



KING EDWARD VI  
HANDSWORTH GRAMMAR  
SCHOOL FOR BOYS



KING EDWARD VI  
ACADEMY TRUST  
BIRMINGHAM

# Year 10

## 2024 Mathematics 2025

### Unit 16 Booklet – Part 1

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_



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# Year 10

## 2024 Mathematics 2025

### Unit 16 Booklet – Part 2

HGS Maths



Tasks



Dr Frost Course



Name: \_\_\_\_\_

Class: \_\_\_\_\_

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# 1 Recurring Decimals

## Worked Example

Express as a decimal:

a)  $\frac{2}{9}$

b)  $\frac{2}{11}$

c)  $\frac{2}{15}$

## Your Turn

Express as a decimal:

a)  $\frac{8}{9}$

b)  $\frac{8}{11}$

c)  $\frac{4}{15}$

## Activity

For each of the following fractions, use your calculator to convert it to a decimal, then decide whether it is terminating or recurring. Now find the denominator as a product of its prime factors. Can you spot any patterns?

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$			
$\frac{1}{7}$			
$\frac{1}{8}$			
$\frac{1}{9}$			
$\frac{1}{10}$			
$\frac{1}{11}$			

Fraction	Decimal using Calculator	Terminating or Recurring	Denominator as Product of Prime Factors
$\frac{1}{12}$			
$\frac{1}{13}$			
$\frac{1}{14}$			
$\frac{1}{15}$			
$\frac{1}{16}$			
$\frac{1}{17}$			
$\frac{1}{18}$	0.0 $\dot{5}$	Recurring	$2 \times 3 \times 3$
$\frac{1}{19}$			
$\frac{1}{20}$			
$\frac{1}{21}$			

## Activity

	Fraction	Factorised	The Law of Cancellation	Simplest Form	Factors of Denominator	Kind of Decimal
(i)	$\frac{8}{12}$					
(ii)	$\frac{3}{16}$					
(iii)	$\frac{9}{27}$					
(iv)	$\frac{12}{30}$					
(v)	$\frac{7}{32}$					
(vi)	$\frac{15}{21}$					
(vii)	$\frac{3}{10}$					
(viii)	$\frac{3}{18}$					
(ix)	$\frac{6}{33}$					
(x)	$\frac{3}{75}$					

### Worked Example

Express as a simplified fraction:

$0.\dot{4}$

### Your Turn

Express as a simplified fraction:

$0.\dot{7}$



**Worked Example**

Express as a simplified fraction:

$0.\dot{5}\dot{4}$

**Your Turn**

Express as a simplified fraction:

$0.\dot{2}\dot{7}$

**Worked Example**

Express as a simplified fraction:

$0.\dot{2}7\dot{9}$

**Your Turn**

Express as a simplified fraction:

$0.\dot{8}3\dot{7}$

### Worked Example

Express as a simplified fraction:

$0.\overline{789}$

### Your Turn

Express as a simplified fraction:

$0.5\overline{79}$

**Worked Example**

Express as a simplified fraction:

$3.\overline{7654}$

**Your Turn**

Express as a simplified fraction:

$7.\overline{5309}$

## Fill in the Gaps

$x$ as recurring decimal	Write out multiples of $x$	Subtract	$x$ as a fraction
$x = 0.\dot{7}$	$10x = 7.\dot{7} = 7.77777 \dots$ $x = 0.\dot{7} = 0.77777 \dots$	$9x = 7$	$x = \frac{7}{9}$
$x = 0.\dot{2}$	$10x =$ $x =$		
$x = 0.\dot{3}\dot{5}$	$100x = 35.\dot{3}\dot{5} = 35.3535 \dots$ $x = 0.\dot{3}\dot{5} = 0.3535 \dots$	$99x = 35$	
$x = 0.\dot{4}\dot{1}$	$100x =$ $x =$		
$x = 0.\dot{2}\dot{7}$			
$x = 0.\dot{6}1\dot{3}$	$1000x =$		
$x = 0.0\dot{2}$	$100x = 2.\dot{2} = 2.22222 \dots$ $10x =$		
$x = 0.1\dot{4}\dot{3}$			
$x = 0.9\dot{3}\dot{2}$			
$x = 0.9\dot{3}\dot{2}$			
$x = 0.0\dot{0}\dot{5}$			

### Worked Example

Write the fraction  $0.1\dot{3}\dot{6} \times 0.\dot{5}$  as a fraction in its simplest form

### Your Turn

Write the fraction  $0.6\dot{8}\dot{1} \times 0.\dot{1}$  as a fraction in its simplest form

## Extra Notes

## 3 Parallel and Perpendicular Lines



## Parallel Lines

### Worked Example

- a) Write down the equation of a line parallel to  $y = 2x - 3$
- b) Write down the equation of the line that is parallel to  $y = 6x + 1$  and passes through  $(0, 8)$

### Your Turn

- a) Write down the equation of a line parallel to  $y = -2x + 3$
- b) Write down the equation of the line that is parallel to  $y = -6x - 1$  and passes through  $(0, -8)$

### Worked Example

Write down the equation parallel to  $y = 4x + 1$  which passes through  $(2, 17)$

### Your Turn

Write down the equation parallel to  $y = 8x + 5$  which passes through  $(2, 26)$

### Worked Example

Find the equation of the line parallel to  $y = -\frac{1}{3}x - 4$  that passes through  $(-2, 5)$

### Your Turn

Find the equation of the line parallel to  $y = -\frac{1}{2}x - 3$  that passes through  $(-2, 5)$

## Perpendicular Lines

## Worked Example

Write the negative reciprocals of:

- a) 6
- b)  $\frac{1}{6}$
- c)  $\frac{5}{6}$
- d)  $1\frac{5}{6}$
- e) 1.2

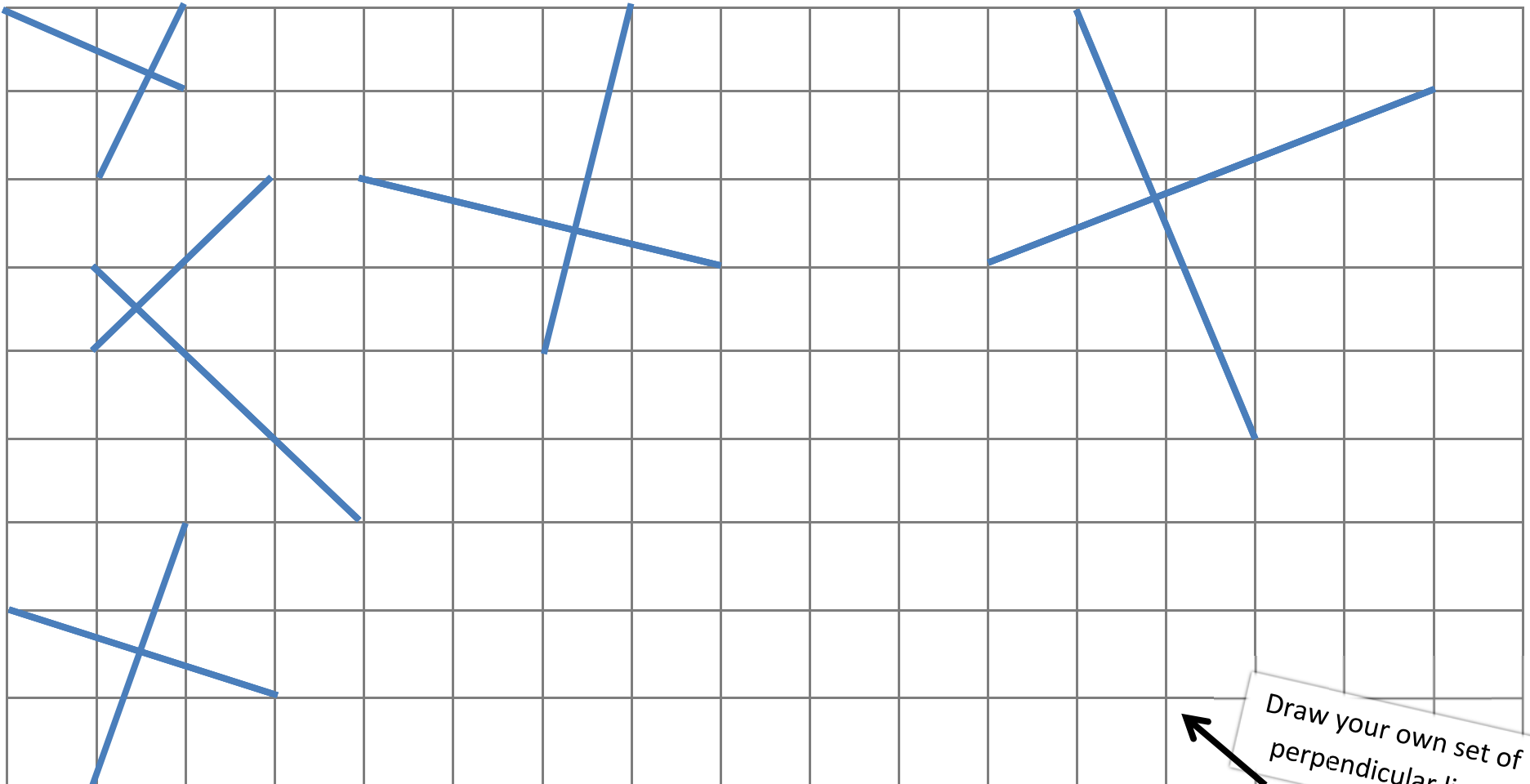
## Your Turn

Write the negative reciprocals of:

- a) 7
- b)  $\frac{1}{7}$
- c)  $\frac{2}{7}$
- d)  $1\frac{2}{7}$
- e) 3.5

# Fluency Practice

Calculate the gradient of each of these pairs of perpendicular lines. Simplify your answers. What do you notice?



Draw your own set of perpendicular lines

### Worked Example

- a) Write down the equation of a line perpendicular to  $y = 2x - 3$
- b) Write down the equation of the line that is perpendicular to  $y = \frac{1}{2}x + 3$  and passes through  $(0, -1)$

### Your Turn

- a) Write down the equation of a line perpendicular to  $y = -2x + 3$
- b) Write down the equation of the line that is perpendicular to  $y = -\frac{1}{2}x + 3$  and passes through  $(0, 1)$



### Worked Example

Find the equation of the line perpendicular to  $y = \frac{1}{2}x - 4$  that passes through  $(-2, 5)$

### Your Turn

Find the equation of the line perpendicular to  $y = -\frac{4}{3}x + 3$  that passes through  $(-12, -5)$

### Worked Example

Write down the equation perpendicular to  $y = 4x + 1$  which passes through  $(8, 17)$

### Your Turn

Write down the equation perpendicular to  $y = 8x + 5$  which passes through  $(16, 26)$

### Worked Example

Find the equation of the line perpendicular to  $3x + 2y = 5$  which passes through the point  $(3, 7)$

### Your Turn

Find the equation of the line perpendicular to  $2x + 3y = 5$  which passes through the point  $(4, 7)$

### Worked Example

The line  $L_1$  has equation  $-3x + 4y = 8$

The line  $L_2$  has equation  $4x + 3y = -5$

Determine whether  $L_1$  and  $L_2$  are perpendicular.

### Your Turn

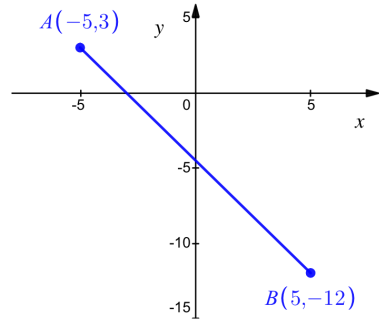
The line  $L_1$  has equation  $-2x + y = 6$

The line  $L_2$  has equation  $-3x + 2y = 4$

Determine whether  $L_1$  and  $L_2$  are perpendicular.

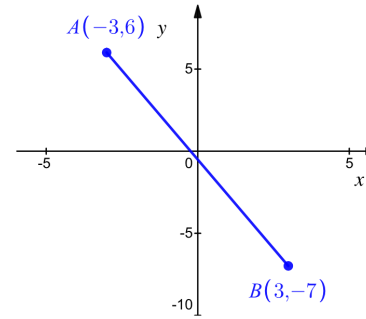
## Worked Example

Find the midpoint of the line segment  $AB$  where  $A(-5, 3)$  and  $B(5, -12)$ . The line segment is plotted below.



## Your Turn

Find the midpoint of the line segment  $AB$  where  $A(-3, 6)$  and  $B(3, -7)$ . The line segment is plotted below.



### Worked Example

Find the midpoint of the line segment between  $(-2,4)$  and  $(-9,9)$

### Your Turn

Find the midpoint of the line segment between  $(2, -4)$  and  $(11,8)$

### Worked Example

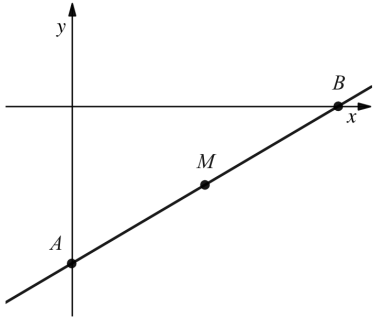
$X$  is the point  $(-10, a)$   
 $Y$  is the point  $(0, -8)$   
 $M$  is the point  $(b, -4)$   
 $M$  is the midpoint of  $XY$ .  
Find the value of  $a$  and the value of  $b$ .

### Your Turn

$X$  is the point  $(-6, a)$   
 $Y$  is the point  $(2, -2)$   
 $M$  is the point  $(b, 2)$   
 $M$  is the midpoint of  $XY$ .  
Find the value of  $a$  and the value of  $b$ .

## Worked Example

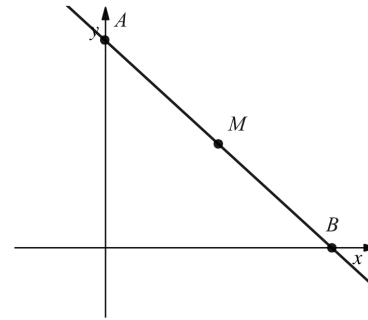
A sketch of the line with equation  $3x - 8y = 24$  is shown below. The line passes through the points  $A$ ,  $B$  and  $M$ , where  $M$  is the midpoint of the line  $AB$ .



Calculate the coordinates of  $M$ .

## Your Turn

A sketch of the line with equation  $6x + 5y = 30$  is shown below. The line passes through the points  $A$ ,  $B$  and  $M$ , where  $M$  is the midpoint of the line  $AB$ .



Calculate the coordinates of  $M$ .



## Fill in the Gaps

Fill in the missing information in the table.

Point A	Point B	<u>Midpoint</u> of the line segment AB	<u>Length</u> of the line segment AB	<u>Gradient</u> of the line segment AB	<u>Equation</u> of the line through A and B.
(1,3)	(5,11)				
(-3,2)	(5,-6)				
$\left(\frac{-7}{3}, \frac{-22}{3}\right)$	$\left(\frac{11}{3}, \frac{-4}{3}\right)$				
	(-7,11)	$\left(-11, \frac{7}{2}\right)$			
	(-2,-4)		$2\sqrt{5}$		$x + 2y + 10 = 0$
		(4,1)	20	$\frac{3}{4}$	
(4,1)			$4\sqrt{13}$		$2x + 3y - 11 = 0$

**To consider:**

- Which of these have multiple possible answers?
- If you were not given either point A or point B, what is the minimum information required to complete the row?

### Worked Example

The point  $M$  lies on the line segment  $AB$  where  $A(-1, -3)$  and  $B(8, 3)$ .  
Given that  $AM : MB = 2 : 1$ , find the coordinates of  $M$ .

### Your Turn

The point  $M$  lies on the line segment  $AB$  where  $A(-5, -1)$  and  $B(7, 3)$ .  
Given that  $AM : MB = 3 : 1$ , find the coordinates of  $M$ .

## Fill in the Gaps

Ratio $AX:XB$	Point $A$	Point $X$	Point $B$
2 : 1	(2,4)		(8,16)
4 : 2	(2,4)		(8,16)
1 : 2	(2,4)		(8,16)
1 : 2	(2,4)	(8,16)	
1 : 2		(2,4)	(8,16)
	(2,4)	(8,16)	(26,52)
	(26,52)	(8,16)	(2,4)
	(13,26)	(4,8)	(1,2)
3 : 1		(4,7)	(0,1)
4 : 1		(4,7)	(0,1)
5 : 1		(4,7)	(0,1)
1 : 1		(4,7)	(0,1)

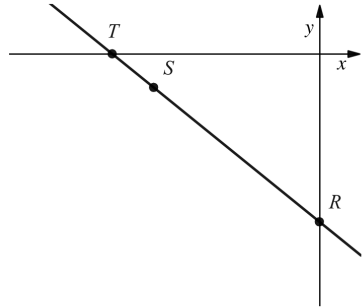
## Fill in the Gaps

Point C divides the line segment AB in the given ratio.

Point A	Point B	AC : CB	Point C	Midpoint of AB
(0,0)	(3,6)	2 : 1		(1.5, 3)
(1,1)	(7,4)	1 : 2		
(10,5)	(0,0)	4 : 1		
(0,0)	(10,5)	2 : 3		
(-1,0)	(11,8)	3 : 1		
(4,7)	(8,-5)	1 : 3		
(2.3, -5.1)	(4.8, 2.4)	3 : 2		
(9,0)	(-5,-7)	5 : 2		
(0,0)		3 : 1		(2,4)
	(5,2)	1 : 2		(3.5, 0.5)
(-1,6)		3 : 2	(5,3)	
(11,-5)	(-3,2)		(5,-2)	
(0,2a)		2 : 1	(-4a, 4a)	
$(-\frac{9}{10}, \frac{2}{3})$	$(\frac{1}{2}, 5\frac{1}{3})$		$(-\frac{1}{10}, \frac{10}{3})$	
		2 : 1	(6,-3)	(4,-2)
		2 : 5	$(\frac{11b}{14}, -2b)$	(b, b)

## Worked Example

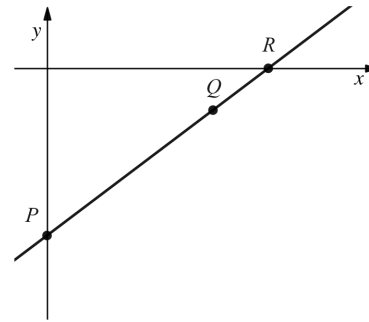
A sketch of  $2x + 3y = -30$  is shown below.  
The line passes through the points  $R$ ,  $S$  and  $T$ , where  $RS : ST$  is in the ratio  $4 : 1$ .



Work out the coordinates of  $S$ .

## Your Turn

A sketch of  $3x - 5y = 60$  is shown below.  
The line passes through the points  $P$ ,  $Q$  and  $R$ , where  $PQ : QR$  is in the ratio  $3 : 1$ .



Work out the coordinates of  $Q$ .

### Worked Example

$PQR$  is a straight line.  
 $P$  has coordinates  $(a, 1)$   
 $Q$  has coordinates  $(5a, 4)$   
 $R$  has coordinates  $(45, b)$   
 $PQ : QR$  is  $2 : 5$   
Find  $a$  and  $b$

### Your Turn

$PQR$  is a straight line.  
 $P$  has coordinates  $(3, b)$   
 $Q$  has coordinates  $(6, 9b)$   
 $R$  has coordinates  $(a, 7)$   
 $PQ : QR$  is  $2 : 3$   
Find  $a$  and  $b$

## Fill in the Gaps

Q	$a$	$b$	$m$	$a$ to $m$ : $m$ to $b$	$a$ to $m$ : $a$ to $b$	$m$ is $\frac{?}{?}$ along the line segment $ab$
1	(1, 5)	(13, 11)		1 : 1		
2		(12, 10)	(7, 8)		1 : 2	
3	(2, 6)		(8, 9)			$\frac{1}{2}$
4	(2, 6)	(14, 12)		1 : 2		
5	(2, 6)	(14, 12)	(10, 10)			
6		(14, 12)	(9, 9)			$\frac{2}{3}$
7		(14, 12)	(9, 9)		3 : 4	
8	(-6, 0)		(9, 9)			$\frac{3}{5}$
9	(-6, 0)	(14, 10)		3 : 2		
10	(-6, 0)	(26, 16)	(6, 6)			
11	(-6, 0)		(-6, -6)			$\frac{3}{8}$
12	(-12, 0)	(-12, -32)	(-12, -12)			

### Worked Example

$A$  is the point  $(3, 8)$   
 $B$  is the point  $(1, -2)$   
 $C$  is the midpoint of  $AB$

Find the equation of the line perpendicular to  $AB$  which passes through  $C$

### Your Turn

$A$  is the point  $(3, 8)$   
 $B$  is the point  $(1, 4)$   
 $C$  is the midpoint of  $AB$

Find the equation of the line perpendicular to  $AB$  which passes through  $C$



## Worked Example

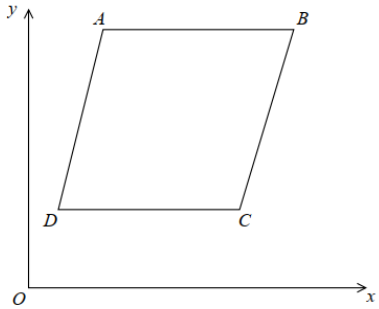
ABCD is a rhombus.

A has coordinates (5, 10)

The equation of  $DB$  is

$$y = \frac{1}{2}x + 5$$

Find an equation of diagonal  $AC$



## Your Turn

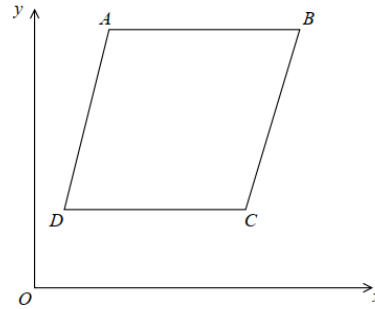
ABCD is a rhombus.

