1 Similarity with Area & Volume

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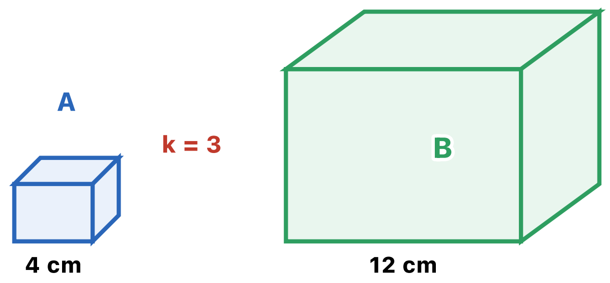
THE THREE SCALE FACTORS

- If the **lengths** multiply by k...
- ...the **areas** multiply by k^2...
- ...and the **volumes** multiply by k^3.
- It works for **surface area** and for **mass** too, if the material is the same.

LEARN THIS TABLE

Length SF	Area SF	Volume SF
× 2	× 4	× 8
× 3	× 9	× 27
× 4	× 16	× 64
× k	× k^2	× k^3

SIMILAR CUBOIDS



A → B: lengths **× 3**, areas **× 9**, volumes **× 27**.

WORKED EXAMPLE — SURFACE AREA

Cuboids A and B are similar, with lengths 4 cm and 12 cm. The surface area of A is 72 cm^2. Find the surface area of B.

$k = 12 \div 4 = 3$

Area scale factor = $k^2 = 9$

$72 \times 9 =$ **648 cm^2**

WORKED EXAMPLE — VOLUME BACKWARDS

The volume of B is 432 cm^3. Find the volume of A.

B → A, so $k = 4 \div 12 = \frac{1}{3}$

Volume scale factor = $k^3 = \frac{1}{27}$

$432 \div 27 =$ **16 cm^3**

Going to the **smaller** shape, so the answer must be **smaller**.

WHICH WAY ROUND?

1. Decide which shape you are going **to**.
2. $k = \frac{\text{length you want}}{\text{length you have}}$
3. **Square** it for area, **cube** it for volume.
4. Check the answer is bigger or smaller as expected.

GOING BETWEEN AREA AND VOLUME

$$\text{area SF} = k^2 \rightarrow k = \sqrt{(\text{area SF})}$$

$$\text{volume SF} = k^3 \rightarrow k = \sqrt[3]{(\text{volume SF})}$$

- You can never go **straight** from an area SF to a volume SF.
- Always come **back to the length** scale factor first.

WORKED EXAMPLE — AREA TO VOLUME

Similar solids A and B have surface areas 100 cm^2 and 64 cm^2. Find the ratio of their volumes.

Area SF = $\frac{64}{100}$

$k = \sqrt{\frac{64}{100}} = \frac{8}{10} = \frac{4}{5}$

Volume SF = $k^3 = \frac{64}{125}$

Volume A : Volume B = **125 : 64**

WORKED EXAMPLE — MASS

A and B are similar solids of the same material. Surface areas 62.5 cm^2 and 22.5 cm^2. The mass of A is 375 g. Find the mass of B.

Area SF = $22.5 \div 62.5 = 0.36$

$k = \sqrt{0.36} = 0.6$

Mass follows **volume**: $k^3 = 0.216$

$375 \times 0.216 =$ **81 g**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using the **length** scale factor for an area or a volume.
- ✕ Doubling the area SF instead of **squaring** the length SF.
- ✕ Going straight from an area SF to a volume SF without finding **k**.
- ✕ Getting k **upside down**, so the answer grows when it should shrink.
- ✕ Forgetting that **mass** scales with volume, not with area.
- ✕ Cubing an area SF, or squaring a volume SF.
- ✕ Rounding k too early — keep it exact until the last line.
- ✕ Giving a ratio the **wrong way round** — check which shape is bigger.

Revise it: a shape is enlarged by k = 5 — what happens to its area and volume?

