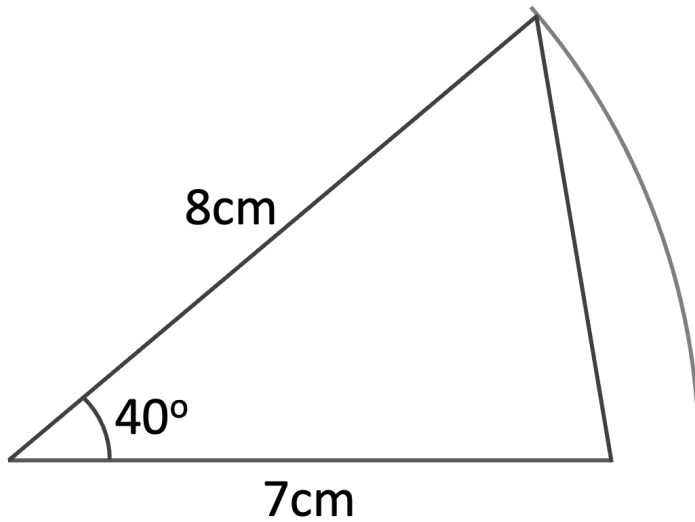
- A side length of 5 *cm*
- A side length of 3 *cm*
- A side length of 4 *cm*

## SAS

Using a ruler, compass and protractor, construct the following SAS triangle accurately.



- 1) Draw a  $7\text{cm}$  line with a ruler.
- 2) Draw an arc with length  $8\text{cm}$ .
- 3) Measure an angle of  $40^\circ$ .
- 4) Draw a line through the angle to the arc.
- 5) Join up the end of the lines.



## Worked Example

Construct a triangle with:

- A side length of 10 *cm*
- An angle of  $30^\circ$
- A side length of 8 *cm*

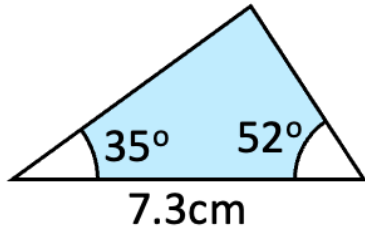
## Your Turn

Construct a triangle with:

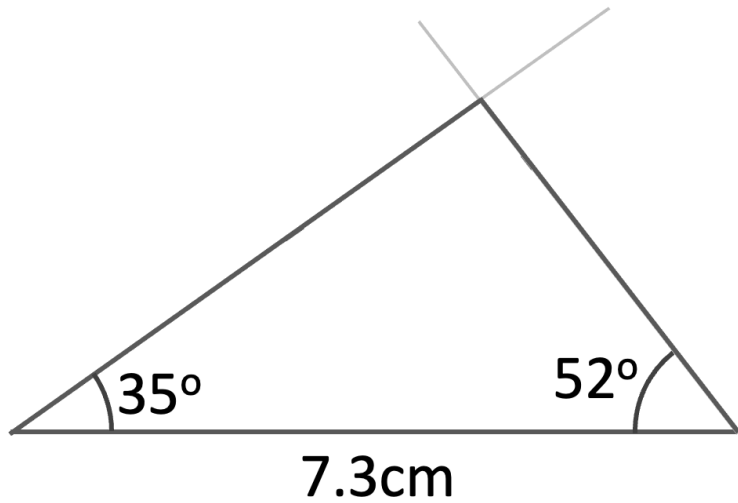
- A side length of 5 *cm*
- An angle of  $30^\circ$
- A side length of 4 *cm*

## ASA

Using a ruler, compass and protractor, construct the following ASA triangle accurately.



- 1) Draw a 7.3cm line with a ruler.
- 2) Measure both angles.
- 3) Draw a faint line through each angle and label them.
- 4) Draw a solid line over each faint line up to the intersection.





## Worked Example

Construct a triangle with:

- An angle of  $30^\circ$
- A side length of  $10\text{ cm}$
- An angle of  $45^\circ$

## Your Turn

Construct a triangle with:

- An angle of  $30^\circ$
- A side length of  $5\text{ cm}$
- An angle of  $60^\circ$

## Loci

# Fluency Practice

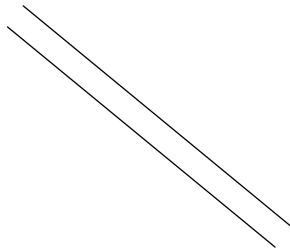
Complete as many of the following challenges as you can, as a group, making a note of the shapes you produce for each one. You will also be expected to demonstrate one of these shapes to the rest of the class.