A has coordinates (5, 11)

The equation of  $DB$  is

$$y = \frac{1}{2}x + 6$$

Find an equation of diagonal  $AC$



## Fill in the Gaps

Equation	Point on the Line (1)	Point on the Line (2)	Gradient	y intercept	The parallel line that goes through (2, 5)	Gradient of all perpendicular lines
$y = 2x + 8$						
$y = 4x - 1$						
	(1, 5)	(3, 11)				
	(5, 9)	(8, 12)				
	(4, 6)	(6, 2)				
	(4, 3)		-3			
	(2, 9)		6			
	(-1, 2)		3			
	(2, 10)			(0, 4)		
	(3, 11)				$y = 5x - 5$	
	(4, 3)					-2

## Fill in the Gaps

Equation of line	Point on the line (1)	Point on the line (2)	Gradient	y - intercept	x - intercept	Gradient of a perpendicular line
$y = 2x + 1$	$(-2, \square)$	$(2, \square)$				
$y = -1 - x$	$(\square, 1)$	$(\square, -2)$				
	$(4, -4)$	$(4, 3)$				
	$(-1, 5)$	$(2, -4)$				
	$(-3, -2)$	$(-8, -2)$				
	$(-3, 5)$	$(3, \square)$	$-\frac{4}{3}$			
	$(4, 0)$	$(\square, -6)$	$\frac{3}{4}$			
$2y = 3x - 5$	$(1, \square)$	$(3, \square)$				
$3y = 4x - 7$	$(\square, -1)$	$(\square, 3)$				

## Fill in the Gaps

$(x_1, y_1)$	$(x_2, y_2)$	Gradient $m$	Perpendicular Gradient	Midpoint of Line	Length of Line	Equation of Line
(0, 3)	(2, 7)	$\frac{7-3}{2-0} = 2$	$-\frac{1}{2}$	(1, 5)	$\sqrt{2^2 + 4^2}$ $= 4.47$	$y = 2x + 3$
(0, 2)	(4, 14)		$-\frac{1}{3}$			
(0, 5)	(3, 8)				$\sqrt{3^2 + 3^2}$ $= 4.24$	
(2, 1)	(0, 9)					
(3, 6)	(1, 10)					
(3, 3)	(2, -1)					
(3, 7)	(6, 8)					
(5, 11)				(4, 9)		
	(2, 9)		1		$\sqrt{8}$	

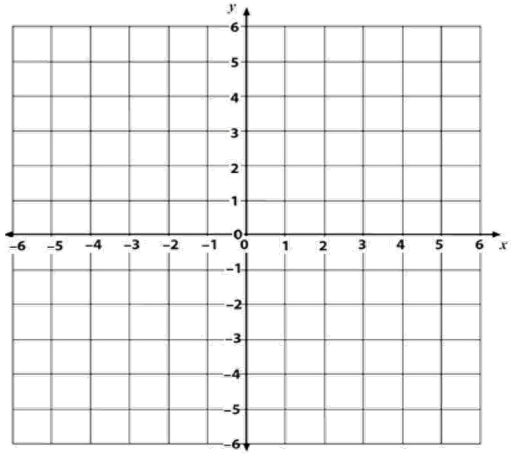
## Extra Notes

## 4 Graphical Inequalities

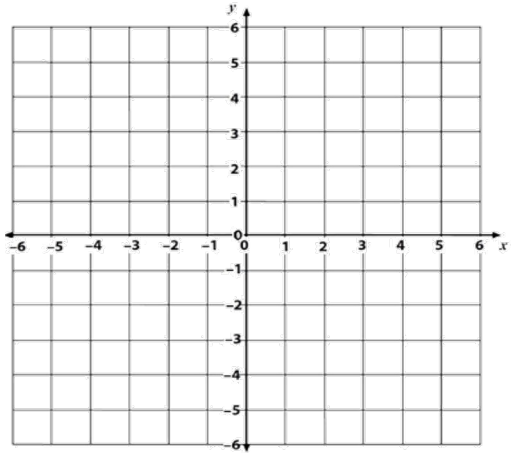
## Worked Example

Shade the region which satisfies the inequality:

a)  $x > 3$



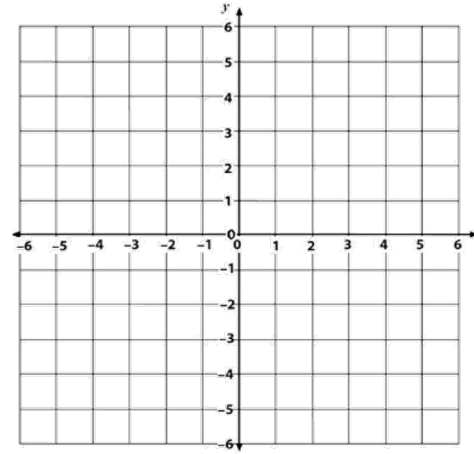
b)  $y \leq -2$



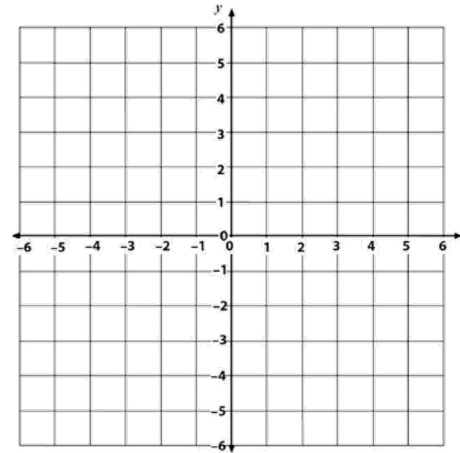
## Your Turn

Shade the region which satisfies the inequality:

a)  $x < 5$

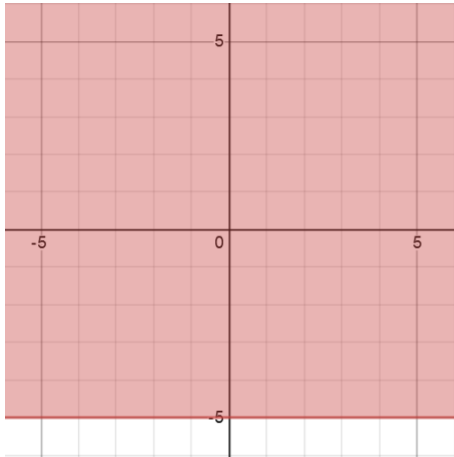
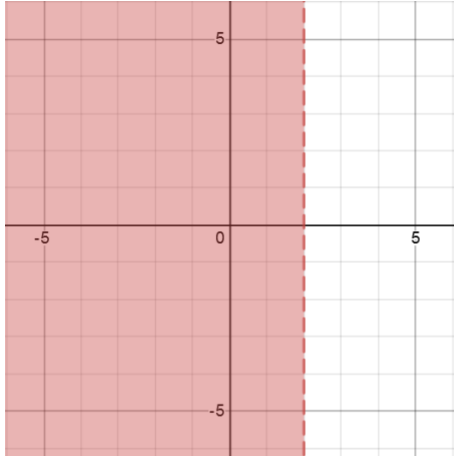


b)  $y \geq -4$



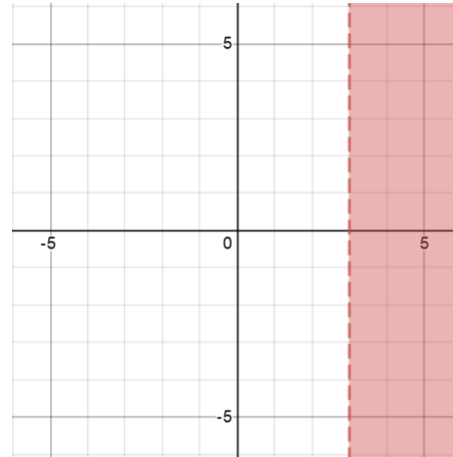
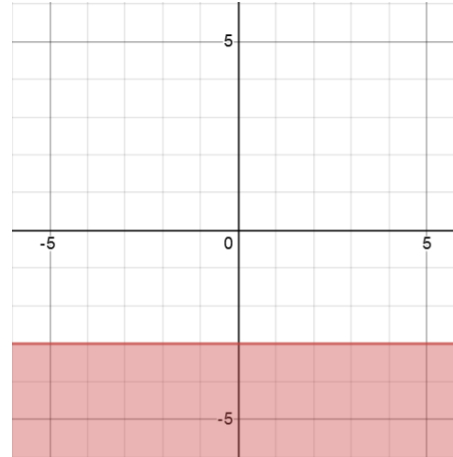
## Worked Example

Write the inequality that defines the red region:



## Your Turn

Write the inequality that defines the red region:

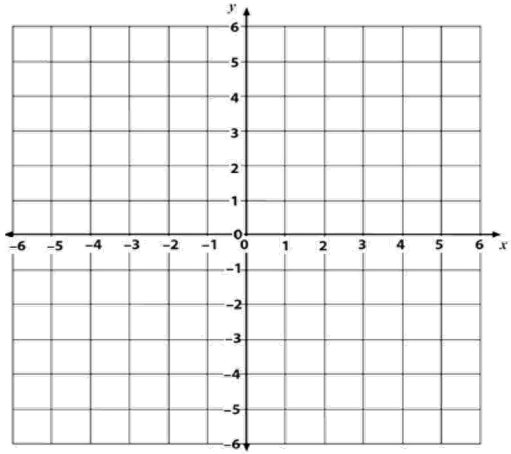




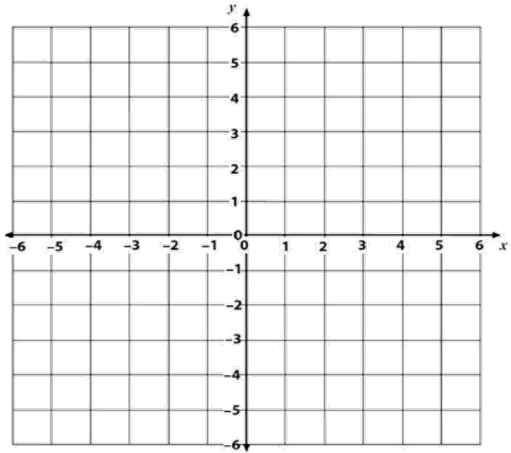
## Worked Example

Shade the region which satisfies the inequality:

a)  $-2 \leq x < 5$



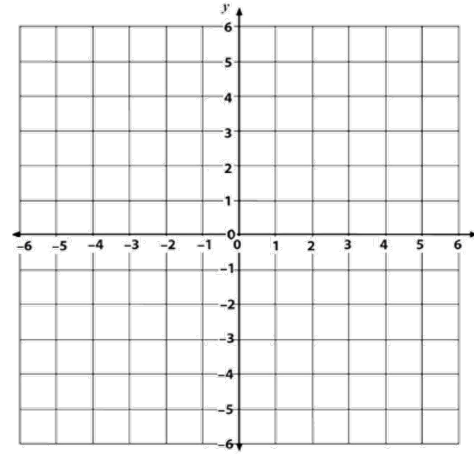
b)  $-2 \leq y < 5$



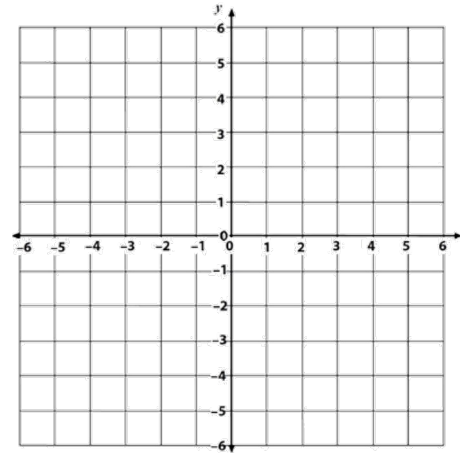
## Your Turn

Shade the region which satisfies the inequality:

a)  $-4 < x \leq 3$



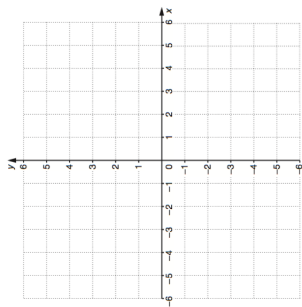
b)  $-4 < y \leq 3$



# Fluency Practice

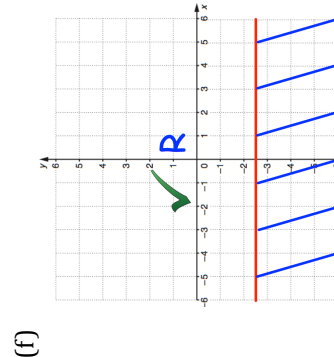
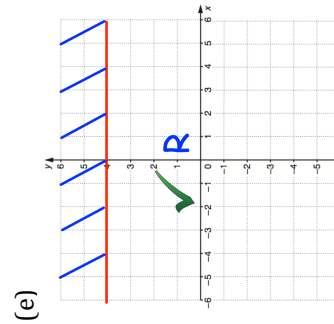
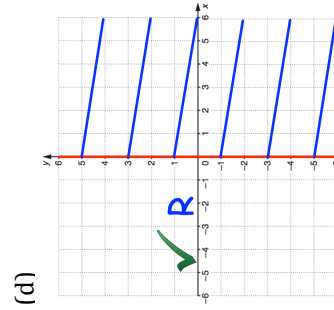
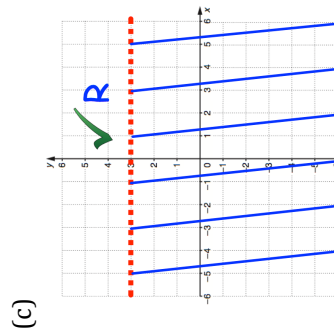
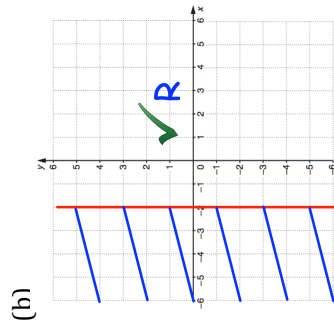
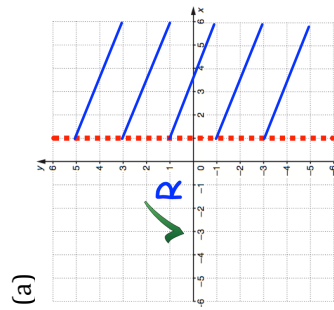
\*There are templates for questions 1, 3, and 5 at the end of this exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.



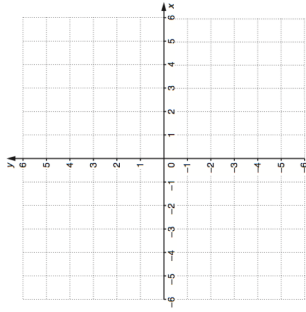
- (a)  $x > 2$     (b)  $x < 4$     (c)  $x \leq -1$     (d)  $x > 0$
- (e)  $x \geq -3$     (f)  $y < 1$     (g)  $y \geq -2$     (h)  $y \leq 4$
- (i)  $y > 2$     (j)  $x \geq 3$     (k)  $y < 0$     (l)  $x < -5$

Question 2: Write down the inequality represented in each diagram below.



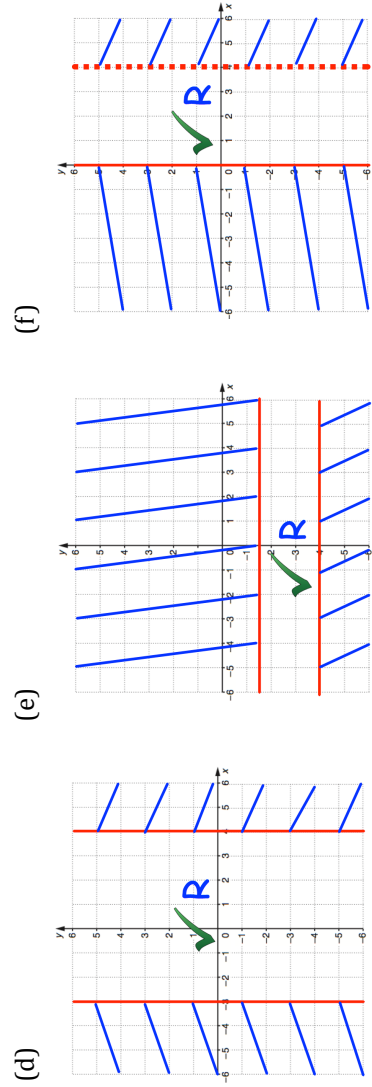
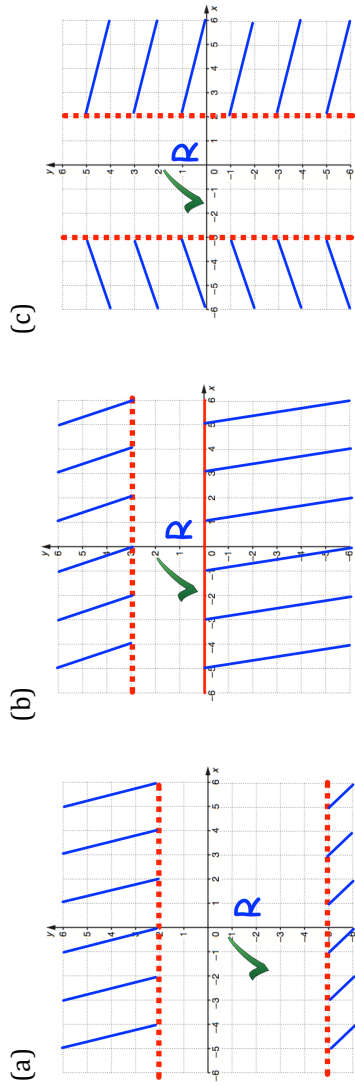
# Fluency Practice

Question 3: On copies of the grid below, clearly indicate the region that satisfies each inequality.



- (a)  $-4 < x < 1$
- (b)  $0 \leq x \leq 5$
- (c)  $-3 \leq x < 3$
- (d)  $-5 \leq y \leq -2$
- (e)  $-1 < y < 4$
- (f)  $-1 < y \leq 2.5$
- (g)  $-2 < x \leq 3$
- (h)  $-4 \leq y \leq 2$
- (i)  $-2 \leq y < 2$

Question 4: Write down the inequality represented in each diagram below.

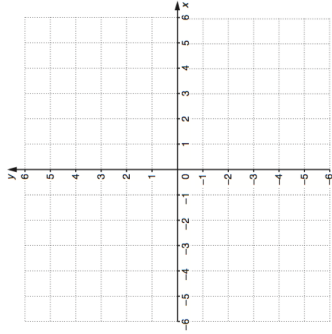


Question 5: On a grid, clearly indicate the region that satisfies the following inequalities.

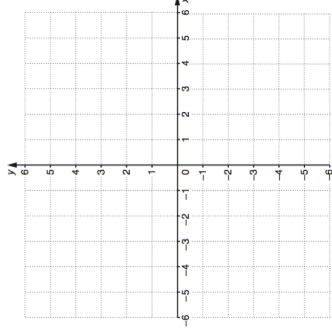
- (a)  $-2 < x < 3$  and  $y \geq -1$
- (b)  $-5 \leq y \leq 1$  and  $x < 3$
- (c)  $1 < x \leq 3$  and  $-2 \leq y < 0$

# Templates

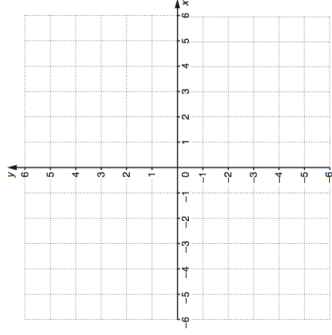
Question 1(a)



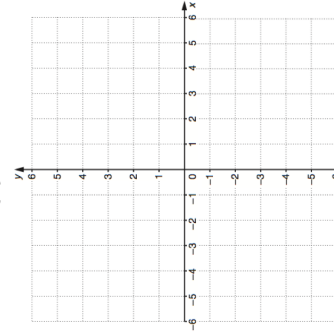
1(b)



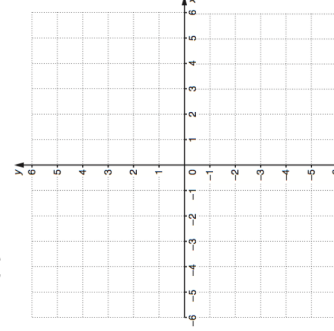
1(c)



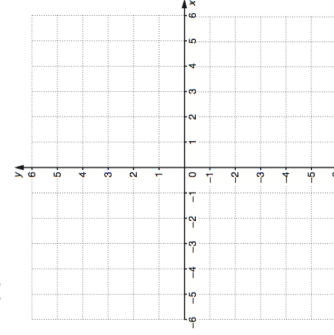
Question 1(d)



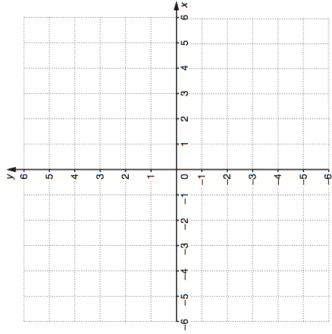
1(e)



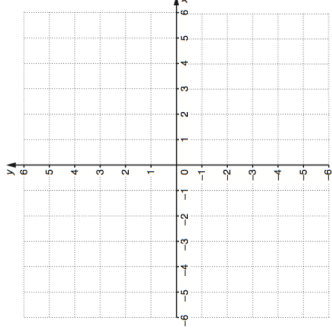
1(f)



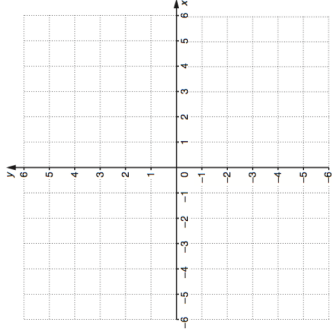
Question 1(g)



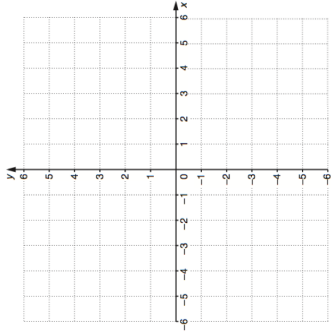
1(h)



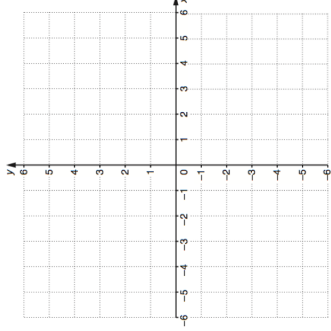
1(i)



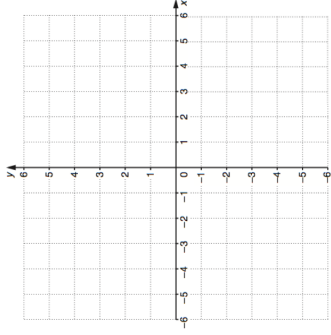
Question 1(j)



1(k)

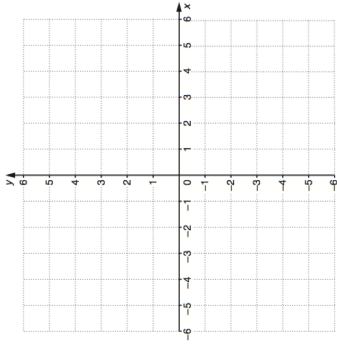


1(l)