2 Arcs, Sectors & Segments

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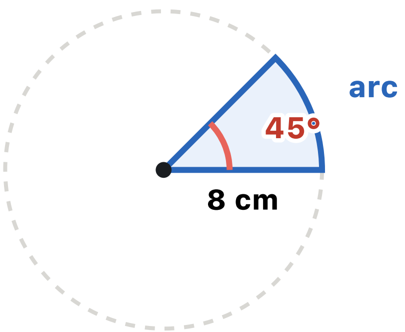
A SECTOR IS A FRACTION OF A CIRCLE

$\frac{\theta}{360}$ **of the whole circle**

- **Arc length** = $\frac{\theta}{360} \times \pi \times$ **diameter**
- **Sector area** = $\frac{\theta}{360} \times \pi \times$ **radius^2**
- **Perimeter** = arc + **2 radii**.

THE PARTS OF A SECTOR



WORKED EXAMPLE — ARC AND PERIMETER

A sector has radius 8 cm and angle 45°. Find the arc length and the perimeter.

Arc = $\frac{45}{360} \times \pi \times 16$

= $\frac{1}{8} \times 16\pi = 2\pi =$ **6.28 cm**

Perimeter = 6.28 + 8 + 8 = **22.3 cm**

The perimeter needs **both** straight radii as well as the arc.

WORKED EXAMPLE — SECTOR AREA

Find the area of the same sector.

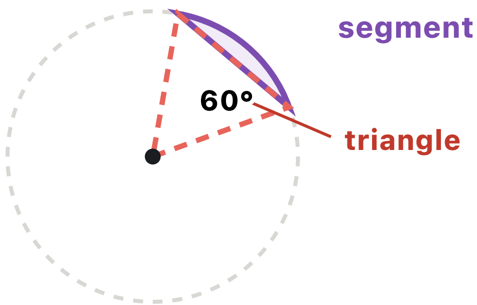
Area = $\frac{45}{360} \times \pi \times 8^2$

= $\frac{1}{8} \times 64\pi$

= 8π = **25.1 cm^2**

Area uses **r^2**; arc length uses the **diameter**.

A SEGMENT IS A SECTOR MINUS A TRIANGLE



Take the **triangle** away from the **sector**.

SEGMENT AREA — THE METHOD

1. Find the **sector** area: $\frac{\theta}{360} \times \pi r^2$.
2. Find the **triangle** area: $\frac{1}{2}ab \sin C$.
3. **Subtract** the triangle from the sector.
4. Keep **full accuracy** until the last line.

WORKED EXAMPLE — SEGMENT AREA

A segment is cut off by a 60° angle in a circle of radius 8 cm. Find its area.

Sector: $\frac{60}{360} \times \pi \times 64$

= $\frac{1}{6} \times 64\pi = 33.5103...$

Triangle: $\frac{1}{2} \times 8 \times 8 \times \sin 60°$

= 27.7128...

33.5103 − 27.7128 = **5.80 cm^2**

Both radii are sides of the triangle, so **a = b = r**.

WORKING BACKWARDS

A sector of radius 8 cm has area 25.1 cm^2. Find the angle.

$\frac{\theta}{360} \times \pi \times 64 = 25.1$

$\theta \times 64\pi = 25.1 \times 360$

$\theta = \frac{25.1 \times 360}{64\pi} =$ **45°**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using the **radius** in the arc formula — arc length needs the **diameter**.
- ✕ Using the diameter in the **area** formula, which needs r^2.
- ✕ Giving the arc when the question asks for the **perimeter** — add the two radii.
- ✕ Finding the sector but forgetting to **subtract** the triangle for a segment.
- ✕ Using $\frac{1}{2}bh$ for the triangle when there is no perpendicular height.
- ✕ Rounding the sector before subtracting, which loses accuracy.
- ✕ Leaving the calculator in **radians** when working out sin C.
- ✕ Writing cm instead of **cm^2** for an area.

