1 Graph Transformations

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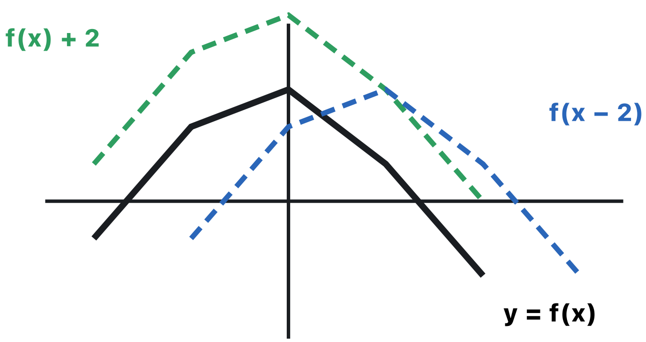
TRANSLATIONS — INSIDE OR OUTSIDE?

- **Outside** the bracket changes **y** — the graph moves **up or down**, as you expect.
- **Inside** the bracket changes **x** — the graph moves **left or right**, the **opposite** way to the sign.

LEARN THESE FOUR

Equation	Translation by
$y = f(x - a)$	$\begin{pmatrix} a \\ 0 \end{pmatrix}$ — right a
$y = f(x + a)$	$\begin{pmatrix} -a \\ 0 \end{pmatrix}$ — left a
$y = f(x) + a$	$\begin{pmatrix} 0 \\ a \end{pmatrix}$ — up a
$y = f(x) - a$	$\begin{pmatrix} 0 \\ -a \end{pmatrix}$ — down a

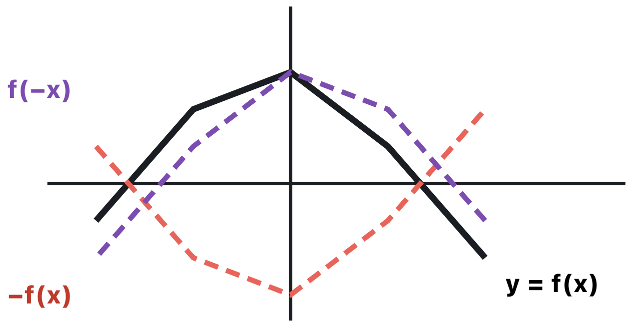
TRANSLATIONS



REFLECTIONS — WHICH SIGN MOVED?

- **y = −f(x)** — the **y** values are negated, so it reflects in the **x-axis**.
- **y = f(−x)** — the **x** values are negated, so it reflects in the **y-axis**.
- The minus sits with whichever letter is being flipped.

REFLECTIONS



WHAT HAPPENS TO A POINT

Transformation	P(2, 5) becomes
$y = f(x) + 3$	(2, 8)
$y = f(x + 3)$	(−1, 5)
$y = -f(x)$	(2, −5)
$y = f(-x)$	(−2, 5)

WORKED EXAMPLE — TWO STEPS AT ONCE

P(2, 5) is on y = f(x). Find where it goes under y = −f(x) + 3.

First **−f(x)**: the y value is negated

(2, 5) → (2, −5)

Then **+ 3**: move up 3

(2, −5) → **(2, −2)**

Do the **reflection first**, then the translation.

WORKED EXAMPLE — TRIGONOMETRIC GRAPHS

Describe y = cos(x) + 1 and y = sin(x − 45°).

cos(x) + 1: translation by $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

The whole wave moves **up 1**, so it now runs from 0 to 2.

sin(x − 45°): translation by $\begin{pmatrix} 45 \\ 0 \end{pmatrix}$

The wave moves **right 45°**.

WORKED EXAMPLE — TRANSFORMING AN EQUATION

The curve $y = 2x^3 + 3x$ is translated by $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$. Find the new equation.

Up 4 means **add 4** to the whole right-hand side

$y = 2x^3 + 3x + 4$

Now translate y = f(x) by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

Right 3 means replace **every x** with **(x − 3)**

$y = 2(x - 3)^3 + 3(x - 3)$

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

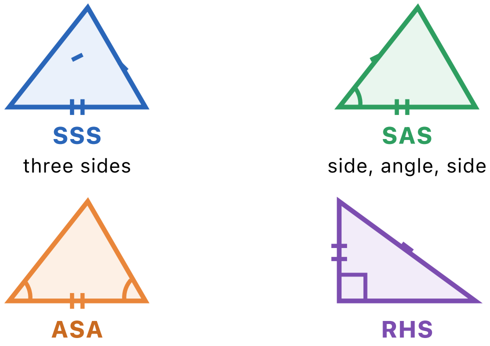
- ✕ Moving the graph **right** for f(x + a) — inside the bracket it goes the **opposite** way.
- ✕ Muddling **−f(x)** with **f(−x)**.
- ✕ Reflecting in the **y**-axis for −f(x) — it is the **x**-axis.
- ✕ Writing the translation vector the wrong way round, e.g. $\begin{pmatrix} 0 \\ 3 \end{pmatrix}$ for a move right.
- ✕ Applying the translation **before** the reflection.
- ✕ Replacing only **one** x when substituting (x − 3) into an equation.
- ✕ Saying "it moves" without giving the **vector** or the axis.
- ✕ Forgetting that a reflection in the x-axis leaves the **x-intercepts** where they are.

Revise it: where does (3, −2) go under y = f(x − 1) and under y = −f(x)?

CONGRUENT OR SIMILAR?

- **Congruent** — exactly the **same shape and size**.
- **Similar** — same shape, **different size**.
- Congruent shapes are similar with scale factor **1**.
- A shape stays congruent after a **reflection** or **rotation**.