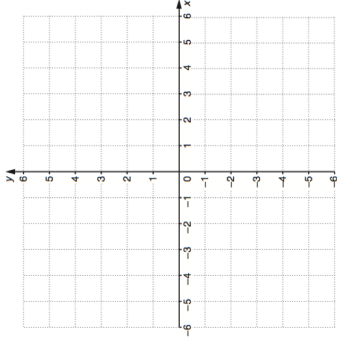


# Templates

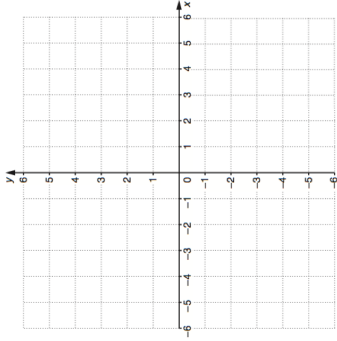
Question 3(a)



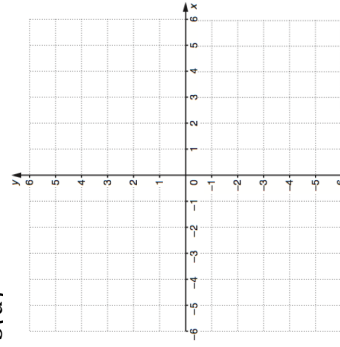
3(b)



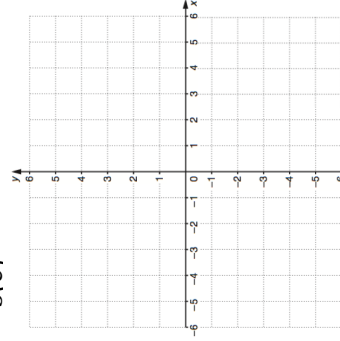
3(c)



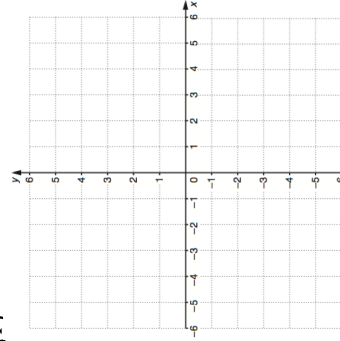
3(d)



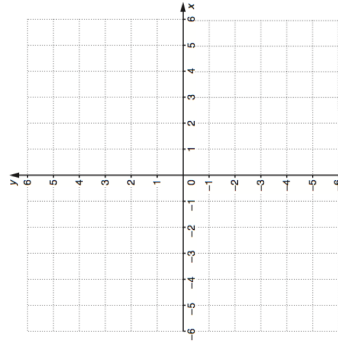
3(e)



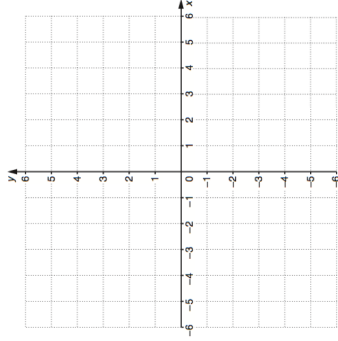
3(f)



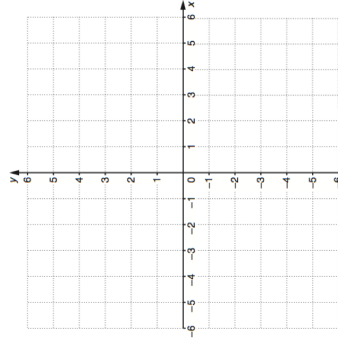
3(g)



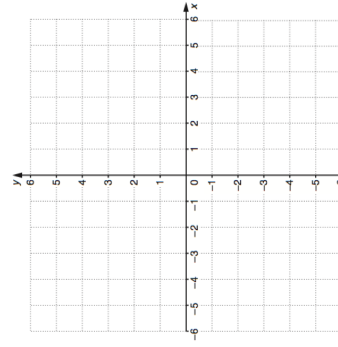
3(h)



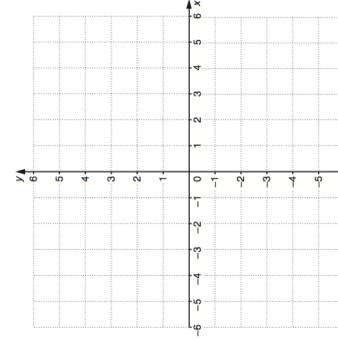
3(i)



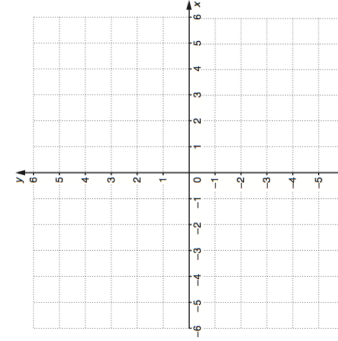
5(a)



5(b)

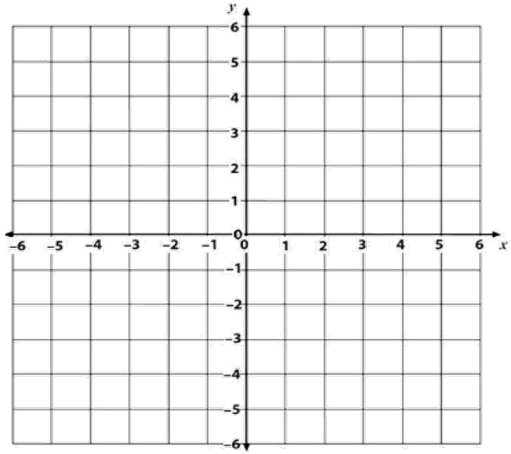


5(c)

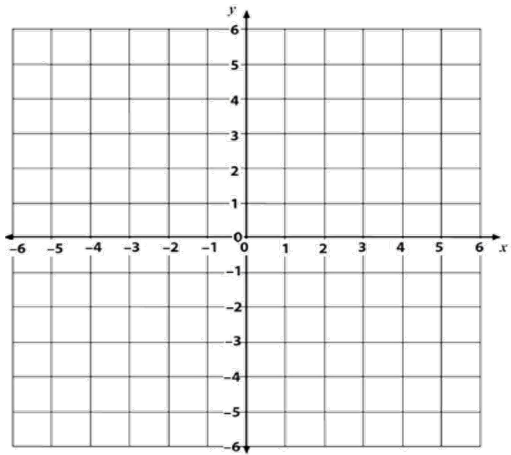


## Worked Example

Shade the region which satisfies the inequality:  
 $y > 2x + 3$

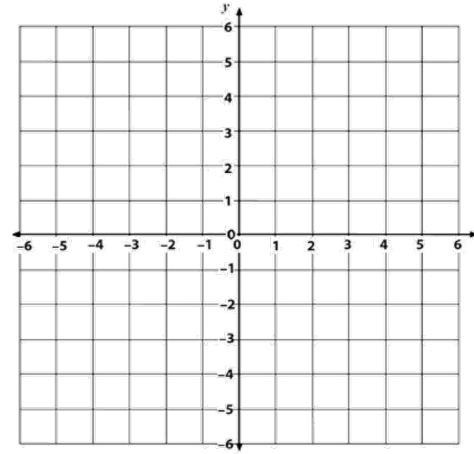


$y < -2x + 3$

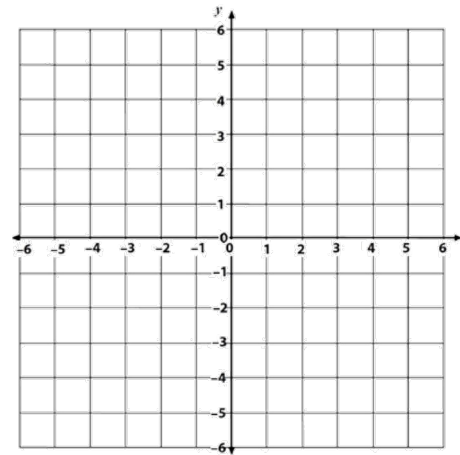


## Your Turn

Shade the region which satisfies the inequality:  
 $y < 4x - 1$

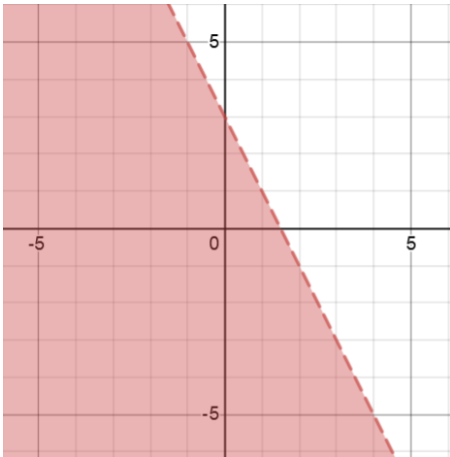
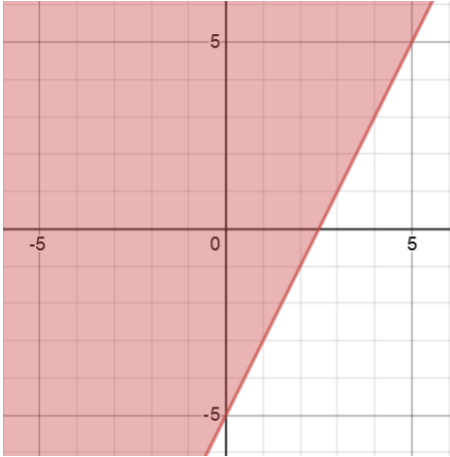


$y > -4x - 1$



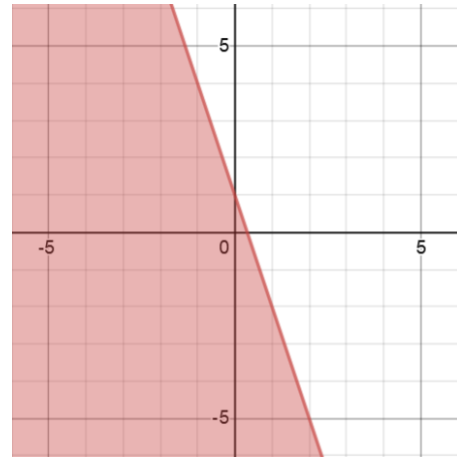
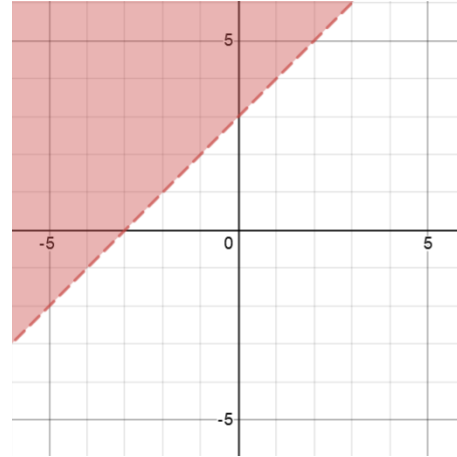
## Worked Example

Write the inequality that defines the red region:



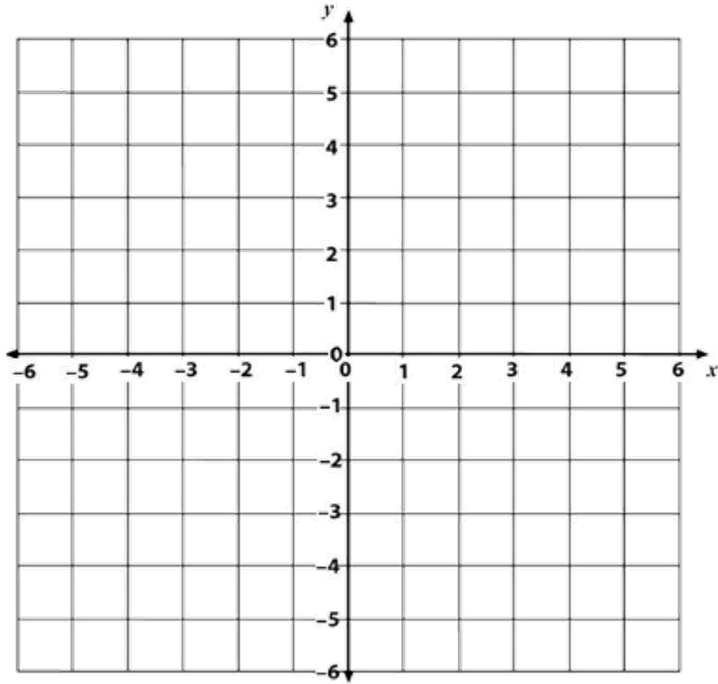
## Your Turn

Write the inequality that defines the red region:



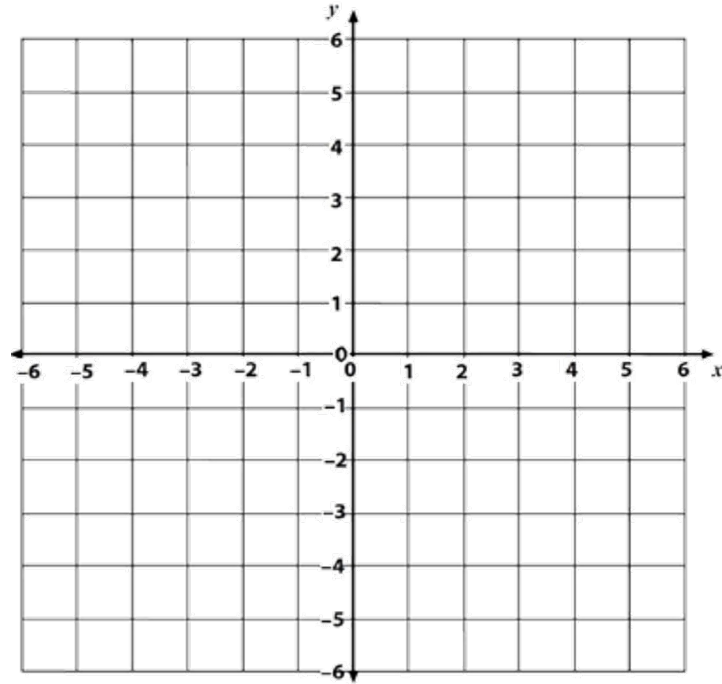
## Worked Example

Shade the region which satisfies the inequalities. Label it R.  
 $x \leq 3$ ,  $y > 1$  and  $y \geq x + 3$



## Your Turn

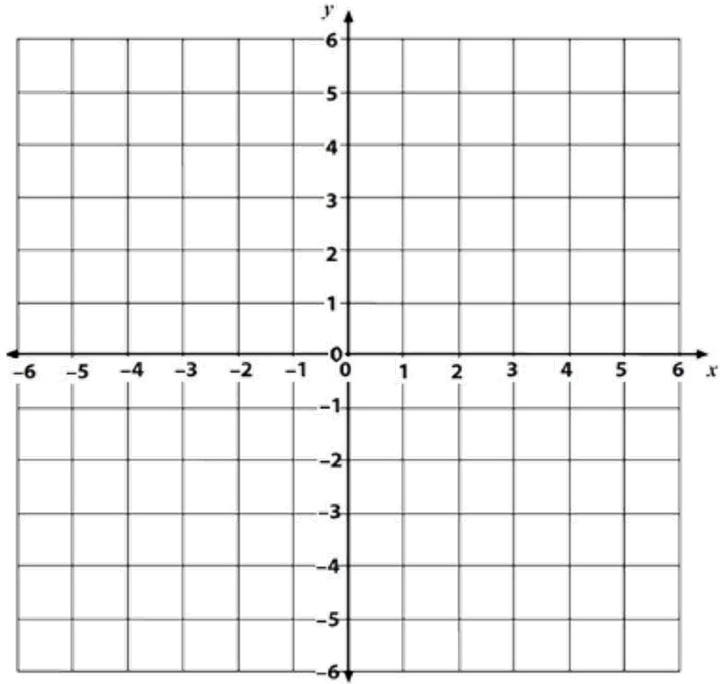
Shade the region which satisfies the inequalities. Label it R.  
 $x < 4$ ,  $y \geq 3$ ,  $y \geq x + 2$





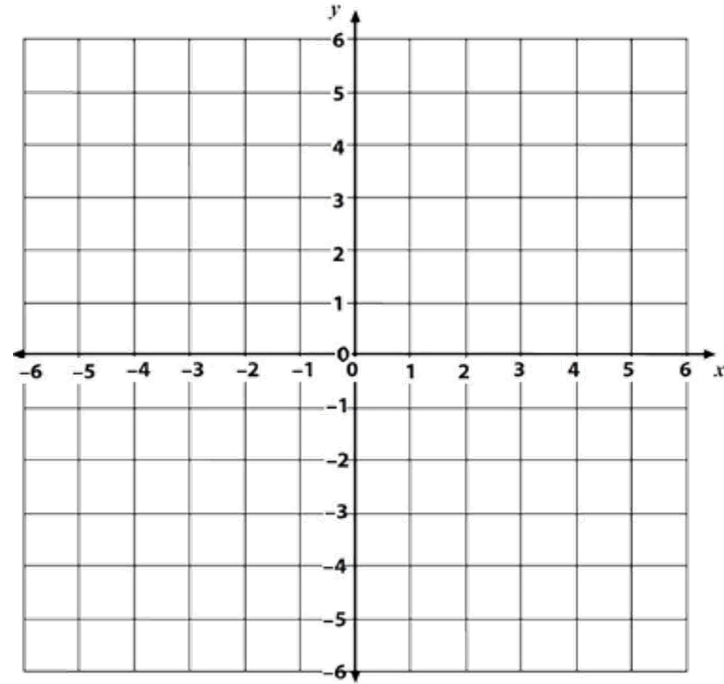
## Worked Example

Shade the region which satisfies the inequalities. Label it R.  
 $x \geq 2, y > -1$  and  $x + y \leq 5$



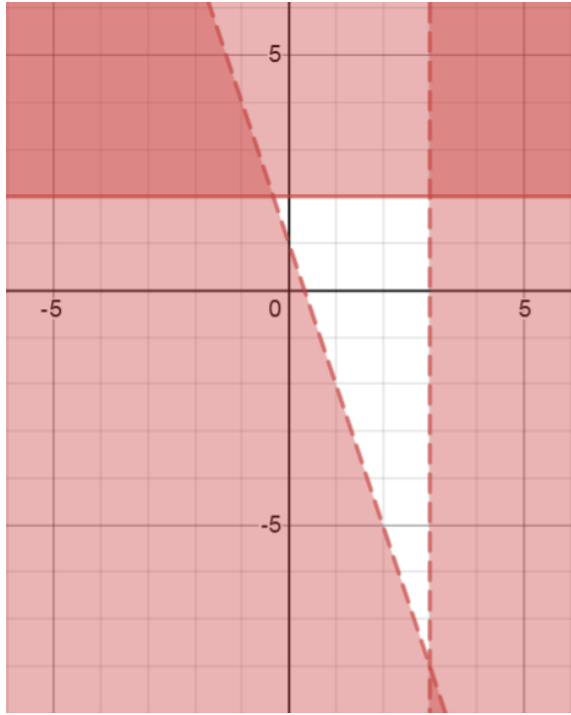
## Your Turn

Shade the region which satisfies the inequalities. Label it R.  
 $x \geq 2, y > 1$  and  $x + y \leq 6$



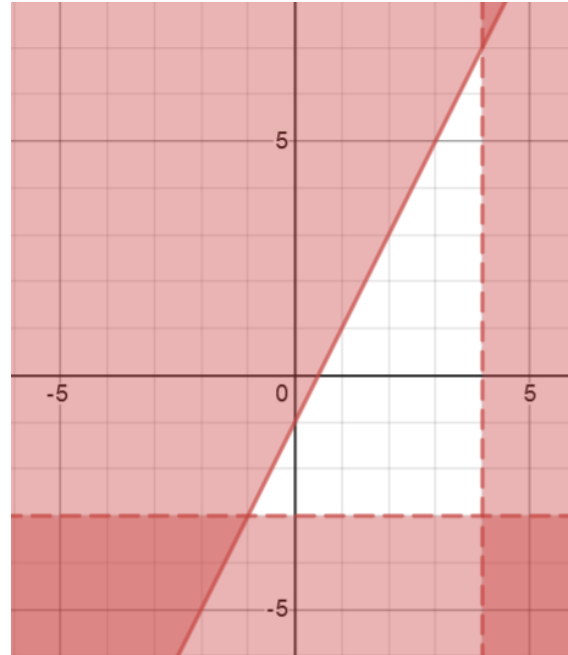
## Worked Example

Write the inequalities that define the unshaded region:



## Your Turn

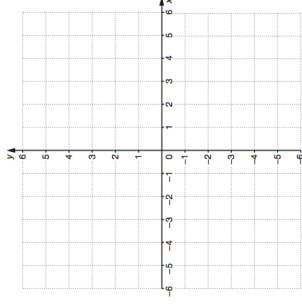
Write the inequalities that define the unshaded region:



# Fluency Practice

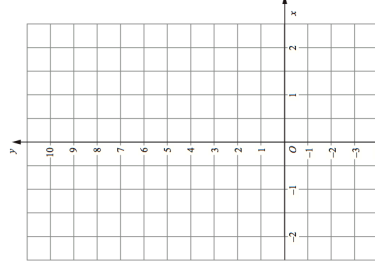
\*There are templates for the questions workout 1, 2, 4 and apply 4, 5 at the end of exercise

Question 1: On copies of the grid below, clearly indicate the region that satisfies each inequality.



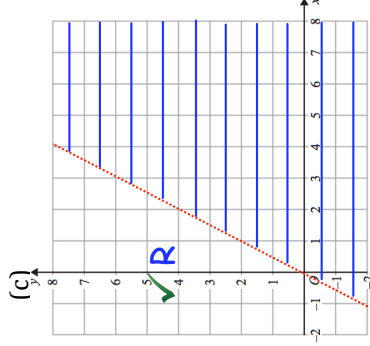
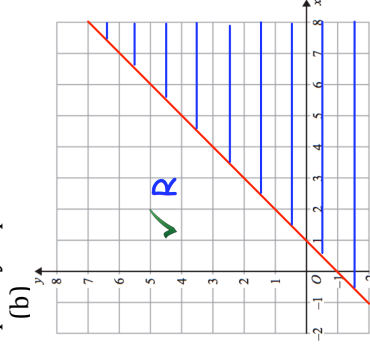
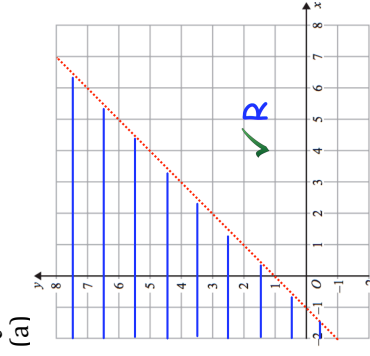
- (a)  $y < x + 1$       (b)  $y \leq 2x + 2$       (c)  $y > 3x - 1$
- (d)  $y \geq x + 3$       (e)  $y > 2x$       (f)  $y \leq 4x$
- (g)  $y < -2x + 1$       (h)  $y \geq \frac{1}{2}x + 2$       (i)  $x + y < 4$

Question 2: On copies of the grid below, clearly indicate the region that satisfies each inequality.