1. In your group, stand **exactly 2m** from one member of your group.  
Draw and describe the shape you have created:



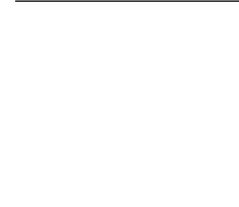
*This is the locus of points a fixed distance from a point.*

2. In your group, stand **exactly 1m** away from a straight wall.  
Draw and describe the shape you have created:



*This is the locus of points a fixed distance from a line.*

3. In your group, stand **exactly 2m** a wall around a corner.  
Draw and describe the shape you have created:



*This can give the locus of points a fixed distance from a rectangle.*

4. In your group, stand **exactly the same distance** away from two members of your group.  
Draw and describe the shape you have created:



*This is the locus of points equidistant from two fixed points.*

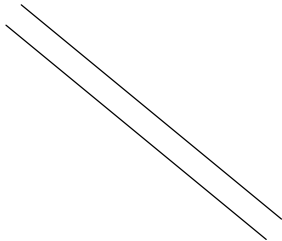
# Fluency Practice

5. In your group, stand **within 2m** of one member of your group.  
Draw and describe the area you have created:



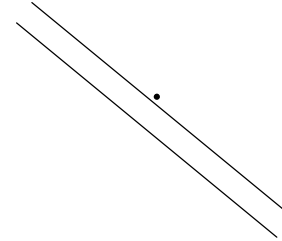
*This is the locus of points within a given distance of a point.*

6. In your group, stand **no further than 1m** away from a straight wall.  
Draw and describe the area you have created:



*This is the locus of points within a given distance of a line.*

7. In your group, stand **at least 1m** away from a straight wall, and **within 2m** of a person standing beside the wall.  
Draw and describe the area you have created:



*This is the locus of points which satisfy both conditions.*

8. Design your own conditions, either by combining those used in these challenges or creating new ones altogether.  
Draw and describe the area you have created:

## Worked Example

Construct the locus of points 1 *cm* away from a point.

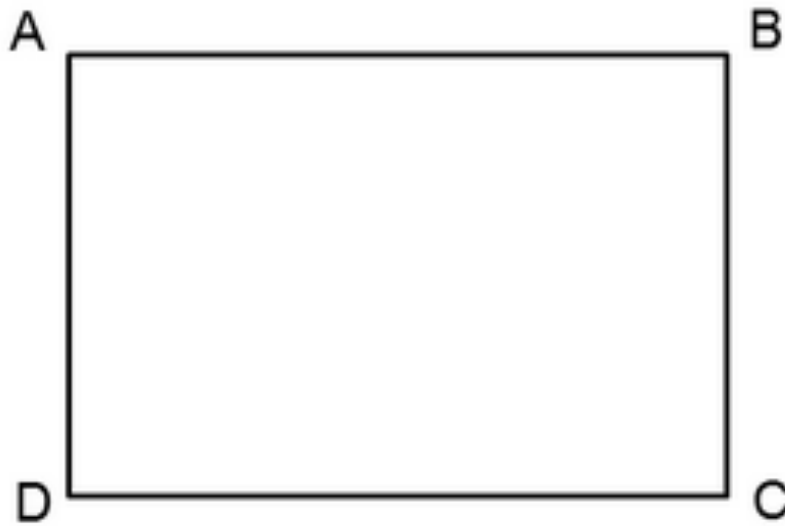
## Your Turn

Construct the locus of points 2 *cm* away from a point.

## Worked Example

Construct the locus of points which is:

- More than 3 *cm* from A

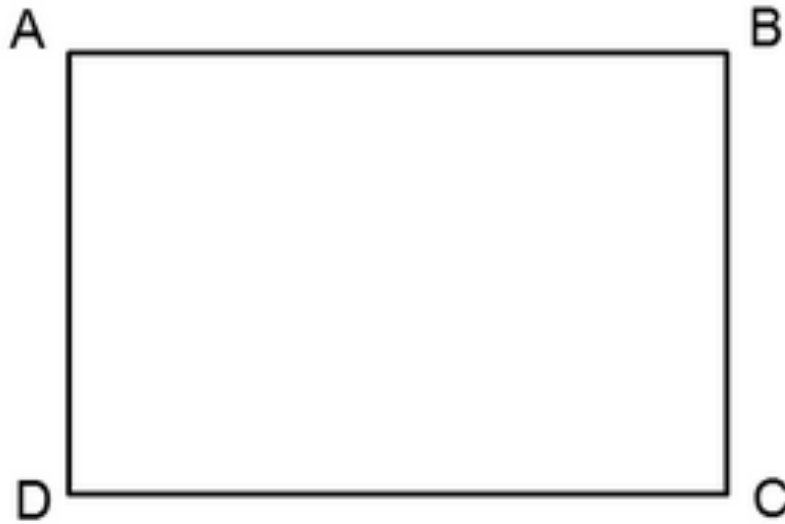




## Your Turn

Construct the locus of points which is:

- More than 5 *cm* from B



## Worked Example

Construct the locus of points equidistant from two points.