Revise it: find the perimeter and area of a 120° sector of radius 9 cm.

3 Volume & Surface Area of Non-Prisms

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SPHERES

$$V = \frac{4}{3}\pi r^3 \quad \cdot \quad SA = 4\pi r^2$$

- A **hemisphere** is half a sphere — halve the volume.
- The surface area of a **solid** hemisphere is $2\pi r^2 + \pi r^2 =$ **$3\pi r^2$**.

FRUSTUMS — CHOP THE TOP OFF

A cone of radius 6 cm and height 12 cm has a similar cone of half the size removed. Find the volume of the frustum.

Large: $\frac{1}{3}\pi(36)(12) = 144\pi$

Small: $\frac{1}{3}\pi(9)(6) = 18\pi$

$144\pi - 18\pi = 126\pi$

= **396 cm^3 (3 s.f.)**

$k = \frac{1}{2}$, so the small cone is $\frac{1}{8}$ of the large one.

WORKED EXAMPLE — CONE

A cone has radius 5 cm and vertical height 12 cm. Find its volume and total surface area.

$V = \frac{1}{3} \times \pi \times 25 \times 12 = 100\pi$

= **314 cm^3 (3 s.f.)**

$l = \sqrt{(5^2 + 12^2)} = \sqrt{169} = 13$

$TSA = \pi(5)(13) + \pi(5)^2$

$= 65\pi + 25\pi = 90\pi$

= **283 cm^2 (3 s.f.)**

WORKED EXAMPLE — SPHERE

A sphere has radius 6 cm. Find its volume and surface area.

$V = \frac{4}{3} \times \pi \times 6^3$

$= \frac{4}{3} \times \pi \times 216 = 288\pi$

= **905 cm^3 (3 s.f.)**

$SA = 4 \times \pi \times 36 = 144\pi$

= **452 cm^2 (3 s.f.)**

CONES

$$V = \frac{1}{3}\pi r^2 h$$

$$CSA = \pi r l \quad \cdot \quad TSA = \pi r l + \pi r^2$$

- Volume uses the **vertical height h**.
- Surface area uses the **slant height l**.

PYRAMIDS

$$V = \frac{1}{3} \times \text{base area} \times \text{height}$$

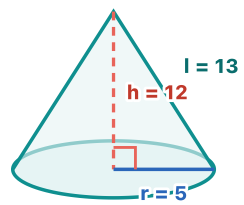
$$TSA = \frac{1}{2}Ps + A$$

P = base perimeter, s = slant height, A = base area.

COMPOSITE SOLIDS

- **Split** the shape into the solids you know.
- Work out each **separately**, then **add**.
- For surface area, leave out any face that is **joined** and no longer on the outside.
- A cone on a hemisphere: surface = $\pi r l$ + **2**πr^2 — no circle in the middle.

CONE — HEIGHT AND SLANT HEIGHT



$l^2 = r^2 + h^2$

$13^2 = 5^2 + 12^2$

Volume uses h

Surface area uses l

"Curved" leaves the base off.

WORKED EXAMPLE — SQUARE PYRAMID

A pyramid has a 6 cm square base and vertical height 4 cm. Find its volume and total surface area.

$V = \frac{1}{3} \times 36 \times 4 =$ **48 cm^3**

$s = \sqrt{(4^2 + 3^2)} = 5$ (3 = **half** the base side)

$TSA = \frac{1}{2} \times 24 \times 5 + 36$

$= 60 + 36 =$ **96 cm^2**

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Using the **slant height** in the cone volume formula — it needs the vertical height.
- ✕ Using the **vertical height** in $\pi r l$ — surface area needs the slant height.
- ✕ Forgetting the $\frac{1}{3}$ in a cone or pyramid volume.
- ✕ Giving the **curved** surface area when the question asks for the **total**.
- ✕ Using the whole base side instead of **half** of it to find the slant height.
- ✕ Adding a base to a **hollow** hemisphere, or leaving it off a solid one.
- ✕ Rounding π to 3.14 early, which loses the third significant figure.
- ✕ Subtracting the small cone's **height** rather than its **volume** for a frustum.