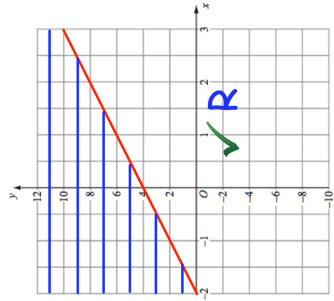
- (a)  $y > 3x + 4$       (b)  $y \geq 5x - 1$
- (c)  $y \leq 4x + 1$       (d)  $y < -2x + 5$
- (e)  $x + y < 2$       (f)  $y > -x - 2$
- (g)  $y \geq 5 - 2x$       (h)  $x + y \geq 7$
- (i)  $3x + y > 3$       (j)  $5x + 2y > 4$

Question 3: Write down the inequality represented in each diagram below.

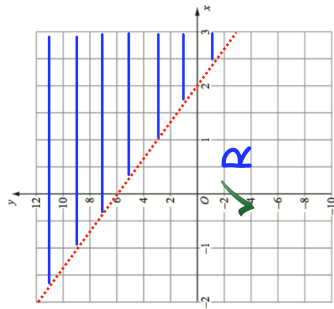


# Fluency Practice

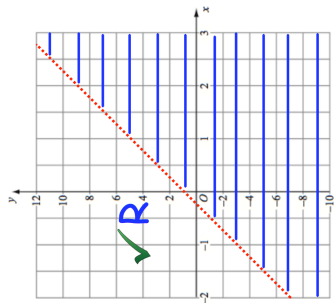
(d)



(e)

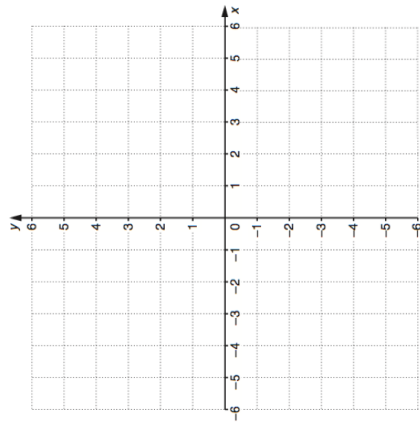


(f)



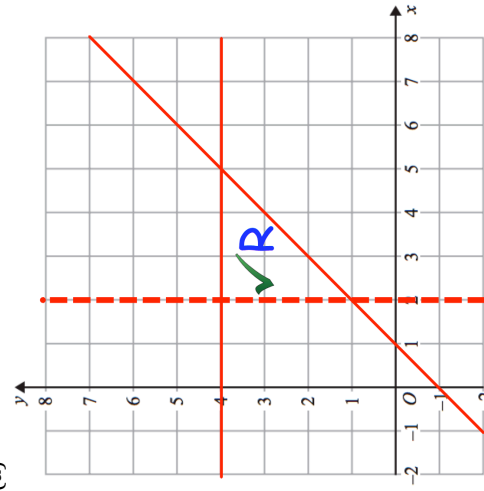
Question 4: On copies of the grid below, clearly indicate the region that satisfies the following inequalities.

- (a)  $y > x - 1$ ,  $x \geq -2$  and  $y < 2$
- (b)  $y \leq 2x$ ,  $x \leq 2$  and  $y > -4$
- (c)  $y \leq -2x + 2$ ,  $x \geq 0$  and  $y > x - 4$
- (d)  $x + y < 3$ ,  $-2 \leq x < 3$  and  $y \geq 0$
- (e)  $y \leq 5x - 4$ ,  $y > x - 4$  and  $y \leq -\frac{1}{2}x + 2$
- (f)  $y \leq -2x + 4$ ,  $y < 2x - 6$  and  $-4 < y < -3$

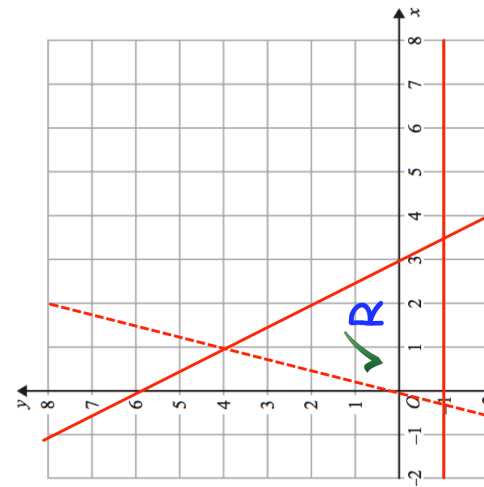


Question 5: State the inequalities that the region labelled R satisfies.

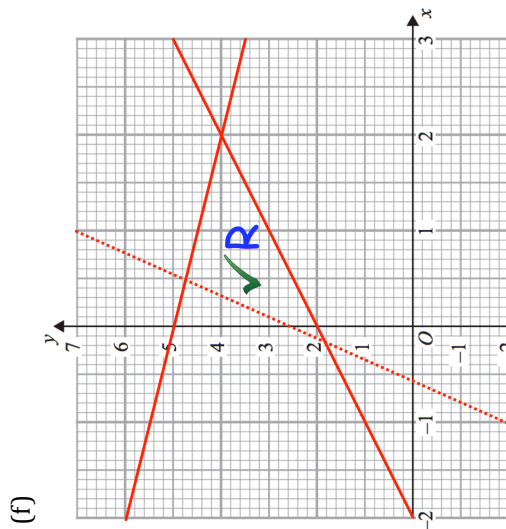
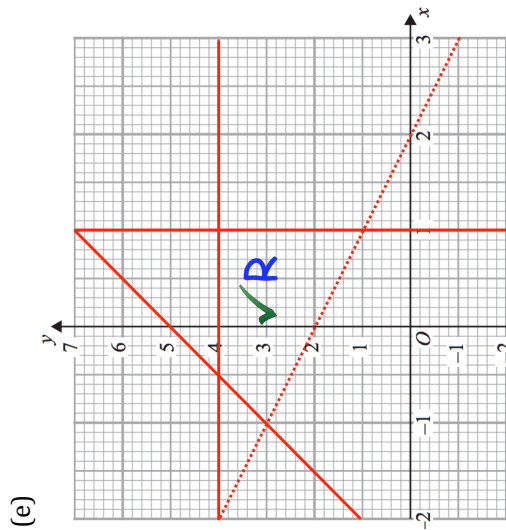
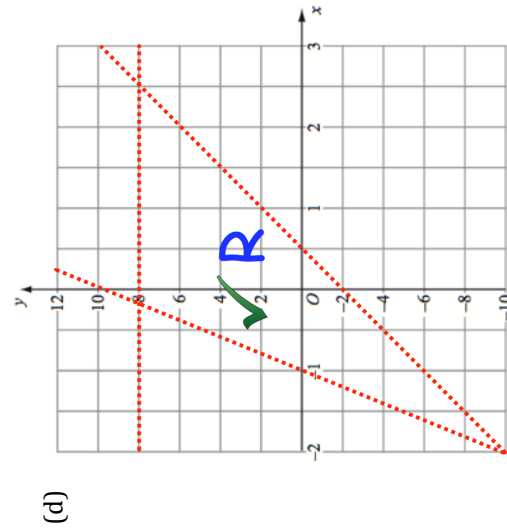
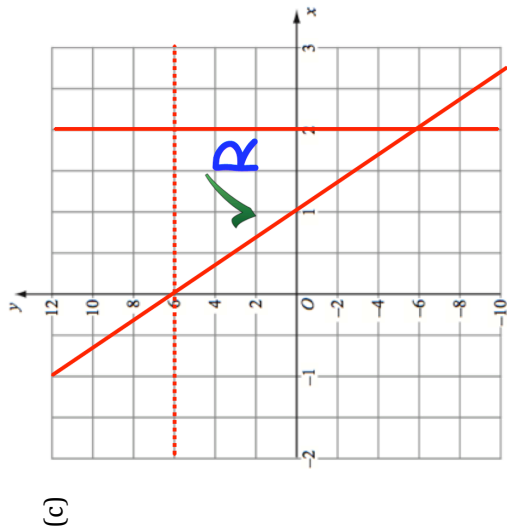
(a)



(b)

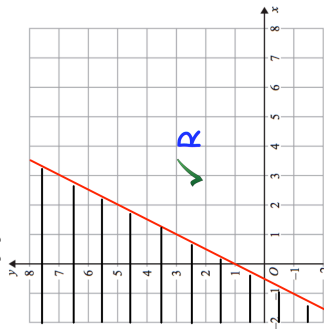


# Fluency Practice



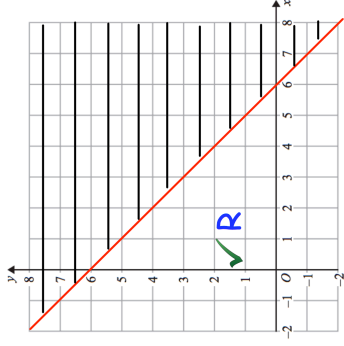
Apply

Question 1: Taylor has been asked to represent graphically  $y \geq 2x + 1$ . Can you spot her mistake?

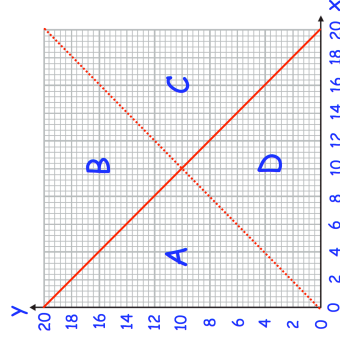


# Fluency Practice

Question 2: Conor has been asked to represent graphically  $x + y < 6$ . Can you spot his mistake?



Question 3: At a fitness class, the maximum number of people who can attend is 20. There are more men than women that attend the fitness class.  
 $y$  = number of men that attend the fitness class.  
 $x$  = number of women that attend the fitness class.  
 Which region A, B, C or D represents the information above?

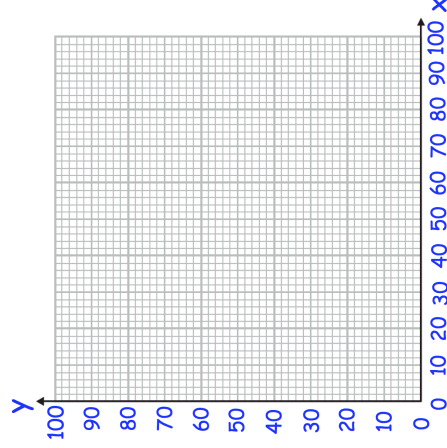


Question 4: A greengrocer sells apples and oranges.

- One morning day he sells
  - up to 50 apples
  - up to 60 oranges
  - no more than a total of 90 pieces of fruit.

Let  $x$  be the number of apples sold.  
 Let  $y$  be the number of orange sold.

Show the region that satisfies these inequalities

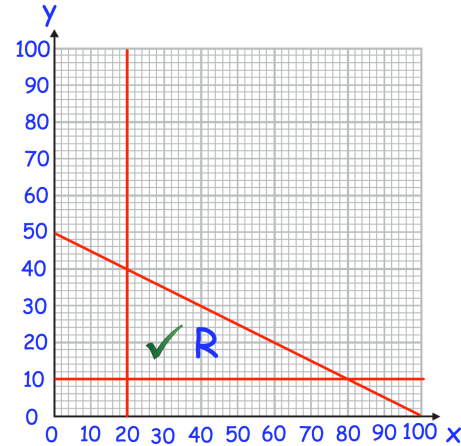


## Fluency Practice

Question 5: The region below shows information about the number first class passengers and the number of economy passengers on a flight.

$x$  = number of economy passengers and  $y$  = number of first class passengers

- (a) Can 15 first class and 60 economy passengers be on the flight?
- (b) Can 30 economy and 40 first class passenger be on the flight?



The profit made by the airline for each economy passenger is £90 and for each first class passenger is £200.

- (c) What is the maximum profit the airline can make on one flight?

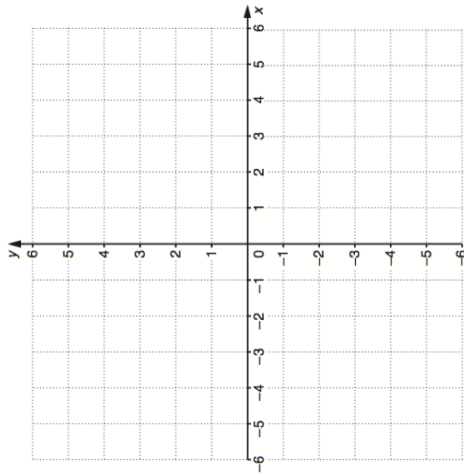
Question 6: A football stadium holds a maximum of 1000 fans.  
 Adult tickets cost £5 each and child tickets cost £2 each.  
 The football club needs to raise at least £3000 to cover costs.  
 The football club aims to sell at least one child ticket for two adult tickets sold  
 Let  $x$  = number of child tickets sold and  $y$  = number of adult tickets sold

Explain why: (a)  $x + y \leq 1000$       (b)  $2x + 5y \geq 3000$       (c)  $y \leq 2x$

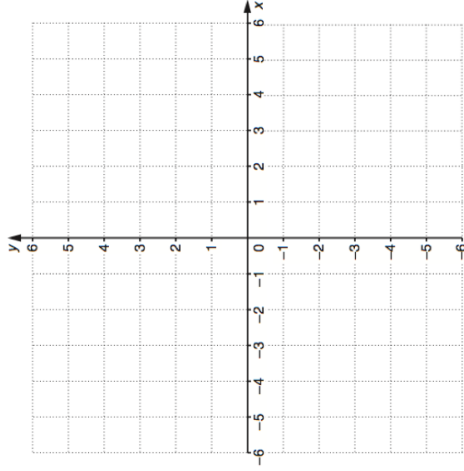
- (d) Represent this information on a graph.

# Templates

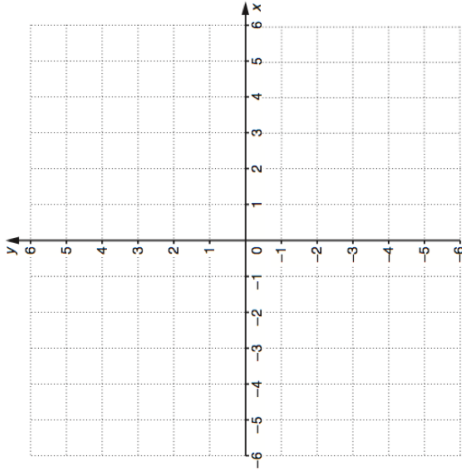
1(a)



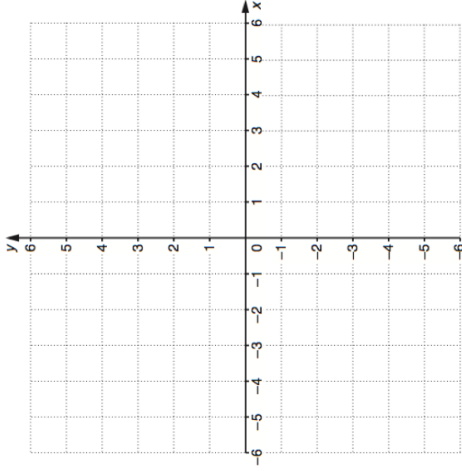
1(b)



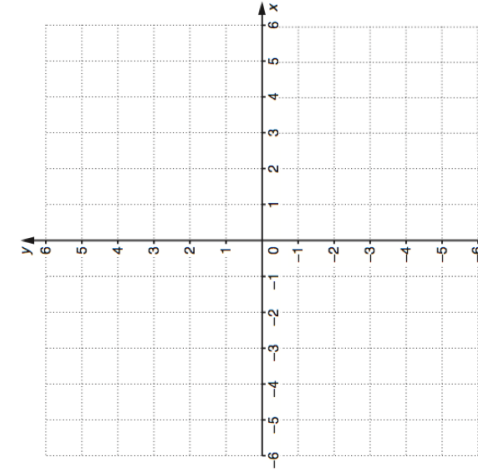
1(c)



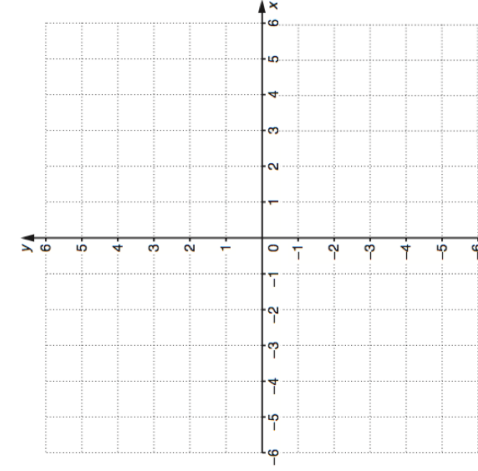
1(d)



1(e)



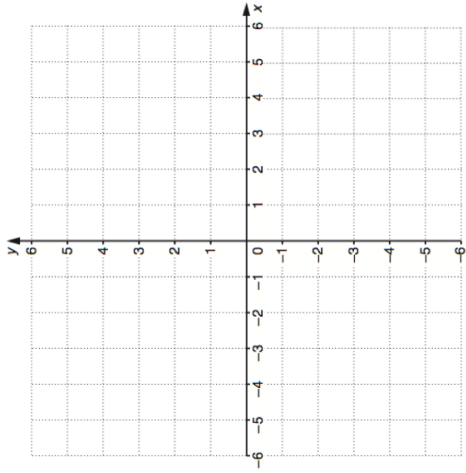
1(f)



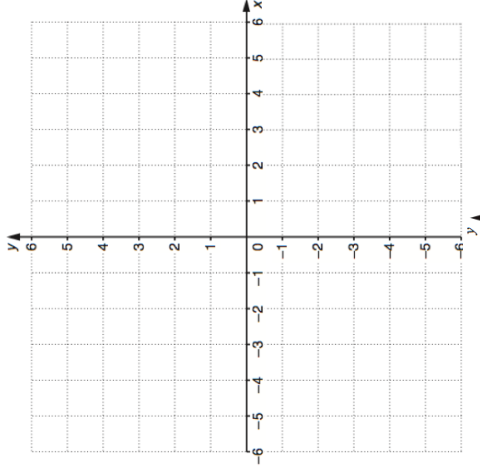


# Templates

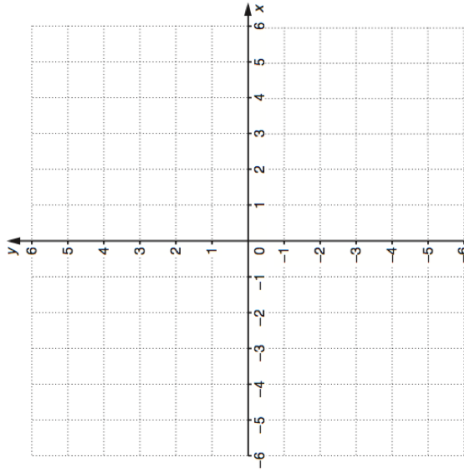
1(g)



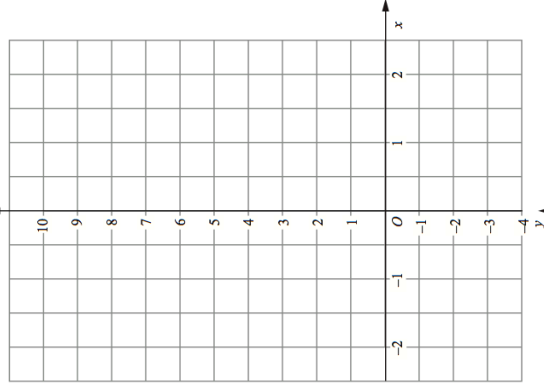
1(h)



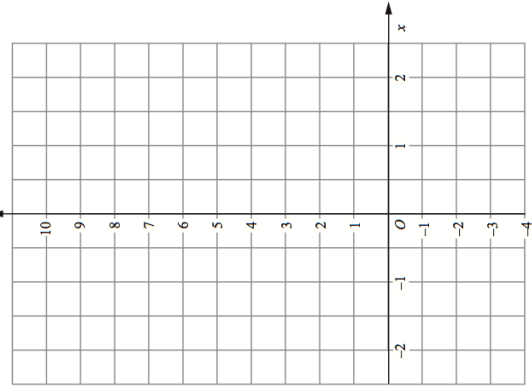
1(i)



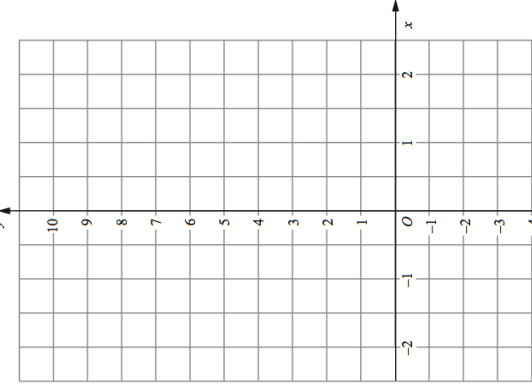
2(a)



2(b)

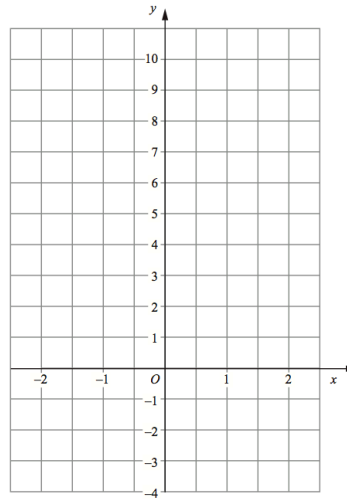


2(c)

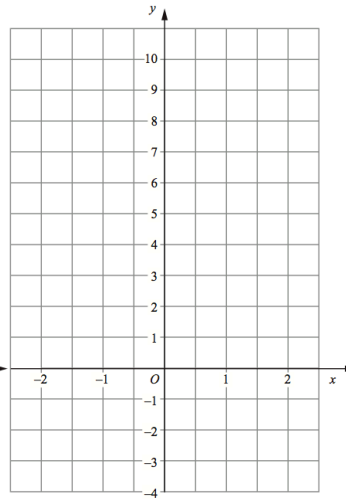


# Templates

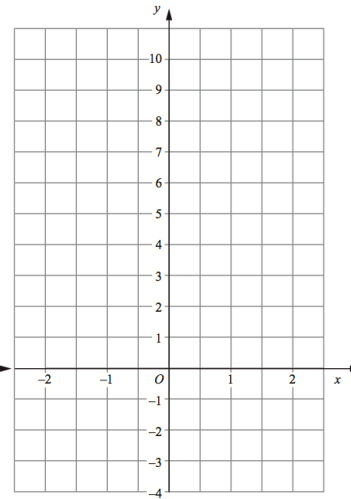
2(d)