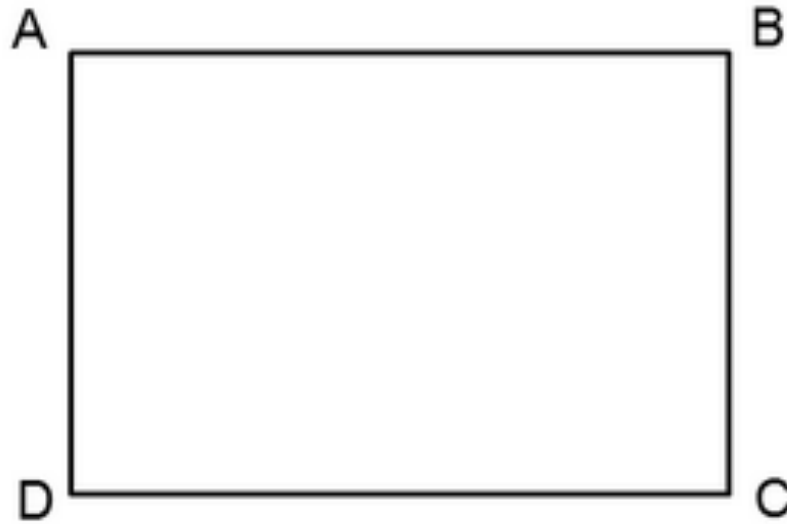
## Your Turn

Construct the locus of points equidistant from two points.

## Worked Example

Construct the locus of points which are:

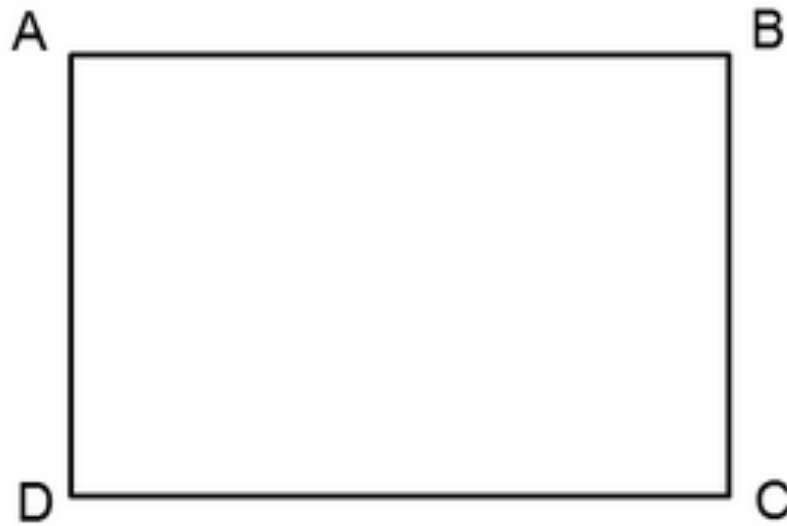
- Closer to B than A
- Closer to C than D



## Your Turn

Construct the locus of points which are:

- Closer to C than B
- Closer to D than A



## Worked Example

Construct the locus of points equidistant from two intersecting lines.

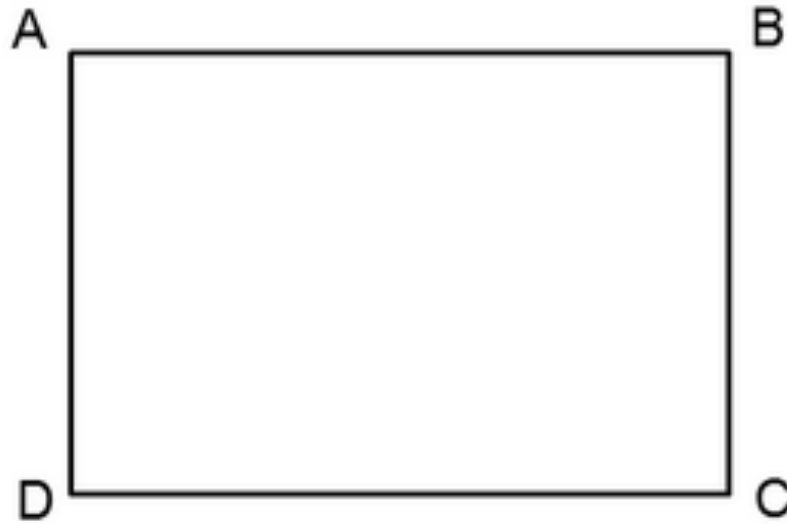
## Your Turn

Construct the locus of points equidistant from two intersecting lines.