THE FOUR CONGRUENCE CONDITIONS



SETTING OUT A CONGRUENCE PROOF

1. Find **three** matching facts.
2. Write each on its **own line** with a **reason**.
3. Name the pair of sides or angles **precisely**, e.g. AB = DE.
4. Finish by naming the **condition**: SSS, SAS, ASA or RHS.

AAA is not a condition — it only proves the triangles are **similar**.

REASONS YOU CAN QUOTE

- Given in the question.
- **Common** side — the two triangles share it.
- **Alternate** or **corresponding** angles, parallel lines.
- **Vertically opposite** angles are equal.
- Sides of a **square**, **rhombus** or **parallelogram**.
- The **midpoint** splits a line into two equal parts.
- Base angles of an **isosceles** triangle are equal.

WORKED EXAMPLE — A CONGRUENCE PROOF

AE and BD are straight lines crossing at C, with AB parallel to DE and C the midpoint of AE. Prove triangle ABC is congruent to triangle EDC.

AC = EC (C is the **midpoint** of AE)

∠BAC = ∠DEC (**alternate angles**, AB parallel to DE)

∠ACB = ∠ECD (**vertically opposite** angles)

So the triangles are congruent by **ASA**

The equal side lies **between** the two equal angles, so it is ASA.

SIMILAR TRIANGLE PROOFS

1. Show **two** pairs of equal angles.
2. Give a **reason** for each.
3. Say the third pair must match, since angles in a triangle add to 180°.
4. Conclude the triangles are **similar**.

Or show all three pairs of sides are in the **same ratio**.

WORKED EXAMPLE — A SIMILARITY PROOF

BC is parallel to DE, and ABD and ACE are straight lines. Prove triangle ABC is similar to triangle ADE.

∠ABC = ∠ADE (**corresponding angles**, BC parallel to DE)

∠BAC = ∠DAE (the **same** angle, shared)

Two pairs of angles are equal, so the third pair is too

So the triangles are **similar**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Giving **AAA** as a congruence condition — it only proves **similarity**.
- ✕ Writing **SSA**, which does not fix the triangle.
- ✕ Listing three facts but never naming the **condition**.
- ✕ Giving no **reason** beside each fact.
- ✕ Naming vertices in the **wrong order**, so the pairs do not match up.
- ✕ Using "they look the same" or measuring from the diagram.
- ✕ Calling the shared side "the same length" instead of "**common**".
- ✕ Proving similarity when the question asked for **congruence**.

Circle Theorem Proofs

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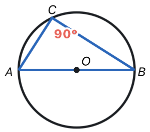
EVERY PROOF STARTS THE SAME WAY

1. Draw the **radii** — equal radii make an **isosceles** triangle.
2. Mark the equal **base angles** with letters.
3. Use **angles in a triangle = 180°**, then rearrange.
4. Give a **reason on every line**.

Proofs **3** and **4** may quote proof **2** — just name it.

1 — ANGLE IN A SEMICIRCLE



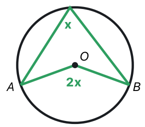
OA = OC (**radii**) → $\angle OAC = \angle OCA = a$

OB = OC (**radii**) → $\angle OBC = \angle OCB = b$

In ΔABC: $a + (a + b) + b = 180$

$2a + 2b = 180 \rightarrow a + b =$ **90°**

2 — ANGLE AT THE CENTRE



Join CO and continue it past O.

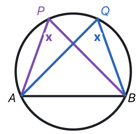
OA = OC (**radii**) → $\angle OCA = \angle OAC = x$

That angle at O is an **exterior angle**, so it is 2x.

Likewise the other part is 2y.

Centre = $2x + 2y = 2(x + y)$ = **twice** the circumference angle.

3 — ANGLES IN THE SAME SEGMENT



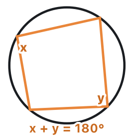
P and Q stand on the **same chord AB**.

$\angle AOB = 2\angle APB$ and $\angle AOB = 2\angle AQB$

Both are **half** the same centre angle:

$\angle APB =$ **$\angle AQB$**

4 — CYCLIC QUADRILATERAL



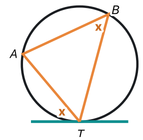
Let the angle at A be **x** and at C be **y**. Join OB and OD.

Angle at the centre on C's side = **2x**; the other = **2y**.

Angles at a point: $2x + 2y = 360$

$x + y =$ **180°**

5 — ALTERNATE SEGMENT



Tangent–chord angle at T = **x**. Draw diameter **TD**.

$\angle DTC = 90°$ (**tangent ⊥ radius**) → $\angle ATD = 90 - x$

$\angle TAD = 90°$ (**angle in a semicircle**)

In ΔTAD: $\angle ADT = 180 - 90 - (90 - x) = x$

$\angle ABT = \angle ADT$ (**same segment**) = **x**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Never **drawing in the radii**, so no isosceles triangle appears.
- ✕ Saying a triangle is isosceles without giving **"radii are equal"** as the reason.
- ✕ Using the **result** you are trying to prove as a step in the proof.
- ✕ Measuring the diagram instead of arguing with **angle facts**.
- ✕ Working with **numbers** rather than letters — one case is not a proof.
- ✕ Leaving out the **final line** that states what has been shown.
- ✕ Forgetting that $\angle ACB$ is made of **both** a and b.
- ✕ In the alternate segment proof, not drawing the **diameter** from the point of contact.

Revise it: write out all five proofs from memory, with a reason on every line.