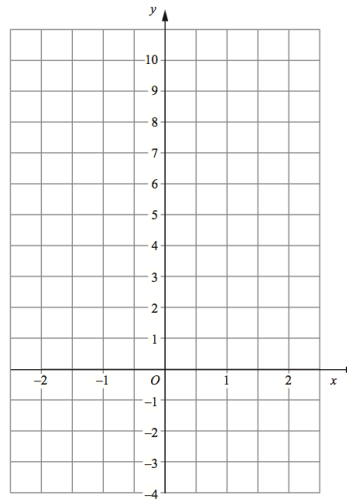
2(e)



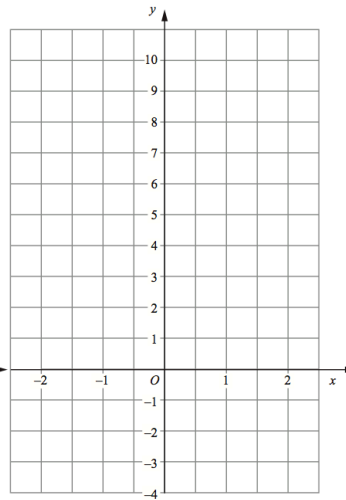
2(f)



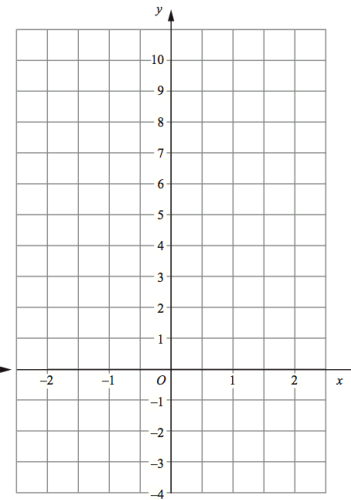
2(g)



2(h)

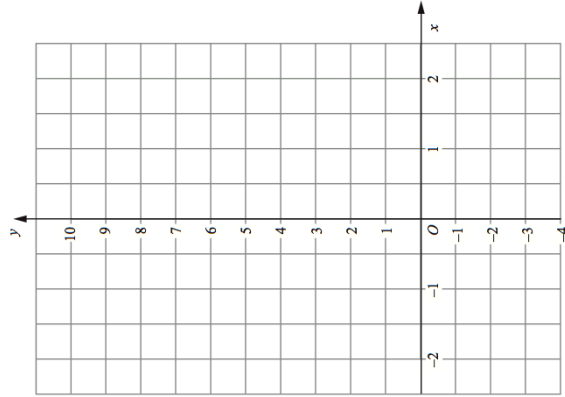


2(i)

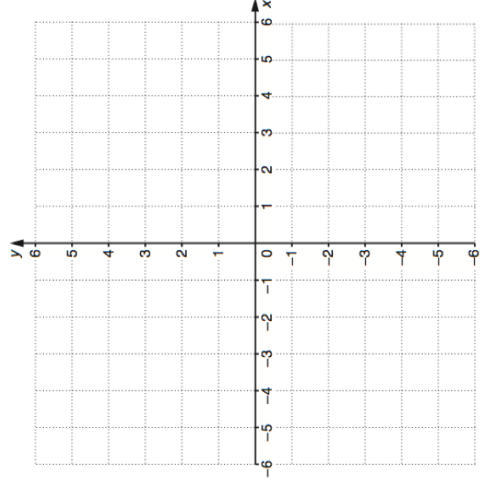


# Templates

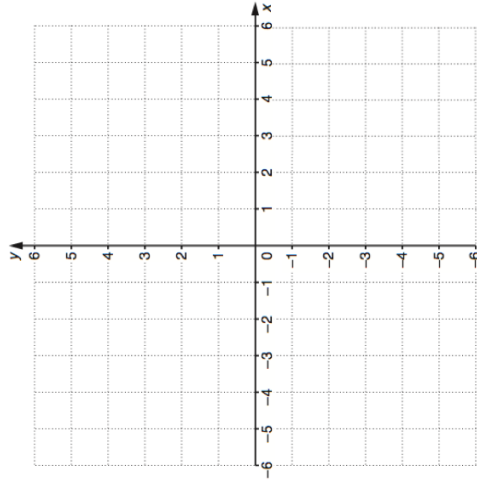
2(f)



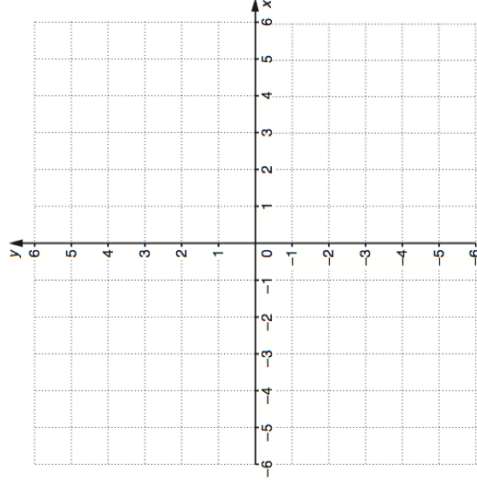
3(a)



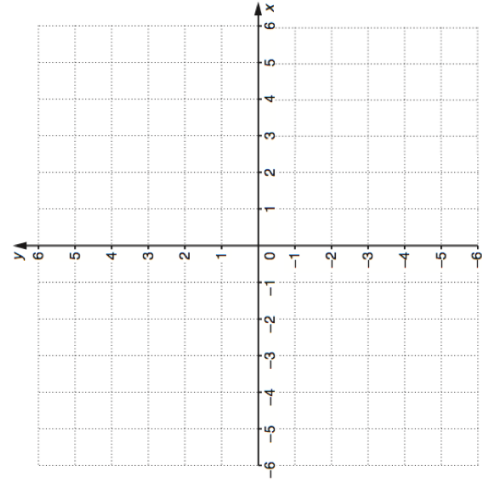
3(b)



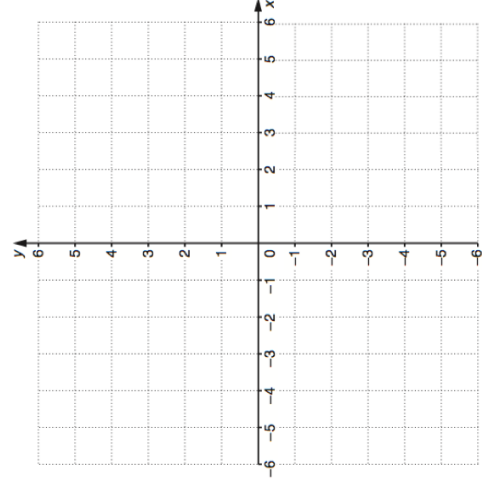
3(c)



3(d)

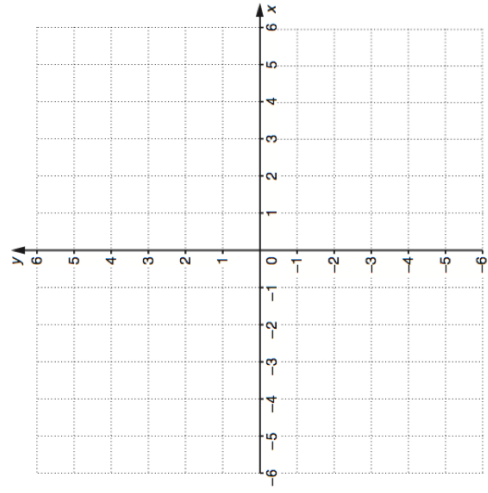


3(e)

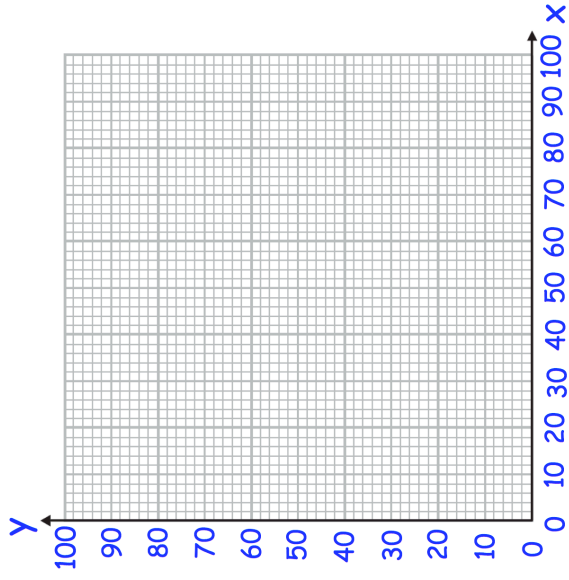


# Templates

3(f)



Apply 4



Apply 5



## Extra Notes

## 5 Non-Linear Graphs

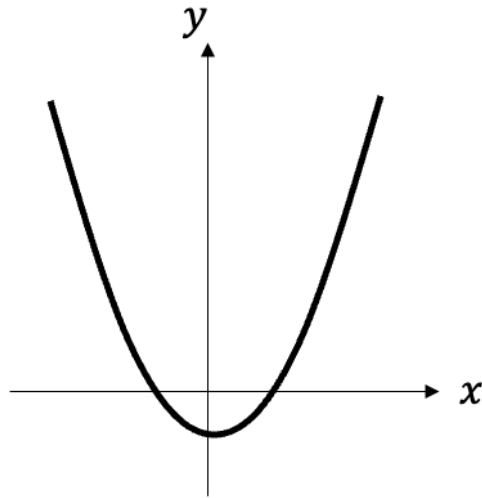
## Quadratic Graphs

A quadratic graph is produced from an equation of the form

$$y = ax^2 + bx + c$$

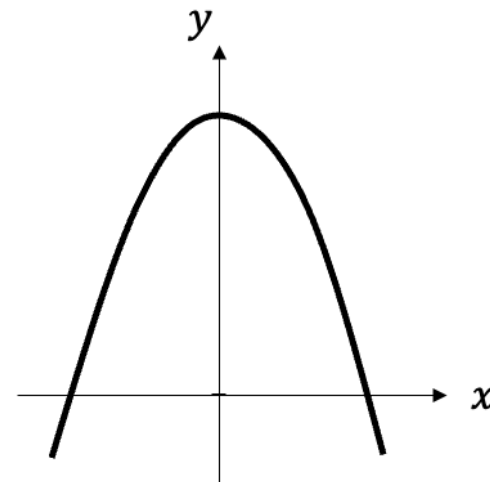
where  $a$ ,  $b$  and  $c$  are constants and  $a$  is non-zero.

When  $a > 0$



For a positive  $x^2$  term,  
we get a U-shaped  
parabola

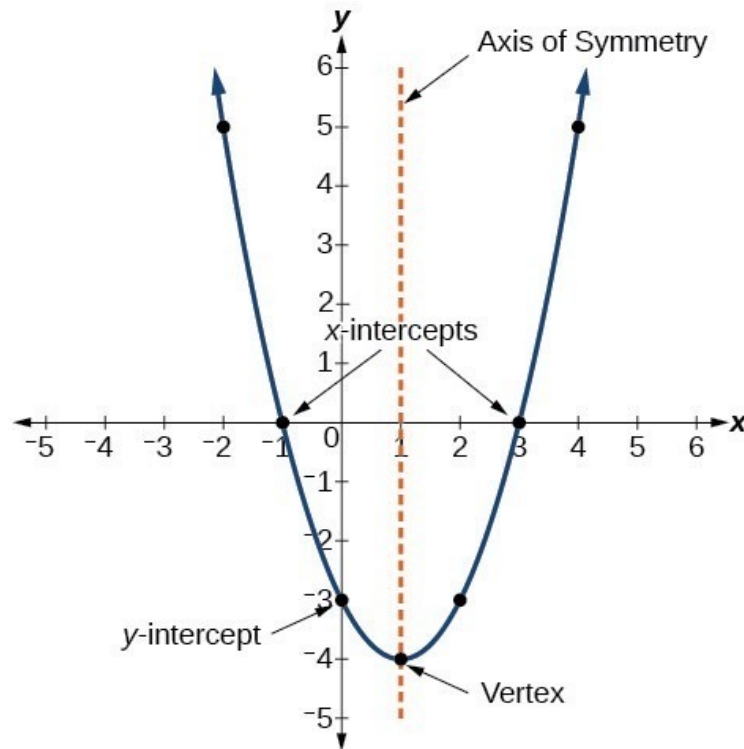
When  $a < 0$



For a negative  $x^2$  term,  
we get a  $\cap$ -shaped  
inverted parabola

## Interpreting Quadratic Graphs

- **y-intercept** – where the graph intercepts the  $y$ -axis
- **x-intercept** or **root** or **solution** – where the graph intercepts the  $x$ -axis
- **Turning point** or **vertex** or **minimum/maximum** – where the graph stops decreasing and starts increasing or vice-versa



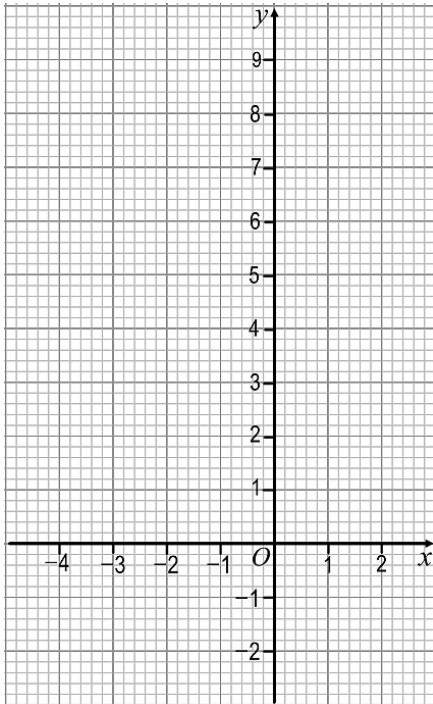


## Worked Example

- a) Complete the table and draw the graph of  $y = x^2 + 2x$  for  $x = -4$  to  $x = 2$
- b) Write down the equation of the line of symmetry of your graph
- c) Use your graph to find:
- the value of  $y$  when  $x = 0.5$
  - the values of  $x$  when  $y = 6$

Here is a table of values for  $y = x^2 + 2x$ .

$x$	-4	-3	-2	-1	0	1	2
$y$	8		0	-1			8

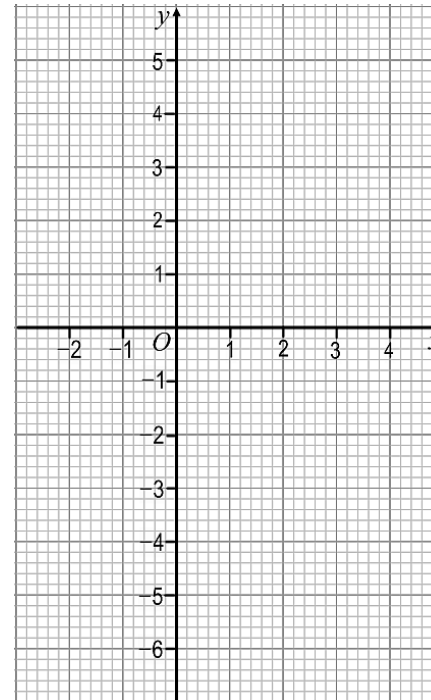


## Worked Example

- a) Complete the table and draw the graph of  $y = x^2 - 2x - 4$  for  $x = -2$  to  $x = 4$
- b) Write down the equation of the line of symmetry of your graph
- c) Write down the values of  $x$  where the graph crosses the  $x$ -axis

Here is a table of values for  $y = x^2 - 2x - 4$ .

$x$	-2	-1	0	1	2	3	4
$y$		-1	-4			-1	



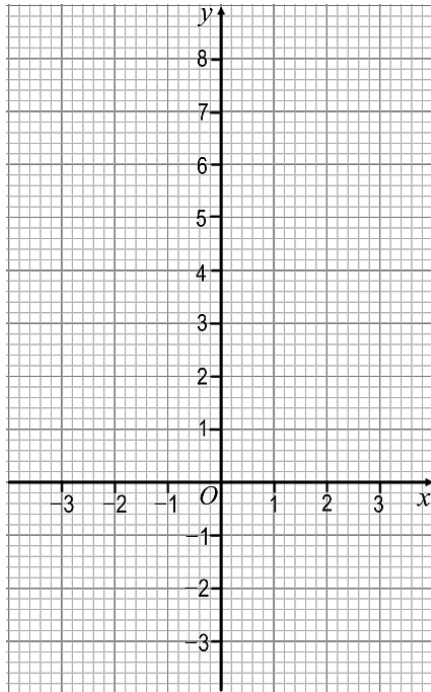
## Fluency Practice

1. Here is a table of values for  $y = x^2 - 2$ .

<b><math>x</math></b>	-3	-2	-1	0	1	2	3
<b><math>y</math></b>	7		-1	-2			7

a) Complete the table of values.

b) On the grid, draw the graph of  $y = x^2 - 2$  for  $x = -3$  to  $x = 3$ .



c) Write down the equation of the line of symmetry of your graph.

d) Write down the coordinates of the minimum point.

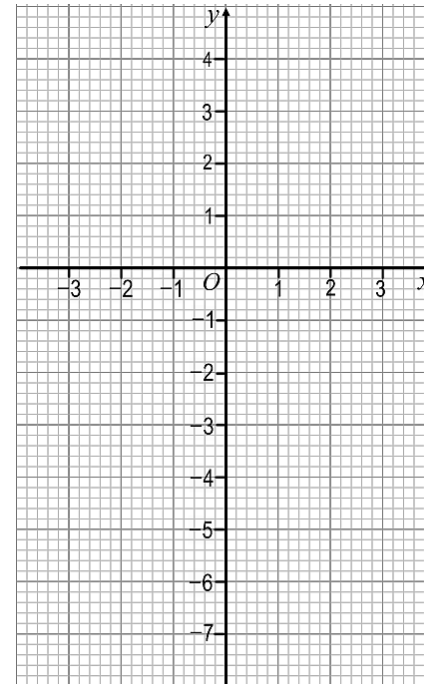
## Fluency Practice

2. Here is the table of values for  $y = 3 - x^2$ .

<b><math>x</math></b>	-3	-2	-1	0	1	2	3
<b><math>y</math></b>	-6		2	3		-1	

a) Complete the table of values.

b) On the grid, draw the graph of  $y = 3 - x^2$  for  $x = -3$  to  $x = 3$ .



c) Write down the coordinates of the maximum point.

d) Write down the values of  $x$  where the graph crosses the  $x$ -axis.

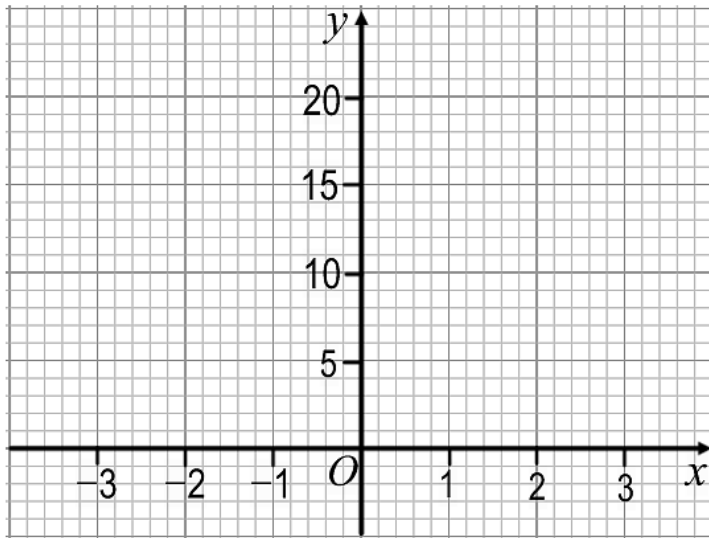
## Fluency Practice

3. Here is a table of values for  $y = 2x^2 + 1$ .

<b><i>x</i></b>	-3	-2	-1	0	1	2	3
<b><i>y</i></b>		9		1	3	9	

a) Complete the table of values.

b) On the grid, draw the graph of  $y = 2x^2 + 1$  for  $x = -3$  to  $x = 3$ .



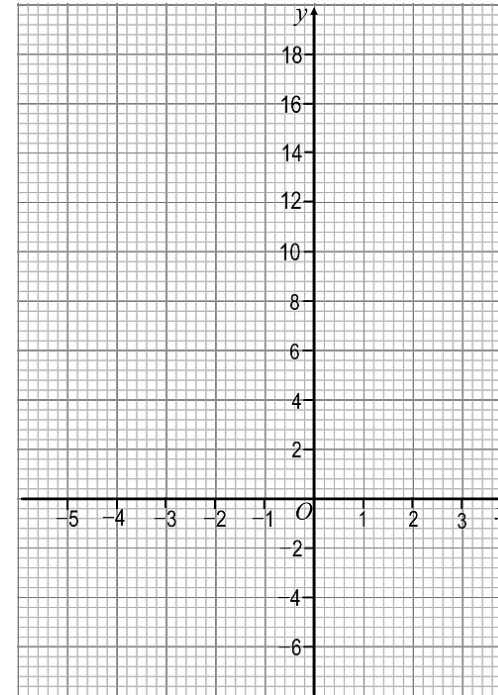
c) Use your graph to find:

i) the value of  $y$  when  $x = -2.5$

ii) the two values of  $x$  when  $y = 6$ .