## Worked Example

Construct the locus of points which is:

- Closer to AD than AB

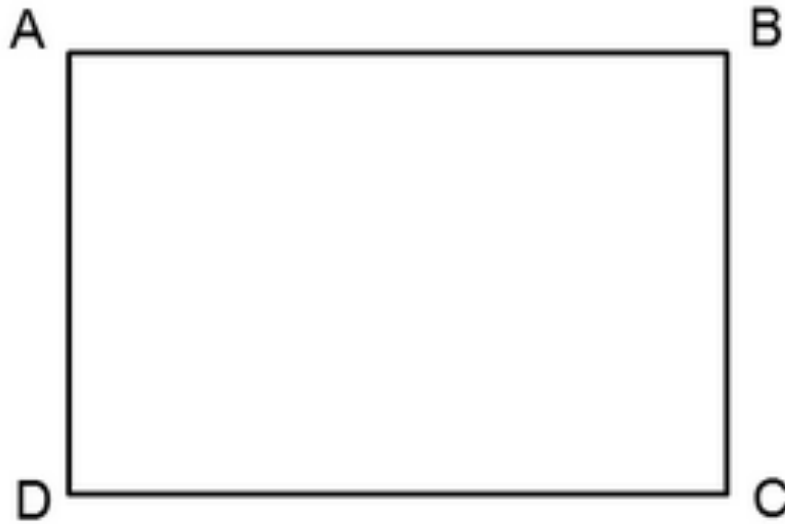




## Your Turn

Construct the locus of points which is:

- Closer to BC than DC



## Worked Example

Construct the locus of points 1 *cm* away from the line.



## Your Turn

Construct the locus of points 1 *cm* away from the line.



## Worked Example

Construct the locus of points equidistant from a line.

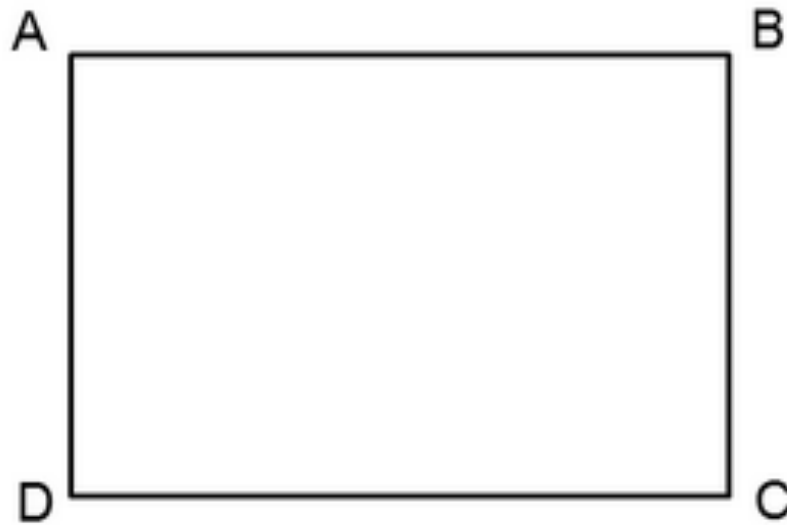
## Your Turn

Construct the locus of points equidistant from a line.

## Worked Example

Construct the locus of points which are:

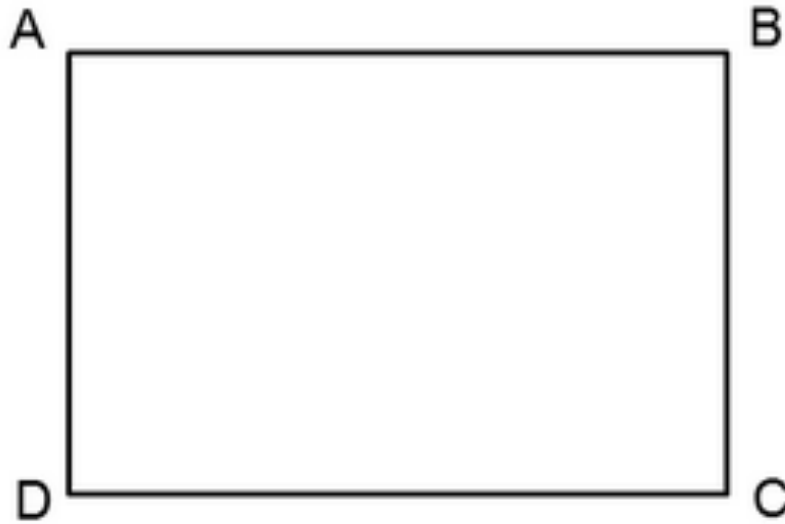
- More than 3 *cm* from AB
- More than 4 *cm* from AD



## Your Turn

Construct the locus of points which are:

- More than 5 *cm* from AB
- More than 3 *cm* from AD



## Worked Example

Construct the locus of points which are:

- Closer to B than C
- More than 3 *cm* from A





## Your Turn

Construct the locus of points which are:

- Closer to C than A
- Less than 5 *cm* from B

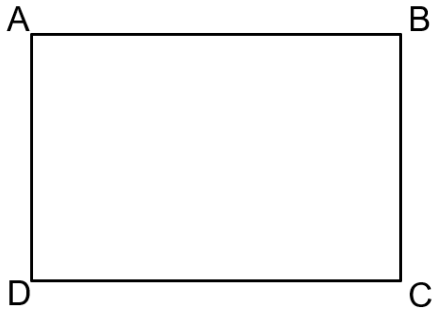


# Fluency Practice

**Loci and Regions** – Shade the region inside the rectangle which satisfies the conditions given.

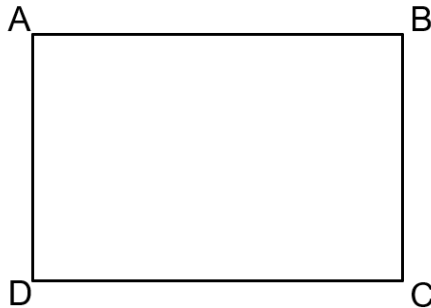
**(a)**

Less than 3 cm from A



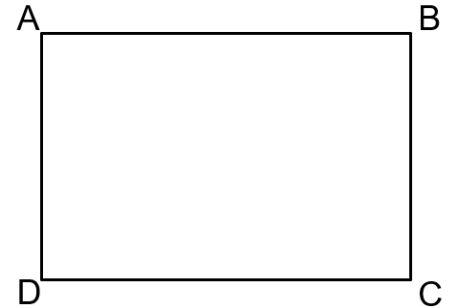
**(b)**

More than 3 cm from D



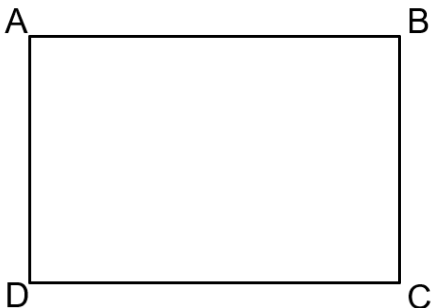
**(c)**

More than 2 cm from AB



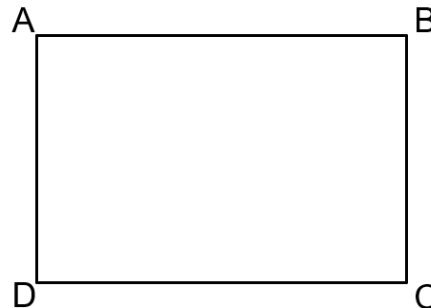
**(d)**

Less than 3 cm from BC



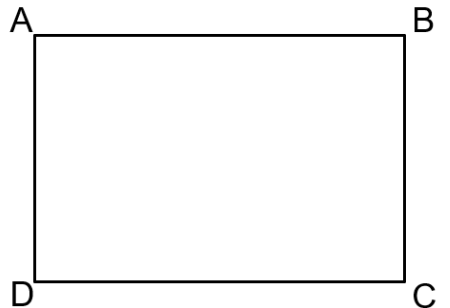
**(e)**

Closer to AB than AD



**(f)**

Closer to A than B

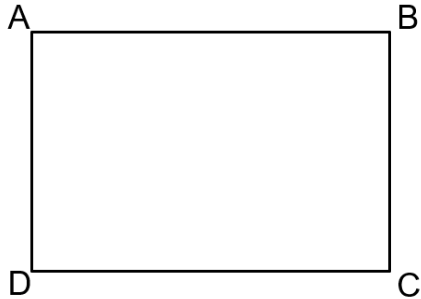


# Fluency Practice

**Harder Loci and Regions** – Shade the region inside the rectangle which satisfies the conditions given.

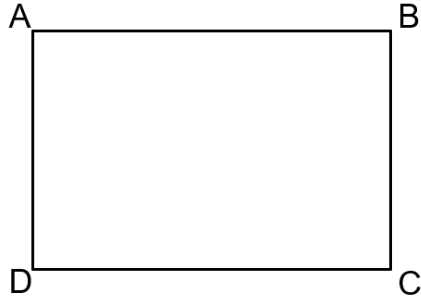
**(a)**

Less than 3 cm from A and more than 2 cm from AB



**(b)**

More than 3 cm from B and closer to AB than BC



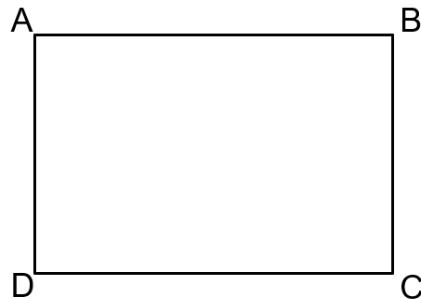
**(c)**

Closer to AD than AB and less than 2 cm from AB



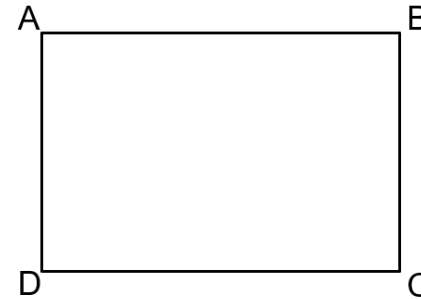
**(d)**

Less than 3 cm from BC and less than 2 cm from CD



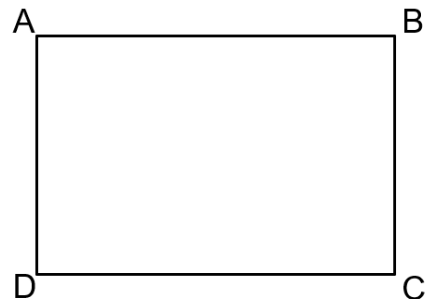
**(e)**

Closer to A than B and more than 4 cm from A



**(f)**

More than 3 cm from D and more than 4 cm from B

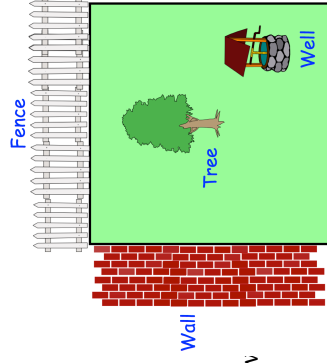


# Fluency Practice

\*There is a template for each question at the end of this exercise

## Question 1:

The diagram shows Lindsey's garden. Lindsey is deciding where to place a bench



Show the possible positions of the bench for each rule below