Revise it: write out the six formulae from memory, then check them.

Bearings

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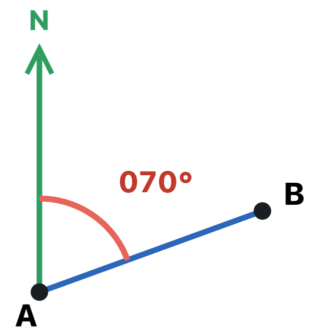
THE THREE RULES OF A BEARING

1. Measure from **north**.
2. Turn **clockwise**.
3. Always write **three figures**.

- So 70° is written **070°** and 5° is written **005°**.
- Bearings run from **000°** to **360°**.

THE BEARING OF B FROM A



Start at **A**, face **north**, then turn clockwise until you face **B**.

"OF" AND "FROM"

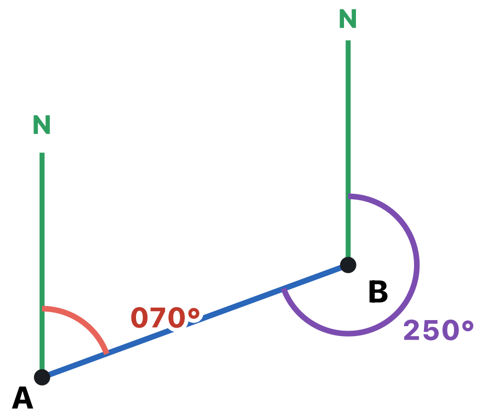
- "The bearing of **B** from **A**" — stand at **A**.
- The **second** place named is where you stand.
- Draw the north line at **that** point first.

BACK BEARINGS

under 180° → + 180 · 180° or more → – 180

- The north lines at A and B are **parallel**.
- The two bearings are **allied (co-interior)** angles plus a straight line, so they differ by **180°**.
- Keep the answer between **000°** and **360°**.

WHY IT IS 180° APART



Bearing of B from A is **070°**, so the bearing of A from B is **250°**.

WORKED EXAMPLE — BACK BEARING

The bearing of B from A is 070°. Find the bearing of A from B.

070° is **less than** 180°

070 + 180 = **250°**

The bearing of Q from P is 210°. Find the bearing of P from Q.

210° is **more than** 180°

210 – 180 = **030°**

BEARINGS WITH ANGLE FACTS

- North lines are **parallel**, so use **alternate**, **corresponding** and **allied (co-interior)** angles.
- Angles round a point at one place add to **360°**.
- Give the **reason** at each step — it earns a mark.

WORKED EXAMPLE — TWO LEGS

A ship sails on a bearing of 070°, then turns 40° clockwise. What is its new bearing?

070 + 40 = **110°**

It then turns 150° anticlockwise.

110 – 150 = –40

–40 + 360 = **320°**

If the answer goes below 000°, **add 360°**; above 360°, **take 360° off**.

⚠ COMMON EXAM MISTAKES (recurring errors flagged in examiners' reports)

- ✕ Writing 70° instead of **070°** — a bearing always has three figures.
- ✕ Measuring **anticlockwise**, or from the east instead of the north.
- ✕ Drawing the north line at the **wrong** point — it goes where you stand.
- ✕ Muddling "the bearing of B **from** A" with the bearing of A from B.
- ✕ Adding 180° when the bearing is already **more** than 180°.
- ✕ Giving an answer **above 360°** or below 000°.
- ✕ Measuring the angle **inside** the triangle rather than from north.
- ✕ Giving no **reason** when parallel north lines were used.

Revise it: the bearing of Y from X is 145° — find the bearing of X from Y.