## Fluency Practice

4. a) On the grid, draw the graph of  $y = x^2 + 3x - 2$  for the values of  $x$  from  $-5$  to  $3$ .



b) Use your graph to:

i) write down the values of  $x$  when the graph crosses the  $x$ -axis

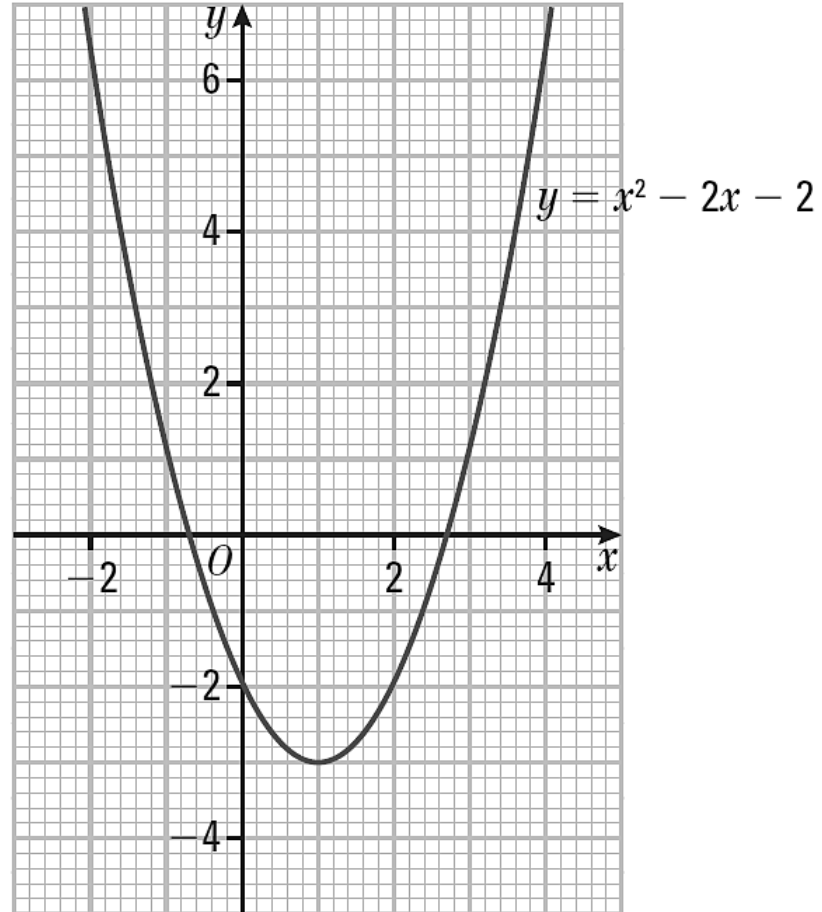
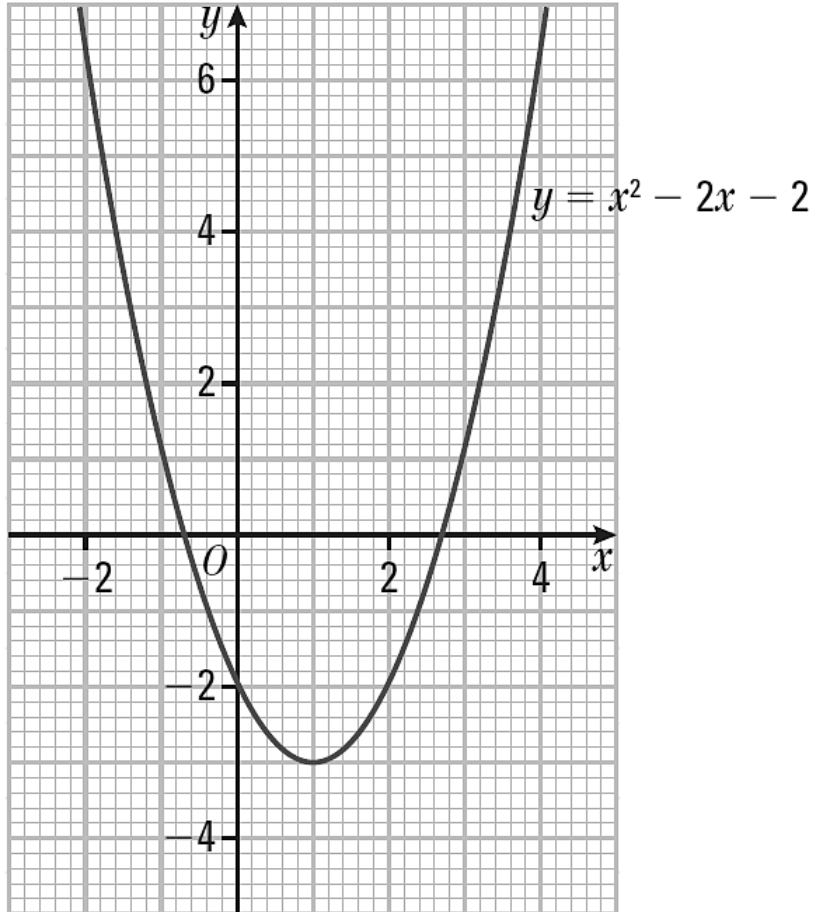
ii) draw in and write down the equation of the line of symmetry.

## Worked Example

Use this graph to solve these equations:

a)  $x^2 - 2x - 2 = 0$

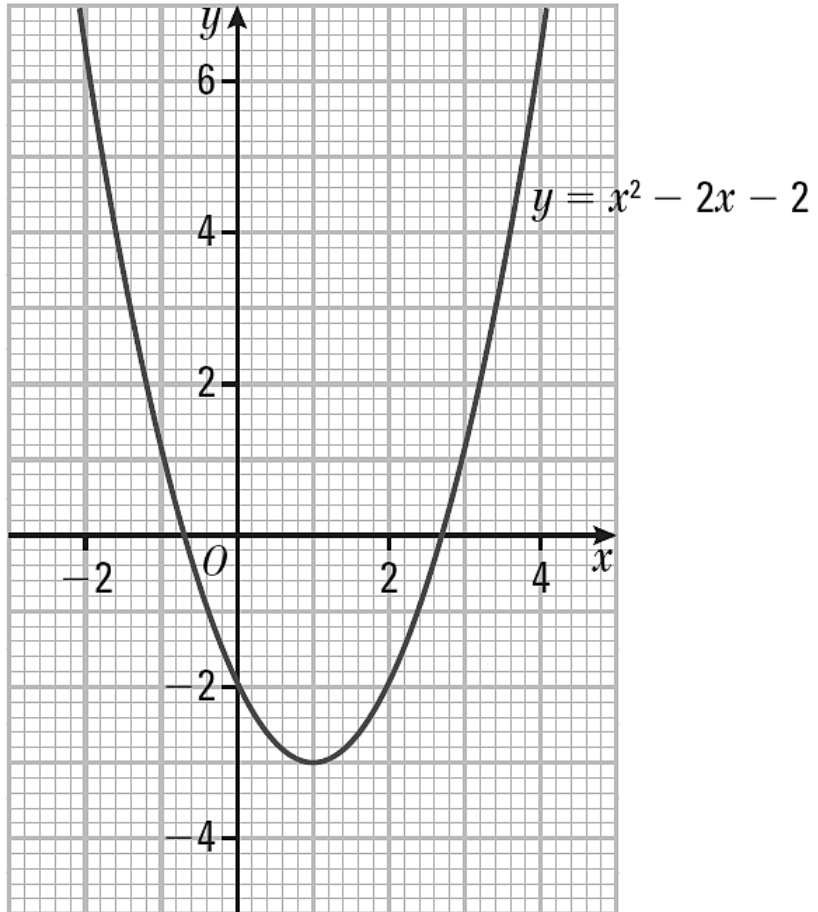
b)  $x^2 - 2x - 5 = 0$



## Worked Example

Use this graph to solve these equations:

c)  $x^2 - 2x - 2 = x$

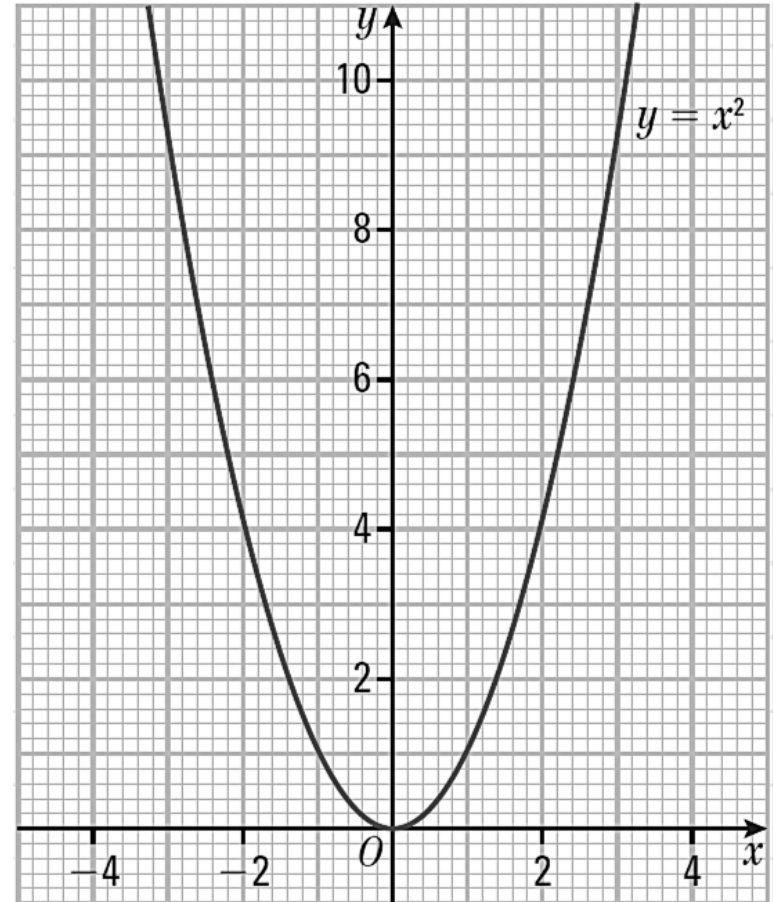
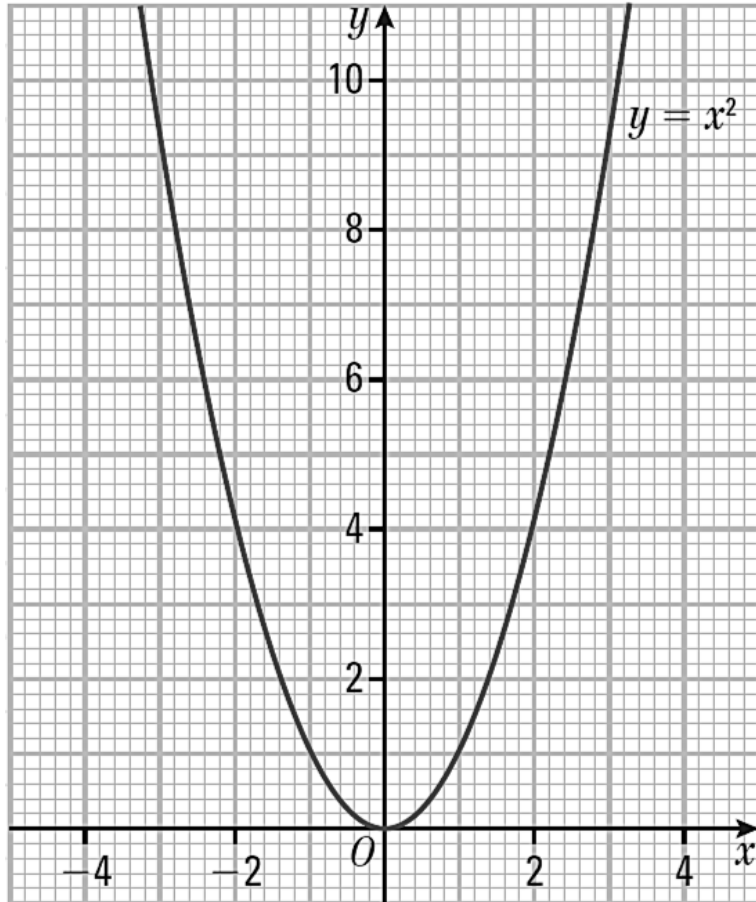


## Worked Example

Use this graph to solve these equations:

a)  $x^2 = 2x + 3$

b)  $x^2 = x + 4$

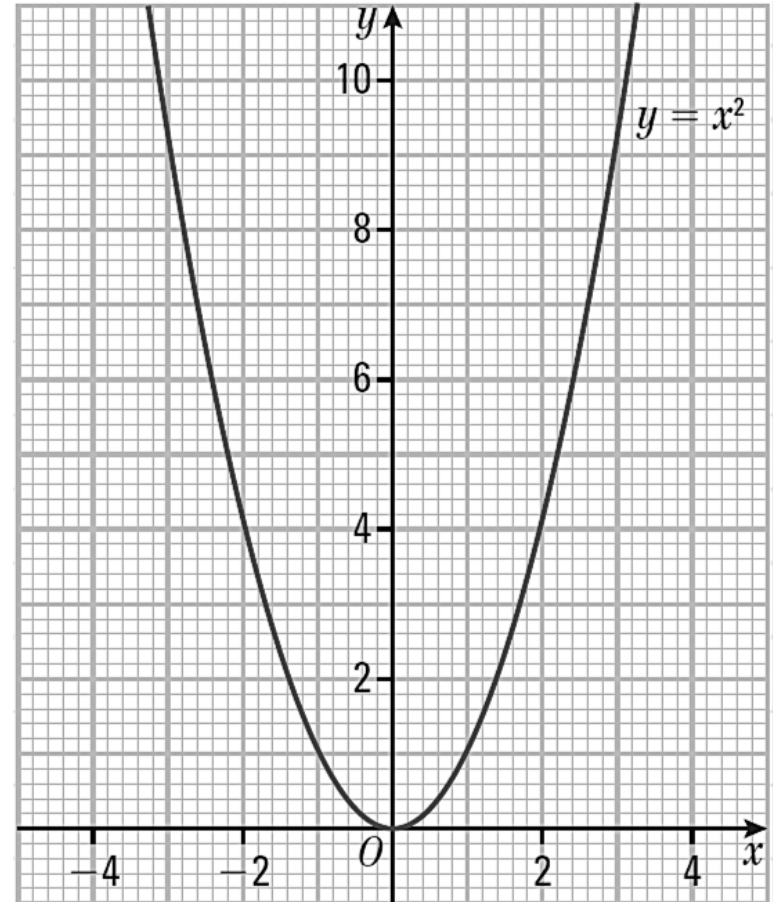
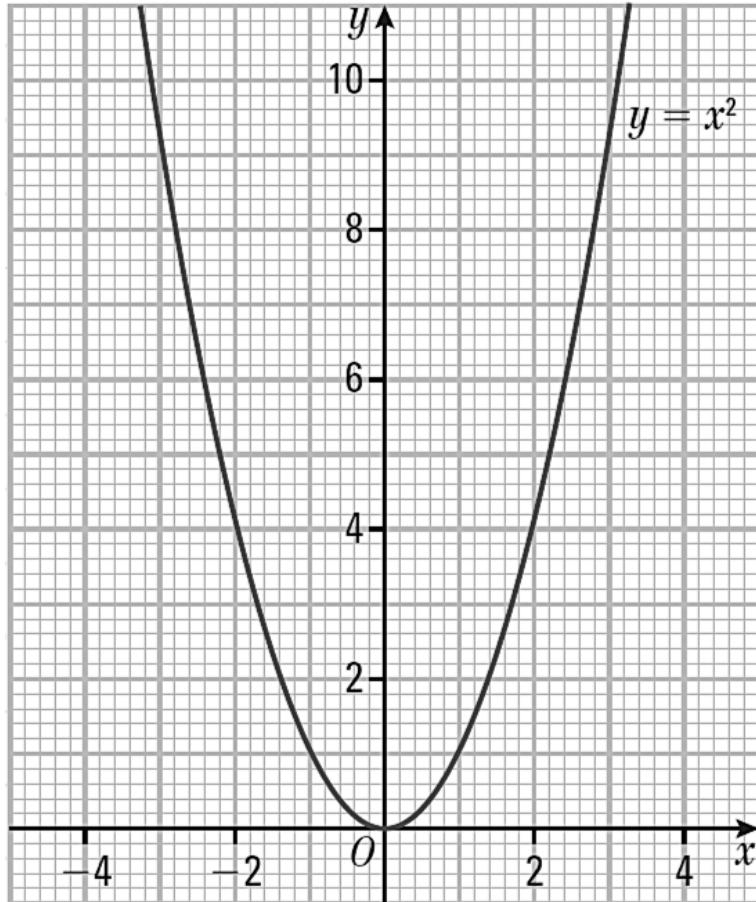


## Worked Example

Use this graph to solve these equations:

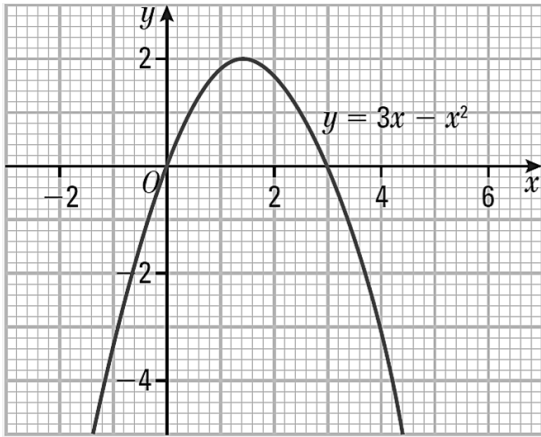
c)  $x^2 + x - 1 = 0$

d)  $x^2 - 2x - 1 = 0$



## Fluency Practice

1. Use this graph to solve the equations.

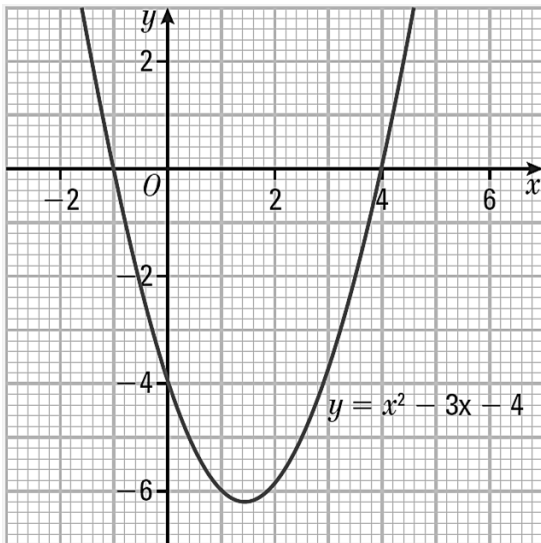


a)  $3x - x^2 = 0$

b)  $3x - x^2 = 1$

c)  $3x - x^2 = -4$

2. Use this graph to solve the equations.



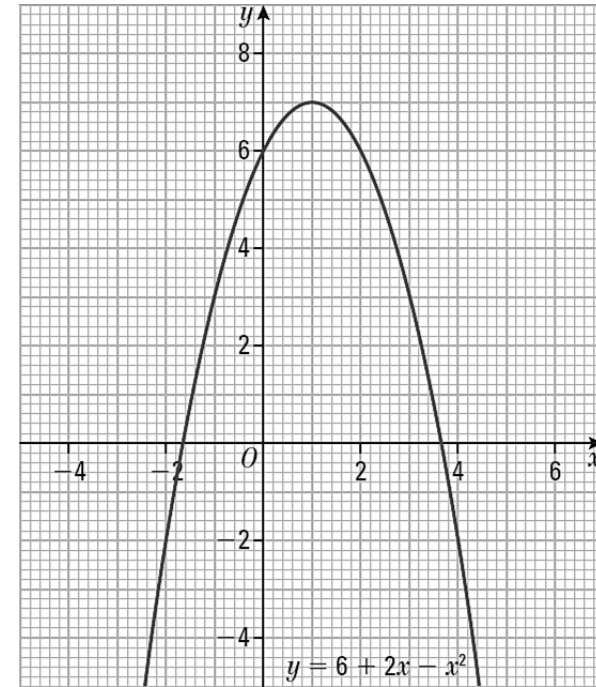
a)  $x^2 - 3x - 4 = 0$

b)  $x^2 - 3x - 4 = 2$

c)  $x^2 - 3x - 4 = -5$

## Fluency Practice

3. Use this graph to solve the equations.



a)  $6 + 2x - x^2 = 0$

b)  $4 + 2x - x^2 = 0$

c)  $6 + 2x - x^2 = x$

d)  $3 + 3x - x^2 = 0$



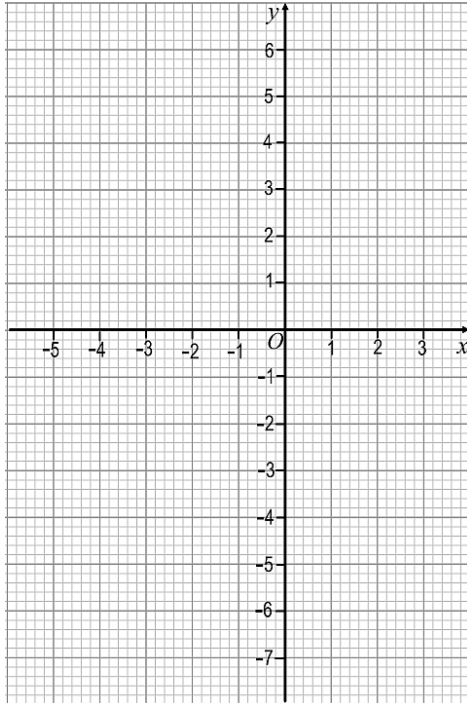
## Fluency Practice

4. Here is a table of values for  $y = x^2 + 3x - 4$ .

$x$	-5	-4	-3	-2	-1	0	1	2
$y$	6	0		-6		-4		

a) Complete the table of values.

b) On the grid, draw the graph of  $y = x^2 + 3x - 4$ .

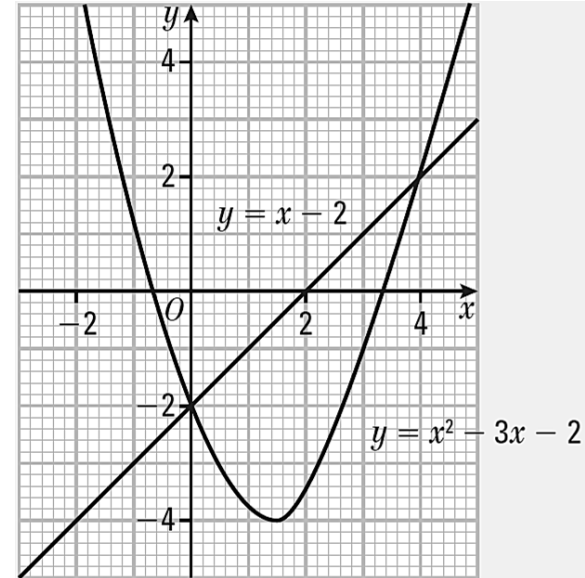


c) Use your graph to solve the equation  $x^2 + 3x - 4 = 2$ .

d) By drawing a suitable straight line on your graph, solve the equation  $x^2 + 3x - 4 = x + 1$ .

## Fluency Practice

5. The graphs  $y = x^2 - 3x - 2$  and  $y = x - 2$  are shown below.



a) Show that the equation  $x^2 - 3x - 2 = x - 2$  can be rewritten as  $x^2 - 4x = 0$ .