- (a) The bench is 1 metre away from the wall.
- (b) The bench is less than 2 metres away from the fence.
- (c) The bench is 2 metres away from the tree.
- (d) The bench is less than 2.5 metres away from the tree.
- (e) The bench is an equal distance (equidistant) from the tree and the well.
- (f) The bench is closer to the well than the tree.
- (g) The bench is an equal distance from the wall and the fence
- (h) The bench is closer to the wall than the fence.
- (i) The bench is less than 2 metres from the wall **and** less than 3 metres from the tree
- (j) The bench is closer to the fence than the wall **and** more than 1 metre from the wall.

## Question 2:

Shown is a map of an island.

Show the possible position of each item.

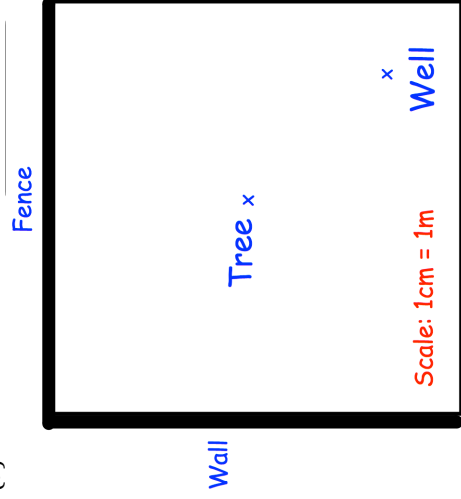


- (a) The stadium is equidistant from Redville and Newtown
- (b) A ship is less than 2 miles from the harbour
- (c) A helicopter is 6 miles from the harbour **and** 4 miles from the airport.
- (d) A church is closer to Redville than Newtown **and** is more than 3 miles from the airport
- (e) A lighthouse is equidistant from Newtown and the harbour **and** is on the coast.
- (f) A windmill is less than 4 miles from Redville **and** less than 4 miles from Newtown **and** less than 4 miles from the Airport.
- (g) A hot air balloon is between 5 miles and 7 miles from the harbour
- (h) A farm is equidistant from the airport and Redville **and** equidistant from the airport and the harbour.

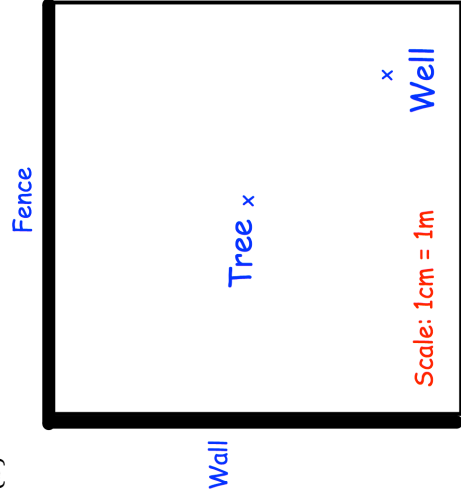
# Templates

Question 1:

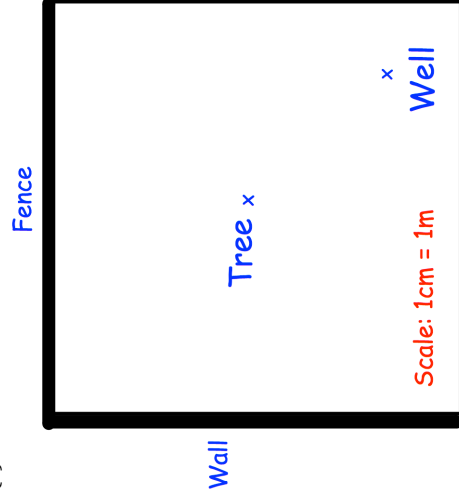
(a)



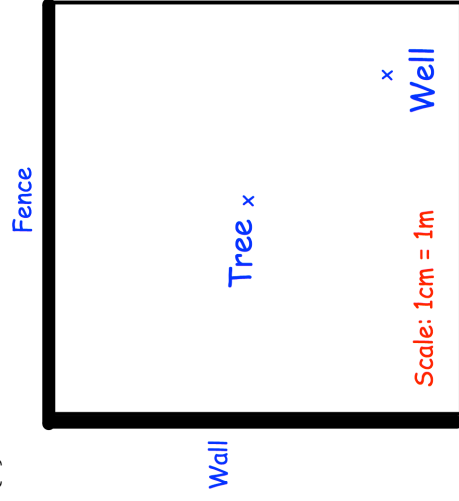
(b)



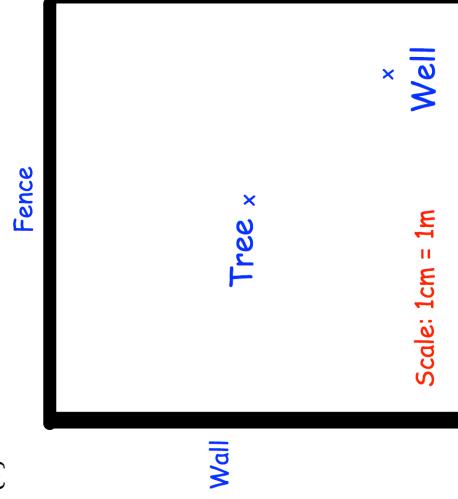
(c)



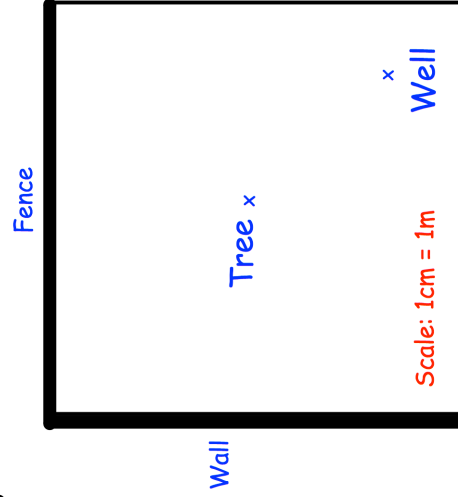
(d)



(e)

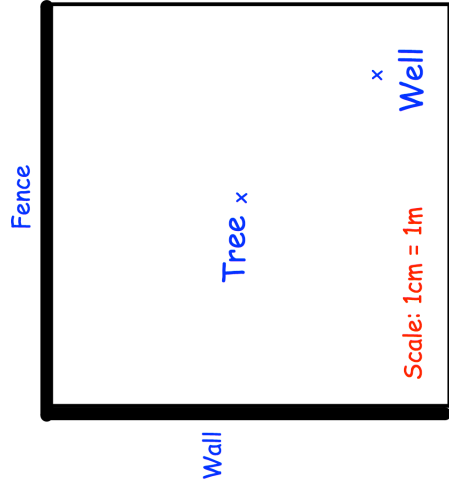


(f)

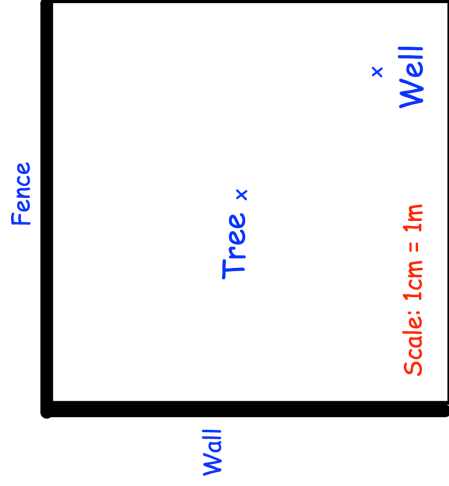


# Templates

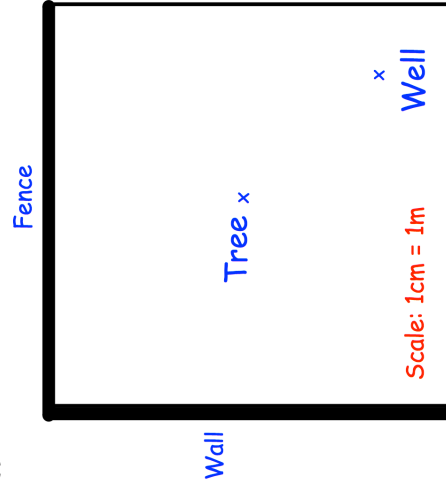
(g)