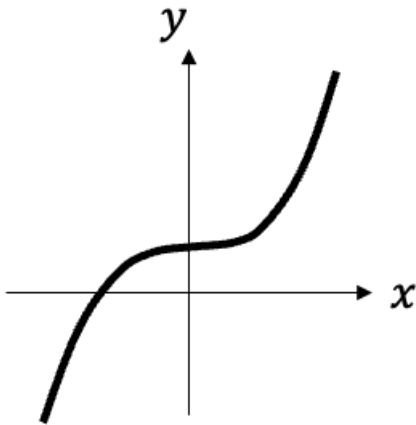
b) Solve the equation  $x^2 - 4x = 0$ .

c) The equation  $x^2 - 2x - 4 = 0$  can be solved by drawing a suitable straight line on the graph. Find the equation of this straight line and solve the equation  $x^2 - 2x - 4 = 0$ .

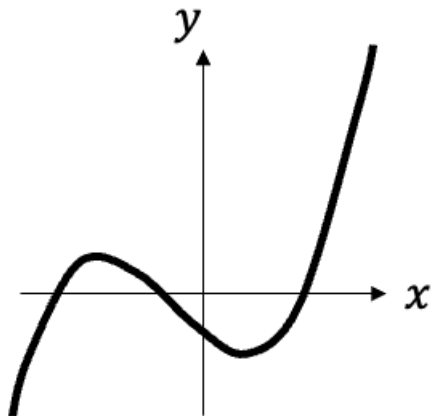
## Cubic Graphs

For a cubic graph  $y = ax^3 + bx^2 + cx + d$

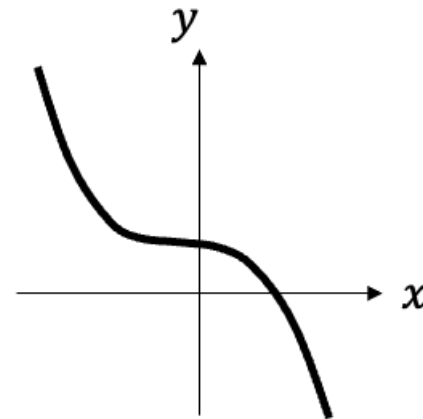
When  $a > 0$



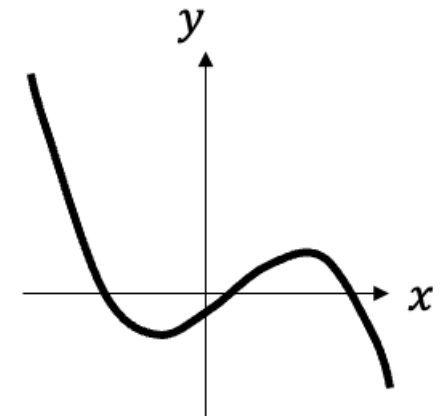
Positive cubics always go from bottom left to top right



When  $a < 0$



Negative cubics always go from top left to bottom right

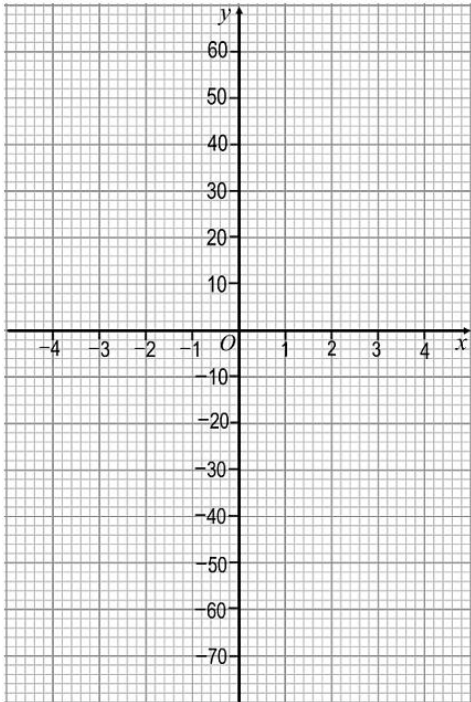


## Worked Example

- a) Complete the table and draw the graph of  $y = x^3 - 4$  for  $x = -4$  to  $x = 4$   
 b) Use the graph to find the value of  $y$  when  $x = 4$

Here is a table of values for  $y = x^3 - 4$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$									

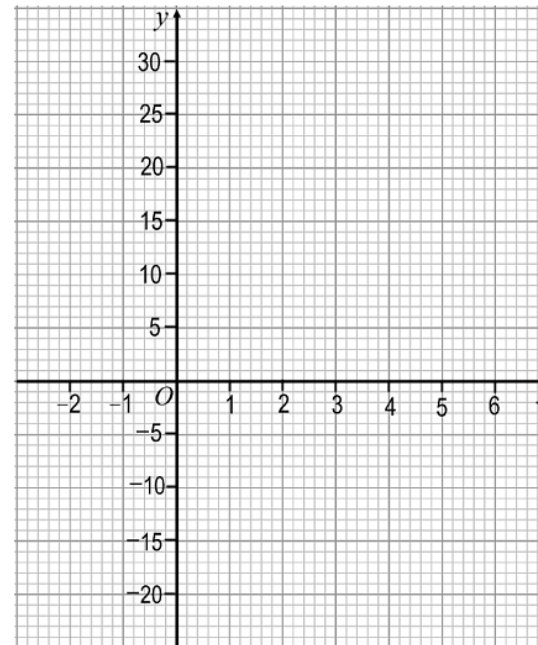


## Worked Example

- a) Complete the table and draw the graph of  $y = x^3 - 4x^2 + 5$  for  $x = -2$  to  $x = 5$   
 b) Use your graph to find the solutions to:  
 i)  $x^3 - 4x^2 + 5 = 0$   
 ii)  $x^3 - 4x^2 - x + 5 = 0$

Here is a table of values for  $y = x^3 - 4x^2 + 5$ .

$x$	-2	-1	0	1	2	3	4	5
$y$	-19		5			-4	5	



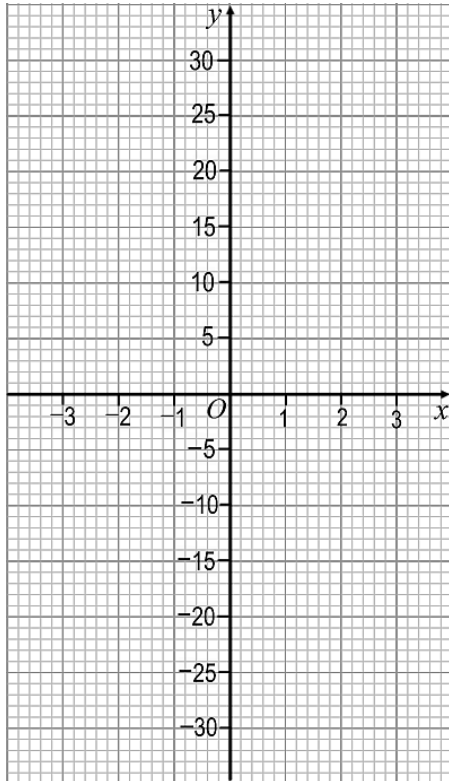
## Fluency Practice

1. Here is a table of values for  $y = x^3 + 1$ .

$x$	-3	-2	-1	0	1	2	3
$y$							

a) Complete the table of values.

b) On the grid, draw the graph of  $y = x^3 + 1$  for  $-3 \leq x \leq 3$ .



c) Use your graph to find the value of  $y$  when  $x = 1.5$ .

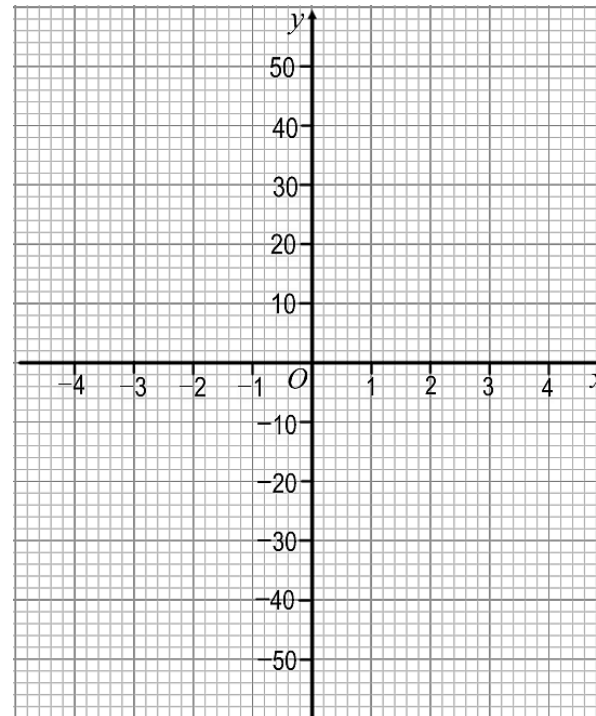
## Fluency Practice

2. Here is the table of values for  $y = x^3 - 5x$ .

$x$	-4	-3	-2	-1	0	1	2	3	4
$y$		-12			0	-4		12	44

a) Complete the table of values.

b) On the grid, draw the graph of  $y = x^3 - 5x$  for  $-4 \leq x \leq 4$ .



c) Use your graph to find the solutions to the equation  $x^3 - 5x = 0$ .

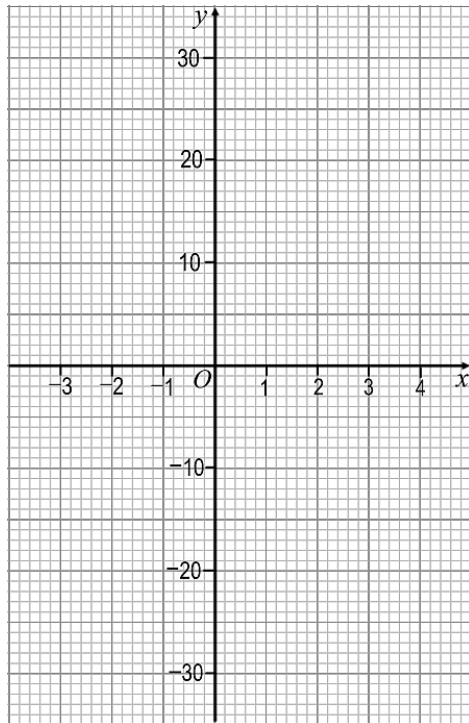
## Fluency Practice

3. Here is a table of values for  $y = 6x + x^2 - x^3$ .

$x$	-3	-2	-1	0	1	2	3	4
$y$		0	-4			8	0	

a) Complete the table of values.

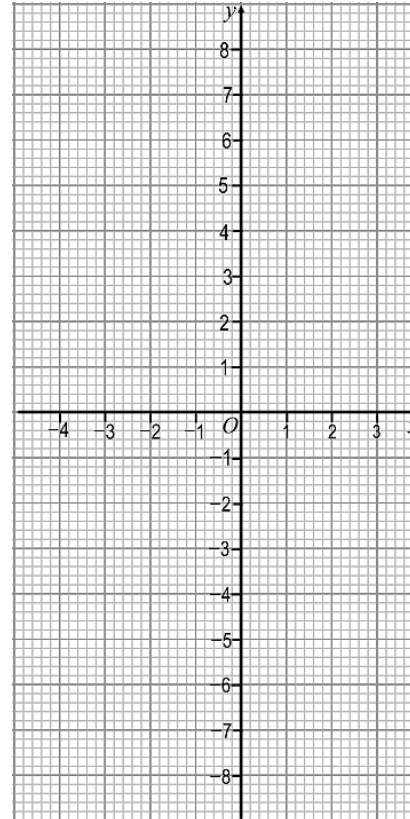
b) On the grid, draw the graph of  $y = 6x + x^2 - x^3$  for  $-3 \leq x \leq 4$ .



c) By drawing a suitable line on your diagram, solve the equation  $6x + x^2 - x^3 = x - 2$ .

## Fluency Practice

4. a) On the grid, draw the graph of  $y = x^3 + x^2 - 4x - 2$  for the values of  $x$  from  $-3$  to  $2$ .



b) By drawing a suitable line on your diagram, solve the equation  $x^3 + x^2 - 5x - 2 = 0$ .

## Reciprocal Graphs

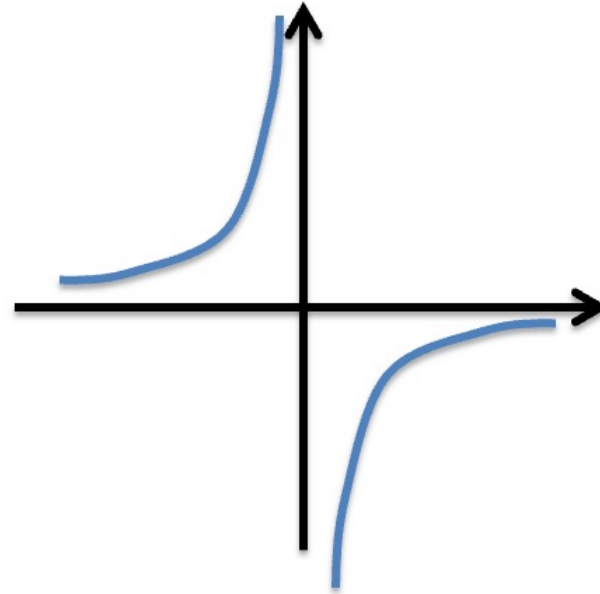
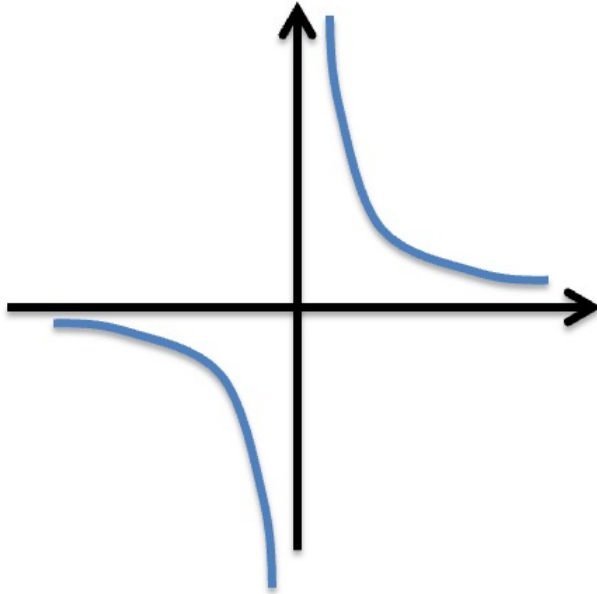
$$y = \frac{a}{x}$$

When  $a > 0$

$a$  is a constant while  $x$  is a variable, so we might have  $y = \frac{3}{x}$

$$y = \frac{a}{x}$$

When  $a < 0$



The lines  $x = 0$  and  $y = 0$  are called asymptotes.  
**An asymptote is a straight line which the curve approaches at infinity.**

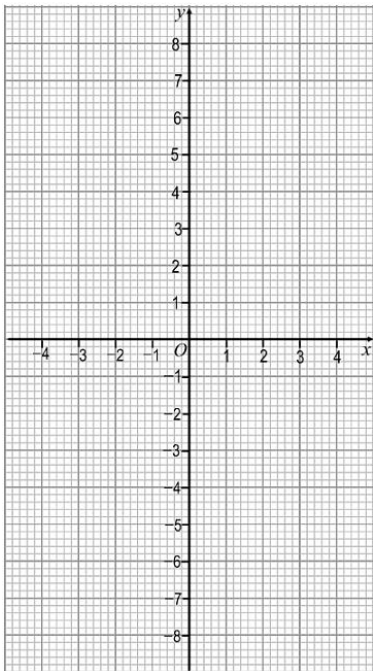
## Worked Example

Complete the tables and draw the graph of  $y = \frac{2}{x}$  for  $x = -5$  to  $x = 5$

Here is a table of values for  $y = \frac{2}{x}$ .

$x$	0.25	0.4	0.5	0.8	1	2	4	5
$y$								

$x$	-0.25	-0.4	-0.5	-0.8	-1	-2	-4	-5
$y$								



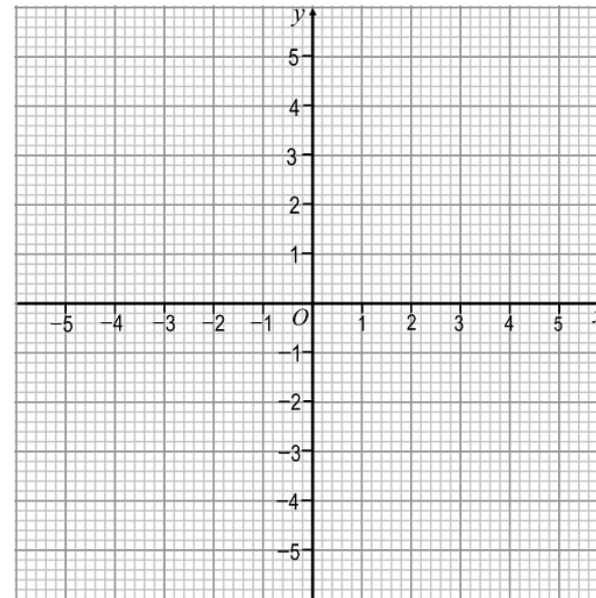
## Worked Example

Complete the tables and draw the graph of  $y = -\frac{1}{x}$  for  $x = -5$  to  $x = 5$

Here is a table of values for  $y = -\frac{1}{x}$ .

$x$	0.2	0.4	0.5	0.8	1	2	3	4	5
$y$									

$x$	-0.2	-0.4	-0.5	-0.8	-1	-2	-3	-4	-5
$y$									



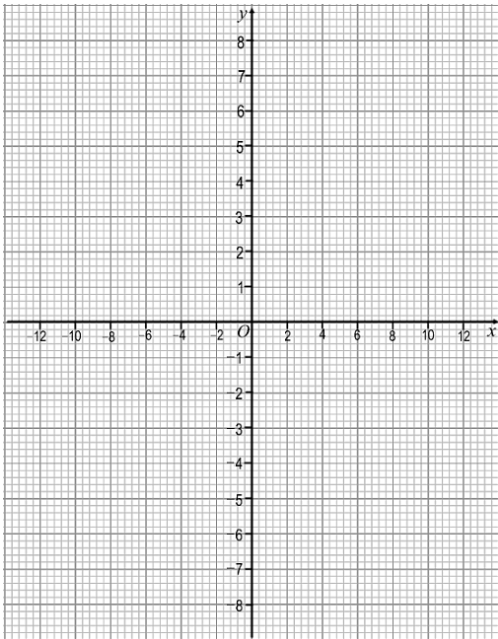
## Worked Example

Complete the tables and draw the graph of  $y = \frac{4}{x-1}$  for  $x = -12$  to  $x = 12$

Here is a table of values for  $y = \frac{4}{x-1}$ .

$x$	1.5	2	3	5	6	9	11
$y$							

$x$	0.5	0	-1	-3	-4	-7	-9
$y$							





## Fluency Practice

1. Here are some table of values for  $y = \frac{4}{x}$ .

$x$	0.2	0.4	0.5	1	2	4	5	8	10
$y$		10		4	2		0.8		

$x$	-10	-8	-5	-4	-2	-1	-0.5	-0.4	-0.2
$y$									

a) Complete the table of values.

b) On your additional sheet, draw the graph of  $y = \frac{4}{x}$  for  $-10 \leq x \leq 10$ .

c) Use your graph to find an estimate for the solutions of  $\frac{4}{x} = 4 - x$ .

2. On your additional sheet, draw the graph of  $y = -\frac{3}{x}$  for  $-10 \leq x \leq 10$ .

## Fluency Practice

3. a) Here are some table of values for  $y = \frac{8}{x+2}$ .

$x$	-12	-10	-7	-6	-4	-3	-1	0	2	3	6	8
$y$												

b) On your additional sheet, draw the graph of  $y = \frac{8}{x+2}$  for  $-12 \leq x \leq 12$ .

c) For which values of  $x$  is  $y = \frac{8}{x+2}$  not defined?

4. a) Complete the table of values for  $y = 3 - \frac{2}{x}$ ,  $x \neq 0$ .

$x$	-3	-2	-1	-0.5	-0.1	0.1	0.5	1	2	3
$y$										

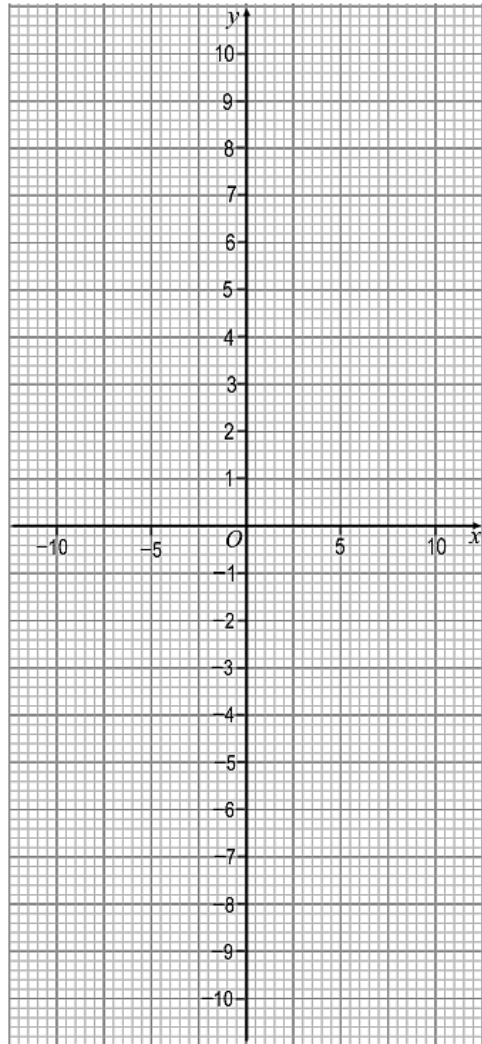
b) On your additional sheet, draw the graph of  $y = 3 - \frac{2}{x}$  for  $-3 \leq x \leq 3$ .

c) This graph approaches two lines without touching them. These lines are called asymptotes. Write down the equation of each of these two lines.