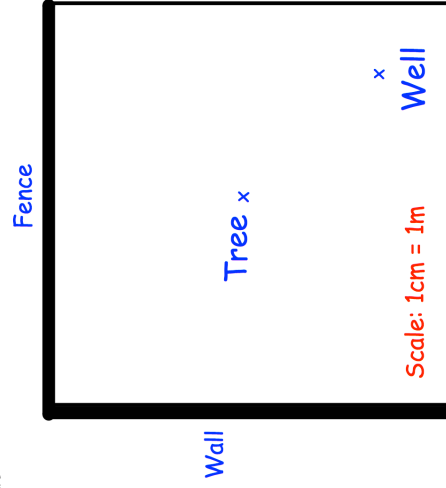
(h)



(i)

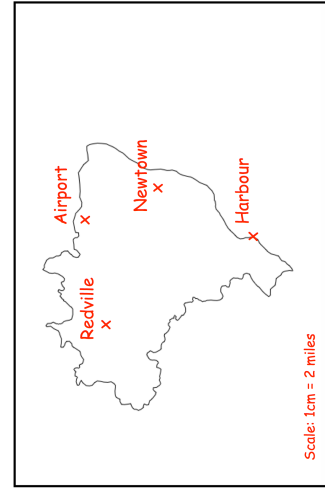


(j)

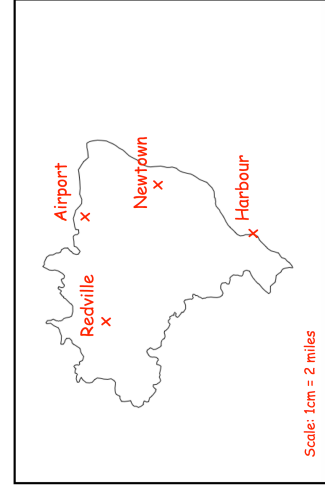


Question 2

(a)

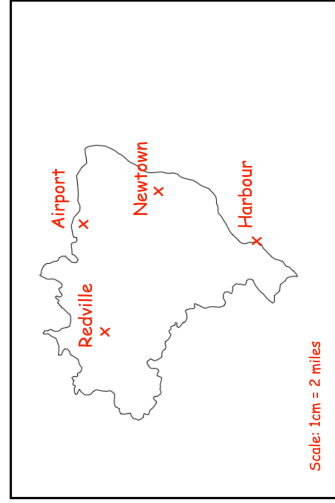


(b)

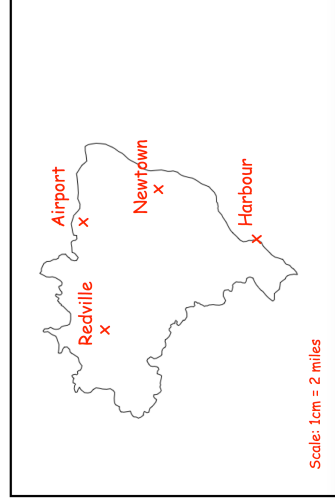


# Templates

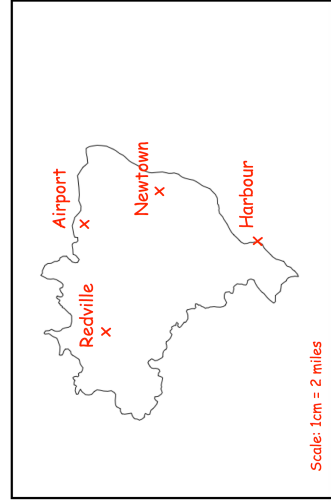
(c)



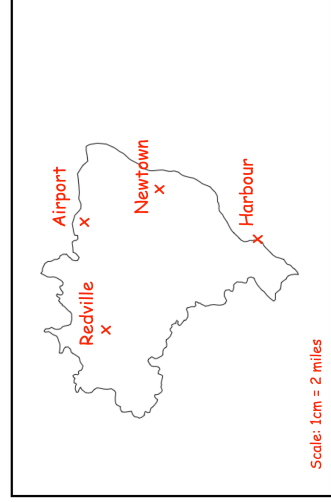
(d)



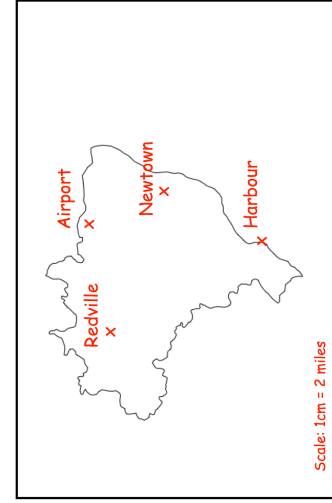
(e)



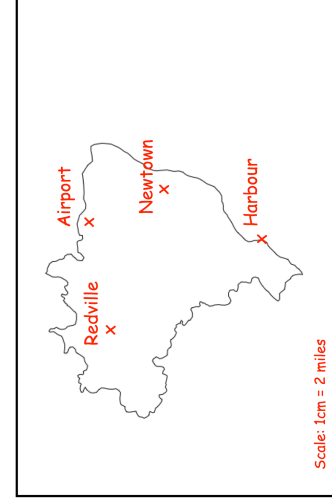
(f)



(g)



(h)



## Extra Notes