## Fluency Practice

Question 1

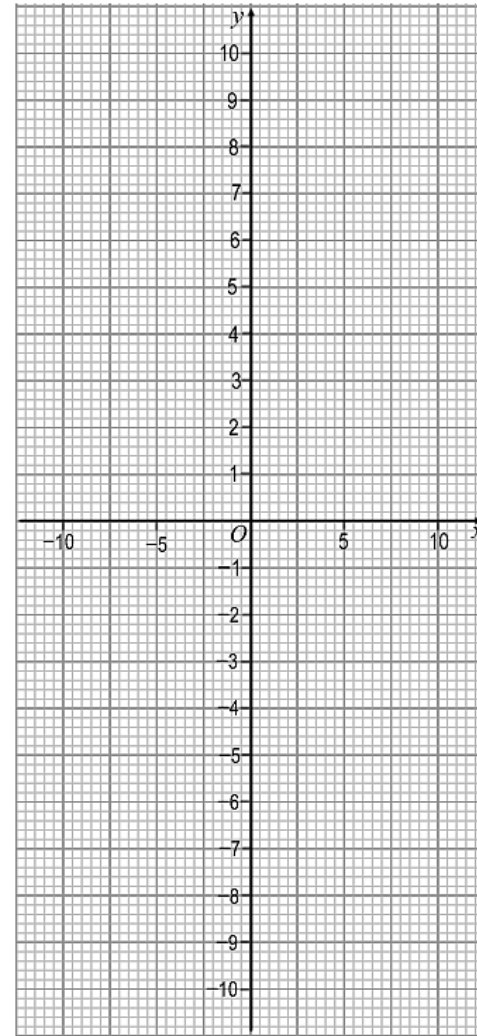
$$y = \frac{4}{x}$$



## Fluency Practice

Question 2

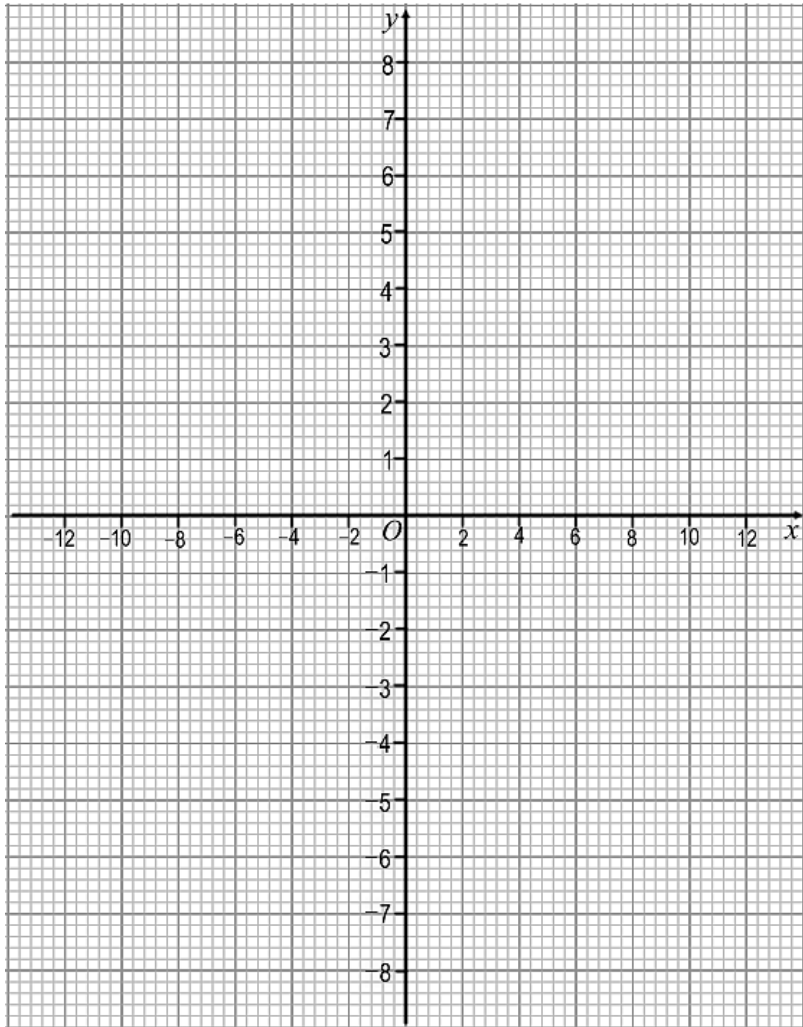
$$y = -\frac{3}{x}$$



## Fluency Practice

Question 3

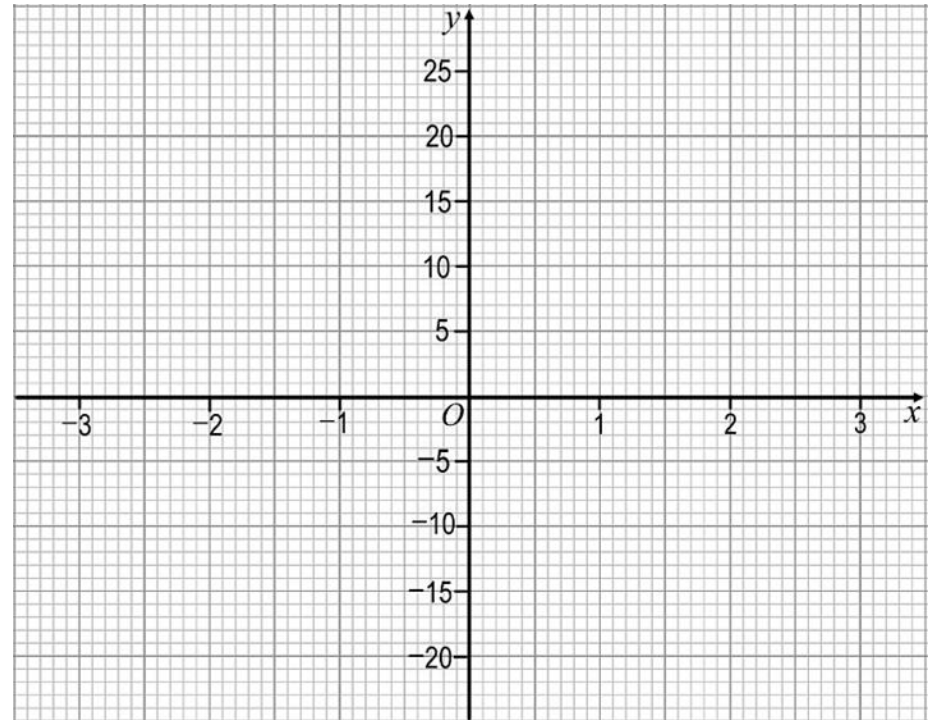
$$y = \frac{8}{x+2}$$



## Fluency Practice

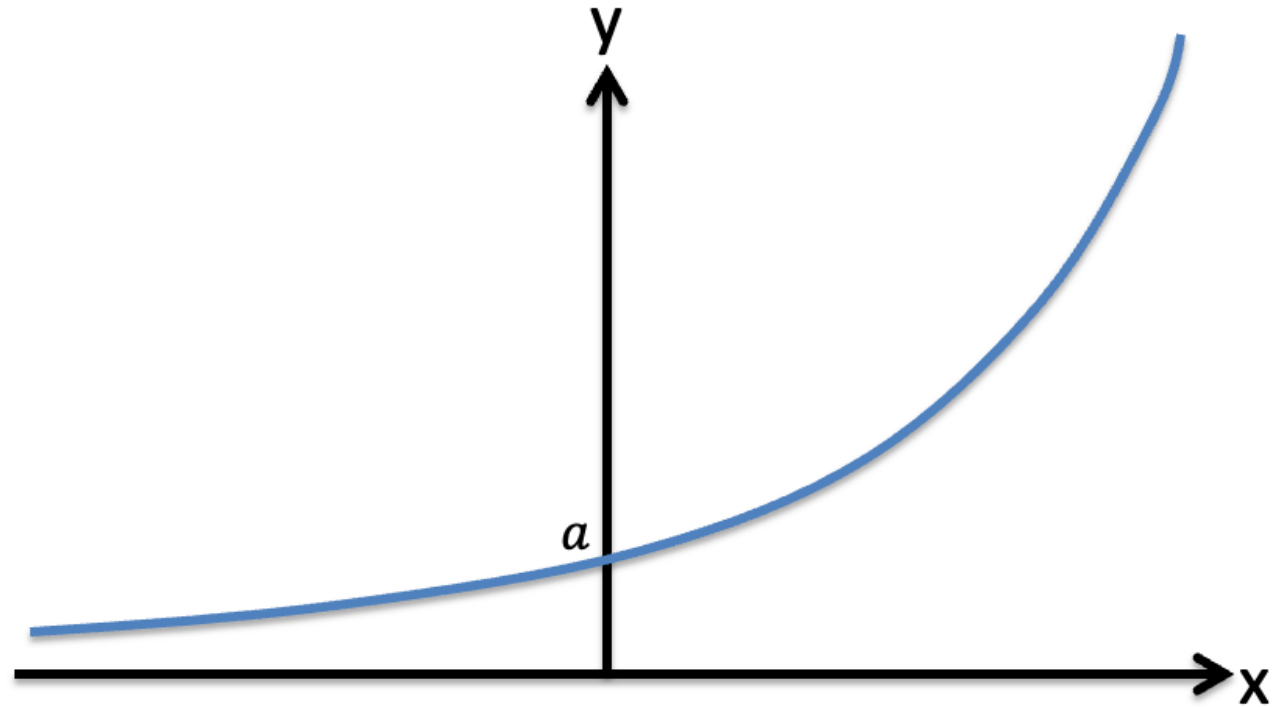
Question 4

$$y = 3 - \frac{2}{x}$$



## Exponential Graphs

$$y = a \times b^x$$



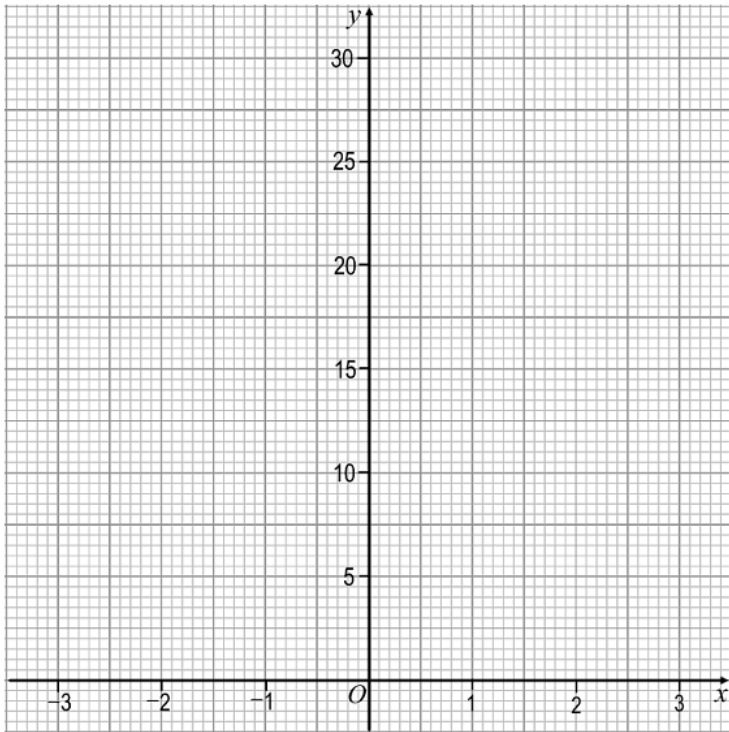
The  $y$ -intercept is  $a$  because  $a \times b^0 = a \times 1 = a$ .  
(unless  $a = 0$ , but let's not go there!)

## Worked Example

- a) Complete the tables and draw the graph of  $y = 3^x$  for  $x = -3$  to  $x = 3$   
 b) Use your graph to estimate the solution to  $3^x = 20$

Here is a table of values for  $y = 3^x$ .

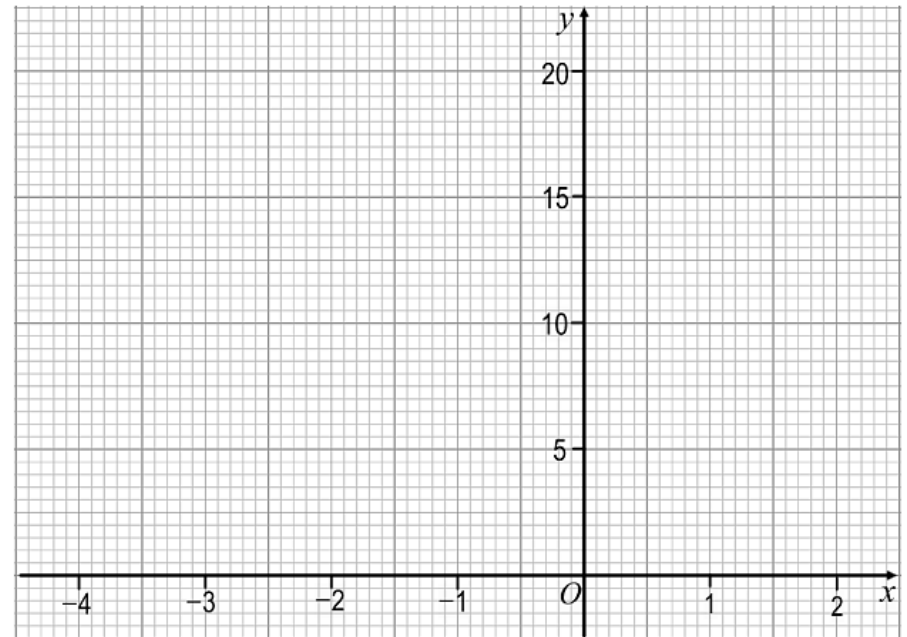
$x$	-3	-2	-1	0	1	2	3
$y$							



- a) Complete the tables and draw the graph of  $y = 2^{-x}$  for  $x = -4$  to  $x = 2$   
 b) Use your graph to estimate  
 i) the value of  $y$  when  $x = 0.5$   
 ii) the solution to the equation  $2^{-x} = 10$

Here is a table of values for  $y = 2^{-x}$ .

$x$	-4	-3	-2	-1	0	1	2
$y$							

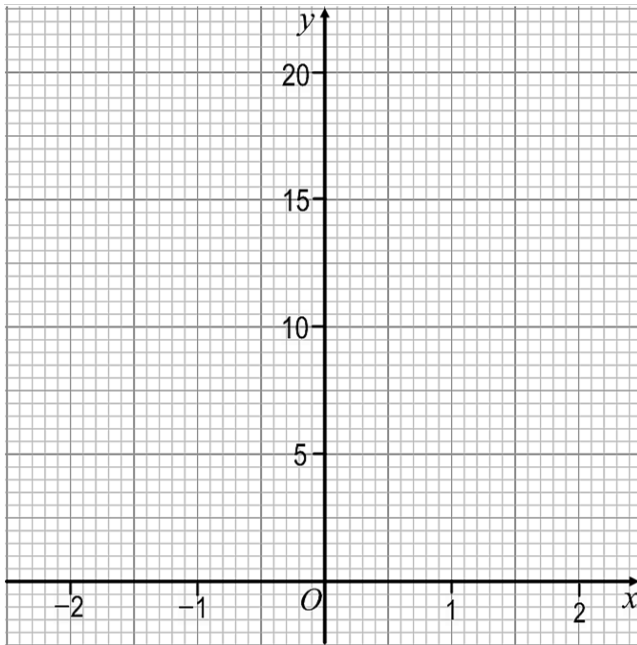


## Fluency Practice

1. Here is a table of values for  $y = 4^x$ .

$x$	-2	-1	0	1	2
$y$					

- a) Complete the table of values.  
 b) On the grid, draw the graph of  $y = 4^x$  for  $-2 \leq x \leq 2$ .



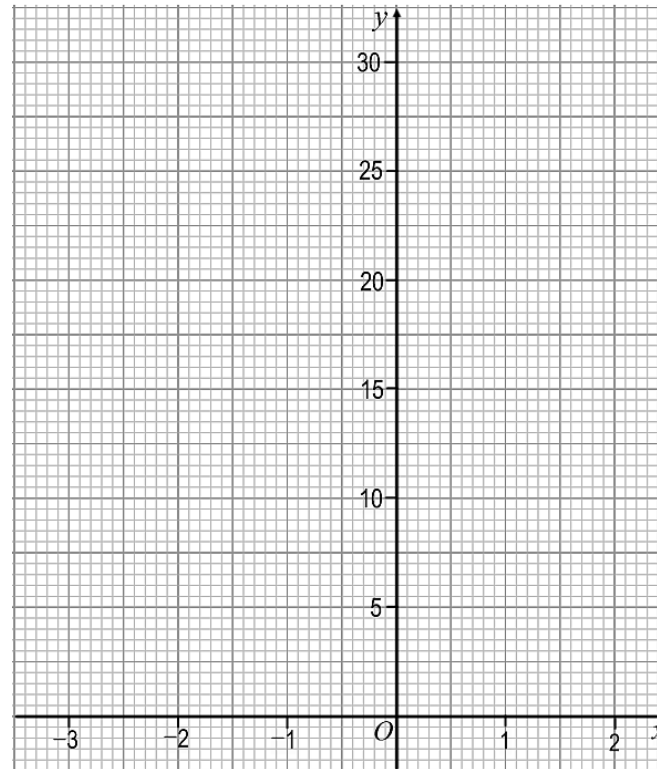
- c) Use your graph to find an estimate for:
- i) the value of  $y$  when  $x = 1.5$
  - ii) the value of  $x$  when  $y = 11$

## Fluency Practice

2. Here is the table of values for  $y = 3^{-x}$ .

$x$	-3	-2	-1	0	1	2
$y$						

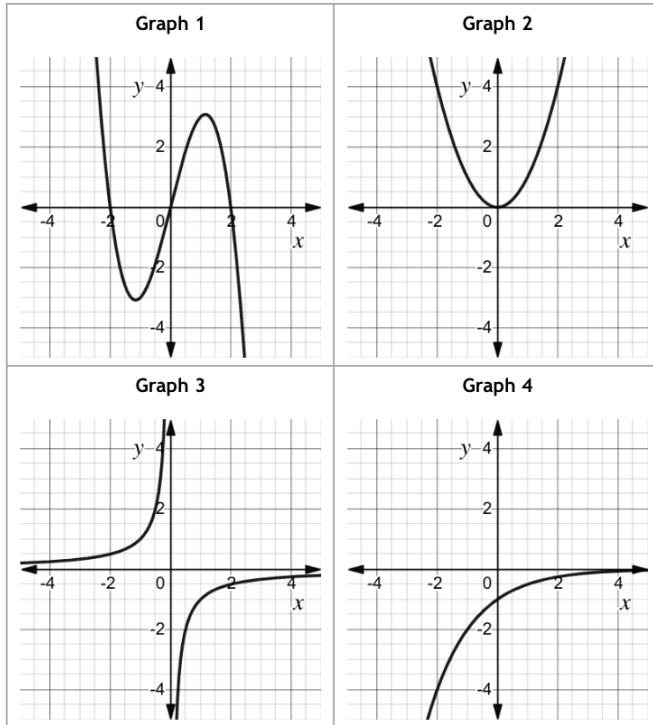
- a) Complete the table of values.  
 b) On the grid, draw the graph of  $y = 3^{-x}$  for  $-3 \leq x \leq 2$ .



- c) Use your graph to find the solution to the equation  $3^{-x} = 7$ .

## Worked Example

Four graphs are sketched below.

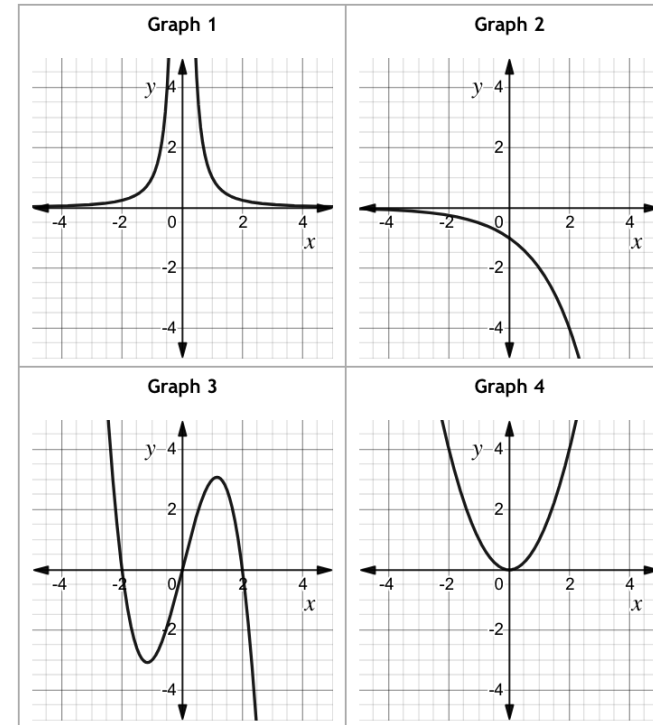


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	<input type="text"/>
$y = x(-x + 2)(x + 2)$	<input type="text"/>
$y = \frac{-1}{x}$	<input type="text"/>
$y = -0.5^x$	<input type="text"/>

## Your Turn

Four graphs are sketched below.

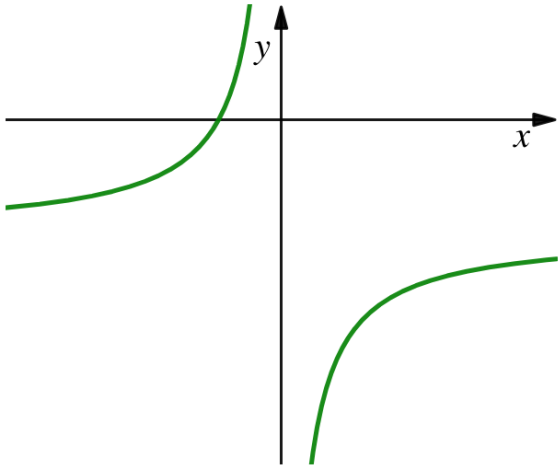


Match each equation in the table with a graph above.

Equation	Graph Number
$y = x^2$	<input type="text"/>
$y = x(-x + 2)(x + 2)$	<input type="text"/>
$y = \frac{1}{x^2}$	<input type="text"/>
$y = -2^x$	<input type="text"/>

## Worked Example

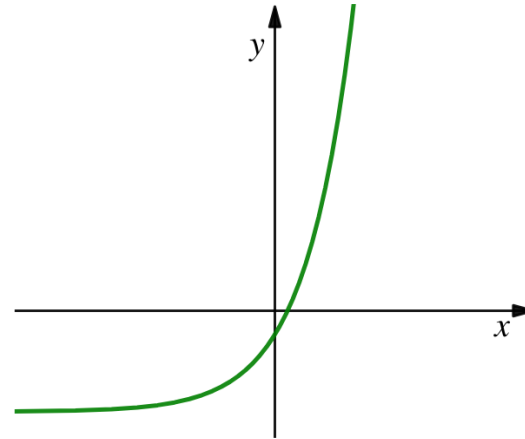
Select the equation of the graph sketched below.



- $y = 2x^3 + 2x^2 + 4x - 4$
- $y = 3x + 3$
- $y = 4^x + 1$
- $y = 4 \times 2^x + 2$
- $y = -\frac{4}{x} - 3$
- $y = 4x + 4$

## Your Turn

Select the equation of the graph sketched below.



- $y = -3x^2 - 2x - 3$
- $y = -x^2 - 4x + 2$
- $y = 2x^2 - 3x + 2$
- $y = 3 \times 3^x - 4$
- $y = -\frac{4}{x} - 3$
- $y = 2x + 2$



## Extra Notes